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New Retail versus Traditional Retail in E-commerce: Channel Establishment, Price Competition, and Consumer Recognition

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Abstract The concept "new retail" in e-commerce is to establish an offline channel and integrate it with the online retail channel. The development of new retail encounters three main problems: locations of the offline stores, the price competition with the traditional online retail, and the difficulty in consumer recognition in the two channels. In this paper, we present a duopoly model consisting of a new retail firm and an online firm, which sell the same product in two periods. The two firms compete for the market share using the behavior-based pricing (BBP), which means that in the second period each firm offers different prices to consumers with different purchasing histories/behaviors in the first period. We also solve the benchmark pricing model, where the histories/behaviors are not considered.

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The results of this paper provide valuable insights to the development of new retail in e-commerce. In the Nash equilibrium, each price of the new retail firm is higher than the corresponding price of the online firm due to a higher channel cost for the offline stores and high-speed deliveries. Under certain condition, the new retail firm will establish an offline channel with a larger hassle cost, which is a measure of the easiness of reaching the offline stores by the consumers, in the BBP model than that in the benchmark model. Interestingly, the difficulty in consumer recognition results in that the new retail firm occupies more market share and may obtain higher profit.

Keywords New retail · Behavior-based pricing · Price competition · Consumer recognition

1 Introduction

E-commerce has reached its development bottleneck in recent years. The growth of sales amount slows down all over the world. In China, the growth rate declined from 50% in 2011 to 21.3% in 2017 (see Table 1), and is forecasted to decrease continuously in the future years. E-commerce has to find a new way to achieve further growth.

Table 1: Growth Rate of E-commerce Sales (‘E’ means expected).

Year	2011	2012	2013	2014	2015	2016	2017	2018E	2019E
Sales (Trillion)	0.8	1.2	1.9	2.8	3.8	4.7	5.7	6.6	7.5
Growth Rate (%)	-	50.0	58.3	47.3	35.7	23.7	21.3	15.8	13.6

In response to this situation, a concept of “New Retail” was proposed in 2016 by Alibaba, the biggest e-commerce company in China. The “new retail” concept, which is taken as a threat to the other largest global player Amazon, is basically the integration of online and offline retailing channels. The two channels complement each other in three aspects: (1) touching the intangible online products in offline physical stores, (2) solving the offline showrooming problem by setting the same price in online and offline channels, (3) delivering the online orders from nearby offline stores to speed up the deliveries.

The consumers will have a refreshing purchasing experience buying from a firm with new retail. Experiencing the product offline eliminates their product value uncertainty. With the same price online and offline, they do not need to compare prices in the same firm's different channels. They can purchase online after experiencing, and wait for a shorter time for the product to come than from a traditional e-commerce firm. The excellent purchasing experience will definitely change the consumers' purchasing behaviors, and yield benefits to the new retail firms.

1.1 Real Cases Discussion

Some real cases are given below to show the practical actions of the e-commerce firms to establish new retail.

Amazon Books

Amazon Books is the physical extension of Amazon.com. 13 physical bookstores have been settled down in cities such as California, New York, and more stores are coming soon. Amazon Books is not competing with traditional bookstores such as Barnes & Noble, but contains products in 35 categories except for the books, such as devices like Echo and book readers for trial. Besides, the bookstores also serve as distribution warehouses and help Amazon in the delivery.

JD.com & Yonghui Supermarket

JD.com, which is the second largest e-commerce firm in China, cooperates with Yonghui Supermarket to transfer a part of the supermarkets as its offline selling channel. Moreover, JD invites Tencent, the largest Internet service provider in China, to jointly support Yonghui Supermarket on the fresh foods new retail establishment. The offline stores named "Super Species" provide fresh foods experience and the deliveries of orders from the supermarket APP.

Alibaba

Alibaba exhibits its online clothes in clothes shops named “Simple Style” to provide try-on service. The value uncertainty in online clothes purchasing is always very high, but in the offline shops, consumers can try the clothes on and eliminate the uncertainty. They can buy the clothes immediately from the offline shops, or they can buy it online through the QR code on the clothes and wait at home for only a few hours. Alibaba also builds fresh food stores named “HEMA” for consumers to experience, and provides both the online and offline purchasing ways.

Although the cases above seem to go very well, the truth is that the offline stores in the real cases are very few. They are only trials for the later process of the complete development of new retail. There are three main difficulties that the e-commerce firms will encounter:

(1) There is a tradeoff between the channel building cost and the distances of the stores to the consumers. Closer distances to consumers will be easier for consumers to come, and consequently attract more demand. However, to cover a fixed market area, closer distances require more offline stores, leading to a higher channel building cost.

(2) The online competitors will compete with them for the market share. When the new retail firm establishes the offline channel, the well-developed or newly-built online competitors will not wait for it to finish.

(3) The combination of the online and offline channels gives rise to difficulty in recognizing consumers. Online consumers are easy to recognize by their accounts, but identifying a consumer in both channels needs high technology support, and should avoid the violation of consumer privacy at the same time.

In our model, we properly capture these three problems perspectively and find the solutions to these problems, which will be elaborated in the next subsection.

1.2 The Model and Results

To investigate the problems above, we develop a duopoly model with a new retail firm and an online firm, who sell the same product during two selling periods. The new retail firm offers the same price in online and offline channels, a certain product value in offline stores and high-speed deliveries for online orders from offline stores. The online firm provides product with value uncertainty, and normal-speed deliveries.

The channel cost of the new retail firm is higher than that of the online firm, including the cost of channel establishment, logistics, etc. Here, we capture the problem of channel building tradeoff between the hassle cost to consumers and the channel cost. When the hassle cost decreases, the channel cost will quadratically increase.

The consumers of the new retail firm resolve the value uncertainty offline in the first period, and can purchase the product online in the second period. The consumers of the online firms buy the product with uncertainty for the first time, and if they buy the product for the second time, the product value is certain because of the first-time purchasing. The consumers are heterogenous in the sensitivity to the hassle cost of purchasing, which is measured by the average hassle distance between the consumers and the nearest offline stores. The consumers will compare the utilities from the two firms and maximize their overall utilities across the two selling periods.

The new retail firm and the online firm seek to occupy the market share, for which price competition is commonly used. We adopt the behavior-based pricing (BBP) competition model, in which the two firms offer different prices in the second period to consumers with different purchasing behaviors/histories (which firm they buy from) in the first period. In the BBP model, after the two firms announce prices in the first period, they will offer a pair of prices in the second period: one to its previous consumers for retaining them, and one to the competitor's previous consumers for attracting them. The consequence is that a part of consumers switch from their first-period chosen firm to the other firm. We solve for the Nash equilibrium of the two-period game. In addition, we also consider a benchmark pricing model, in which the consumer behaviors/histories are out of consideration.

One necessary matter of using BBP is the consumer recognition on their historical purchasing behaviors/histories. Consumers with different behaviors will be offered different prices from the firms. However, in new retail, consumer may not be completely recognized. To capture the problem of consumer recognition, we consider the situation that the new retail firm cannot recognize all its previous consumers, to investigate the influence of the difficulty of consumer recognition.

The BBP model is not new, but our model has unique novelties. First, we are the first to properly use it in a new retail and online competition problem. In our model, the second period is not the repeat of the first period, but fits the features of purchasing from the new retail firm and the online firm. We also consider the situations using behavior-based pricing (BBP) and without consideration of consumer behaviors/histories, to compare the new retail firm's decisions on channel building. Second, we consider different channel costs of the two firms. The new retail firm's choice on how well to build its offline stores to balance the channel cost and consumers' distances (or hassle costs) to offline stores is a significant problem. Third, considering the difficulty in consumer recognition, we analyze the situation when consumers are not completely recognized.

The results we obtain provide valuable insights to the new retail development in e-commerce. Each price of the new retail firm, which is the first-period price, the second-period price to own consumers or the second-period price to competitor's consumers, is higher than the corresponding price of the traditional online firm. The reason is that the new retail firm has a higher channel cost in building offline stores and high-speed deliveries from the stores. Under certain condition, the new retail firm will establish a larger hassle cost to consumers in the BBP model than that in the benchmark model. Interestingly, the difficulty of consumer recognition leads the new retail firm to occupying market share. Meanwhile, it may benefit the new retail firm.

The rest of the paper is organized as follows. We present the literature review following the Introduction. In Sections 3 and 4, we introduce the model setup and present the benchmark model without consideration of consumer behaviors. In Section 5, we solve for the Nash equilibrium with BBP competition. In Sections 6 and 7, we analyze the impact of the BBP competition and consumer recognition. We conclude the managerial implications and directions for future research in Section 8. All proofs appear in the Appendix.

2 Literature review

The new retail is the integration of the online and offline channels. In the retailing channel studies, there is a literature stream studying the competition between the manufacturer and its independent retailers. Chiang et al. (2003) construct a price setting game to show that direct marketing of the manufacturer helps in improving its profit. Meanwhile, the participation of manufacturer may not hurt the retailer because of a lower wholesale price. Cattani et al. (2006) study the pricing matching between the manufacturer and the retailer. They find that when the direct channel of manufacturer is not convenient enough for the consumers, the equal-pricing is preferred. Otherwise, the equal-pricing policy will be abandoned. Liu and Zhang (2006) conclude that the retailer is worse off when it can personalized pricing. Other aspects, e.g., sales effort (Tsay and Agrawal 2004), service decisions and competition (Hu and Li 2012, Chen et al. 2008), and drop-shipping (Netessine and Rudi 2006), are also studied.

Recently, Li et al. (2017) focus on the impact of the retailer's risk-averse behavior and the selling cost of manufacturer on the optimal supply chain decisions. Li et al. (2016) also consider the retailer's risk-averse and present an improved risk-sharing contract for the supply chain cooperation. Amrouche and Yan (2016) investigate the wholesale pricing of the manufacturer when it simultaneously manages an online channel and a traditional retail channel. Soleimani et al. (2016) study the pricing strategies of dual-channel under centralization and decentralization. Ding et al. (2016) consider a hierarchical pricing decision process and find the optimal pricing strategy in a dual-channel supply chain with a manufacturer as the leader and a retailer as the follower. In the same setting, Huang et al. (2018) extend to consider an optimization problem under stochastic demand.

Omni-channel management is also broadly studied, the goal of which is to provide all available channels to consumers without barriers. Ansari et al. (2008) empirically study the customer migration between channels. Ofek et al. (2011) explore the impact of product returns in the omni-channel setting. Gao and Su (2016) develop a theoretical framework to study the impact of Buy-Online-and-Pick-up-in-Store on the store operations. Unlike the concept of omni-channel, the concept of new retail in our paper focuses on the integration of online and offline channels. The new retail does not consider all the available channels, and is also an improvement of omni-channel in the consumer data collection (recognition).

Our work is also closely related to a literature stream of the behavior-based pricing. Fudenberg and Villas-Boas (2006) present a comprehensive review. Villas-Boas (2004) considers a monopoly model in which the firms live infinitely and each consumer lives for two periods. He shows that the monopolist is worse off than if it could not recognize its previous customers. Villas-Boas (1999) studies in the similar setup and shows that in a duopoly market, the firms offer lower prices to attract the competitor's consumers. Fudenberg and Tirole (2000) study the duopoly poaching under both short-term and long-term contracts when consumer's brand preferences are fixed or independent over time. Zhang (2011) shows that behavior-based personalization damages the product differentiation and intensifies the price competition. Shaffer and Zhang (2000) find that price discrimination leads to lower prices. When the demand is symmetric (asymmetric), charging lower prices to the competitor's (own) consumers is optimal. Gehrig et al. (2011) consider a model in which a new firm shows up in the market but does not have the access to consumer purchasing histories, and implies that the use of BBP is for exploitation, not exclusion.

The literature also examines when the firm should conduct the BBP. Acquisti and Varian (2005) study the situation when firms can commit to a pricing policy with price discrimination or not, and conclude that it is never optimal for the firm to distinguish the high-value and low-value consumers. Pazgal and Soberman (2008) examine the competitive effect when firms are able to commit about whether to conduct behavior-based pricing, and conclude that firms' profits of conducting BBP are always lower than those without BBP. Esteves (2010) extends this finding when the consumers are myopic. Shin and Sudhir (2010) attempts to answer the firms' dilemma that when firms should conduct behavior-based pricing, considering two features of consumers: heterogeneity in consumer value and changing preference. Chen (2008) studies the situation when the market consists of a stronger firm and a weaker firm, and concludes that when the weaker firm can exist in the market, and BBP benefits the consumers. Gehrig et al. (2012) develop asymmetric duopoly model to show that the uniform pricing is better off than history-based pricing, and the latter benefits the consumers.

Some studies focus on the impact on the BBP decisions of other factors. Li and Jain (2015) considers the consumer unfairness with behavior-based pricing, and studies the impact of fairness concerns on firms' prices, profits, consumer surplus and social welfare. Jing (2016) studies the firms' quality differentiation and profits in BBP model, when the product qualities are exogenous or endogenous. Rhee and Thomadsen (2016) study the asymmetric BBP model with vertical differentiation, and highlight the quality-adjusted cost difference between firms, and consumer discounting and firm discounting. Colombo (2016) develops a BBP model with incomplete information on consumers' purchasing histories and shows that the impact of information accuracy on profits is non-monotonic. Esteves and Cerqueira (2017) are the first considering the firms' advertising efforts on prices to target consumers in a BBP model. Caillaud and De Nijs (2014) consider the pricing discrimination with loyalty reward, which helps the firm in extracting more surplus from consumers who reveal strong preferences and in recognizing new consumers.

3 The model

We consider a duopoly market, which consists of a new retail firm (n-firm) and an online firm (o-firm), selling the same product during two selling periods. The production cost of the product is c, which is assumed to be zero without loss of generality. The base value of the product is v. Following the literature (Li and Jain 2015, Jing 2016, Rhee and Thomadsen 2016), v is assumed to be constant and sufficiently high, so the market is fully covered.

The n-firm combines the online and offline channels, and offers the same price p_n in the two channels. We assume that one consumer to the new retail firm will first go to the offline stores to certain the value v with hassle cost t_n (transportation cost) and pay p_n. After that, he will buy the product online with the certain value v with a hassle cost h_n (paying shipping fees, waiting for the product to come). As the hassle costs in two periods are the costs of getting the product, by self immediately with t_n or by waiting for a high-speed delivery with h_n, the consumer experiences do not differ much. We assume that $t_n = h_n$ for the model efficiency. The combined channel cost of selling one product is c_n.

The online firm only provides online channel. One consumer to the online firm will buy with value uncertainty (discount) of αv at price p_o, and the hassle cost is h_o. We assume that all consumers are homogenous in the value uncertainty. After the first trial in the first period, the consumer will buy with certain value v with the same price p_o and hassle cost h_o for the second time. The channel cost of the online firm for selling one product is c_o.

With shorter distances to offline stores and higher speed deliveries, the n-firm is more convenient for consumers to purchase from than the o-firm. Hence, the consumer hassle cost of buying from the n-firm is less than that of the o-firm, that is, $h_n < h_o$. Meanwhile, the n-firm builds two combined channels and the o-firm has one online channel. Thus, the channel cost of the n-firm is higher than that of the o-firm, that is, $c_n > c_o$. Both firms seek to maximize their overall profits across the two periods.

We assume that consumers are heterogeneous in the sensitivity of hassle cost. The sensitivity level θ is assumed to be uniformly distributed at $[0, 1]$. The consumer utility of purchasing from the n-firm in each period is $v - p_n - \theta h_n$. For the o-firm, the utility of a first-time consumer is $\alpha v - p_o - \theta h_o$, and the utility of a second-time consumer is $v - p_o - \theta h_o$. The consumers compare the utilities of purchasing from the two firms and maximize their overall utility across the two periods.

We adopt the behavior-based pricing (BBP) model, in which both firms try to occupy the other firm's previous consumers with the tool of price. In the second period, they discriminate consumers by offering different prices to their previous consumers and its competitor's consumers. Thus, they decide a set of prices: the first-period price $p_i, i \in [n, o]$, the second-period

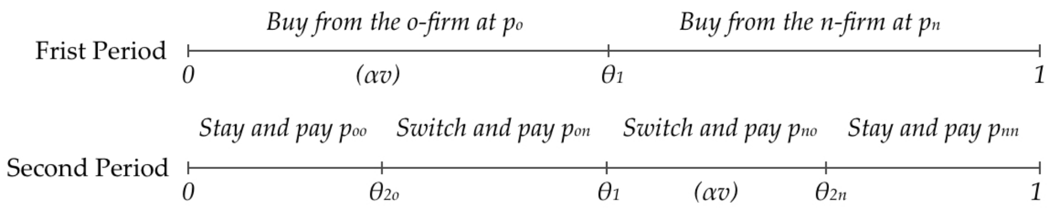

Fig. 1: The market structure of the behavior-based pricing model.

price to its first-period consumers p_{ii} and the second-period price to the competitor's first-period consumers $p_{ji}(j \neq i, j \in [n,o])$. The consequence is that in the second period, the two firms will poach the other firm's first-period market share.

The market structure in the two selling periods is illustrated in Fig. 1. In the first period, the consumers with $0 < \theta < \theta_1$ buy from the o-firm with product value αv, and the left consumers buy from the n-firm with certain product value v. In the second period, the o-firm's first-period market share is split into two market shares of two firms by θ_{2o}. The n-firm's first-period market share is also split by θ_{2n}, but the difference is that for consumers in $[\theta_1, \theta_{2n}]$, the product value is αv because it is the first time for them to purchase from the o-firm.

4 Benchmark without consideration of consumer behaviors

Before proceeding, we present a benchmark model in which consumer behaviors/histories are not taken into account. In this case, the prices offered by one firm in two periods are the same.

The consumers make purchasing decisions to maximize their utilities across the two periods. As shown in Fig. 2, the marginal consumer who is indifferent in buying from the two firms locates at θ'. Consumers with $\theta < \theta'$ choose the o-firm because they are less sensitive to hassle cost, while consumers with $\theta > \theta'$ choose the n-firm. And in the first period, the o-firm's consumers purchase with uncertain value αv. Solving $(\alpha v - p'_o - \theta' h_o) + (v - p'_o - \theta' h_o) = 2(v - p'_n - \theta' h_n)$, we have

$$\theta' = \frac{2(p'_n - p'_o) - v(1-\alpha)}{2(h_o - h_n)} \tag{1}$$

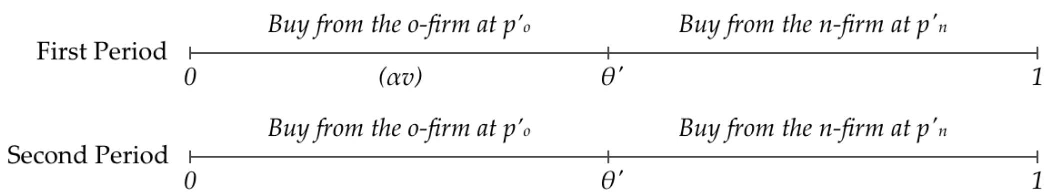

Fig. 2: The market structure without consideration of consumer behaviors.

The o-firm maximizes its profit $\pi_o' = 2(p_o' - c_o)\theta'$, while the n-firm maximizes its profit $\pi_n' = 2(p_n' - c_n)(1-\theta')$. The Nash equilibrium of the two firms are given in the following proposition.

Proposition 1 *(Equilibrium without consideration of consumer behaviors) The prices of the two firms are* $p_o' = [2(c_n + 2c_o) + 2(h_o - h_n) - v(1-\alpha)]/6, p_n' = [2(2c_n + c_o) + 4(h_o - h_n) + v(1-\alpha)]/6$*. It is obvious that* $p_n' > p_o'$*. The market cutoff* $\theta' = [2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha)]/[6(h_o - h_n)]$*.*

The market share of the o-firm is $s_o = \theta'$ *and the n-firm is* $s_n = 1 - \theta'$*.* $s_o < (\geq) s_n$ *when* $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$*.*

The profits of the two firms are $\pi_o' = [2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha)]^2/[36(h_o - h_n)], \pi_n' = [4(h_o - h_n) - 2(c_n - c_o) + v(1-\alpha)]^2/[36(h_o - h_n)]$*.* $\pi_o < (\geq) \pi_n$ *when* $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$*.*

In addition, the n-firm can exist in the market only when $\theta' < 1$*, that is,* $2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha) < 0$*. The o-firm can exist in the market when* $\theta' > 0$*, that is* $2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha) > 0$*.*

The proposition shows that the n-firm provides a higher price than that of the o-firm. The reason is that the n-firm has a higher channel cost, and it provides the consumers with lower hassle cost and no value uncertainty. The market shares and profits of the two firms are influenced by the differences between the channel costs, hassle costs, and value uncertainties of the two firms: $c_n - c_o$, $h_o - h_n$, and $v(1-\alpha)$. When $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$, the market share and the profit of the o-firm are smaller (larger) than those of the n-firm. When $2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha) < 0$, the n-firm can survive in the market, or the o-firm will occupy all the market share. When $2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha) > 0$, the o-firm can survive in the market, or the n-firm will occupy all the market share.

In this paper, we emphasize the three problems faced by the new retail firm (n-firm): the choice on the distance of offline stores to consumers, the price decision in price competition, and the difficulty of consumer recognition. We now discuss the first problem: the distance of offline stores to consumers. As a shorter distance between the consumers and the offline stores indicates smaller transportation cost, shipping fee and waiting time, that is, a smaller hassle cost to the consumers. Therefore, the distance problem can be reflected by the hassle cost h_n. To provide a lower average hassle cost (shorter distance) to consumers in a market area, the n-firm must build more offline stores with a higher cost. Given the existing online hassle cost h_o and the corresponding channel cost c_o, the n-firm decreases the hassle cost to h_n with a higher channel cost c_n.

We assume that the relation between the reduced hassle cost and increased channel cost is quadratic: $c_n - c_o = A(h_o - h_n)^2$. Transforming it into $c_n = A(h_o - h_n)^2 + c_o$, this formula follows the commonly used quadratic cost function $c = aq^2 + bq + d$ in the operations management research with $b = 0$. $\frac{\partial c_n}{\partial (h_o - h_n)} = 2A(h_o - h_n) > 0$ satisfies the property that the marginal channel cost of reducing consumer hassle cost increases as $h_o - h_n$ increases, which corresponds to the reality.

Substituting $c_n - c_o = A(h_o - h_n)^2$ into the n-firm's profit $\pi'_n = [2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha)]^2/[36(h_o - h_n)]$, we solve for the optimal h_n^* and the result is given in the following proposition.

Proposition 2 *The optimal hassle cost $h_n'^*$ to consumers of the n-firm is*

(1) if $v(1-\alpha) \leq \frac{2}{3A}$, $(h_o - h_n)^* = \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$, *so* $h_n'^* = \left(h_o - \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}\right)^+$;

(2) if $v(1-\alpha) > \frac{2}{3A}$, $(h_o - h_n)^* = \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$, *so* $h_n'^* = \left(h_o - \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}\right)^+$;

, since the hassle cost $h_n'^*$ must be positive.

This proposition indicates that, when the consumer value uncertainty of the o-firm $v(1-\alpha)$ is smaller or larger than $\frac{2}{3A}$, the n-firm will provide different optimal hassle costs to consumers. We can observe that when $h_n < h_n'^*$, as the hassle cost h_n increases, the profit of the n-firm increases. This counter-intuitive result is because although the higher hassle cost has a negative impact on the consumers' purchasing decisions, the benefit due to the quadratical decrease of the channel cost will overweigh the negative impact, and consequently increase the profit.

5 Competition with behavior-based pricing

In this section, we study the price competition with behavior-based pricing. The two firms try to poach the competitor's market share in the second period, by offering different prices to consumers with different purchasing behaviors in the first period.

We solve for the subgame-perfect equilibrium of the two-period game backwards. We will first analyze the consumer choices and the firms' pricing decisions in the second period, taken the first-period market share as given.

5.1 Competition in the second period

In the second period, the market share in the first period is taken as settled. We assume a cutoff value θ_1 such that consumers with $\theta < \theta_1$ purchase from the o-firm in the first period, while consumers with $\theta > \theta_1$ purchase from the n-firm in the first period. Thus, θ_1 also represents the market share of the o-firm in the first period and $1-\theta_1$ represents the market share of the n-firm.

The competitions in the second period occur separately in the market share of the n-firm and the o-firm in the first period. As shown in Fig. 3, consumers who purchase from the o-firm are divided by a new indifferent cutoff value θ_{2o}. Consumers with $\theta < \theta_{2o}$ buy from the o-firm for the second time in the second period, and consumers with $\theta_{2o} < \theta < \theta_1$ switch and buy from the n-firm for the first time in the second period. Thus, θ_{2o} can be found by setting $v - p_{oo} - \theta_{2o}h_o = v - p_{on} - \theta_{2o}h_n$. Solving for θ_{2o} yields

$$\theta_{2o} = \frac{p_{on} - p_{oo}}{h_o - h_n} \tag{2}$$

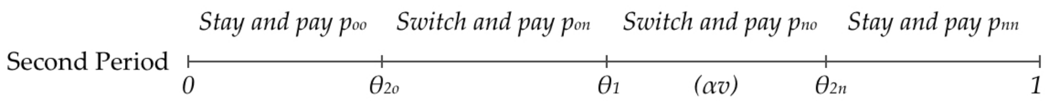

Fig. 3: The two competitive regions in the second period: $\theta < \theta_1$ and $\theta > \theta_1$.

The o-firm sets its price to maximize its profit $\pi^o_{2o} = (p_{oo} - c_o)\theta_{2o}$, while the n-firm maximizes $\pi^n_{2o} = (p_{on} - c_n)(\theta_1 - \theta_{2o})$. The equilibrium is summarized in Lemma 1.

Lemma 1 *(Competition on the o-firm's market share) The equilibrium can be expressed as a function of* θ_1. *The prices are* $p^*_{oo} = [(c_n + 2c_o) + \theta_1(h_o - h_n)]/3$, $p^*_{on} = [(2c_n + c_o) + 2\theta_1(h_o - h_n)]/3$. *The n-firm's market share is split by two firms with* $\theta_{2o} = [(c_n - c_o) + \theta_1(h_o - h_n)]/[3(h_o - h_n)]$.

We then consider the competition in the n-firm's market share in the first period. The consumers with $\theta_1 < \theta < \theta_{2n}$ switch and buy from the o-firm for the first time with value αv, and consumers with $\theta_{2n} < \theta < 1$ buy from the n-firm for the second time. The new cutoff value θ_{2n} is solved by setting $\alpha v - p_{no} - \theta_{2o}h_o = v - p_{nn} - \theta_{2o}h_n$. Thus,

$$\theta_{2n} = \frac{p_{nn} - p_{no} - v(1-\alpha)}{h_o - h_n}. \tag{3}$$

In such a case, the o-firm maximizes its profit $\pi^o_{2n} = (p_{no} - c_o)(\theta_{2n} - \theta_1)$ and the o-firm maximizes $\pi^n_{2n} = (p_{nn} - c_n)(1 - \theta_{2n})$. The equilibrium is summarized in Lemma 2.

Lemma 2 *(Competition on the n-firm's market share) The equilibrium can be expressed as a function of* θ_1. *The prices are* $p^*_{no} = [(c_n + 2c_o) + (1 - 2\theta_1)(h_o - h_n) - v(1-\alpha)]/3$, $p^*_{nn} = [(2c_n + c_o + (2 - \theta_1)(h_o - h_n) + v(1-\alpha)]/3$. *The o-firm's market share is split by the two firms with* $\theta_{2n} = [(c_n - c_o) + (1 + \theta_1)(h_o - h_n) - v(1-\alpha)]/[3(h_o - h_n)]$.

Lemma 1 and 2 present the equilibrium results in the second period in terms of θ_1. Next, we go back to the first period.

5.2 Competition in the first period

In the first period, the consumers are clear that their choices in the first period will affect their prices in the second period. Thus, their decisions are based on the utility of two periods. As shown in Fig. 4, at the cutoff value θ_1, the marginal consumer is indifferent between buying from the o-firm in the first period and the n-firm in the second period, and from the n-firm in the first period and o-firm in the second period, so

$$(\alpha v - p_o - \theta_1 h_o) + (v - p_{on} - \theta_1 h_n) = (v - p_n - \theta_1 h_n) + (\alpha v - p_{no} - \theta_1 h_o). \tag{4}$$

From equation (4) we have

$$p_o - p_n = p_{no} - p_{on}. \tag{5}$$

In Lemma 1 and Lemma 2, we have obtained p^*_{no} and p^*_{on} in the function of θ_1. By substituting p^*_{no} and p^*_{on} , θ_1 can be represented as a function of $p_o - p_n$:

$$\theta_1 = \frac{(c_o - c_n) + (h_o - h_n) - 3(p_o - p_n) - v(1-\alpha)}{4(h_o - h_n)}. \tag{6}$$

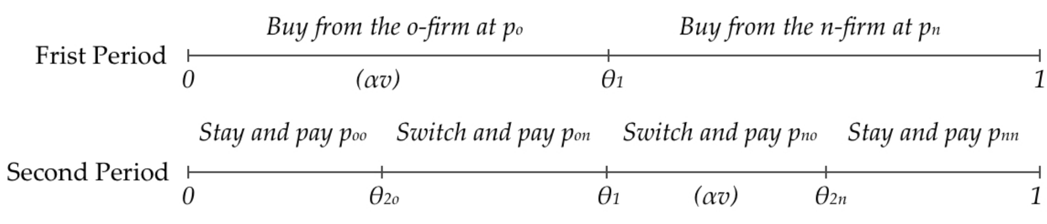

Fig. 4: The competition in the first period.

The profits of the o-firm and n-firm are the overall profits across the two periods, which can be specified using the first-period market shares determined by θ_1 and the second-period equilibrium prices and market shares in Lemma 1 and Lemma 2. Thus, the profits of the o-firm and the n-firm are

$$\pi_o = (p_o - c_o)\theta_1 + (p_{oo} - c_o)\theta_{2o} + (p_{no} - c_o)(\theta_{2n} - \theta_1), \tag{7}$$

$$\pi_n = (p_n - c_n)(1 - \theta_1) + (p_{on} - c_n)(\theta_1 - \theta_{2o}) + (p_{nn} - c_n)(1 - \theta_{2n}). \tag{8}$$

5.3 Equilibrium

The pure-strategy equilibrium of π_o and π_n is found by solving the Nash game between the o-firm and n-firm. $p_{oo}, p_{on}, p_{no}, p_{nn}$ are written by Lemmas 1 and 2, θ_{2o} and θ_{2n} by equations (2) and (3), π_o and π_n by equations (7) and (8). All the decision variables in equilibrium are solved. The results are given in the following proposition.

Proposition 3 *The equilibrium prices p_o and p_n and market cutoff θ_1 in the first period are*

$$\begin{aligned}
p_o =& [(6c_n + 18c_o) + 13(h_o - h_n) - 11v(1-\alpha)]/24, \\
p_n =& [(18c_n + 6c_o) + 19(h_o - h_n) - 5v(1-\alpha)]/24, \\
\theta_1 =& [2(c_n - c_o) + 7(h_o - h_n) - v(1-\alpha)]/[16(h_o - h_n)].
\end{aligned}$$

The prices and market cutoffs in the second period are

$$\begin{aligned}
p_{oo} =& [(18c_n + 30c_o) + 7(h_o - h_n) - v(1-\alpha)]/48, \\
p_{on} =& [(18c_n + 6c_o) + 7(h_o - h_n) - v(1-\alpha)]/24, \\
p_{no} =& [(6c_n + 18c_o) + (h_o - h_n) - 7v(1-\alpha)]/24, \\
p_{nn} =& [(30c_n + 18c_o) + 25(h_o - h_n) + 17v(1-\alpha)]/48, \\
\theta_{2o} =& [18(c_n - c_o) + 7(h_o - h_n) - v(1-\alpha)]/[48(h_o - h_n)], \\
\theta_{2n} =& [18(c_n - c_o) + 23(h_o - h_n) - 17v(1-\alpha)]/[48(h_o - h_n)].
\end{aligned}$$

The equilibrium profits of the two firms are

$$\pi_o = [(540(c_n-c_o)^2+599(h_o-h_n)^2-610(h_o-h_n)v(1-\alpha)+263v^2(1-\alpha)^2$$
$$+708(h_o-h_n)(c_n-c_o)-540(c_n-c_o)v(1-\alpha)]/[2304(h_o-h_n)],$$
$$\pi_n = [(540(c_n-c_o)^2+1847(h_o-h_n)^2+638(h_o-h_n)v(1-\alpha)+263v^2(1-\alpha)^2$$
$$-1788(h_o-h_n)(c_n-c_o)-540(c_n-c_o)v(1-\alpha)]/[2304(h_o-h_n)].$$

In addition, from $\theta_{2o} < \theta_1 < \theta_{2n}$, we have $7(h_n-h_o)-6(c_n-c_o)-v(1-\alpha) > 0$ and $(h_o-h_n)+6(c_n-c_o)-7v(1-\alpha) > 0$.

The analysis of the equilibrium will be presented in the next section.

6 BBP equilibrium analysis

In this section, we analyze the equilibrium with behavior-based pricing. First, we compare the prices offered by the two firms in the two periods and the results are given in the following proposition.

Proposition 4 *The price of the n-firm is larger than that of the o-firm in the first period, $p_n > p_o$. In the second period, the n-firm also offers higher prices than the o-firm for one firm's first-period consumers, $p_{on} > p_{oo}$ and $p_{nn} > p_{no}$. Meanwhile, both firms offer a lower price to the competitor's consumers than that to its previous consumers, $p_{on} < p_{nn}$ and $p_{no} < p_{oo}$.*

This proposition indicates that, each price offered by the n-firm is larger than the corresponding price of the o-firm in both periods. The reason is that each price of the n-firm minuses the corresponding price of the o-firm is determined to be positive because $c_n > c_o$ (the n-firm has a larger channel cost than that of the o-firm) and $h_n < h_o$ (the n-firm provides a smaller average consumer hassle cost than that of the o-firm). In addition, both firms offer a lower price to its competitor's consumers than that to its previous consumers, which means that they treat its competitor's consumers better than their own consumers. This result is in accord with the literature.

The market share of the o-firm is $so_1 = \theta_1$ in the first period and $so_2 = \theta_{2o} + \theta_{2n} - \theta_1$ in the second period. The market share of the n-firm is $sn_1 = 1 - \theta_1$ in the first period and $sn_2 = \theta_1 - \theta_{2o} + 1 - \theta_{2n}$ in the second period. We compare the market shares and profits of the two firms and give the condition when one firm obtains more market share and profit than the other.

Proposition 5 *When* $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$*, the o-firm's market shares in two periods and profit are all smaller (larger) than those of the n-firm, and vice versa.*

The condition $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$ determines the comparison results of the two firms on market shares in two periods and the profits. Recall that in the benchmark model (see Section 4), the condition for the o-firm's market share and profit to be larger (no larger) than those of the n-firm is also $2(c_n - c_o) - (h_o - h_n) - v(1-\alpha) < (\geq) 0$. The conditions which determine the comparison result of the two firms are the same in the BBP model and benchmark model.

However, the optimal decision of hassle cost h_n in BBP competition will be different from the benchmark model. We still assume that $c_n - c_o = A(h_o - h_n)^2$. By substituting it into the profit of n-firm, we have

$$\begin{aligned}\pi_n = &(h_o - h_n)[540A^2(h_o - h_n)^2 - 1788A(h_o - h_n) + 1847] \\ &- [540A(h_o - h_n) - 638]v(1-\alpha) + \frac{263v^2(1-\alpha)^2}{2304(h_o - h_n)}\end{aligned} \tag{9}$$

We calculate the optimal h_n^* and give the result in the following proposition.

Proposition 6 *The optimal hassle cost choice* h_n^* *to the n-firm with BBP competition is:*
$h_n^* = (h_o - h^*)^+$,
where h^* *satisfies* $(1620A^2 h^{*2} - 3576Ah^* + 1847) - 540Av(1-\alpha) - \frac{263v^2(1-\alpha)^2}{h^{*2}} = 0$ *and is unique.*

Compared with the optimal hassle cost $h_n'^*$ in the benchmark model without consideration of consumer behaviors, we have the following proposition.

Proposition 7 *The comparison result of the hassle costs in the BBP model and benchmark model is:*

(1) if $v(1-\alpha) \leq \frac{53(\sqrt{6817}-2619)}{4096A}$, *then* $h^* \geq \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$, *so* $h_n^* \leq h_n'^*$;

(2) if $v(1-\alpha) > \frac{53(\sqrt{6817}-2619)}{4096A}$, *then* $h^* < \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$ *and* $h^* < \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$, *so* $h_n^* > h_n'^*$.

This proposition indicates that when the value uncertainty (discount) $v(1-\alpha)$ of the o-firm is small enough, the n-firm will establish an offline channel with a smaller hassle cost than that in the benchmark model. While when the value uncertainty of the o-firm is large, the n-firm will establish the offline channel with a larger hassle cost than that in the benchmark model. This result reveals the insight that in the BBP competition, the new retail firm who choose a product with large enough value uncertainty can build a smaller number of online stores than that without the BBP competition. In practice, this insight is applicative to the the new retail firms who choose the products with large value uncertainty, such as fresh foods, clothes.

7 Consumer recognition

We have solved the problems of the n-firm on the store distance to consumers (represented by consumer hassle cost) and price competition, now we focus on the difficulty of consumer recognition. We assume that the n-firm cannot recognize all its previous consumers. For the un-recognized consumers, the difference is that when they purchase from the n-firm in the second period, they are taken as the o-firm's first-period consumers. The price to the un-recognized consumers are p_{on} instead of p_{nn} (noted that $p_{on} < p_{nn}$), which leads to different consumer purchasing decisions.

As shown in Fig. 5, a part of consumers, who would switch to the o-firm in the second period, now stay with n-firm at un-recognized price p_{on}. A new cutoff value θ_u shows up, at which the utility of purchasing from the o-firm $\alpha v - p_{no} - \theta_u h_o$ and the utility from the n-firm

$v - p_{on} - \theta_u h_n$ are equal. So, we have

$$\theta_u = \frac{p_{on} - p_{no} - v(1-\alpha)}{h_o - h_n} = \frac{2(c_n - c_o) + (h_o - h_n) - 3v(1-\alpha)}{4(h_o - h_n)}. \tag{10}$$

Moreover, as shown in Fig. 6, if θ_u is smaller than θ_1, then the market structure is affected more by consumer un-recognition. As all the n-firm's first-period consumers stay with the n-firm because they are un-recognized, their purchasing decisions in the first period will exclude the consideration of the second period. They directly choose between buying from the o-firm at price p_o and from the n-firm at price p_n. Solving $\alpha v - p_o - \theta h_o = v - p_n - \theta h_n$ and based on equation (5) $p_o - p_n = p_{no} - p_{on}$, we obtain

$$\theta = \frac{p_n - p_o - v(1-\alpha)}{h_o - h_n} = \frac{p_{on} - p_{no} - v(1-\alpha)}{h_o - h_n} = \theta_u. \tag{11}$$

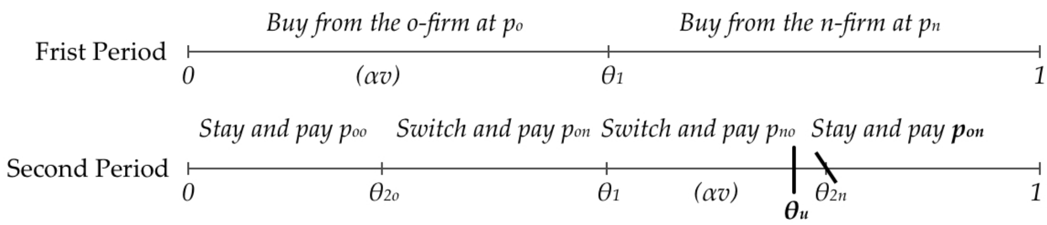

Fig. 5: The market structure with consumer un-recognition ($\theta_u \geq \theta_1$).

Our aim is to explore the impact of consumer recognition on the n-firm's profit. If $\theta_u \geq \theta_1$ (see Fig. 5), the n-firm retains extra consumers in $[\theta_u, \theta_{2n}]$ in the second period. This part of extra market share is $\Delta\theta n = \theta_{2n} - \theta_u$. Although the n-firm retains more consumers, it offers the un-recognized consumers a lower price. The n-firm's profit difference because of consumer un-recognition is $\Delta\pi_n = (p_{on} - c_n)(1 - \theta_u) - (p_{nn} - c_n)(1 - \theta_{2n})$.

If $\theta_u < \theta_1$ (see Fig. 6), the n-firm retains all its first-period consumers, and it also occupies more market share in the first period. The extra market share because of consumer recognition is $\Delta\theta n_1 = \theta_1 - \theta_u$ in the first period and $\Delta\theta n_2 = \theta_{2n} - \theta_1$ in the second period. The profit difference due to consumer recognition is $\Delta\pi_n = (p_n - c_n)(\theta_1 - \theta_u) + (p_{on} - c_n)(1 - \theta_1) - (p_{nn} - c_n)(1 - \theta_{2n})$.

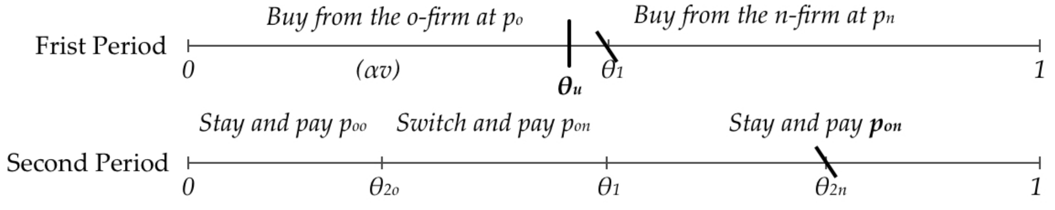

Fig. 6: The market structure with consumer un-recognition ($\theta_u < \theta_1$).

The impact of consumer un-recognition to the n-firm is summarized in the following proposition.

Proposition 8 *(The impact of consumer un-recognition to the n-firm)*

(1) if $6(c_n - c_o) - 3(h_o - h_n) - 11v(1-\alpha) \leq 0,\ \theta_u \geq \theta_1$. *The n-firm occupies extra market share* $\Delta\theta n = \frac{11(h_o-h_n)-6(c_n-c_o)+19v(1-\alpha)}{48(h_o-h_n)}$ *than with complete recognition in the second period. The profit difference of the n-firm because of consumer un-recognition is* $\Delta\pi_n = -\frac{[11(h_o-h_n)-6(c_n-c_o)+19v(1-\alpha)]^2}{2304(h_o-h_n)} < 0$, *which indicates that the profit of the n-firm decreases because of the consumer un-recognition.*

(2) if $6(c_n-c_o)-3(h_o-h_n)-11v(1-\alpha) < 0,\ \theta_u < \theta_1$. *The n-firm occupies extra market share* $\Delta\theta n_1 = \frac{-[6(c_n-c_o)-3(h_o-h_n)-11v(1-\alpha)]}{16(h_o-h_n)}$ *in the first period, and* $\Delta\theta n_2 = \frac{(h_o-h_n)+6(c_n-c_o)-7v(1-\alpha)}{24(h_o-h_n)}$ *in the second period. The profit difference due to consumer un-recognition is* $\Delta\pi_n$, *and* $\Delta\pi_n > 0$ *when* $95(h_o-h_n)^2+372(c_n-c_o)v(1-\alpha)+302(h_o-h_n)v(1-\alpha)-36(c_n-c_o)^2-300(c_n-c_o)(h_o-h_n)-625v^2(1-\alpha)^2 > 0$.

The consumer un-recognition problem results in that the new retail occupies more market share in the BBP model. Under certain condition, it even benefits the n-firm. However, to the o-firm, the consumer un-recognition of the n-firm is harmful. The n-firm's extra market share is the loss of the o-firm, and the o-firm obviously loses the profit on the market share.

8 Conclusions

Some companies in e-commerce have been developing the new retail in recent 2-3 years. The three main challenges are the building of offline stores, the prices in a competition environment, and the difficulty in recognizing consumers. This article seeks to provide management insights to help the e-commerce in developing its new retail mode. We employ the behavior-based pricing (BBP) model, to characterize the price competition between the new retail firm and the traditional online firm. We capture the building of offline stores by the consumer hassle cost, which is a measure of the easiness of reaching the offline stores by the consumers.

We conclude that, in the BBP model, each price offered by the n-firm is higher than the corresponding price of the o-firm. Both the firms offer lower prices to the competitor's consumers than its own consumers. This result is in accord with the common sense and literature. We also provide the optimal decision on the new retail firm's consumer hassle cost. When the consumers' value uncertainty to the online firm is large enough , the n-firm will provide a larger hassle cost to the consumers in the BBP model than that in the benchmark model.

As for the consumer un-recognition to the new retail firm, it is not a weakness in BBP competition, but leads the new retail firm to occupy more market share. The profit difference because of consumer un-recognition may not be positive, moreover, there exists condition under which the new retail firm benefits from the consumer un-recognition.

This article is a real case application of the BBP model. In this article, we consider the competition of a new retail firm and an online firm, as the new retail mode mainly influences the consumer purchasing experience online. However, the new retail also has an impact on the offline market because of the offline stores occupy a part of the offline market. This impact can be a future research direction. In addition, the BBP model works on the premise that consumers can clearly observe prices they are offered. However, consumers need efforts to make prices visible. The impact of this situation is also worth investigating.

References

1. Acquisti, A. and Varian, H. R. (2005). Conditioning prices on purchase history. *Marketing Science*, 24(3):367–381.
2. Amrouche, N. and Yan, R. (2016). A manufacturer distribution issue: how to manage an online and a traditional retailer. *Annals of Operations Research*, 244(2):257–294.
3. Ansari, A., Mela, C. F., and Neslin, S. A. (2008). Customer channel migration. *Journal of marketing research*, 45(1):60–76.
4. Caillaud, B. and De Nijs, R. (2014). Strategic loyalty reward in dynamic price discrimination. *Marketing Science*, 33(5):725–742.
5. Cattani, K., Gilland, W., Heese, H. S., and Swaminathan, J. (2006). Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel. *Production and Operations Management*, 15(1):40.
6. Chen, K.-Y., Kaya, M., and Özer, Ö. (2008). Dual sales channel management with service competition. *Manufacturing & Service Operations Management*, 10(4):654–675.
7. Chen, Y. (2008). Dynamic price discrimination with asymmetric firms. *The Journal of Industrial Economics*, 56(4):729–751.
8. Chiang, W.-y. K., Chhajed, D., and Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management science*, 49(1):1–20.
9. Colombo, S. (2016). Imperfect behavior-based price discrimination. *Journal of Economics & Management Strategy*, 25(3):563–583.
10. Ding, Q., Dong, C., and Pan, Z. (2016). A hierarchical pricing decision process on a dual-channel problem with one manufacturer and one retailer. *International Journal of Production Economics*, 175:197–212.
11. Esteves, R. B. (2010). Pricing with customer recognition. *International Journal of Industrial Organization*, 28(6):669–681.
12. Esteves, R. B. and Cerqueira, S. (2017). Behavior-based pricing under imperfectly informed consumers. *Information Economics and Policy*, 40:60–70.

13. Fudenberg, D. and Tirole, J. (2000). Customer poaching and brand switching. *RAND Journal of Economics*, 31(4):634–657.

14. Fudenberg, D. and Villas-Boas, J. M. (2006). Behavior-based price discrimination and customer recognition. *Handbook on economics and information systems*, pages 377–436.

15. Gao, F. and Su, X. (2016). Omnichannel retail operations with buy-online-and-pick-up-in-store. *Management Science*, 63(8):2478–2492.

16. Gehrig, T., Shy, O., and Stenbacka, R. (2011). History-based price discrimination and entry in markets with switching costs: a welfare analysis. *European Economic Review*, 55(5):732–739.

17. Gehrig, T., Shy, O., and Stenbacka, R. (2012). A welfare evaluation of history-based price discrimination. *Journal of Industry, Competition and Trade*, 12(4):373–393.

18. Hu, W. and Li, Y. (2012). Retail service for mixed retail and e-tail channels. *Annals of operations Research*, 192(1):151–171.

19. Huang, G., Ding, Q., Dong, C., and Pan, Z. (2018). Joint optimization of pricing and inventory control for dual-channel problem under stochastic demand. *Annals of Operations Research*, pages 1–31.

20. Jing, B. (2016). Behavior-based pricing, production efficiency, and quality differentiation. *Management Science*, 63(7):2365–2376.

21. Li, B., Hou, P.-W., Chen, P., and Li, Q.-H. (2016). Pricing strategy and coordination in a dual channel supply chain with a risk-averse retailer. *International Journal of Production Economics*, 178:154–168.

22. Li, K. J. and Jain, S. (2015). Behavior-based pricing: An analysis of the impact of peer-induced fairness. *Management Science*, 62(9):2705–2721.

23. Li, Q., Li, B., Chen, P., and Hou, P. (2017). Dual-channel supply chain decisions under asymmetric information with a risk-averse retailer. *Annals of Operations Research*, 257(1-2):423–447.

24. Liu, Y. and Zhang, Z. J. (2006). Research note?the benefits of personalized pricing in a channel. *Marketing Science*, 25(1):97–105.

25. Netessine, S. and Rudi, N. (2006). Supply chain choice on the internet. *Management Science*, 52(6):844–864.

26. Ofek, E., Katona, Z., and Sarvary, M. (2011). ?bricks and clicks?: The impact of product returns on the strategies of multichannel retailers. *Marketing Science*, 30(1):42–60.

27. Pazgal, A. and Soberman, D. (2008). Behavior-based discrimination: Is it a winning play, and if so, when? *Marketing Science*, 27(6):977–994.

28. Rhee, K. E. and Thomadsen, R. (2016). Behavior-based pricing in vertically differentiated industries. *Management Science*, 63(8):2729–2740.

29. Shaffer, G. and Zhang, Z. J. (2000). Pay to switch or pay to stay: Preference-based price discrimination in markets with switching costs. *Journal of Economics & Management Strategy*, 9(3):397–424.

30. Shin, J. and Sudhir, K. (2010). A customer management dilemma: When is it profitable to reward one's own customers? *Marketing Science*, 29(4):671–689.

31. Soleimani, F., Khamseh, A. A., and Naderi, B. (2016). Optimal decisions in a dual-channel supply chain under simultaneous demand and production cost disruptions. *Annals of Operations Research*, 243(1-2):301–321.

32. Tsay, A. A. and Agrawal, N. (2004). Channel conflict and coordination in the e-commerce age. *Production and operations management*, 13(1):93–110.

33. Villas-Boas, J. M. (1999). Dynamic competition with customer recognition. *The Rand Journal of Economics*, 30(4):604–631.

34. Villas-Boas, J. M. (2004). Price cycles in markets with customer recognition. *RAND Journal of Economics*, 35(3):486–501.

35. Zhang, J. (2011). The perils of behavior-based personalization. *Marketing Science*, 30(1):170–186.

Appendix

Proof of Proposition 1.

The o-firm's profit is $\pi_o' = (p_o' - c_o)\frac{2(p_n'-p_o')-v(1-\alpha)}{2(h_o-h_n)}$. The optimal p_o' is given by the first order condition: $p_o' = \frac{2c_o+2p_n'-v(1-\alpha)}{4}$. The n-firm's profit is $\pi_n' = (p_n' - c_n)(1 - \frac{2(p_n'-p_o')-v(1-\alpha)}{2(h_o-h_n)})$. The optimal p_n' is given by the first order condition: $p_n' = \frac{2c_n-2h_n+2h_o+2p_o'+v(1-\alpha)}{4}$. Solve the two equations we obtain p_o' and p_n'. The cutoff value θ' and the profits of the two firms are obtained by substituting p_o' and p_n'.

Proof of Proposition 2.

Substituting $c_n - c_o = A(h_o - h_n)^2$, conditions $2(c_n - c_o) - 4(h_o - h_n) - v(1-\alpha) < 0$ and $2(c_n - c_o) + 2(h_o - h_n) - v(1-\alpha) > 0$ determine that $\frac{\sqrt{4+2Av(1-\alpha)}-2}{2A} \leq (h_o - h_n) \leq \frac{2+\sqrt{4+2Av(1-\alpha)}}{2A}$.
Substituting $c_n - c_o = A(h_o - h_n)^2$ into the n-firm's profit, we have:
$\pi_n' = \frac{[-2A(h_o-h_n)^2+4(h_o-h_n)+v(1-\alpha)]^2}{36(h_o-h_n)}$. $\frac{\partial \pi_n'}{\partial (h_o-h_n)} = \frac{[-(2(c_n-c_o)-4(h_o-h_n)-v(1-\alpha)]}{36(h_o-h_n)^2}[-6A(h_o - h_n)^2 + 4(h_o - h_n) - v(1-\alpha)]$, and $\frac{[-(2(c_n-c_o)-4(h_o-h_n)-v(1-\alpha)]}{36(h_o-h_n)^2} > 0$. $-6A(h_o - h_n)^2 + 4(h_o - h_n) - v(1-\alpha)$ is positive and then negative as $h_o - h_n$ increases when $v(1-\alpha) \leq \frac{2}{3A}$. Thus, π_n' increases then decreases and $(h_o - h_n)^*$ is at $\frac{\partial \pi_n'}{\partial (h_o-h_n)} = 0$, that is, $(h_o - h_n)^* = \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$.
When $v(1-\alpha) > \frac{2}{3A}$, $-6A(h_o - h_n)^2 + 4(h_o - h_n) - v(1-\alpha)$ is negative, so π_n' decreases as $(h_o - h_n)$ increases, $(h_o - h_n)^* = \frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$.

Proof of Lemma 1.

The o-firm's profit in the o-firm's first-period market share is $\pi_{2o}^o = (p_o - c_o)\frac{p_{on}-p_{oo}}{h_o-h_n}$. The optimal p_{oo} is given by the first order condition: $p_{oo} = \frac{c_o+p_{on}}{2}$. The n-firm's profit is $\pi_{2o}^n = (p_{on} - c_n)(\theta_1 - \frac{p_{on}-p_{oo}}{h_o-h_n})$. The optimal p_{on} is given by the first order condition: $p_{on} = \frac{c_n-(h_n-h_o)\theta_1+p_{oo}}{2}$.
Solving the two equations, we obtain p_{oo}^* and p_{on}^*.

Proof of Lemma 2.

The o-firm's profit in the n-firm's first-period market share is $\pi_{2n}^{o} = (p_{no} - c_o)(\frac{p_{nn}-p_{no}-v(1-\alpha)}{h_o-h_n} - \theta_1)$. The optimal p_{no} is given by the first order condition: $p_{no} = \frac{c_o+p_{nn}-(h_o-h_n)\theta_1-v(1-\alpha)}{2}$. The n-firm's profit is $\pi_{2n}^{n} = (p_{nn} - c_n)(1 - \frac{p_{nn}-p_{no}-v(1-\alpha)}{h_o-h_n})$. The optimal p_{nn} is given by the first order condition: $p_{nn} = \frac{c_n-(h_n-h_o)+p_{no}+v(1-\alpha)}{2}$. Solving the two equations, we obtain p_{no}^{*} and p_{nn}^{*}.

Proof of Proposition 3.

We substitute $\theta_1, p_{oo}^{*}, p_{on}^{*}, p_{no}^{*}, p_{nn}^{*}$ into the two firms' profits. The two variables are p_o and p_n. The profit functions are concave. The first order conditions give: $p_o = \frac{c_n+5c_o+3(h_o-h_n)+p_n-3v(1-\alpha)}{7}$ and $p_n = \frac{5c_n+c_o+5(h_o-h_n)+p_o-v(1-\alpha)}{7}$. Solving the two equations, we obtain p_o and p_n.

Proof of Proposition 4.

$c_n > c_o$ and $h_o > h_n$. So, $p_n - p_o = \frac{2(c_n-c_o)+(h_o-h_n)+v(1-\alpha)}{4} > 0$. $v(1-\alpha) < (h_o - h_n)$, so $p_{on} - p_{oo} = \frac{18(c_n-c_o)+7(h_o-h_n)-v(1-\alpha)}{48} > 0$, $p_{nn} - p_{no} = \frac{18(c_n-c_o)+23(h_o-h_n)+31v(1-\alpha)}{48} > 0$, $p_{no} - p_{oo} = \frac{6(c_n-c_o)+5(h_o-h_n)+13v(1-\alpha)}{48} > 0$, $p_{on} - p_{nn} = \frac{6(c_n-c_o)+11(h_o-h_n)+19v(1-\alpha)}{48} > 0$.

Proof of Proposition 5.

The differences between the market shares of the o-firm and the n-firm in the two periods are: $so_1 - sn_1 = \frac{2(c_n-c_o)-(h_o-h_n)-v(1-\alpha)}{8(h_o-h_n)}$, and $so_2 - sn_2 = \frac{5[2(c_n-c_o)-(h_o-h_n)-v(1-\alpha)]}{8(h_o-h_n)}$. The difference between the profits of the two firms is $\pi_o - \pi_n = \frac{13[2(c_n-c_o)-(h_o-h_n)-v(1-\alpha)]}{24}$.

Proof of Proposition 6.

$\frac{\partial \pi_n}{\partial (h_o-h_n)} = [1620A^2(h_o-h_n)^2 - 3576A(h_o-h_n) + 1847] - 540Av(1-\alpha) - \frac{263v^2(1-\alpha)^2}{(h_o-h_n)^2}$. $\frac{\partial^4 \pi_n}{\partial (h_o-h_n)^4} > 0$. $\frac{\partial^3 \pi_n}{\partial (h_o-h_n)^3}$ changes from negative to positive as $(h_o - h_n)$ increases. $\frac{\partial^2 \pi_n}{\partial (h_o-h_n)^2}$ first decreases from a positive value, then increases, however, we cannot judge the value is positive or negative. We use the compel method. $\frac{\partial \pi_n}{\partial (h_o-h_n)}$ is larger than $\frac{135A^2(h_o-h_n)^2-343A(h_o-h_n)+132}{192}$, and smaller than $\frac{1620A^2(h_o-h_n)^2-3576A(h_o-h_n)+1847}{2304}$. From the monotonicity of the two functions, $\frac{\partial \pi_n}{\partial (h_o-h_n)}$ increases then decreases. Thus, $(h_o-h_n)^*$ satisfies $\frac{\partial \pi_n}{\partial (h_o-h_n)}=0$.

Proof of Proposition 7.

When $v(1-\alpha)\le \frac{2}{3A}$, we substitute $h_o-h_n=\frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$ into $\frac{\partial \pi_n}{\partial (h_o-h_n)}$.
If $\frac{\partial \pi_n}{\partial (h_o-h_n)}\le 0$, that is, $v(1-\alpha)\le \frac{53(\sqrt{6817}-2619)}{4096A}$, $h^*=(h_o-h_n)^*\ge \frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$.
If $\frac{\partial \pi_n}{\partial (h_o-h_n)}>0$, that is, $v(1-\alpha)>\frac{53(\sqrt{6817}-2619)}{4096A}$, $h^*=(h_o-h_n)^*<\frac{2+\sqrt{4-6Av(1-\alpha)}}{6A}$. When $v(1-\alpha)>\frac{2}{3A}$, we substitute $h_o-h_n=\frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$ into $\frac{\partial \pi_n}{\partial (h_o-h_n)}$. $\frac{\partial \pi_n}{\partial (h_o-h_n)}\le 0$, $h^*=(h_o-h_n)^*<\frac{\sqrt{4+2Av(1-\alpha)}-2}{2A}$.

Proof of Proposition 8.

$\Delta\pi_n=\frac{[v(1-\alpha)-3(h_o-h_n)][6(c_n-c_o)-3(h_o-h_n)-11v(1-\alpha)]}{96(h_o-h_n)}-\frac{[11(h_o-h_n)-6(c_n-c_o)+19v(1-\alpha)]^2}{2304(h_o-h_n)}$
>0, which can be transformed to the form in the proposition.