Gender-Related Operational Issues Arising from On-Demand Ride-Hailing Platforms: Safety Concerns and System Configuration

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Abstract: Female user (driver and rider) safety is a serious concern for ride-hailing platforms. One way to address this concern is to migrate from the traditional "pooling" system that matches riders with drivers without considering gender to a "hybrid" system with a "female-only" option. Will such a hybrid system result in a win-win-win outcome for all involved parties (riders, drivers and the platform)? To answer this question, we investigate the performance of the two operational systems: a pooling system, and a hybrid system. For each system, we analyze a two-stage queueing game to determine the equilibrium "joining" and "participating" behavior of riders and drivers, and then derive the platform's optimal pricing and wage decisions. We posit a mismatch cost incurred by a safety-concerned female user when she is matched with a male counterpart in a ride. By comparing the equilibrium outcomes associated with the pooling and the hybrid systems, we draw the following conclusions: when safety-concerned female users' mismatch cost is above a certain level, switching from a pooling system to a hybrid system can result in a win-win outcome for safety-concerned female users and the platform. However, male and safety-unconcerned female users might be worse off due to this change in the system configuration. Our results also help us to rectify some of our intuitions about these two systems. One, in the pooling system, reducing the mismatch cost associated with safety-concerned female drivers may not lead to more female riders joining the pooling system, even though it boosts the platform's profit in general. Two, in the hybrid system, it is not necessary for female riders to pay a higher price when they opt for female drivers instead of male drivers. We also relate our results to certain system configurations adopted by various ride-hailing platforms to address female safety concerns in different countries.

Keywords: Ride-hailing, gender-based safety concerns, pooling system, hybrid system, queueing equilibrium behavior.

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1 Introduction

On-demand ride-hailing platforms provide great convenience for riders who need transportation services and work flexibility for self-regulated drivers who work independently. However, a notable issue facing platforms such as Uber, Lyft and DiDi is safety concerns: some female users (i.e., riders and drivers) have been sexually harassed/assaulted by male counterparts (Feeney 2015). (Hereafter, we refer "users" to as riders and drivers.) Over the past 3 years, there have been a series of reports about female users being sexually assaulted, raped or murdered by male users. For example, Uber reported 2,936 sexual assaults on female users during Uber rides in 2017 and 3,045 in 2018 (Conger 2019). In 2018, a 21-year-old female rider was murdered by a male DiDi driver during her DiDi ride in China (Grothaus 2018), and another 20-year-old woman was raped and killed by a male DiDi driver (Zhang and Munroe 2018). On the driver side, a 20-year-old female DiDi driver was murdered by a male passenger in 2016 (ChinaDaily 2016).

These sex crimes have heightened female user safety concerns about ride-hailing platforms (Fong 2019). A six-country survey revealed that 64% of surveyed women drivers stated that security as a key reason why they do not sign up to become Uber drivers (IFC 2018). To sustain growth, ride-hailing platforms must develop measures to ensure user safety (Dai and Tang 2020). One approach is to change from the current gender-neutral "pooling system" (that matches riders and drivers without considering gender) to a gender-dedicated system (that only matches riders and drivers of the same gender). DiDi announced in 2018 that its carpooling drivers can only pick up riders of the same gender in the early morning (late evening) hours (Al-Heeti 2018). This concern also spawned female-only ride-hailing service startups: SheTaxis, Safr and Chariot for Women (United States), She Cabs (India) and She'Kab (Pakistan) .

Moving to a gender-dedicated system has many challenges. First, there is a severe imbalance between female-rider demand and female-driver supply. For instance, females account for only 2% of drivers but 60% of customers in the taxi and delivery industry (SheRides 2016). In China, only 10% of registered DiDi drivers are female (about 2.3 million), while half of its riders are female (more than 200 million) (Borak 2018, ChinaNews 2018, AsiaSociety 2017). According to DiDi, if only same-gender users are allowed to be matched, then DiDi can only serve 5% of its female riders (DidiPublic 2019). In the United States, 48% of Uber's 41.8 million riders are female, and yet 14% of its 1 million drivers are female (Iqbal 2019, Muchneeded 2019). Thus, while females' safety concerns are lessened (or absent) in a gender-dedicated system, the implementation of such system may lead to a significant loss of female riders due to the limited supply of female drivers and the significant loss of the “pooling” effect in a gender-dedicated system.

Instead of separating users (riders and drivers) by gender, some suggest that ride-hailing companies should create two separate “gender-specific driver” subsystems. At the same time, because of the scarcity of female drivers, the platform should provide “an option” for female riders to choose between these two gender-specific driver subsystems. This way, safety-concerned female riders can join a female-driver subsystem (Buxton 2018) to address their concern even though they may need to wait a bit longer for their rides. This idea has received public support in a MoveOn petition that has collected more than 14,000 signatures (Green and Zimmer 2019). Implementing such an idea requires the establishment of a “hybrid system” that entails a male-driver subsystem and a female-driver subsystem. Male riders can join the male-driver subsystem only; however, female riders can join either. By offering each female rider an option to choose between male- and female-driver system, it can mitigate the loss of the pooling effect in a gender-dedicated system. Also, this hybrid system could exploit the heterogeneity of user safety concerns: not all female riders prefer female drivers and some female riders are not concerned about safety during a ride. According to a survey, 47% of women have no preference between male and female drivers (IFC 2018). With the flexibility to choose between the two subsystems, safety-unconcerned female riders can pick the male-driver subsystem, which allows them to enjoy a shorter waiting time due to a larger male driver pool, while safety-concerned female riders have the option to choose the female-driver subsystem.

The implications of switching from a gender-neutral pooling system to a hybrid system (with two gender-specific driver subsystems) are unclear. On one hand, the hybrid system offers more joining options, attracting more safety-concerned female riders and drivers to join the hybrid system (relative to the gender-neutral pooling system). On the other hand, the pooling effect is weaker in the hybrid system (relative to the gender-neutral pooling system), which may increase the waiting time and discourage riders to join. In view of these trade offs, we study the implications of both ride-hailing service system configurations by examining the following questions:

1. What is the user joining behavior in equilibrium under the pooling and hybrid systems?

2. What are the platform’s optimal pricing and wage decisions in each system?

3. In a pooling system, how would the mismatch costs associated with safety-concerned female users affect the users’ joining incentives and the platform’s profit?

4. Relative to the pooling system, will the hybrid system result in a win-win-win solution for riders, drivers and the platform?

To explore the above research questions, we consider a situation in which riders are price- and waiting-time-sensitive and drivers are wage-sensitive. In our model, male user safety concern is normalized to zero. Female user safety concerns are heterogeneous: some female users exhibit safety concerns when matched with male counterparts and incur a mismatch cost, and the rest have no such concern. By considering these factors, we construct a two-stage queueing game model (Hassin and Haviv 2003, Hassin 2016) to analyze the performance of the following two systems: 1) a pooling system that matches riders and drivers without considering gender, where the platform adopts a *gender-neutral policy* for its pricing and wage decisions; and 2) a hybrid system consisting of two gender-specific driver subsystems – a male-driver subsystem and a female-driver subsystem, where the platform adopts a *subsystem-based pricing and wage policy* for its pricing and wage decisions. In the hybrid system, male riders can join the male-driver subsystem only; however, each female rider has the option to choose between two subsystems. (As discussed earlier, we allow female riders to choose between male-driver and female-driver subsystems in order to address the issue of female-driver scarcity so as to ameliorate the waiting time for female riders.)

For each system, we derive the corresponding equilibrium joining and participating behavior of riders and drivers, respectively and the platform's optimal pricing and wage decisions. For the pooling system, we show that reducing the mismatch cost for those safety-concerned female users has an increasing marginal effect on the platform's profit. This result implies that it is beneficial for the ride-hailing platform to develop initiatives for reducing the mismatch cost and safety concern. This is in line with the initiatives taken by DiDi and Uber.[1] Also, we obtain two seemingly interesting results. First, in the pooling system, we show that reducing the mismatch cost for female drivers does not necessarily lead to more female riders joining the pooling system because of the pricing decision of the platform.

Second, in the hybrid system, female riders do not necessarily pay a higher price when they opt for female drivers instead of male drivers. When the mismatch cost of safety-concerned female riders is either sufficiently high or sufficiently low, we show that female riders will end up paying less when they opt for female drivers. To elaborate, when the mismatch cost is high, safety-concerned female riders prefer the female-driver subsystem, causing its waiting time to increase. Hence, the platform needs to lower the price for those riders to offset their cost of long waiting time in the female-driver subsystem. Also, when the mismatch cost is low, safety-concerned female riders are likely to join the male-driver

[1]To improve rider safety, DiDi has developed a one-click emergency call feature in its app, and an in-trip audio recording to educate drivers regarding various safety measures (EJinsight 2018, Dai 2018). In the same vein, Uber has also installed an in-app emergency button in its safety toolkit (Uber 2019). Both DiDi and Uber now conduct background checks and screen their drivers to improve rider safety (Shen 2018, Bell 2018). Those actions can effectively reduce female users' safety concern and thus decrease the mismatch costs when matched with male counterparts, which in turn could help the platform to increase its profit.

subsystem, causing its waiting time to increase. To persuade female riders to switch from the male-driver subsystem to the female-driver subsystem, the platform has to charge a lower price in its female-driver subsystem.

Finally, we compare the equilibrium outcomes associated with the two systems by focusing on three issues: (1) accessibility of safety-concerned female users, (2) other users' utility, and (3) platform profit. Our comparisons yield the following results. First, relative to the pooling system, the hybrid system reduces the mismatch cost incurred by safety-concerned female users and increases their accessibility. Second, while the hybrid system suffers from a slight loss of the pooling effect, the hybrid system renders a higher profit than the pooling system when the mismatch cost of safety-concerned female users is sufficiently high. Third, from the perspective of those safety-unconcerned users, i.e., all male and safety-unconcerned female users, their utility may deteriorate slightly when the platform switches from the pooling system to the hybrid system. This is because the platform has to adjust its price and wage to "subsidize" those safety-concerned users at the "expense" of those safety-unconcerned users in the hybrid system. These findings imply that the hybrid system benefits safety-concerned female users and the platform. Lastly, by considering different mismatch costs that female users incur in different countries, we illustrate how our analytical results are congruent with different ride-hailing systems adopted in different countries. For example, in Saudi Arabia where female user safety concern is high (Narayan 2018), our results supported the adoption of the hybrid system with a female-driver subsystem.

The remainder of this paper is organized as follows. We review the related literature in Section 2. The model formulation is presented in Section 3. We conduct the game-theoretical analysis of the pooling and hybrid systems, respectively, in Sections 4 and 5. In particular, we derive the associated equilibrium outcomes. We then compare the two systems and conduct the related discussions in Section 6. Section 7 concludes the paper. All of the proofs are relegated to the online Appendix A. We discuss the detailed equilibrium derivation and analysis associated with the pooling system and the hybrid system in online Appendices B and C, respectively.

2 Literature Review

Our paper belongs to the emerging research stream that studies on-demand ride-hailing platforms in a two-sided market. For research on two-sided markets, see Armstrong (2006), Rochet and Tirole (2006), Weyl (2010), Hagiu (2014), Hagiu and Wright (2015), Eisenmann et al. (2006) and the references therein. The literature on ride-hailing platforms has investigated issues such as surge pricing (e.g., Banerjee et al. (2015) and Cachon et al. (2017)), optimal commission contracts (e.g., Hu and Zhou (2017) and Bai et al. (2019)), pricing with cost-sharing consideration (e.g., Jacob and Roet-Green), driver and rider role exchanges (i.e., the roles of riders and drivers are interchangeable, e.g., Gao et al. (2018)), competition between platforms (e.g., Cohen and Zhang (2019)) and matching between different types of users (see e.g., Caldentey et al. (2009), Baccara et al. (2020) and Hu and Zhou (2020)). We refer the reader to the comprehensive reviews provided by Benjaafar and Hu (2020) and Hu (2020). In this stream of work, ours is closely related to Taylor (2018) and Benjaafar et al. (2020). Taylor (2018) investigates how rider delay sensitivity and driver self-regulation affect a platform's optimal pricing and wage decisions. Benjaafar et al. (2020) investigate the effect of the labor pool size on labor welfare. Also, the price discrimination literature is vast; see, e.g., Choudhary et al. (2005), Ferrell et al. (2018), Trégouët (2015), Horstmann and Krämer (2013), Jayaswal et al. (2011), Mitra and Capella (1997) and Chen (2009). In our paper, we also consider price and wage decisions and price discrimination issues; however, ours is the first to investigate the gender-related safety concerns and how these concerns affect the profitability of a ride-hailing platform.

While Kostami et al. (2017) consider users' gender preferences and pricing issues in a club setting, we examine the users' gender preference in a different context (ride-hailing platform) and investigate the platform's optimal price and wage decisions and the joining (and participating) behavior of riders (and drivers). Like Kostami et al. (2017), we examine the externality effect (excluding congestion effect) brought by one gender type users on the other, and the use of dedicated capacities to separate these two gender types of users to mitigate such externality. Unlike Kostami et al. (2017), our model differs in two aspects. One, Kostami et al. (2017) focus on a one-sided market so that externality occurs in the demand-side; however, we deal with a two-sided market so that same-side externality occurs in both the demand- and the supply-side. Also, we have to deal with cross-side externality because the utility of the demand-side riders is affected by the number of the supply-side drivers and vice versa. Two, in Kostami et al. (2017), the capacity is "exogenously given" and it can be arbitrarily allocated between the two gender types of users. However, in our paper, the capacity is "endogenously determined" by the drivers' participating behaviors of both genders. Moreover, the supply of the two gender types of drivers (especially female drivers) is capacitated. Therefore, in contrast to the two dedicated systems proposed in Kostami et al. (2017), we instead consider a "hybrid" system that grants safety-unconcerned female riders the flexibility so that the platform can achieve a better balance between demand and supply. Finally, while Kostami et al. (2017) focus only on profit maximization, we examine additional performance measures: accessibility of safety-concerned female users.

Our paper is also related to the product line design literature when customers are heterogeneous. Chen (2001) considers a manufacturer's product line design (product types and quality of each type) when the market is comprised of green and ordinary customers. Bellos et al. (2017) study how providing car sharing affects a car manufacturer's driving performance design under a setting that customers have different valuations of driving performance. The most closely related paper is Netessine and Taylor (2007) who examine how a manufacturer's product line design is affected by the observed customer type information and production technology. Akin to Netessine and Taylor (2007), our pooling system can be viewed as the system that produces a composite product while the hybrid system produces two different quality products. However, our service system design problem is different from the product line design problem in four ways. First, unlike the product line design problem in which the capacity is directly controlled by the firm, our system capacity is endogenously determined by the self-regulated drivers. Second, unlike the product line design problem without capacity constraint, we deal with limited capacity (especially female drivers). Third, unlike the product line design problem in which externality may occur on the demand-side, our system has different types of aforementioned externalities. Fourth, unlike the product line design problem in which high-quality products can be sold in higher prices, our platform may charge a lower price in the female-driver subsystem than that of the male-driver subsystem, due to the potential longer waiting time (congestion) in the female-only subsystem. Ultimately, our context and our analysis are completely different from that of the product line design problem.

3 Model Preliminaries

Consider an on-demand ride-hailing platform that sets a price rate p (measured in terms of price per service) and a wage rate w (measured in terms of wage per service) to coordinate price- and waiting-time-sensitive riders (i.e., demand) and earning-sensitive independent drivers (i.e., supply) of both genders, female (labeled f) and male (labeled m).

Platform's System Configuration. The platform can adopt two potential operational systems: a gender-neutral pooling system and a hybrid system (consisting of a male-driver subsystem and a female-driver subsystem); see Figure 1 for an illustration. In a pooling system, riders and drivers are matched without considering gender. Hence, the safety concerns of female riders and drivers are present when they are matched with male counterparts. As the pooling system is operated as a single legal entity, gender-based pricing and wages are normally deemed discriminatory and may be illegal. Thus, it suffices to consider a *gender-neutral pricing and wage policy.*

As explained in §1, drivers are divided into two gender-specific subsystems in the hybrid

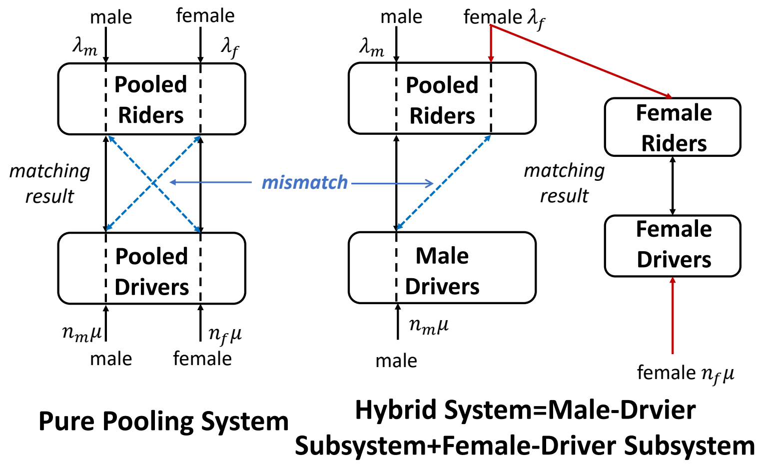


Figure 1: Two Operational Systems for the Platform

system. Also, male riders can join the male-driver subsystem only, while female riders have the option to join either subsystems (in order to address the issue of female-driver scarcity and the issue of long waiting time for female riders). Hence, safety concerns are absent in the female-driver subsystem from the perspective of both female riders and female drivers. Also, we consider the case when the platform operates the two subsystems (in the hybrid system) as two separate legal entities so that the platform could set *subsystem-based prices and wages*. (Our model can easily be extended to examine the case when the pricing and wages are identical in both subsystems by restricting that wages and prices in the subsystems are equal.)

Rider Characteristics. Potential female and male riders may request on-demand ride-hailing service according to independent Poisson processes with rates Λ_f and Λ_m, respectively. The total potential arrival rate $\Lambda = \Lambda_m + \Lambda_f$. Male riders are homogeneous and have less safety concerns about driver's gender. Without loss of generality, we scale male safety concern to zero (i.e., no safety concern). Female riders are heterogeneous regarding safety concerns about driver gender. Specifically, δ_R proportion of them have no safety concern regarding driver gender (IFC 2018); that is, they are *safety-unconcerned female* (labelled f_ϕ) riders with a Poisson arrival rate $\Lambda_{f_\phi} = \delta_R \Lambda_f$. The remaining $(1 - \delta_R)$ proportion are concerned about safety and feel uncomfortable when matched with a male driver; that is, they are *safety-concerned female* (labelled f_c) riders with arrival rate $\Lambda_{f_c} = (1 - \delta_R)\Lambda_f$. The total potential arrival rate of *safety-unconcerned riders* consists of all male and those safety-unconcerned female riders, and we denote it as Λ_ϕ $(= \Lambda_m + \Lambda_{f_\phi})$.

Note that given price p and anticipating waiting cost $c \cdot W$, in which c is the unit-time waiting cost and W is the expected waiting time in the queue, some riders may choose not to request the service. We denote the effective joining rates of female and male riders as λ_f and λ_m, respectively. Then, $\lambda_i \leq \Lambda_i$, $i = f, m$. Furthermore, let λ_{f_c} and λ_{f_ϕ} denote the effective joining rates of safety-concerned and safety-unconcerned female riders, respectively. Thus, the effective total joining rate of female riders is $\lambda_f = \lambda_{f_c} + \lambda_{f_\phi}$.

To simplify our exposition, we assume that both male and female riders receive the same base reward R from the ride-hailing service. However, when matched with male drivers, safety-concerned female riders suffer a "gender-induced mismatch cost" a. Such mismatch cost may result from anxiety and worries during the ride. It measures the degree of female riders' safety concerns when the ride-hailing service is offered by male drivers, that is, to what extent female riders dislike being matched with male drivers. A larger a implies that female riders have a lower safety confidence level towards male drivers.

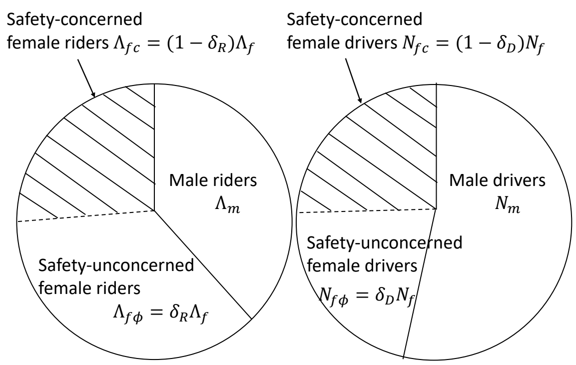


Figure 2: The Composition of Riders and Drivers

Driver Characteristics. There are N_f female and N_m male registered drivers, each of whom can serve a rider according to an exponential distribution with service rate μ (e.g., the number of riders served per unit of time). Male drivers outnumber female drivers, i.e., $N_m > N_f$, which is consistent with actuality (SheRides 2016, Borak 2018, ChinaNews 2018, AsiaSociety 2017). Again, male drivers are homogeneous and less safety-concerned about rider gender so that we scale male safety concerns to zero. Female drivers are heterogeneous regarding safety concerns about rider gender. Among the N_f female drivers, δ_D proportion of them do not have safety concerns about rider gender while the remaining $(1-\delta_D)$ proportion have such safety concerns. That is, the number of safety-concerned and unconcerned female drivers are $N_{f_c} = (1-\delta_D)N_f$ and $N_{f_\phi} = \delta_D N_f$, respectively. Hence, the total number of *safety-unconcerned drivers*, including male and safety-unconcerned female drivers, denoted by N_ϕ can be expressed as $N_\phi = N_m + N_{f_\phi}$. For ease of reference, we summarize the composition of rider and driver types in Figure 2.

All registered drivers, regardless of gender, have the same reservation price (or opportunity cost) r. They participate and serve if the earning rate is no less than r. Denote n_{f_c}, n_{f_ϕ} and n_m as the effective participating number of safety-concerned female drivers, safety-unconcerned female drivers and male drivers, respectively. Then, the effective total participating number of female drivers is $n_f = n_{f_c} + n_{f_\phi}$. When matched with male riders, safety-concerned female drivers suffer a gender-based mismatch cost b. The parameter b measures the degree of female drivers' safety concerns when serving male riders, that is, to what extent female drivers dislike being matched with male riders. A larger b implies that female drivers have a lower safety confidence level towards male riders.

Waiting Time. For tractability, we model the ride-hailing service operation as an M/M/1 queueing system. A similar assumption has been adopted in the literature; see, e.g., Benjaafar et al. (2020). Given an effective rider joining rate, λ, and an effective driver service rate, $n\mu$ (that is, n effective drivers), the expected waiting time in the queue $W(\lambda, n)$ can be shown to be

$$W(\lambda, n) = \begin{cases} \frac{\lambda}{n\mu(n\mu-\lambda)}, & \text{if } \lambda < n\mu \\ +\infty, & \text{otherwise.} \end{cases} \tag{1}$$

Sequence of Events. For both the pooling and hybrid systems, the sequence of events is as follows. First, the platform decides on the price(s) p and wage(s) w (for the entire pooling system or for each gender-specific subsystem in the hybrid system) to maximize its profit

$$\Pi = \lambda(p - w), \tag{2}$$

where λ is the effective joining rate of riders (for the entire pooling system or for each gender-specific subsystem in the hybrid system). Upon observing the price(s) and wage(s), riders and drivers of both genders respectively decide whether to participate based on their own utility functions. Note that the effective joining rates of different types of riders and the effective participating number of different drivers must be jointly solved through an equilibrium analysis of each player's behavior because their payoffs are determined by their joint behavior.

As the ride-hailing system is often *supply-constrained* (Banerjee et al. 2015, Taylor 2018), $\frac{\Lambda}{N\mu} > 1$ is required, where $N := N_f + N_m$. Also, to reflect the reality that on ride-hailing platforms, female riders account for a large proportion of demand but female drivers account for a only small proportion of supply (SheRides 2016, Borak 2018, ChinaNews 2018), we assume that $\frac{\Lambda_f}{N_f\mu} > 1$. This assumption assures that even when all female drivers participate, they cannot serve all female riders in a steady state. Throughout this paper, we restrict our attention to the parameter range within which a platform's expected profit under optimal pricing and wage decisions is strictly positive (Taylor 2018). Table 1 summarizes the key notation used.

Table 1: A List of Key Notation

f, m, f_c, f_ϕ	Female: f; male: m; safety-concerned/-unconcerned female: f_c / f_ϕ
ϕ	Safety-unconcerned users, consisting of males and safety-unconcerned females
$\Lambda_i, i \in \{f_c, f_\phi, f, m\}$	Potential arrival rate of type-i riders with $\Lambda_f = \Lambda_{f_c} + \Lambda_{f_\phi}, \Lambda = \Lambda_m + \Lambda_f$
Λ_ϕ	Potential arrival rate of safety-unconcerned riders with $\Lambda_\phi = \Lambda_m + \Lambda_{f_\phi}$
δ_R	Fraction of safety-unconcerned female riders, $0 < \delta_R = \frac{\Lambda_{f_\phi}}{\Lambda_f} < 1$
$\lambda_i, i \in \{f_c, f_\phi, f, m\}$	Effective joining rate of type-i riders
$N_i, i \in \{f_c, f_\phi, f, m\}$	Number of registered type-i drivers with $N_f = N_{f_c} + N_{f_\phi}, N = N_m + N_f$
N_ϕ	Number of registered safety-unconcerned drivers with $N_\phi = N_m + N_{f_\phi}$
δ_D	Fraction of safety-unconcerned female drivers, $0 < \delta_D = \frac{N_{f_\phi}}{N_f} < 1$
$n_i, i \in \{f_c, f_\phi, f, m, \phi\}$	Effective participating number of type-i drivers
μ	Service rate
r	Reservation price
R	Base service reward per ride
c	Unit-time waiting cost
a	Mismatch cost associated with female riders when being served by male drivers
b	Mismatch cost associated with female drivers when serving male riders
p	Price per service
w	Wage per service

4 Analysis of the Pooling System

In this section, we analyze the system performance associated with a gender-neutral pooling system via backward induction. Below, we first characterize the utilities of riders and drivers, and then we derive their equilibrium joining/participating behavior. By considering the equilibrium behavior, we derive the platform's optimal price and wage decisions by solving (2).

4.1 Users' Utility Functions

In a pooling system, safety-unconcerned female riders behave the same as male riders. Given riders' effective joining rate $\boldsymbol{\lambda} = \lambda_f + \lambda_m = (\lambda_{f_c} + \lambda_{f_\phi}) + \lambda_m$ and drivers' effective participating number $\boldsymbol{n} = n_f + n_m = (n_{f_c} + n_{f_\phi}) + n_m$, the utility of a male rider or a safety-unconcerned female rider joining the ride-hailing service can be written as

$$U_m(\boldsymbol{\lambda}, \boldsymbol{n}) = U_{f_\phi}(\boldsymbol{\lambda}, \boldsymbol{n}) = R - p - cW(\boldsymbol{\lambda}, \boldsymbol{n}), \tag{3}$$

where R is service reward, p is price, $W(\boldsymbol{\lambda}, \boldsymbol{n})$ is the waiting time given in (1), and $cW(\boldsymbol{\lambda}, \boldsymbol{n})$ is the encountered total waiting cost.

Given the number of female and male drivers, n_f and n_m, the probability of a safety-concerned female rider being matched with a male driver is $n_m/(n_f + n_m)$, in which situation she incurs a mismatch cost a. Thus, the utility of a safety-concerned female rider can be derived as

$$U_{f_c}(\boldsymbol{\lambda}, \boldsymbol{n}) = R - p - cW(\boldsymbol{\lambda}, \boldsymbol{n}) - \frac{n_m}{n_f + n_m} \cdot a. \tag{4}$$

Similarly, we can derive the net utilities of male drivers, safety-unconcerned female drivers and safety-concerned female drivers participating in the ride-hailing service as follows:

$$S_m(\boldsymbol{\lambda}, \boldsymbol{n}) = S_{f_\phi}(\boldsymbol{\lambda}, \boldsymbol{n}) = \frac{\lambda_m + \lambda_{f_c} + \lambda_{f_\phi}}{n_{f_c} + n_{f_\phi} + n_m} w - r = \frac{\lambda_m + \lambda_f}{n_f + n_m} w - r, \tag{5}$$

$$S_{f_c}(\boldsymbol{\lambda}, \boldsymbol{n}) = \frac{\lambda_m + \lambda_{f_c} + \lambda_{f_\phi}}{n_{f_c} + n_{f_\phi} + n_m} w - r - \frac{\lambda_m}{\lambda_m + \lambda_{f_c} + \lambda_{f_\phi}} b = \frac{\lambda_m + \lambda_f}{n_f + n_m} w - r - \frac{\lambda_m}{\lambda_m + \lambda_f} b. \tag{6}$$

For notational convenience, we define

$$d_i(n_{f_c}, n_{f_\phi}, n_m) := \frac{\lambda_m + \lambda_f}{n_{f_c} + n_{f_\phi} + n_m}, i = m, f_\phi, f_c. \tag{7}$$

Then, $d_i(n_{f_c}, n_{f_\phi}, n_m)$ can be regarded as the *demand rate* of a type-i driver, $i = m, f_\phi, f_c$.

4.2 Equilibrium Analysis and Optimal Price and Wage Decisions

Below, we first derive the equilibrium joining/participating behaviors of riders/drivers of both genders for the given price and wage. Following the similar approach for analyzing standard queueing games (Hassin and Haviv 2003), the process to determine the equilibrium joining/participating behaviors of 6 groups (male riders and drivers, safety-unconcerned female riders and drivers, safety-concerned female riders and drivers) is rather tedious because of the following reasons. In the pooling system as shown in Figure 1, each of the three groups of drivers (male drivers, safety-concerned and safety-unconcerned female drivers) have to decide whether to participate or balk. Similarly, each of the three groups of riders have to decide whether to join or balk. For each individual within each group, s/he will choose the option that yields the higher utility as defined above. However, the utility of each individual within one group depends on the joining/participating behavior of all other groups due to the (same-side and cross-side) externality effect caused by the waiting time. For this reason, the determination of the equilibrium joining/participating behavior is complex and it involves many intermediate steps.

To reduce the burden, we can utilize a fact (that can be proven) to facilitate our equilibrium analysis: if some safety-concerned female riders join the system, then safety-unconcerned female riders (and all male riders) must "all join." The same rationale holds for drivers. In other words, safety-unconcerned users gain some "privilege" over same-type safety-concerned users.[2]

[2]Changing to a hybrid system could cause safety-unconcerned users to lose such privilege, which will lower their utility under the hybrid system.

After obtaining the equilibrium joining behavior of users, we can derive the platform's optimal pricing and wage decisions p^* and w^* by maximizing the platform's profit stated in (2). Instead of showing the details of the intermediate steps for determining the equilibrium user (rider and driver) behavior and the optimal price and wage in the main text, we focus on the discussion of our key results and their implications and refer interested readers to the online Appendix B for the detailed equilibrium analysis and Table B.1 for the optimal price p^*, wage w^* and platform profit Π^*, as well as the corresponding equilibrium joining/participating behaviors of riders/drivers of both genders.[3] We now present users' equilibrium joining and participating behaviors associated with the platform's optimal (profit-maximizing) price and wage in the following proposition.

Proposition 1. *When the platform charges riders the optimal price and offers drivers the optimal wage in the pooling system, the equilibrium joining rates of riders and the number of participating drivers exhibit the following characteristics:*

1. ***(Demand).*** *All safety-unconcerned riders, i.e., male and safety-unconcerned female riders, join the system. However, only a fraction of safety-concerned female riders join the system. That is,* $\lambda_i^* = \Lambda_i$, $i = m, f_\phi$, *and* $\lambda_{f_c}^* < \Lambda_{f_c}$.

2. ***(Supply).*** *All registered safety-unconcerned drivers, i.e., male and safety-unconcerned female drivers, participate in the system. That is,* $n_i^* = N_i$, $i = m, f_\phi$. *However,*

   (a) *if the number of registered safety-unconcerned drivers* N_ϕ *is sufficiently large such that* $\mu N_\phi > \Lambda_\phi$ *and the mismatch cost* b *is sufficiently high (i.e.,* $b \geq \widehat{b}(a)$*), then all safety-concerned female drivers will balk, i.e.,* $n_{f_c}^* = 0$*;*

   (b) *otherwise, all safety-concerned female drivers will participate, i.e.,* $n_{f_c}^* = N_{f_c}$.

The expressions for $\lambda_{f_c}^*$ *and* $\widehat{b}(a)$ *stated above can be found in Table B.1 and equation* (27) *of the online Appendix B, respectively.*

Proposition 1 indicates that all safety-unconcerned users, including male and safety-unconcerned female riders and drivers, always join the system as they have no safety concerns. Also, safety-concerned female riders join the system with a certain probability; in contrast, safety-concerned female drivers may "all join" or "never join" the system, a result hinging upon the mismatch cost b and the labor pool size of safety-unconcerned drivers N_ϕ. Such

[3] Note that throughout our analyses, we only consider the equilibrium outcomes in which safety-concerned female riders join the system at a non-zero rate. While deriving the equilibrium joining/participating behaviors of riders/drivers, one can easily find some equilibria in which all safety-concerned female riders balk in a pooling/hybrid system. As such equilibrium outcomes deviate from our research motivation, they are not our focus, and we omit such trivial cases.

"all join" or "never join" behavior is due to the *positive participating driver externality*, that is, "the equilibrium demand allocated to a driver strictly increases with the number of participating drivers" (Taylor 2018). Thus, under the optimal price and wage, either all registered drivers work or only safety-unconcerned drivers work. Therefore, when making staffing decisions, the platform has to choose between two options: setting a relatively high wage to attract all drivers or setting a relatively low wage to attract only safety-unconcerned drivers. Proposition 1 implies that when the labor pool size of safety-unconcerned drivers is large enough ($N_\phi \mu > \Lambda_\phi$) and the mismatch cost associated with safety-concerned female drivers is relatively high ($b \geq \widehat{b}(a)$), the latter dominates the former as the profit gained from serving more safety-concerned riders cannot surpass the loss encountered due to higher payments to drivers. Proposition 1 also implies that the platform may use wages as a tool to screen drivers who are concerned about safety.

We next investigate the effects of mismatch costs a and b associated with safety-concerned female users on the system performance. We obtain the following results.

Proposition 2. *When both safety-concerned female riders and drivers join at a non-zero rate in the pooling system (which occurs when either the number of safety-unconcerned drivers is sufficiently small ($\mu N_\phi \leq \Lambda_\phi$) or when safety-concerned female drivers' mismatch cost $b < \widehat{b}(a)$), then we have:*

1. *The optimal price p^* decreases while the optimal wage w^* increases in both a and b.*
2. *The effective joining rate of safety-concerned female riders $\lambda^*_{f_c}$ is decreasing with a but increasing with b.*
3. *The participating number of safety-concerned female drivers is always N_{f_c}, regardless of the magnitude of a and b.*
4. *The platform's profit Π^* is decreasing and convex in both a and b.*

The first statement of Proposition 2 shows that higher mismatch costs of safety-concerned female users (i.e., as a or b increases) would induce the platform to reduce its price and increase the wage. This is because a higher gender-based mismatch cost more likely prevents safety-concerned female users to join the system. Upon closer examination, Proposition 2 has the following implications.

One, to improve profitability, the platform should reduce safety-concerned female users' mismatch costs a and b as much as possible. Proposition 2 implies that enhancing the safety confidence of female users to reduce the mismatch costs when they are matched with a male counterpart can improve the platform's profitability. Statement 4 of Proposition 2 further shows that there is an "increasing marginal improvement" on the platform's profit as a or b is reduced.

Two, reducing female riders' mismatch cost a can entice more joining female riders; however, reducing female drivers' mismatch cost b can result in fewer joining female riders. The former is intuitive, but the latter is not. To elaborate, consider the case when the mismatch cost of safety-concerned female drivers is high so that the platform has to set a sufficiently low price to entice more female riders to join in order to attract more safety-concerned female drivers to participate. However, as the mismatch cost b is reduced, female drivers are more willing to participate and the platform can afford to increase its price and turn away some female riders.

5 Analysis of the Hybrid System

We now turn our attention to analyzing the hybrid system that is comprised of a male-driver subsystem and a female-driver subsystem as shown in Figure 1. As explained earlier, male riders can only join the male-driver subsystem; however, female riders can choose between the male-driver subsystem (labeled "M") and the female-driver subsystem (labeled "F"). By granting this flexibility, the platform can mitigate the loss of the pooling effect (from the rider side) due to a gender-dedicated system.

Female-driver subsystem. By using subscripts M and F to denote the male-driver subsystem and female-driver subsystem, respectively, the effective rider joining rate in the female-driver subsystem with an effective female driver participating number n_f is $\boldsymbol{\lambda}_F = (\lambda_{f_c,F} + \lambda_{f_\phi,F})$ (from safety-concerned and -unconcerned female riders). Because safety concern is absent in the female-driver subsystem as depicted in Figure 1, the joining utility for both safety-concerned or -unconcerned female riders are identical so that

$$U_{f_c,F}(\boldsymbol{\lambda}_F, n_f) = U_{f_\phi,F}(\boldsymbol{\lambda}_F, n_f) = R - p_F - cW(\boldsymbol{\lambda}_F, n_f), \tag{8}$$

where p_F is the price charged in the female-driver subsystem. Similarly, the net utilities of safety-concerned and -unconcerned female drivers participating in the female-driver subsystem are also identical so that

$$S_{f_c,F}(\boldsymbol{\lambda}_F, n_f) = S_{f_\phi,F}(\boldsymbol{\lambda}_F, n_f) = \frac{\lambda_{f_c,F} + \lambda_{f_\phi,F}}{n_f} \cdot w_F - r, \tag{9}$$

where w_F is the wage per service, $\frac{\lambda_{f_c,F}+\lambda_{f_\phi,F}}{n_f}$ is the demand rate, and r is the reservation price of the female driver.

Male-driver subsystem. In a male-driver subsystem with an effective rider joining rate $\boldsymbol{\lambda}_M = (\lambda_m + \lambda_{f_\phi,M} + \lambda_{f_c,M})$ (including male riders and safety-unconcerned and -concerned female riders) and an effective male driver participating number n_m, each safety-unconcerned female rider and each male rider receive the same utility from joining the male-driver subsystem so that

$$U_{f_\phi,M}(\boldsymbol{\lambda}_M, n_m) = U_{m,M}(\boldsymbol{\lambda}_M, n_m) = R - p_M - cW(\boldsymbol{\lambda}_M, n_m), \tag{10}$$

where p_M is the price charged. However, in the male-driver subsystem, mismatch cost a is incurred for each safety-concerned female rider with certainty so that her utility of joining the system is

$$U_{f_c,M}(\boldsymbol{\lambda}_M, n_m) = R - p_M - cW(\boldsymbol{\lambda}_M, n_m) - a, \tag{11}$$

By observing that the demand rate of a male driver equals $\frac{\lambda_{f_c,M}+\lambda_{f_\phi,M}+\lambda_m}{n_m}$ in the male-driver subsystem, each male driver receives the following net utility:

$$S_m(\boldsymbol{\lambda}_M, n_m) = \frac{\lambda_m + \lambda_{f_c,M} + \lambda_{f_\phi,M}}{n_m} w_M - r. \tag{12}$$

Using the same approach as stated in §4, we first analyze the equilibrium joining/participating behaviors of riders/drivers for the given prices and wages of the two subsystems. Unlike the pooling system, the drivers are gender-specific in each subsystem in the hybrid system so that each driver only needs to decide whether to participate in the corresponding gender subsystem or not. Male riders just need to decide whether to join the male-driver subsystem or balk. However, as depicted in Figure 1, (safety-concerned and safety-unconcerned) female riders have three options: join the male-driver subsystem, join the female-driver subsystem, or balk, and they will choose the one that yields the highest utility. The joining/participating behavior of each group affects the joining/participating behavior of other groups due to the externality effect caused by the waiting time. After deriving the equilibrium joining/participating behavior, we then investigate the platform's optimal pricing and wage decisions by maximizing the platform's profit stated in (2).

Again, to avoid getting bogged down in the details of various intermediate steps of our analysis, we relegate the detail analysis to online Appendix C for the equilibrium analysis, and Table C.1 for the expression of the optimal prices $\widetilde{p}_s^*$ and wages $\widetilde{w}_s^*$ of subsystem $s \in \{M, F\}$, the platform's profit $\widetilde{\Pi}^*$, and the corresponding equilibrium joining/participating behavior of riders/drivers, where $\widetilde{\cdot}$ is intended to denote the equilibrium outcomes associated with the hybrid system. By considering the equilibrium outcomes, we get:

Proposition 3. *When the platform charges riders the optimal price and offers drivers the optimal wage in the hybrid system, the equilibrium outcomes exhibit the following characteristics:*

1. ***(Demand).*** *Suppose that the number of registered male drivers N_m is sufficiently large compared to the potential arrival rate of safety-unconcerned riders Λ_ϕ (i.e., $\mu N_m > \Lambda_\phi$) and that the safety-concerned female riders' mismatch cost a is sufficiently low (i.e., $a \leq \widehat{a}$, where $\widehat{a}$ can be found in (31) of the online Appendix C). Then,*

   (a) all safety-unconcerned riders (i.e., all male and safety-unconcerned female riders) join the male-driver subsystem.

   (b) some safety-concerned female riders join the male-driver subsystem, some join the female-driver subsystem, and the rest balk. More formally, $\widetilde{\lambda}^*_m = \Lambda_m$, $\widetilde{\lambda}^*_{f_\phi,M} = \Lambda_{f_\phi}$, $\widetilde{\lambda}^*_{f_\phi,F} = 0$, $\widetilde{\lambda}^*_{f_c,M} > 0$, $\widetilde{\lambda}^*_{f_c,F} > 0$, and $\widetilde{\lambda}^*_{f_c,M} + \widetilde{\lambda}^*_{f_c,F} < \Lambda_{f_c}$.

   If the above conditions do not hold, then some male riders balk. For safety-unconcerned female riders, some join the male-driver subsystem, some join the female-driver subsystem, and the rest balk. For safety-concerned female riders, they either join the female-driver subsystem or balk. More formally, $0 < \widetilde{\lambda}^*_m + \widetilde{\lambda}^*_{f_\phi,M} < \Lambda_m + \Lambda_{f_\phi}$, $0 < \widetilde{\lambda}^*_{f_\phi,F} + \widetilde{\lambda}^*_{f_\phi,M} \leq \Lambda_{f_\phi}$, and $\widetilde{\lambda}^*_{f_c,M} = 0$ and $0 < \widetilde{\lambda}^*_{f_c,F} < \Lambda_{f_c}$.

2. ***(Supply).*** *All registered male drivers participate in the male-driver subsystem and all registered female drivers participate in the female-driver subsystem. That is, $\widetilde{n}^*_m = N_m$, $\widetilde{n}^*_f = N_f$.*

Statement 2 of Proposition 3 reveals that all registered drivers participate in the hybrid system, which is different from the pooling system in which safety-concerned drivers may not participate (see Proposition 1). This difference is caused by the fact that drivers are gender-specific in the subsystems so that female drivers, regardless of whether they are safety-concerned, are now matched with female riders only in the female-driver subsystem. Without incurring the mismatched cost b, all female drivers will participate in the female-driver subsystem.

From the riders' perspective, statement 1 of Proposition 3 shows that safety-concerned female riders may also join the male-driver subsystem when their mismatch cost a is low ($a \leq \widehat{a}$) and the labor pool size of male drivers is large (i.e., $\mu N_m > \Lambda_\phi$) (so that the platform can offer a lower wage and charge a lower price). However, when either one or both conditions do not hold, safety-concerned female riders only join the female-driver subsystem.[4] In this case, the hybrid system essentially operates as "two dedicated queues": the male-driver subsystem consists of only safety-unconcerned users while the female-driver subsystem

[4]We note that multiple equilibria exist; however, they do not affect the platform's optimal pricing and wage decisions. We refer interested readers to the online Appendix C for the detail.

consists of only safety-concerned female users. Consequently, the gender-based mismatch costs play no role in this situation.

By using the equilibrium joining and participating behavior of riders and drivers, we can determine the optimal prices and wages by solving the platform's problem (2) associated with each subsystem. Recall that the platform can set subsystem-specific prices and wages as two legal entities. At the same time, knowing female drives are scarce $N_f < N_m$, we wonder if the platform will charge a higher price in the female-driver subsystem so that it can offer a higher wage to female drivers. The corollary below specifies the conditions under which the optimal price is higher in the female-driver subsystem so that $\widetilde{p}_F^* \geq \widetilde{p}_M^*$.

Corollary 1. *Consider the case when the platform adopts the hybrid system. Suppose that the number of registered male drivers N_m is large ($N_m\mu > \Lambda_\phi$) and safety-concerned female riders' mismatch cost a satisfies $\widetilde{a} \leq a \leq \widehat{a}$ (where $\widehat{a}$ is defined in Proposition 3 and $\widetilde{a}$ is characterized by* (16) *in the online Appendix A). Then, the optimal price is lower in the male-driver subsystem so that $\widetilde{p}_M^* \leq \widetilde{p}_F^*$. Otherwise, the optimal price is lower in the female-driver subsystem; i.e., $\widetilde{p}_M^* > \widetilde{p}_F^*$.*

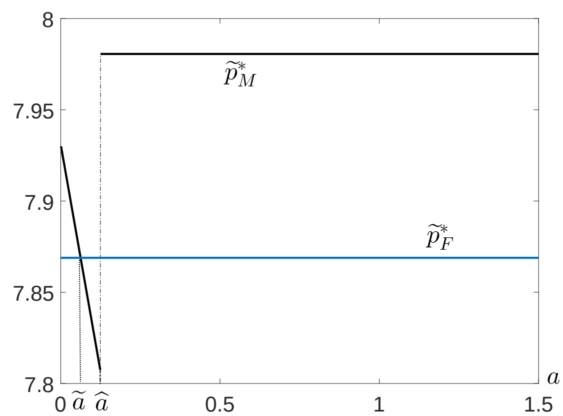


Figure 3: Optimal Prices in Two Subsystems: $N_m = 1100$, $\Lambda_m = 1000$, $N_f = 300$, $\Lambda_f = 1500$, $\mu = 1.5$, $r = 2$, $c = 1$, $R = 8$, $\delta_R = 40\%$, $\delta_D = 50\%$

Before we interpret the result stated in Corollary 1, let us illustrate our result graphically in Figure 3. Observe from Figure 3 that the optimal price $\widetilde{p}_F^*$ of the female-driver subsystem is independent of the mismatch cost a. This is because there is no mismatch in the female-driver subsystem. Next, notice that the optimal price is higher in the female-driver subsystem ($\widetilde{p}_F^* \geq \widetilde{p}_M^*$) when the safety-concerned female riders' mismatch cost a satisfies $\widetilde{a} \leq a \leq \widehat{a}$. Given the scarcity of female drivers, this result is as expected because the platform has to charge more so that it can pay female drivers more to entice more female drivers to participate. However, it is interesting to note that it is not always true that the optimal price in the female-driver subsystem is always higher. As shown in Corollary 1 and Figure 3, the optimal price for female-driver subsystem can be lower than that of the male-driver subsystem when a is sufficiently low or sufficiently high.

This seemingly counter-intuitive result can be explained as follows. First, consider the case when $a > \widehat{a}$ (so that the condition as stated in Proposition 3 does not hold), more safety-concerned female riders would prefer the female-driver subsystem, which will cause the waiting time of the female-driver subsystem to increase (due to female-driver scarcity). To compensate for this increase in waiting time, the platform has to charge a lower price in the female-driver subsystem. Second, consider the case when $a < \widetilde{a}$ (so that the condition as stated in Proposition 3 holds). Specifically, when a is sufficiently low, female riders have very little safety concerns and behave as if they are safety-unconcerned. Some safety-concerned female riders will join the male-driver subsystem (along with all safety-unconcerned riders as stated in statement 1(a) of Proposition 3). This will cause the waiting time of the male-driver subsystem to increase (especially when both subsystems exhaust all drivers as shown in statement 2 of Proposition 3). To channel some safety-concerned female riders from the male-driver subsystem to the female-driver subsystem, the platform has the incentive to charge a higher price in the male-driver subsystem.

6 System Comparison and Discussion

By using the results established in §4 and §5 for the pooling and hybrid systems, we now compare the equilibrium outcomes of these two systems. Specifically, we wonder if and when the hybrid system dominates the pooling system in terms of user utility and platform profit. We end this section by relating our results to the actual system adopted by various ride-hailing platforms in different countries/regions.

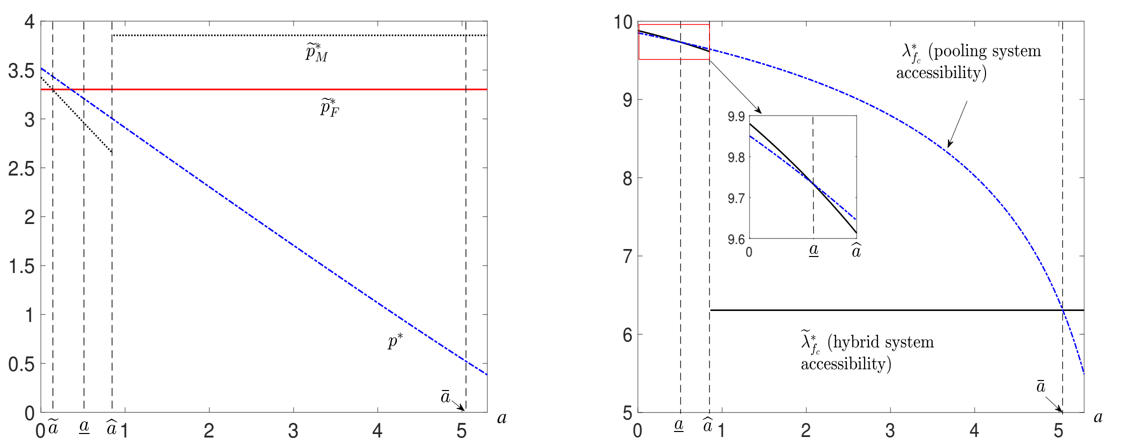
6.1 Pooling versus Hybrid

Because the motivation for considering the hybrid system is to serve more female riders and to support more female drivers due to their safety concerns, we shall first compare the joining rate of safety-concerned female riders and the participation rate of safety-concerned female-drivers between the pooling and the hybrid systems. Then we compare the utility of different groups of users and the platform's profit between these two systems.

Proposition 4 (**Safety-concerned Female Riders' Accessibility**). *The equilibrium joining rate of safety-concerned female riders in the hybrid system is (weakly) larger than that of the pooling system when (a) the number of male drivers is sufficiently large such that*

$N_m\mu > \Lambda_\phi$; and (b) the mismatch costs of female users $(a, b) \in \Theta_1 \equiv \{(a, b) : (a < \underline{a}$ or $a > \bar{a})$ and $b \geq \widehat{b}(a)\}$; i.e., when female driver mismatch cost b is high and female rider mismatch cost a is either sufficiently high or sufficiently low, where the expressions for $\underline{a}$ and $\bar{a}$ and the property that $\underline{a} \leq \widehat{a} \leq \bar{a}$ are provided in (17) and (18) of the online Appendix A.

Recall from Proposition 1 (statement 1(a)) and Proposition 3 (statement 2) that, when the safety-concerned female driver's mismatch cost $b > \widehat{b}(a)$, all safety-concerned female drivers balk in the pooling system and yet they all participate in the female-driver subsystem within the hybrid system. Also, recall from statement 1 of Proposition 3 that safety-concerned female riders will consider joining the female-driver subsystem only when their mismatch cost $a > \widehat{a}$. Hence, as more female drivers participate and female safety concerns are absent in the female-driver subsystem, it is intuitive to expect more safety-concerned female riders to join the hybrid system than the pooling system when their mismatch cost $a > \bar{a}$.



(a) Platform's Pricing

(b) Safety-concerned Female Riders' Accessibility

Figure 4: Illustration of Proposition 4: $N_m = 13$, $\Lambda_m = 6.5$, $N_f = 8$, $\Lambda_f = 15$, $\mu = 1$, $r = 0.2$, $c = 3$, $R = 4$, $\delta_R = 5\%$, $\delta_D = 85\%$

However, it is less obvious why the hybrid system can attract more safety-concerned female riders when their mismatch cost $a < \underline{a}$. To gain a better understanding, we conduct a numerical study to illustrate our result in Figure 4. Before we explain the result, let us first examine the optimal prices that the platform will charge under both systems. For the pooling system, statement 1 of Proposition 2 indicates that the platform's optimal price p^* is continuous and the platform can afford to charge a higher price as the mismatch cost a becomes lower. However, for the hybrid system, we note that there is a downward "jump" in the optimal price $\widetilde{p}_M^*$ for the male-driver subsystem when a becomes lower than $\widehat{a}$, as depicted in Figure 4(a). Specifically, when $a \leq \widehat{a}$, Figure 4(a) reveals that $\widetilde{p}_M^* < \widetilde{p}_F^*$ (on the range $a \in [\widetilde{a}, \underline{a}]$ according to Corollary 1) and $\widetilde{p}_M^* < p^*$. Hence, when the mismatch cost $a < \underline{a}$, safety-concerned female riders can enjoy a lower price $\widetilde{p}_M^*$ by joining the male-driver subsystem, which is lower than the price they would have to pay in the pooling system (if they were to join). As a consequence, more safety-concerned female riders will join the hybrid system than the pooling system, as depicted in Figure 4(b).

Next, let us consider the participation rate of safety-concerned female drivers in both systems. Because female driver's safety concern is absent in the female-driver subsystem, more female drivers are eager to participate in the hybrid system. More formally, we have:

Proposition 5 (**Safety-concerned Female Drivers' Accessibility**). *The equilibrium participating rate of safety-concerned female drivers is always (weakly) larger in the hybrid system than that of the pooling system.*

We now compare the utility of different groups of users (male, safety-concerned female and safety-unconcerned female) in both systems. To do so, we first determine the utility of each type-i rider U_i ($\widetilde{U}_i$)and the utility for each type-i driver S_i ($\widetilde{S}_i$), where $i = m, f_c, f_\phi$, for the pooling system (the hybrid system) as defined in §4.1 (in §5) by substituting the joining rate and participation rate presented in Proposition 1 (Proposition 3). Through the direct comparison, we get:

Proposition 6 (**User Utility**). *Under optimal pricing and wage decisions,*

1. *(a) Safety-concerned female riders obtain the same individual utility in the hybrid system as in the pooling system, i.e., $U_{f_c} = \widetilde{U}_{f_c}$;*

   (b) Both male and safety-unconcerned female riders obtain a higher individual utility in the hybrid system than in the pooling system, i.e., $\widetilde{U}_i > U_i$, $i = f_\phi, m$, if and only if: (a) the number of registered male drivers is sufficiently large such that $\mu N_m > \Lambda_\phi$; and (b) the safety-concerned female riders' mismatch cost is low ($a \leq \widehat{a}$).

2. *(a) Participating safety-concerned female drivers obtain the same individual utility in the hybrid system as in the pooling system, i.e., $S_{f_c} = \widetilde{S}_{f_c}$;*

   (b) Participating male and safety-unconcerned female drivers obtain a weakly lower individual utility in the hybrid system than in the pooling system, i.e., $S_i \geq \widetilde{S}_i, i = f_\phi, m$.

Proposition 6 reveals that transitioning from the pooling system to the hybrid system will affect the utility of different user groups as follows. From statements 1(a) and 2(a), the safety-concerned female user's utility remains the same under both systems in equilibrium.

However, from statements 2(b) and 1(b), safety-unconcerned drivers are always worse off, and safety-unconcerned riders are worse off (when the conditions in statement 1(b) do not hold) under the hybrid system. The hybrid system hurts these two groups for two reasons. First, safety-unconcerned users are worse off because of the weakened pooling effect in the hybrid system. Second, safety-unconcerned users cannot benefit from the hybrid system than they would have in the pooling system. To elaborate, in the pooling system, the optimal pricing and wage decisions are intended to entice safety-concerned female users to join (or participate), which benefits the safety-unconcerned users.

Next, we compare the platform profit under both systems in the following result:

Proposition 7 (**Platform Profitability**). *The platform's profit is higher in the hybrid system than in the pooling system; i.e.,* $\widetilde{\Pi}^* \geq \Pi^*$, *if and only if the mismatch costs of safety-concerned female users* $(a, b) \in \Theta_2 \equiv \{(a, b) : a \geq \check{a}(b)\}$.[5]

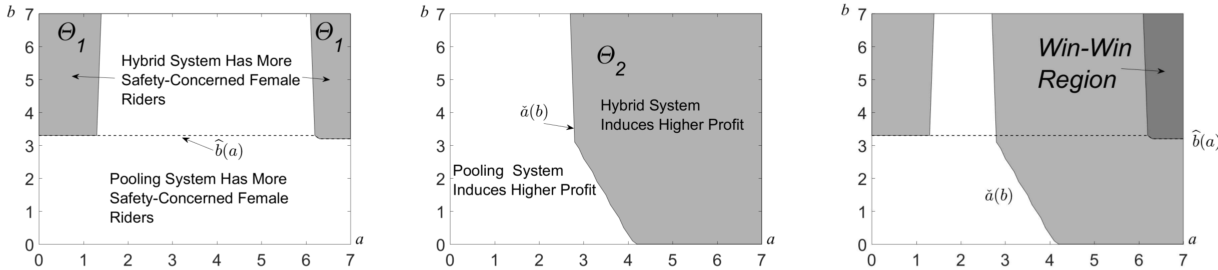
Proposition 7 reveals that the hybrid system generates a higher profit for the platform when safety-concerned female riders' mismatch cost is relatively high ($a \geq \check{a}(b)$). Although the pooling effect is weakened, the hybrid system lessens the safety concerns arising from the gender mismatch for safety-concerned female users, which enables it to expand the supply and demand pools. Recall that the platform's pricing and wage decisions are intended to entice safety-concerned female users to join (or participate). In this case, as the platform can customize its prices and wages for each subsystem, it enables the platform to extract more surplus from safety-unconcerned female users and thus obtain a higher profit under the hybrid system.

By combining the results obtained from Propositions 4, 5 and 7, we can identify conditions under which the hybrid system is the dominant system in terms of safety-concerned female users' accessibility and the platform's profit.

Corollary 2. *When safety-concerned female users' mismatch costs* $(a, b) \in \Theta_1 \cap \Theta_2$, *the hybrid system dominates the pooling system.*

Recall from Proposition 5 that the hybrid system entices more safety-concerned female drivers. This implies that when the condition of Corollary 2 is satisfied, the hybrid system increases participation of both safety-concerned female riders and drivers and improves the platform's profit. To illustrate the results as stated in Propositions 4, 7 and Corollary 2, we conduct some numerical studies. Figure 5 depicts the win-win regions characterized by Θ_1, Θ_2, and $\Theta_1 \cap \Theta_2$ as stated in Propositions 4, 7 and Corollary 2, respectively. Observe

[5] While Proposition 7 focuses on the effect of female riders' mismatch cost a, we can draw a similar conclusion if we vary female drivers' mismatch cost b. To avoid repetition, we omit details here.



(a) Safety-concerned Female Riders

(b) Platform's Profit

(c) Win-Win Region

Figure 5: When a Hybrid System Can Achieve Win-Win Compared to a Pooling System: $N_m = 110$, $\Lambda_m = 100$, $N_f = 40$, $\Lambda_f = 150$, $\mu = 1.5$, $r = 0.5$, $c = 1$, $R = 8$, $\delta_R = 20\%$ and $\delta_D = 50\%$ $(\mu N_\phi > \Lambda_\phi)$

from Figure 5(a) that the hybrid system can entice more safety-concerned female riders joining when $b \geq \hat{b}(a)$ as stated in Proposition 4. Figure 5(b) confirms that the hybrid system renders a higher profit for the platform when the mismatch cost $a \geq \check{a}(b)$ as stated in Proposition 7. Finally, Figure 5(c) reveals the win-win region by considering the intersection of those regions as shown in Figures 5(a) and (b), which occurs when both mismatch costs for female riders and drivers (a and b) are sufficiently high.

6.2 Discussion: Linking Results to Practice

Corollary 2 reveals that when female users' mismatch costs a and b are high, the hybrid system dominates; otherwise, the pooling system is preferred. Our analytic result may help us explain why different ride-hailing systems have been adopted in different countries, depending on the mismatch costs in the corresponding country. For instance, female safety is a serious concern in some countries: Thomson Reuters Foundation reported that the top 10 most dangerous countries for women include India (1st), Saudi Arabia (5th), Pakistan (6th) and the United States (10th) (Narayan 2018). Due to severe female safety concerns, gender-dedicated ride-hailing services are now provided in certain countries, such as Chariot for Women (United States), She Cabs (India) and She'Kab (Pakistan). In Saudi Arabia, a hybrid system was adopted by Uber which allows its female drivers to serve only female passengers in the female-driver subsystem (Kumar 2019).

Some countries are considered relatively safe for women. For example, according to the 2019 Global Wealth Migration Review conducted by New World Wealth (a global market research group), the five safest countries for women are Australia, Malta, Iceland, New Zealand and Canada (Perper 2019). A pooling system is often adopted by ride-hailing platforms in those countries as females usually have high safety confidence, such as Uber in

Australia and Canada and Ola in New Zealand (Barratt et al. 2018, Brail and Donald 2018, Kashyap 2018).

Table 2: Current System Adoption across Countries

Category	Country	Ride-hailing Platforms Examples
Pooling System	China	DiDi, DidaChuxing
	Australia	Uber, DiDi
	Canada	Uber, DiDi, Lyft, Grab, Yandex
	New Zealand	Uber, Ola
Hybrid System	Saudi Arabia	Uber Arabia
Dedicated System	India/Pakistan/United States	She Cabs/She'Kab/Chariot for Women
	China	DiDi (early morning and late night)

Notes: Dedicated system contains two gender-specific subsystems in which only users of the same gender are matched.

In other countries, such as China, females have a moderate safety concern and thus their mismatch costs are moderate. Different systems are used by ride-hailing platforms in different time slots. For example, DiDi provides the gender-dedicated service in the early morning hours (during which female users' safety concern is high so that their mismatch cost is also high). However, in other hours, DiDi operates as a pooling system (Al-Heeti 2018). Table 2 summarizes the operation of ride-hailing platforms in the aforementioned countries.

Recently, we also note that many ride-hailing platforms have begun to provide different services, which may exhibit different degrees of safety, leading to different levels of mismatch costs. For example, DiDi runs three business services: DiDi Premier, DiDi Express and DiDi Carpool (Hitch). Platforms can adopt the pooling system for services that are regarded as safer, such as DiDi premier, and consider a hybrid system for services that are regarded as less safe, such as DiDi carpool.

7 Conclusion

Some female riders/drivers have safety concern when they are matched with male counterparts. In this paper, we have presented a model to examine the interplay between gender-related safety concerns and two operational systems: a pooling system in which riders and drivers are matched without considering gender and a hybrid system in which females riders can select between a male-driver subsystem and a female-driver subsystem. We have shown that a pooling system is preferred when safety-concerned female users incur low mismatch costs. For the hybrid system, we have found that male and safety-unconcerned female users can be worse off under such a system. Despite this shortcoming, we show that the hybrid system can dominate the pooling system in terms of the safety-concerned female user's utility and the platform's profit when the mismatch costs lie within a certain region. By considering different levels of female safety concerns in different countries, we have discussed how our results are consistent with the actual system adopted in different countries.

Our paper represents an initial attempt to examine ways to address female safety concerns. We admit that our work has limitations. For example, except the gender-based safety concerns, there are other concerns due to racism that deserve further exploration. There are reports that Black riders/drivers are rejected by their counterparts in ride-hailing. This is an important issue to explore in the future. Besides concerns over safety and rejection, there is chronic shortage of female drivers, limiting the growth of ride-hailing platforms. Therefore, it is of interest to explore ways to encourage/entice more female drivers to participate in ride-hailing services in the future. Furthermore, motivated by Uber Saudi Arabia, we consider a hybrid system that is comprised of two gender-specific driver subsystems. In practice, the platform is usually profit-driven. Letting all the users go to a single system (i.e., the pooling configuration) usually induces a higher profit than separating them into two subsystems. As shown in our paper, only if the mismatch costs of the safety-concerned female users are high enough, implicating the hybrid system may benefit the platform. However, in reality, the platform usually could not obtain a very exact information about the female users' mismatch costs. This practical situation prevents the implication for the hybrid system. It is possible to consider a different hybrid system configuration that is comprised of a "pooling subsystem" and a female-driver subsystem so that both female riders and female drivers can choose between the pooling subsystem and the female-driver subsystem. But such a hybrid system involves two female driver queues in both subsystems that are interdependent. This inter-dependency makes the analysis intractable and we shall relegate to future research. In our paper, we do not consider the order cancellation issue. In practice, a safety-concerned female user may selectively cancel the ride. However, such cancellation often has conditions, and a cancellation fee may apply if a rider cancels the order *after* he/she is matched with a driver. If a rider cancels repeatedly in a short time, he/she will be penalized (UberHelp 2020). Similar policy applies to the driver side; see, e.g., Campbell (2018) and Smith (2018). Finally, in our study, we focus on examining the impact of gender-based safety concerns on the operational system design by considering a single platform, in which the equilibrium analysis that involves two types of riders and two types of drivers are already complex. Extending our current setting to a duopoly setting with platform competition is much more complicated and challenging and we would like to leave it to future research.

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Online Appendices

“Gender-Related Operational Issues Arising from On-Demand Ride-Hailing Platforms: Safety Concerns and System Configuration”

In this online appendix, we provide the proofs as well as the detailed equilibrium analysis for both the pooling system and the hybrid system.

Appendix A Proofs of Propositions

We prove the propositions stated in the main manuscript based on the optimal solutions derived in the Appendices B (for the pooling system) and C (for the hybrid system).

Proof of Proposition 1 : This proposition can be easily obtained based on Propositions B.1, B.2, B.3 and B.4 stated in Appendix B.

Proof of Proposition 2 : Based on Propositions B.1, B.2, B.3 and B.4, when either $N_\phi \mu \leq \Lambda_\phi$ or $b < \widehat{b}(a)$, in a pooling system, we know that the interior optimal total effective arrival rate λ^* satisfies the following first-order condition:

$$\frac{d\Pi(\lambda)}{d\lambda}\Big|_{\lambda=\lambda^*} = R - \frac{N_m a}{N} - \frac{c\lambda(2\mu N - \lambda)}{\mu N(\mu N - \lambda)^2} + \frac{b\Lambda_m N}{\lambda^2} = 0. \tag{13}$$

Then, according to the implicit function theorem, we have

$$\begin{aligned} \frac{d\lambda^*(a,b)}{da} &= -\frac{\frac{\partial^2\Pi(\lambda)}{\partial\lambda\partial a}}{\frac{\partial^2\Pi(\lambda)}{\partial\lambda^2}}\Big|_{\lambda=\lambda^*} = -\frac{-N_m/N}{\frac{d^2\Pi(\lambda)}{d\lambda^2}}\Big|_{\lambda=\lambda^*} < 0, \text{and} \\ \frac{d\lambda^*(a,b)}{db} &= -\frac{\frac{\partial^2\Pi(\lambda)}{\partial\lambda\partial b}}{\frac{\partial^2\Pi(\lambda)}{\partial\lambda^2}}\Big|_{\lambda=\lambda^*} = -\frac{\Lambda_m N/\lambda^2}{\frac{d^2\Pi(\lambda)}{d\lambda^2}}\Big|_{\lambda=\lambda^*} > 0, \end{aligned}$$

where the inequalities hold due to the concavity of the profit function, that is,

$$\frac{d^2\Pi(\lambda)}{d\lambda^2}\Big|_{\lambda=\lambda^*} = -\frac{2c\mu N}{(\mu N - \lambda^*)^3} - \frac{2b\Lambda_m N}{\lambda^{*3}} < 0,$$

which is derived from (25) of the Appendix B. Thus, $\lambda^*(a,b)$ decreases in a but increases in b. Since $\lambda^*_{fc}(a,b) = \lambda^*(a,b) - \Lambda_\phi$, it must have $\lambda^*_{fc}(a,b)$ decreases in a but increases in b.

Since $p^* = R - \frac{N_m a}{N} - \frac{c\lambda^*}{\mu N(\mu N - \lambda^*)}$, we can show that

$$\begin{aligned} \frac{dp^*(a,b)}{da} &= -\frac{N_m}{N} - \frac{c}{\mu N} \cdot \frac{\mu N}{(\mu N - \lambda^*)^2} \cdot \frac{d\lambda^*}{da} \\ &= \frac{-N_m}{N} \cdot \frac{\frac{2c\mu N}{(\mu N - \lambda^*)^3} - \frac{c}{(\mu N - \lambda^*)^2} + \frac{2b\Lambda_m N}{\lambda^{*3}}}{\frac{2c\mu N}{(\mu N - \lambda^*)^3} + \frac{2b\Lambda_m N}{\lambda^{*3}}} < 0, \end{aligned}$$

because $\frac{2c\mu N}{(\mu N-\lambda^*)^3} - \frac{c}{(\mu N-\lambda^*)^2} = \frac{c(\mu N+\lambda^*)}{(\mu N-\lambda^*)^3} > 0$. Besides, we have

$$\frac{dp^*(a,b)}{db} = -\frac{c}{(\mu N - \lambda^*)^2} \cdot \frac{d\lambda^*}{db} < 0.$$

That is, the optimal price p^* decreases in both a and b.

Recall that $w^* = \frac{\left(r+\frac{\Lambda_m b}{\lambda^*}\right)N}{\lambda^*}$, based on which we have

$$\frac{dw^*(a,b)}{da} = \left(-\frac{r}{\lambda^{*2}} - 2\frac{\Lambda_m b}{\lambda^{*3}}\right) N \cdot \frac{d\lambda^*}{da} > 0,$$

which concludes that the optimal wage w^* increases in a. Next, we analyze the sign of $\frac{dw^*(a,b)}{db}$. First, for a platform whose profit is positive, $p^* > w^*$ is requested, which is equivalent to

$$R - \frac{N_m a}{N} - \frac{c\lambda^*}{\mu N(\mu N - \lambda^*)} - \frac{\left(r + \frac{\Lambda_m b}{\lambda^*}\right) N}{\lambda^*} > 0. \tag{14}$$

Furthermore, (13) stated above can be rewritten as

$$R - \frac{N_m a}{N} - \frac{c\lambda^*}{\mu N(\mu N - \lambda^*)} - \frac{\left(r + \frac{\Lambda_m b}{\lambda^*}\right) N}{\lambda^*} + \frac{rN}{\lambda^*} - \frac{c\lambda^*}{(\mu N - \lambda^*)^2} = 0.$$

Combining this equation and inequality 14, it must have that

$$\frac{rN}{\lambda^*} - \frac{c\lambda^*}{(\mu N - \lambda^*)^2} < 0.$$

By noting that

$$\frac{c\lambda^*}{(\mu N - \lambda^*)^2} < \frac{c\lambda^*\mu N}{(\mu N - \lambda^*)^3} < \frac{2c\lambda^*\mu N}{(\mu N - \lambda^*)^3},$$

one can easily conclude that

$$\frac{r}{\lambda^{*2}} < \frac{2c\mu}{(\mu N - \lambda^*)^3}. \tag{15}$$

Since

$$\begin{aligned}\frac{dw^*(a,b)}{db} &= -\left(\frac{r}{\lambda^{*2}} + \frac{2b\Lambda_m}{\lambda^{*3}}\right) N \cdot \frac{d\lambda^*}{db} + N\frac{\Lambda_m}{\lambda^{*2}} \\ &= N\frac{\Lambda_m}{\lambda^{*2}}\left(1 - \frac{\frac{r}{\lambda^{*2}} + \frac{2b\Lambda_m}{\lambda^{*3}}}{\frac{2c\mu}{(\mu N-\lambda^*)^3} + \frac{2b\Lambda_m}{\lambda^{*3}}}\right),\end{aligned}$$

where the second equality results from plugging the expression of $\frac{d\lambda^*}{db}$ into the first equality. We then have $\frac{dw^*(a,b)}{db} > 0$ according to (15). In a word, the optimal wage w^* increases in b.

Regarding the profit $\Pi(\lambda) = \left(R - \frac{N_m a}{N} - \frac{c\lambda}{\mu N(\mu N-\lambda)}\right)\lambda - \left(r + \frac{\Lambda_m b}{\lambda}\right)N$, by the envelope theorem, it can be derived that

$$\frac{d\Pi^*}{da} = \frac{\partial\Pi(\lambda,a)}{\partial a}\Big|_{\lambda=\lambda^*} = -\frac{N_m}{N}\lambda^* < 0, \text{ and } \frac{d\Pi^*}{db} = \frac{\partial\Pi(\lambda,b)}{\partial b}\Big|_{\lambda=\lambda^*} = \frac{-\Lambda_m N}{\lambda^*} < 0.$$

Moreover,

$$\frac{d^2\Pi^*}{da^2} = -\frac{N_m}{N}\cdot\frac{d\lambda^*}{da} > 0, \text{ and } \frac{d^2\Pi^*}{db^2} = \frac{\Lambda_m N}{\lambda^{*2}}\cdot\frac{d\lambda^*}{db} > 0.$$

That is, the optimal profit is decreasing and convex in both a and b.

Proof of Proposition 3: This proposition can be directly obtained based on Propositions C.1, C.3 and C.4 stated in the Appendix C.

Proof of Corollary 1: We compare the optimal prices in the two subsystems based on two cases. (1) When $a > \widehat{a}$ or $N_m\mu \le \Lambda_\phi$, under which situation $(\widetilde{p}^*_M, \widetilde{w}^*_M; \widetilde{p}^*_F, \widetilde{w}^*_F) = (\widetilde{p}^*_{M_1}, \widetilde{w}^*_{M_1}; \widetilde{p}^*_{F_1}, \widetilde{w}^*_{F_1})$, from the expression listed in (30), we know that $\widetilde{p}^*_M = \widetilde{p}^*_{M_1} \ge R + \frac{c}{N_m\mu} - \sqrt{\frac{cR}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}$. Recall from the proof of Proposition C.2, $R + \frac{c}{N_m\mu} - \sqrt{\frac{cR}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2} > R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}$. Therefore, $\widetilde{p}^*_M = \widetilde{p}^*_{M_1} > \widetilde{p}^*_F = \widetilde{p}^*_{F_1} = R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}$. (2) When $a \le \widehat{a}$ and $N_m\mu > \Lambda_\phi$, under which situation $(\widetilde{p}^*_M, \widetilde{w}^*_M; \widetilde{p}^*_F, \widetilde{w}^*_F) = (\widetilde{p}^*_{M_2}, \widetilde{w}^*_{M_2}; \widetilde{p}^*_{F_2}, \widetilde{w}^*_{F_2})$, we have $\widetilde{p}^*_{M_2} = \min\left\{R - a - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu - \Lambda_\phi)}, R - a + \frac{c}{N_m\mu} - \sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}\right\}$, which can be easily shown is decreasing in a. Besides, $\widetilde{p}^*_{F_2} = R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}$ is independent of a. Let $\widetilde{a}$ be the unique solution of

$$\left(\widetilde{p}^*_{M_2} - \widetilde{p}^*_{F_2}\right)\Big|_{a=\widetilde{a}} = 0 \tag{16}$$

if it exists. If $\left(\widetilde{p}^*_{M_2} - \widetilde{p}^*_{F_2}\right)\Big|_{a\to 0} < 0$, then $\widetilde{a} = 0$; and if $\left(\widetilde{p}^*_{M_2} - \widetilde{p}^*_{F_2}\right)\Big|_{a\to\widehat{a}} > 0$, then $\widetilde{a} = \widehat{a}$. Based on the above discussions, we conclude that when $\widetilde{a} \le a \le \widehat{a}$, $\widetilde{p}^*_{M_2} \le \widetilde{p}^*_{F_2}$; and when $0 < a < \widetilde{a}$, $\widetilde{p}^*_{M_2} > \widetilde{p}^*_{F_2}$.

Summarizing the discussions in (1) and (2), we obtain that when $N_m\mu > \Lambda_\phi$ and $\widetilde{a} \le a \le \widehat{a}$ hold simultaneously, $\widetilde{p}^*_M \le \widetilde{p}^*_F$; otherwise, $\widetilde{p}^*_M > \widetilde{p}^*_F$.

Proof of Proposition 4: Note that when $N_m\mu > \Lambda_\phi$, it is possible that some safety-concerned female riders join the male-driver system, in which situation the accessibility of safety-concerned female riders in the hybrid system is large and so that it has a high possibility that the accessibility of safety-concerned female riders is larger in the hybrid system than that in the pooling system. According to Proposition 1 and Table B.1, when $b \ge \widehat{b}(a)$, in the pooling system, the equilibrium effective joining rate of safety-concerned female riders under the optimal price and wage is

$$\lambda^*_{f_c} = \mu N_\phi - c\Big/\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)} - \Lambda_\phi.$$

In the hybrid system, we have the following two cases.

Case (1): When $a > \widehat{a}$, according to Propositions C.3 and C.4, in the hybrid system, multiple equilibria exist and safety-concerned female riders only join the female-driver subsystem. Below, we consider the equilibrium outcome that induces the highest joining rate of safety-concerned female riders; that is,

$$\widetilde{\lambda}^*_{f_c} = \frac{N_f^2\mu^2(R-p^*_{F_1})}{c+N_f\mu(R-p^*_{F_1})} = \mu N_f - \frac{c}{\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}}.$$

Let $F_{d_2}(\alpha) = \widetilde{\lambda}^*_{f_c} - \lambda^*_{f_c} = (N_f - N_m - N_{f_\phi})\mu - \frac{c}{\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}} + \Lambda_\phi + c\Big/\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}.$

It can be easily shown that $F_{d_2}(\alpha)$ is increasing in a. Let $\bar{a}_0$ be the solution of $F_{d_2}(a) = 0$ if it exists, that is,

$$F_{d_2}(a)\Big|_{a=\bar{a}_0} = (N_f - N_m - N_{f_\phi})\mu - \frac{c}{\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}} + \Lambda_\phi + \frac{c}{\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}} = 0.$$

If $F_{d_2}(a)\big|_{a\to 0} > 0$, we let $\bar{a}_0 = 0$. Then $F_{d_2}(a) > 0$ if and only if $a > \bar{a}_0$. Thus, the hybrid system induces a higher effective joining rate for safety-concerned female riders than the pooling system when $a > \max\{\bar{a}_0, \widehat{a}\}$.

Case (2): When $a \le \widehat{a}$. According to Propositions C.3 and C.4 and (30), in the hybrid system, the total effective joining rate of safety-concerned female riders is

$$\begin{aligned}\widetilde{\lambda}^*_{f_c} &= \widetilde{\lambda}^*_{f_c,F_2} + \widetilde{\lambda}^*_{f_c,M_2} = \frac{N_m^2\mu^2(R-a-\widetilde{p}^*_{M_2})}{c+N_m\mu(R-a-\widetilde{p}^*_{M_2})} - \Lambda_\phi + \frac{N_f^2\mu^2(R-\widetilde{p}^*_{F_2})}{c+N_f\mu(R-\widetilde{p}^*_{F_2})}\\ &= N\mu - \Lambda_\phi - \frac{c}{\sqrt{\frac{c(R-a)}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2}} - \frac{c}{\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}}.\end{aligned}$$

Let $F_{d_1}(a) = \widetilde{\lambda}^*_{f_c} - \lambda^*_{f_c} = N_{f_c}\mu - \frac{c}{\sqrt{\frac{c(R-a)}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2}} - \frac{c}{\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}} + c\Big/\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}.$

It can be shown that

$$\begin{aligned}\frac{dF_{d_1}}{da} &= \frac{c^2\cdot\left(-\frac{c}{N_m\mu}\right)}{\frac{c(R-a)}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2} + \frac{c\cdot\frac{cN_m}{\mu N_\phi^2}}{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R-\frac{N_m a}{N_\phi}\right)}\\ &= \frac{c^2}{\mu}\left(\frac{-N_m}{\frac{c^2}{\mu^2}+\frac{cRN_m}{\mu}-\frac{caN_m}{\mu}} + \frac{N_m}{\frac{c^2}{\mu^2}+\frac{cRN_\phi}{\mu}-\frac{caN_m}{\mu}}\right) < 0.\end{aligned}$$

That is, $F_{d_1}(a)$ is decreasing in a. Let $\bar{a}_1$ be the solution of $F_{d_1}(a) = 0$ if it exists, that is,

$$F_{d_1}(a)\Big|_{a=\bar{a}_1} = N_{f_c}\mu - \frac{c}{\sqrt{\frac{c(R-a)}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2}} - \frac{c}{\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}} + c\Big/\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R-\frac{N_m a}{N_\phi}\right)} = 0.$$

Thus, $F_{d_1}(a) > 0$ if and only if $a < \bar{a}_1$. In other words, $\widetilde{\lambda}^*_{f_c} > \lambda^*_{f_c}$ when $a < \min\{\bar{a}_1, \widehat{a}\}$.

In summary, when $N_m\mu > \Lambda_\phi$ holds, $\widetilde{\lambda}^*_{f_c} > \lambda^*_{f_c}$ if $b > \widehat{b}(a)$ and ($a < \min\{\bar{a}_1, \widehat{a}\}$ *or* $a > \max\{\bar{a}_0, \widehat{a}\}$). For ease of reference, we define

$$\underline{a} := \min\{\bar{a}_1, \widehat{a}\} \tag{17}$$

and

$$\bar{a} := \max\{\bar{a}_0, \widehat{a}\}. \tag{18}$$

Proof of Proposition 5: The result can be easily obtained by directly comparing the female drivers' equilibrium participating rates in the pooling and hybrid systems as summarized in Tables B.1 and C.1 of the online Appendices B and C.

Proof of Proposition 6: First, consider the case that $N_\phi\mu \le \Lambda_\phi$. Based on Propositions B.1 and B.2, we know that the safety-concerned female riders' joining utility is zero in equilibrium, that is,

$$U^*_{f_c} = R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; N_{f_c}, N_{f_\phi}, N_m) - \frac{N_m}{N}a = 0.$$

As to male riders and safety-unconcerned female riders, they all join the system and each obtains:

$$U^*_{f_\phi} = U^*_m = R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; N_{f_c}, N_{f_\phi}, N_m) = \frac{N_m}{N}a + U^*_{f_c} = \frac{N_m}{N}a.$$

Regarding the hybrid system, since $N_\phi\mu \le \Lambda_\phi$, it must have that $N_m\mu = N_\phi\mu - N_{f_\phi}\mu < \Lambda_\phi$. Based on Proposition C.1 and its proof, we can easily know that all the joining riders obtain a zero utility, that is, $\widetilde{U}^*_{f_c} = \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$. Based on the above analysis, we can easily obtain that when $N_\phi\mu \le \Lambda_\phi$, $U^*_{f_c} = \widetilde{U}^*_{f_c} = 0$, and $U^*_{f_\phi} = U^*_m > \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$.

We now analyze the utility of each participating driver. Based on Propositions B.1 and C.1, (22) and (29), we can know that each safety-concerned female driver obtains zero utility in both the pooling and the hybrid system. That is, $S^*_{f_c} = \widetilde{S}^*_{f_c} = r$. Regarding male drivers and safety-unconcerned female drivers, in the pooling system, each obtains the following utility:

$$S^*_{f_\phi} = S^*_m = \frac{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}{N} \cdot w^* - r = \frac{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}{N} \cdot \frac{\left(r + \frac{\Lambda_m b}{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}\right) N}{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}} - r > 0.$$

While in the hybrid system, each obtains $\widetilde{S}^*_{f_\phi} = \widetilde{S}^*_m = r$. Therefore, $S^*_i > \widetilde{S}^*_i$ for $i = f_\phi, m$.

Next, we consider the case $N_\phi\mu > \Lambda_\phi$. In the pooling system, according to Propositions B.3 and B.4 stated in Appendix B, we then have that:

(a). When $b < \widehat{b}(a)$, under the platform's optimal wage and price decision, the system behaves the same as those when $N_\phi \mu \leq \Lambda_\phi$, and the users' joining and participating behaviors are the same as those shown above as well. Thus, we have

$$U^*_{f_c} = 0, U^*_{f_\phi} = U^*_m = \frac{aN_m}{N}, S^*_{f_c} = 0, S^*_{f_\phi} = S_m = \frac{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}{N} \cdot \frac{\left(r + \frac{\Lambda_m b}{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}\right) N}{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}} - r > 0.$$

(b). When $b \geq \widehat{b}(a)$, no safety-concerned female drivers participate to work and in equilibrium, safety-concerned female riders' utility is zero; that is,

$$U^*_{f_c} = R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; 0, N_{f_\phi}, N_m) - \frac{N_m \cdot a}{N_\phi} = 0.$$

Safety-concerned female drivers get the reservation price $\mathbb{U}^*_{f_c} = r$. Each safety-unconcerned female rider and each male rider obtain the following utility:

$$U^*_{f_\phi} = U^*_m = R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; 0, N_{f_\phi}, N_m) = \frac{N_m \cdot a}{N_\phi} + U^*_{f_c} = \frac{N_m \cdot a}{N_\phi}.$$

As only safety-unconcerned drivers participate to work, we can easily get that $S^*_{f_\phi} = S^*_m = 0$.

In the hybrid system, according to Propositions C.3 and C.4 of the online Appendix C.2, if and only if when $a \leq \widehat{a}$ and $N_m \mu \leq \Lambda_\phi$, under the platform's optimal wage and price decision, the participating drivers obtain the reservation cost r and thus they obtain zero utility, i.e., $\widetilde{S}^*_{f_\phi} = \widetilde{S}^*_{f_c} = \widetilde{S}^*_m = 0$. It can be easily shown that the safety-concerned female riders obtain zero utility and safety-unconcerned female riders and male riders obtain the following utility

$$\widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = a + \widetilde{U}^*_{f_c} = a.$$

Otherwise, the system behaves the same as those when $N_\phi \mu \leq \Lambda_\phi$. Thus, all the joining riders obtain a zero utility, that is, $\widetilde{U}^*_{f_c} = \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$. And all drivers obtain the reservation cost r.

Based on above discussions, we can conclude that when $N_\phi \mu > \Lambda_\phi$, at the driver side, $S^*_{f_c} = \widetilde{S}^*_{f_c} = 0$ and $S^*_i \geq \widetilde{S}^*_i$, $i = f_\phi, m$. At the rider side, $U^*_{f_c} = \widetilde{U}^*_{f_c} = 0$. As both $a > \frac{N_m}{N_\phi} a$ and $a > \frac{N_m \cdot a}{N}$, we have that when $N_m \mu > \Lambda_\phi$ and $a \leq \widehat{a}$, $\widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m > U^*_{f_\phi} = U^*_m$; otherwise, $U^*_{f_\phi} = U^*_m > \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$.

Proof of Proposition 7: We consider the following two cases.

One, $\mu N_\phi \leq \Lambda_\phi$. According to Tables B.1 and C.1 and Proposition 2, the profit in the pooling system Π^* is decreasing with both a and b while the profit in the hybrid system $\widetilde{\Pi}^*$ is independent of both a and b. Thus, the profit difference between the pooling system and the hybrid system, $\Pi^* - \widetilde{\Pi}^*$, is decreasing in a for any given b.

Two, $\mu N_\phi > \Lambda_\phi$. According to Table C.1 and Proposition C.4, the platform's profit in the hybrid system $\widetilde{\Pi}^*$ is independent of b because all the female drivers are only allowed to join the female-driver subsystem, where they have no chances to be matched with a male rider. $\widetilde{\Pi}^*$ is dependent of a only when safety-concerned female riders also join the male-driver subsystem at a non-zero rate, that is, when $N_m\mu > \Lambda_\phi$ and $a \le \widehat{a}$, under which situation $(\widetilde{p}^*_M, \widetilde{w}^*_M; \widetilde{p}^*_F, \widetilde{w}^*_F) = \left(\widetilde{p}^*_{M_2}, \widetilde{w}^*_{M_2}; \widetilde{p}^*_{F_2}, \widetilde{w}^*_{F_2}\right)$ and $\widetilde{p}^*_{M_2} = R - a + \frac{c}{N_m\mu} - \sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}$. Otherwise, the platform's profit in the hybrid system $\widetilde{\Pi}^*$ is independent of a while the platform's profit in the pooling system Π^* is decreasing in a (refer to the proof of Proposition B.3 in online Appendix B.2). Then, in this situation, the profit difference between the pooling system and the hybrid system, $\Pi^* - \widetilde{\Pi}^*$, is decreasing in a.

Below, we show that when $\mu N_m > \Lambda_\phi$ and $a \le \widehat{a}$, the profit difference between the pooling system and the hybrid system, $\Pi^* - \widetilde{\Pi}^*$, is decreasing in a as well. Note that for the optimal profit in the hybrid system, $\widetilde{\Pi}^*$, we have

$$\frac{d\widetilde{\Pi}^*}{da} = -\mu N_m \cdot \frac{\sqrt{(R-a)\mu N_m + c} - \sqrt{c}}{\sqrt{(R-a)\mu N_m + c}}, \quad \text{when} \quad \widetilde{p}^*_{M_2} = R - a + \frac{c}{N_m\mu} - \sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}.$$

Case (1): When $b < \widehat{b}(a)$, according to Table B.1, $\Pi^* = \Pi_1^*$. Then based on the proof of Proposition 2, we know that $\frac{d\Pi^*}{da} = -\frac{N_m}{N}\lambda^*$. Recall that the optimal arrival rate λ^* satisfies

$$R - \frac{N_m a}{N} - \frac{c\lambda(2\mu N - \lambda)}{\mu N(\mu N - \lambda)^2} + \frac{b\Lambda_m N}{\lambda^2} = 0.$$

In other words, $R - \frac{N_m a}{N} - \frac{c\lambda^*(2\mu N - \lambda^*)}{\mu N(\mu N - \lambda^*)^2} < 0$, from which we can derive that

$$\lambda^* > \frac{2\mu N(\mu N(R - \frac{N_m a}{N}) + c) - \sqrt{4\mu^2 N^2 c(c + \mu N(R - \frac{N_m a}{N}))}}{2(c + \mu N(R - \frac{N_m}{N}a))} = \mu N\left(1 - \frac{\sqrt{c}}{\sqrt{c + \mu N R - \mu N_m a}}\right).$$

Besides, note that $\mu N\left(1 - \frac{\sqrt{c}}{\sqrt{c + \mu N R - \mu N_m a}}\right) > \mu N\left(1 - \frac{\sqrt{c}}{\sqrt{c + \mu N_m R - \mu N_m a}}\right)$, so we have

$$\lambda^* > \mu N\left(1 - \frac{\sqrt{c}}{\sqrt{c + \mu N_m R - \mu N_m a}}\right).$$

Hence,

$$\begin{aligned} \frac{d\left(\Pi^* - \widetilde{\Pi}^*\right)}{da} &= \frac{d\Pi^*}{da} - \frac{d\widetilde{\Pi}^*}{da} = -\frac{N_m\lambda^*}{N} + \mu N_m \cdot \frac{\sqrt{(R-a)\mu N_m + c} - \sqrt{c}}{\sqrt{(R-a)\mu N_m + c}} \\ &= -\frac{N_m}{N}\left(\lambda^* - \mu N\left(1 - \frac{\sqrt{c}}{\sqrt{c + \mu N_m R - \mu N_m a}}\right)\right) < 0. \end{aligned}$$

Case (2): When $b \geq \widehat{b}(a)$, according to Table B.1, $\Pi^* = \Pi_2^*$. We can show that

$$\Pi^* = \Pi_2^* = \left(\sqrt{(R-a)\mu N_\phi + c} - \sqrt{c}\right)^2 - rN_\phi,$$

based on which we obtain $\frac{d\Pi^*}{da} = -\mu N_\phi \cdot \frac{\sqrt{(R-a)\mu N_\phi + c} - \sqrt{c}}{\sqrt{(R-a)\mu N_\phi + c}}$. Define

$$f(x) = x \cdot \frac{\sqrt{(R-a)x + c} - \sqrt{c}}{\sqrt{(R-a)x + c}}.$$

It can be easily shown that $f'(x) = \frac{\sqrt{(R-a)x+c} - \sqrt{c}}{\sqrt{(R-a)x+c}} + \frac{x\sqrt{c}(R-a)}{2((R-a)x+c)\sqrt{(R-a)x+c}} > 0$. Therefore,

$$\frac{d\Pi^*}{da} - \frac{d\widetilde{\Pi}^*}{da} = -\mu N_\phi \cdot \frac{\sqrt{(R-a)\mu N_\phi + c} - \sqrt{c}}{\sqrt{(R-a)\mu N_\phi + c}} + \mu N_m \cdot \frac{\sqrt{(R-a)\mu N_m + c} - \sqrt{c}}{\sqrt{(R-a)\mu N_m + c}} = f(\mu N_m) - f(\mu N_\phi) < 0.$$

Based on the above discussion, we can conclude that the profit difference between the two systems $\Pi^* - \widetilde{\Pi}^*$ is always decreasing in a. Let $\check{a}(b)$ be the solution of

$$\left(\Pi^* - \widetilde{\Pi}^*\right)\Big|_{a=\check{a}(b)} = 0 \tag{19}$$

if it exists. $\Pi^* > \widetilde{\Pi}^*$ only when $a < \check{a}(b)$.

Last, we consider a special case when both drivers and riders have no mismatch costs, that is, $a = b \to 0$. Under such a situation, all the drivers shall join the pooling system as well as the hybrid system due to the abundant demand. Thus, the pooling system is an $M/M/1$ queue with capacity $(N_f + N_m)\mu$ while the hybrid system is a system consisting of two $M/M/1$ queues with capacity $N_m\mu$ and $N_f\mu$, respectively.

For the pooling system, we can derive that

$$\lambda^e(p) = \frac{\mu^2 N^2 (R-p)}{c + \mu N(R-p)}, \text{ and } w(p) = \frac{rN}{\lambda^e(p)}.$$

The platform maximizes

$$\Pi(p) = (p - w(p))\lambda^e(p),$$

which is can be easily shown concave in p and the first order condition is

$$\frac{d\Pi(p)}{dp} = \mu^2 N^2 \cdot \frac{\mu N(R-p) + c(R-2p)}{(c + \mu N(R-p))^2} = 0.$$

It can be easily shown that the optimal price $p^* = R + \frac{c}{\mu N} - \sqrt{(\frac{c}{\mu N})^2 + \frac{cR}{\mu N}}$. Then we can get the total effective joining rate under optimal price is $\lambda_f^* + \lambda_m^* = \mu N - \frac{c}{\sqrt{(\frac{c}{\mu N})^2 + \frac{cR}{\mu N}}}$.

As for the hybrid system, we can show that the two subsystem adopt the optimal prices

$$\widetilde{p}_M^* = R + \frac{c}{\mu N_m} - \sqrt{(\frac{c}{\mu N_m})^2 + \frac{cR}{\mu N_m}} \text{ and } \widetilde{p}_F^* = R + \frac{c}{\mu N_f} - \sqrt{(\frac{c}{\mu N_f})^2 + \frac{cR}{\mu N_f}}.$$

The corresponding equilibrium joining rates in the two subsystems are respectively

$$\lambda_M^e(\widetilde{p}_M^*) = \mu N_m - \frac{c}{\sqrt{(\frac{c}{\mu N_m})^2 + \frac{cR}{\mu N_m}}}, \text{ and } \lambda_F^e(\widetilde{p}_F^*) = \mu N_f - \frac{c}{\sqrt{(\frac{c}{\mu N_f})^2 + \frac{cR}{\mu N_f}}}.$$

Since the riders' effective joining rate in the hybrid system $\widetilde{\lambda}_f^* + \widetilde{\lambda}_m^* = \lambda_M^e(\widetilde{p}_M^*) + \lambda_F^e(\widetilde{p}_F^*)$, we have

$$\begin{aligned}
\lambda_f^* + \lambda_m^* - \left(\widetilde{\lambda}_f^* + \widetilde{\lambda}_m^*\right) &= \mu N - \frac{c}{\sqrt{(\frac{c}{\mu N})^2 + \frac{cR}{\mu N}}} - \left(\mu N_m - \frac{c}{\sqrt{(\frac{c}{\mu N_m})^2 + \frac{cR}{\mu N_m}}} + \mu N_f + \frac{c}{\sqrt{(\frac{c}{\mu N_f})^2 + \frac{cR}{\mu N_f}}}\right) \\
&= \frac{c\mu N_m}{\sqrt{c^2 + cR\mu N_m}} + \frac{c\mu N_f}{\sqrt{c^2 + cR\mu N_f}} - \frac{c\mu N}{\sqrt{c^2 + cR\mu N}} \\
&> \frac{c\mu N_m}{\sqrt{c^2 + cR\mu N}} + \frac{c\mu N_f}{\sqrt{c^2 + cR\mu N}} - \frac{c\mu N}{\sqrt{c^2 + cR\mu N}} = 0.
\end{aligned}$$

We then can show that

$$\begin{aligned}
\Pi^* - \widetilde{\Pi}_M^* - \widetilde{\Pi}_F^* &= p^*(\lambda_f^* + \lambda_m^*) - rN - (\widetilde{p}_M^* \lambda_M^e(\widetilde{p}_M^*) - rN_m + \widetilde{p}_F^* \lambda_F^e(\widetilde{p}_F^*) - rN_f) \\
&> p^* \left(\lambda_M^e(\widetilde{p}_M^*) + \lambda_F^e(\widetilde{p}_F^*)\right) - \widetilde{p}_M^* \lambda_M^e(\widetilde{p}_M^*) - \widetilde{p}_F^* \lambda_F^e(\widetilde{p}_F^*) > 0,
\end{aligned}$$

because $p^* > \widetilde{p}_j^*, j = F, M$. Thus, $\Pi^* > \widetilde{\Pi}_M^* + \widetilde{\Pi}_F^* = \widetilde{\Pi}^*$.

Appendix B The Pooling System: Detailed Analyses

We first present an implication that is useful to understand the equilibrium behaviors of riders and drivers in a pooling system (the logic of this implication can be applied to the male-driver subsystem in a hybrid system).

In the pooling system, safety-unconcerned female and male riders and safety-concerned female riders continue to join the system until their utility U_i given in (3) and (4) hits zero, where $i = f_c, f_\phi, m$. A close look at (3) and (4) implies that $U_m = U_{f_\phi} \geq U_{f_c}$. Similarly, because safety-unconcerned female and male drivers have no safety concerns, a close look at (5) and (6) implies that $S_m = S_{f_\phi} \geq S_{f_c}$. These observations yield the following implications.

Implication B.1. *In a pooling system, if some safety-concerned female riders/drivers join the system, then all safety-unconcerned riders/drivers will join the system.*

Note that safety-unconcerned riders and drivers, which contains all males and a fraction of safety-unconcerned females, are more eager to join the system than their safety-concerned female counterparts. It is likely to have all $\Lambda_\phi (= \Lambda_m + \Lambda_{f_\phi})$ safety-unconcerned riders and all $N_\phi (= N_m + N_{f_\phi})$ safety-unconcerned drivers joining the system before their safety-concerned female counterparts. Also note that throughout our analyses, we only consider the equilibrium outcomes in which the safety-concerned female riders join the system at a non-zero rate. While deriving the equilibrium joining/participating behaviors of riders/drivers, one can easily find some equilibria in which all the safety-concerned female riders balk in a pooling/hybrid system. As such equilibrium outcomes deviate from our research motivation, they are not our focus and thus we omit such trivial cases. When some safety-concerned female riders join the system, then all Λ_ϕ safety-unconcerned male and female riders join the system (due to Implication B.1). Then, some safety-concerned female drivers must participate in the service to ensure the stability of the queuing system when the number of safety-unconcerned drivers are not sufficiently high to serve even just the safety-unconcerned riders, that is, when $\mu N_\phi \leq \Lambda_\phi$. Then, we have the following implication.

Implication B.2. *When $\mu N_\phi \leq \Lambda_\phi$, if some safety-concerned female riders join the system, then all safety-unconcerned drivers must participate in the system.*

Below, we derive the equilibrium joining (and participating) behaviors of riders (and drivers) under the two exhaustive and exclusive cases.

B.1 When the number of safety-unconcerned drivers is low: $N_\phi \mu \leq \Lambda_\phi$

Based on Implication B.2, we can conclude that when $N_\phi \mu \leq \Lambda_\phi$, in order for the platform to retain safety-concerned female riders in the pooling system, it must be the case that all safety-unconcerned riders (and drivers) and some safety-concerned female riders (and drivers) join (and participate in) the system at their potential arrival rates, respectively. This allows us to focus on deriving the joining and participating behaviors of only the female safety-concerned riders and drivers.

Denote λ_i^e as the effective joining rate of type-i riders and n_i^e as the number of participating type-i drivers in equilibrium, $i = f_c, f_\phi, m$. Then, $\lambda_f^e = \lambda_{f_c}^e + \lambda_{f_\phi}^e$. By focusing on the equilibrium outcome that some safety-concerned female riders join the system, we now develop the conditions under which this equilibrium will exist in the following proposition.

Proposition B.1. *In a pooling system, if $N_\phi \mu \leq \Lambda_\phi$, the platform sets the price $p \leq \bar{p}_1 := R - \frac{c\Lambda_\phi}{\mu N(\mu N - \Lambda_\phi)} - \frac{N_m a}{N}$ and the wage $w \geq \underline{w}_1(p) := \frac{rN + \Lambda_m b(c+\mu(N(R-p)-N_m a))/(\mu^2(N(R-p)-N_m a))}{(\mu^2 N((R-p)N - N_m a))/(c+\mu((R-p)N-N_m a))}$ to ensure the joining of the safety-concerned female riders. Then, in equilibrium, all registered* *drivers participate (i.e, $n_m^e = N_m$, $n_{f_\phi}^e = N_{f_\phi}$ and $n_{f_c}^e = N_{f_c}$) and all safety-unconcerned riders (that is, male riders and safety-unconcerned female riders) join the system so that $\lambda_m^e = \Lambda_m$ and $\lambda_{f_\phi}^e = \Lambda_{f_\phi}$. Some safety-concerned female riders join the system and the others balk with an effective joining rate $\lambda_{f_c}^e(p, N_{f_c}) = \frac{\mu^2 N^2 (R-p-\frac{N_m a}{N})}{c+\mu N(R-p-\frac{aN_m}{N})} - \Lambda_\phi$.*

Proof of Proposition B.1. Based on Implications B.1 and B.2, we know that when $N_\phi \mu \leq \Lambda_\phi$, if some safety-concerned female riders join the system, then in equilibrium, safety-unconcerned drivers "all participate" and safety-unconcerned riders "all join"; that is, $\lambda_m^e = \Lambda_m$, $\lambda_{f_\phi}^e = \Lambda_{f_\phi}$, $n_m^e = N_m$ and $n_{f_\phi}^e = N_{f_\phi}$. We now analyze the joining and participating behavior of safety-concerned female riders and drivers.

Given n_{f_c}, the number of participating safety-concerned female drivers, to ensure that safety-concerned female riders are willing to join, we should have

$$U_{f_c}(0, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = R - p - cW(0, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) - \frac{N_m}{N_m + N_{f_\phi} + n_{f_c}} a \geq 0, \tag{20}$$

where $W(0, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = \frac{\Lambda_\phi}{(N_m + N_{f_\phi} + n_{f_c})\mu\left((N_m + N_{f_\phi} + n_{f_c})\mu - \Lambda_\phi\right)}$, where $\Lambda_\phi = \Lambda_m + \Lambda_{f_\phi}$. Recall that $\mu N < \Lambda$. Thus, in equilibrium, some safety-concerned female riders must balk. The equilibrium effective joining rate of safety-concerned female riders can be obtained by solving

$$U_{f_c}(\lambda_{f_c}, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = R - p - cW(\lambda_{f_c}, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) - \frac{N_m}{N_m + N_{f_\phi} + n_{f_c}} a = 0,$$

where $W(\lambda_{f_c}, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = \frac{\Lambda_\phi + \lambda_{f_c}}{(N_m + N_{f_\phi} + n_{f_c})\mu\left((N_m + N_{f_\phi} + n_{f_c})\mu - \Lambda_\phi - \lambda_{f_c}\right)}$. It can be shown that safety-concerned female riders' equilibrium joining rate

$$\lambda_{f_c}^e(p, n_{f_c}) = \frac{((N_m + N_{f_\phi} + n_{f_c})\mu)^2 (R - p - \frac{N_m}{N_m + N_{f_\phi} + n_{f_c}} a)}{c + (N_m + N_{f_\phi} + n_{f_c})\mu(R - p - \frac{N_m}{N_m + N_{f_\phi} + n_{f_c}} a)} - \Lambda_m - \Lambda_{f_\phi}.$$

Then, we have

$$\begin{aligned} d_{f_c}(n_{f_c}, N_{f_\phi}, N_m) &= \frac{\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c}^e(p, n_{f_c})}{n_{f_c} + N_{f_\phi} + N_m} - \frac{\Lambda_m}{\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c}^e(p, n_{f_c})} b \\ &= \frac{\mu^2 \left((n_{f_c} + N_{f_\phi} + N_m)(R - p) - N_m a\right)}{c + \mu(n_{f_c} + N_{f_\phi} + N_m)(R - p) - \mu N_m a} - \frac{\Lambda_m \cdot b}{\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c}^e(p, n_{f_c})} \end{aligned} \tag{21}$$

Taking the first order derivative with respect to n_{f_c}, we get

$$\frac{\partial d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)}{\partial n_{f_c}} = \frac{\mu^2 c(R-p)}{(c + \mu n(R-p) - \mu N_m a)^2} + \frac{\Lambda_m b \frac{c(2n(R-p) - N_m a) + \mu(n(R-p) - N_m a)^2}{(c + \mu(R-p) - \mu N_m a)^2}}{\left(\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c}^e(p, n_{f_c})\right)^2} > 0$$

due to $p < R$ (otherwise, no rider is willing to join), where $n = n_{f_c} + N_{f_\phi} + N_m$. That is, the safety-concern-adjusted demand rate $d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)$ is increasing in the number of participating safety-concerned female drivers n_{f_c}.

Recall that a safety-concerned female driver is willing to participate if and only if her net utility given in (6),

$$S_{f_c}(\lambda^e_{f_c}(p, n_{f_c}), \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = \frac{\Lambda_m + \lambda^e_{f_c}(p, n_{f_c}) + \Lambda_{f_\phi}}{n_{f_c} + N_{f_\phi} + N_m} w - r - \frac{\Lambda_m b}{\Lambda_m + \lambda^e_{f_c}(p, n_{f_c}) + \Lambda_{f_\phi}} \geq 0.$$

And we just show that $d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)$ increases in n_{f_c}. Following the same logic stated in the Lemma 1 of Taylor (2018), we can get the following result:

$$n^e_{f_c} = \begin{cases} N_{f_c}, & \text{if and only if} \quad w \geq \frac{\left(r + \frac{\Lambda_m b}{\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})}\right)(N_m + N_{f_\phi} + N_{f_c})}{\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})}, \\ 0, & \text{otherwise.} \end{cases}$$

Note that when $n^e_{f_c} = 0$, no safety-concerned female drivers participate in the system. Implication B.2 then implies that under this situation, no safety-concerned female riders join the system. Thus, to ensure the joining of safety-concerned female riders, the platform shall set the wage $w \geq \frac{\left(r + \frac{\Lambda_m b}{\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})}\right)(N_m + N_{f_\phi} + N_{f_c})}{\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})}$, under which $n^e_{f_c} = N_{f_c}$. Plugging $n^e_{f_c} = N_{f_c}$ into inequality (20), we then have that $p \leq R - \frac{c\Lambda_\phi}{\mu N(\mu N - \Lambda_\phi)} - \frac{N_m a}{N}$ is required. Under such a situation, all drivers participate in the service, i.e., $n^e_i = N_i$, $i \in \{f_c, f_\phi, m\}$. The corresponding equilibrium effective joining rate of safety-concerned female riders for any given price p is $\lambda^e_{f_c}(p, N_{f_c}) = \frac{\mu^2 N^2 (R - p - \frac{N_m a}{N})}{c + \mu N (R - p - \frac{a N_m}{N})} - \Lambda_\phi$. Thus, $w \geq \frac{\left(r + \frac{\Lambda_m b}{\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})}\right)(N_m + N_{f_\phi} + N_{f_c})}{\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})} = \frac{rN + \Lambda_m b(c + \mu(N(R-p) - N_m a))/(\mu^2(N(R-p) - N_m a))}{(\mu^2 N((R-p)N - N_m a))/(c + \mu((R-p)N - N_m a))}$ is required. □

We now examine the platform's optimal pricing and wage decisions with an aim to maximize its profitability, subject to the constraints $p \leq \bar{p}_1 := R - \frac{c\Lambda_\phi}{\mu N(\mu N - \Lambda_\phi)} - \frac{N_m a}{N}$ and the wage $w \geq \underline{w}_1(p) := \frac{rN + \Lambda_m b(c + \mu(N(R-p) - N_m a))/(\mu^2(N(R-p) - N_m a))}{(\mu^2 N((R-p)N - N_m a))/(c + \mu((R-p)N - N_m a))}$ (which ensures the joining of safety-concerned female riders in the system). Note that $\underline{w}_1(p)$ is the required minimum wage for any given price $p \in (0, \bar{p}_1)$. Clearly, there is no incentive for the platform to offer a wage that is above $\underline{w}_1(p)$. Hence, for a given price p, a rational platform shall set

$$w(p) = \underline{w}_1(p) = \frac{rN + \Lambda_m b\left(c + \mu(N(R-p) - N_m a)\right) / (\mu^2(N(R-p) - N_m a))}{(\mu^2 N\left((R-p)N - N_m a\right)) / \left(c + \mu\left((R-p)N - N_m a\right)\right)}. \tag{22}$$

Combining this along with the result stated in Proposition B.1, we can formulate the platform's optimization problem as follows:

$$\Pi^* = \max_{\underline{w}_1(p) < p < \bar{p}_1} \Pi(p) = \max_{\underline{w}_1(p) < p < \bar{p}_1} (p - \underline{w}_1(p))(\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})),$$

where $\lambda^e_{f_c}(p, N_{f_c}) = \frac{\mu^2 N^2 (R-p-\frac{N_m a}{N})}{c+\mu N(R-p-\frac{aN_m}{N})} - \Lambda_\phi$. Note that $\lambda^e_{f_c}(p, N_{f_c}) = \frac{\mu^2 N^2 (R-p-\frac{N_m a}{N})}{c+\mu N(R-p-\frac{aN_m}{N})} - \Lambda_\phi$ is equivalent to $p = R - \frac{N_m a}{N} - \frac{c\lambda}{\mu N(\mu N - \lambda)}$, where $\lambda = \Lambda_\phi + \lambda_{f_c}$[6]. Then the above optimization problem regarding the price can be rewritten as a problem regarding the effective arrival rate λ, which is as follows:

$$\Pi^* = \max_{\underline{w}_1(\lambda)<p(\lambda)<\bar{p}_1;\lambda<\Lambda} \Pi(\lambda) = \max_{\underline{w}_1(\lambda)<p(\lambda)<\bar{p}_1;\lambda<\Lambda} (p(\lambda) - \underline{w}_1(\lambda))\lambda,$$

where $p(\lambda) = R - \frac{N_m a}{N} - \frac{c\lambda}{\mu N(\mu N-\lambda)}$ and $\underline{w}_1(\lambda) = \frac{(r+\Lambda_m b/\lambda)N}{\lambda}$.

Proposition B.2. *In a pooling system, the platform's profit function $\Pi(\lambda)$ is concave in total effective arrival rate λ. Let λ^* be the solution of the first-order condition*

$$\frac{d\Pi(\lambda)}{d\lambda} = R - \frac{N_m a}{N} - \frac{c\lambda(2\mu N - \lambda)}{\mu N(\mu N - \lambda)^2} + \frac{b\Lambda_m N}{\lambda^2} = 0.$$

Then, λ^* is an interior optimal solution if and only if

$$\frac{d\Pi(\lambda)}{d\lambda} |_{\lambda\to\Lambda_\phi} = \eta(a, b) > 0$$

where the detailed expressions of $\eta(a, b)$ is provided in (26) in the following proof. The optimal effective arrival rate of safety-concerned female riders $\lambda^*_{f_c} = \lambda^* - \Lambda_\phi$.

Proof of Proposition B.2. Plugging $p(\lambda) = R - \frac{N_m a}{N} - \frac{c\lambda}{\mu N(\mu N-\lambda)}$ and $\underline{w}_1(\lambda) = \frac{(r+\Lambda_m b/\lambda)N}{\lambda}$ into $\Pi(\lambda) = (p(\lambda) - \underline{w}_1(\lambda))\lambda$, we get

$$\Pi(\lambda) = \left(R - \frac{N_m a}{N} - \frac{c\lambda}{\mu N(\mu N - \lambda)}\right)\lambda - \left(r + \frac{\Lambda_m b}{\lambda}\right)N. \tag{23}$$

Then we can derive that

$$\frac{d\Pi(\lambda)}{d\lambda} = R - \frac{N_m a}{N} - \frac{c\lambda(2\mu N - \lambda)}{\mu N(\mu N - \lambda)^2} + \frac{b\Lambda_m N}{\lambda^2}, \tag{24}$$

and

$$\frac{d^2\Pi(\lambda)}{d\lambda^2} = -\frac{2c\mu N}{(\mu N - \lambda)^3} - \frac{2b\Lambda_m N}{\lambda^3} < 0, \tag{25}$$

where $\mu N > \lambda$ must hold in a steady state. That is, $\Pi(\lambda)$ is concave in λ. Recall that $\lambda = \Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c} = \Lambda_\phi + \lambda_{f_c}$ based on the Proposition B.1, therefore, there must have an interior optimal solution in the range (Λ_ϕ, Λ) if and only if [7]

$$\frac{d\Pi(\lambda)}{d\lambda} |_{\lambda\to\Lambda_\phi} > 0.$$

[6]Here, with a little abuse of notations, we use λ_{f_c} and $\lambda^e_{f_c}$ interchangeably.

[7]Here, note that when $\lambda \to \Lambda$, it must have $\Pi(\lambda) \to -\infty$ according to (1).

For ease of notation, let

$$\eta(a,b) = \frac{d\Pi(\lambda)}{d\lambda} \,|_{\lambda\to\Lambda_\phi} = R - \frac{N_m a}{N} - \frac{c\Lambda_\phi(2\mu N - \Lambda_\phi)}{\mu N(\mu N - \Lambda_\phi)^2} + \frac{b\Lambda_m N}{\Lambda_\phi}. \tag{26}$$

Moreover, based on above equation, we have

$$\frac{\partial\eta(a,b)}{\partial a} = -\frac{N_m}{N} < 0 \text{ and } \frac{\partial\eta(a,b)}{\partial b} = \frac{\Lambda_m N}{\Lambda_\phi} > 0.$$

That is, $\eta(a,b)$ decreases in a but increases in b. We can construct ranges of a and b under which the interior optimal solution exists by applying this property of $\eta(a,b)$. □

We can then derive the optimal price $p^* = R - \frac{N_m a}{N} - \frac{c\lambda^*}{\mu N(\mu N - \lambda^*)}$ and optimal wage $w^* = \frac{(r+\Lambda_m b/\lambda^*)N}{\lambda^*}$. The corresponding optimal profit $\Pi^* = (p^* - w^*)\lambda^*$.

B.2 When the number of safety-unconcerned drivers is large: $N_\phi\mu > \Lambda_\phi$

We now analyze the case when $N_\phi\mu > \Lambda_\phi$. Similar to the previous subsection, we first characterize the joining behaviors of drivers and riders for the given price and wage. We then analyze the platforms's optimal price and wage decisions. Again, we shall focus on the equilibrium outcome in which some safety-concerned female riders join the system. We now develop the conditions under which this equilibrium exists in the following proposition.

Proposition B.3. *When $N_\phi\mu > \Lambda_\phi$, the safety-concerned female riders join the system at a non-zero rate in equilibrium under the following two cases:*

1. ***(Case $\mathcal{P}1$):*** *the platform sets the price $p \le \bar{p}_1 := R - \frac{c\Lambda_\phi}{\mu N(\mu N - \Lambda_\phi)} - \frac{N_m a}{N}$ and the wage $w \ge \underline{w}_1(p) := \frac{rN + \Lambda_m b(c+\mu(N(R-p)-N_m a))/(\mu^2(N(R-p)-N_m a))}{(\mu^2 N((R-p)N - N_m a))/(c+\mu((R-p)N-N_m a))}$, under which the equilibrium outcome stated in Proposition B.1 is the equilibrium outcome here.*

2. ***(Case $\mathcal{P}2$):*** *the platform sets the price $p \le \bar{p}_2 := R - \frac{c\Lambda_\phi}{\mu N(\mu N - \Lambda_\phi)} - \frac{N_m a}{N_\phi}$ and the wage $w \ge \underline{w}_2(p) := \frac{r\left(c+\mu\left(N_\phi(R-p)-N_m a\right)\right)}{\mu^2\left(N_\phi(R-p)-N_m a\right)}$. Then, in equilibrium, all the safety-unconcerned drivers participate in the system but all the safety-concerned female drivers balk, i.e., $n_m^e = N_m$, $n_{f_\phi}^e = N_{f_\phi}$ and $n_{f_c}^e = 0$. All the safety-unconcerned riders join the system so that $\lambda_m^e = \Lambda_m$ and $\lambda_{f_\phi}^e = \Lambda_{f_\phi}$. Some safety-concerned female riders join the system and the others balk with an effective joining rate $\lambda_{f_c}^e(p,0) = \frac{\mu^2 N_\phi^2(R-p-N_m a/N_\phi)}{c+\mu N_\phi(R-p-N_m a/N_\phi)} - \Lambda_\phi$.*

Proof of Proposition B.3. Here, we adopt the same logic of proof as that for the proof of Proposition B.1. Again, we only focus on the cases where safety-concerned female riders join at a non-zero rate. Then, it must be the case that all the safety-unconcerned riders have joined, that is, $\lambda_m^e = \Lambda_m$ and $\lambda_{f\phi}^e = \Lambda_{f_\phi}$. Since $N_\phi \mu > \Lambda_\phi$, it is possible that no safety-concerned female drivers participate to work in equilibrium. According to the proof of Proposition B.1, we know that if the platform sets a wage

$$w \geq \underline{w}_1(p) = \frac{\left(r + \frac{\Lambda_m b}{\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c}^e(p, N_{f_c})}\right)(N_m + N_{f_\phi} + N_{f_c})}{\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c}^e(p, N_{f_c})},$$

then all the registered drivers would participate to work. Similarly, following the proof of Proposition B.1, we can show that if the platform sets a wage

$$w \geq \underline{w}_2(p) := \frac{r(N_m + N_{f_\phi})}{\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c}^e(p, 0)},$$

where $\lambda_{f_c}^e(p, 0) = \frac{\mu^2 N_\phi^2 (R - p - N_m a / N_\phi)}{c + \mu N_\phi (R - p - N_m a / N_\phi)} - \Lambda_\phi$, then all the safety-unconcerned drivers would participate to work.

If $\underline{w}_2(p) \geq \underline{w}_1(p)$, the platform has no incentives to set $w = \underline{w}_2(p)$ because setting this higher wage can only attract a fraction of drivers to participate in the system. If $\underline{w}_2(p) < \underline{w}_1(p)$, then we get the following result, which has the similar structure with that shown in the proof of Proposition B.1:

$$(n_m^e, n_{f_\phi}^e, n_{f_c}^e) = \begin{cases} (N_m, N_{f_\phi}, N_{f_c}), & \text{if and only if} \quad w \geq \underline{w}_1(p); \\ (N_m, N_{f_\phi}, 0), & \text{if and only if} \quad \underline{w}_1(p) > w \geq \underline{w}_2(p); \\ (0, 0, 0), & \text{if and only if} \quad w < \underline{w}_2(p). \end{cases}$$

When $w \geq \underline{w}_1(p)$, the equilibrium outcome of case $\mathcal{P}1$ can be shown to be exactly the same as that stated in Proposition B.1. When $\underline{w}_1(p) > w \geq \underline{w}_2(p)$, by adopting the same logic of proof for case $\mathcal{P}1$ stated in Proposition B.1, we can easily obtain the equilibrium outcome for case $\mathcal{P}2$. □

Proposition B.3 indicates that there may exist two equilibrium outcomes which differ from each other regarding the participating behaviors of safety-concerned female drivers when the given price and wage satisfy both conditions stated in cases $\mathcal{P}1$ and $\mathcal{P}2$. Under such a situation, we follow Taylor (2018) and assume that all the parties (riders, drivers and the platform) work together to coordinate on the equilibrium that has most drivers participating in the system.

We now proceed to analyze the platform's pricing and wage decision. Note that the platform's optimization problem under case $\mathcal{P}1$ is exactly the same as that presented in §B.1. Thus, all the analysis and results stated in Proposition B.2 hold. Let Π_1^* denote the optimal profit under case $\mathcal{P}1$ and (p_1^*, w_1^*) the associated optimal price and wage.

As to case $\mathcal{P}2$, when the price and wage satisfy its conditions, for a given price p, a rational platform shall set

$$w(p) = \underline{w}_2(p) = \frac{r\left(c + \mu\left(N_\phi(R-p) - N_m a\right)\right)}{\mu^2\left(N_\phi(R-p) - N_m a\right)},$$

as increasing the wage above $\underline{w}_2(p)$ has no impact on the joining behaviors of drivers. Then, the platform's optimization problem under case $\mathcal{P}2$ can be formulated as

$$\Pi_2^* = \max_{\underline{w}_2(p)<p<\bar{p}_2} \Pi(p) = \max_{\underline{w}_2(p)<p<\bar{p}_2} (p - \underline{w}_2(p))(\Lambda_\phi + \lambda_{fc}^e(p,0)),$$

where $\lambda_{fc}^e(p,0) = \frac{\mu^2 N_\phi^2(R-p-N_m a/N_\phi)}{c+\mu N_\phi(R-p-N_m a/N_\phi)} - \Lambda_\phi$. Substituting $\underline{w}_2(p)$ and $\lambda_{fc}^e(p,0)$ into $\Pi(p)$, we can derive that

$$\Pi(p) = p \cdot \frac{\mu^2 N_\phi^2(R-p-N_m a/N_\phi)}{c+\mu N_\phi(R-p-N_m a/N_\phi)} - rN_\phi.$$

We can show that

$$\frac{d\Pi(p)}{dp} = \frac{\mu^2 N_\phi^2\left(c(R-2p-\frac{N_m a}{N_\phi}) + \mu N_\phi(R-p-\frac{N_m a}{N_\phi})^2\right)}{\left(c+\mu N_\phi(R-p-\frac{N_m a}{N_\phi})\right)^2}, \text{ and}$$

$$\frac{d^2\Pi(p)}{dp^2} = \frac{-2\mu^2 N_\phi^2\left(c^2 + c\mu N_\phi(R-\frac{N_m a}{N_\phi})\right)}{\left(c+\mu N_\phi(R-p-\frac{N_m a}{N_\phi})\right)^2} < 0$$

Thus, $\Pi(p)$ is concave in p. Denote $(p_2^*, w_2^* = \underline{w}_2(p_2^*))$ as the corresponding optimal price and wage under this case. Then, the optimal p_2^* shall be the solution of $\frac{d\Pi(p)}{dp} = 0$. It can be shown that $p_2^* = R - \frac{N_m a}{N_\phi} + \frac{c}{\mu N_\phi} - \sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}$. Correspondingly, $w_2^* = \frac{rN_\phi}{\mu N_\phi - c/\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R-\frac{N_m a}{N_\phi}\right)}}$ and $\lambda_{fc}^e(p_2^*,0) = \mu N_\phi - c/\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R-\frac{N_m a}{N_\phi}\right)} - \Lambda_\phi$. Besides, one can easily show that $\Pi_2^* = p_2^*\lambda_{fc}^e(p_2^*,0) - rN_\phi$ is decreasing in a as $\lambda_{fc}^e(p_2^*,0)$ is obviously decreasing in a and $\frac{dp_2^*}{da} = \frac{N_m}{N_\phi} \cdot \frac{\frac{c}{\mu N_\phi} - 2\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R-\frac{N_m a}{N_\phi}\right)}}{2\sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R-\frac{N_m a}{N_\phi}\right)}} < 0$.

The platform then compares its profits under the two cases, case $\mathcal{P}1$ and case $\mathcal{P}2$ and chooses the one that has a higher profit. That is, the optimal profit of the platform is $\Pi^* = \max\{\Pi_1^*, \Pi_2^*\}$. Π_1^* is decreasing in b (stated in Proposition 2) and one can easily check that Π_2^* is independent of b. Let $\widehat{b}(a)$ is the unique solution of

$$(\Pi_1^* - \Pi_2^*)\big|_{b=\widehat{b}(a)} = 0, \tag{27}$$

if it exists. If $(\Pi_1^* - \Pi_2^*)\,|_{b\to 0} < 0$, we let $\widehat{b}(a) = 0$. We then have the following result.

Proposition B.4. *In a pooling system, when $N_\phi \mu > \Lambda_\phi$, there exists a threshold value $\widehat{b}(a)$ such that if the safety-concerned female drivers' mismatch cost $b < \widehat{b}(a)$, the platform sets the optimal price and wage $(p^*, w^*) = (p_1^*, w_1^*)$, the one characterized by Proposition B.2 and equation (22). Otherwise, the platform sets $(p^*, w^*) = (p_2^*, w_2^*)$, where $p_2^* = R - \frac{N_m a}{N_\phi} + \frac{c}{\mu N_\phi} - \sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}$ and $w_2^* = \frac{r N_\phi}{\mu N_\phi - c \Big/ \sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}}$.*

For ease of reference, we summarize the platform's optimal price and wage decisions and the corresponding equilibrium user joining behaviors in a pooling system in Table B.1.

Table B.1: Equilibrium Price, Wage and User Joining Behaviors in a Pooling System

<table>
<tr><th colspan="2">Market Condition / Players' Decision</th><th>$N_\phi \mu \leq \Lambda_\phi$</th><th>$N_\phi \mu > \Lambda_\phi$, $b < \widehat{b}(a)$</th><th>$N_\phi \mu > \Lambda_\phi$, $b \geq \widehat{b}(a)$</th></tr>
<tr><td rowspan="2">Platform</td><td>optimal price</td><td>p_1^*</td><td>p_1^*</td><td>p_2^*</td></tr>
<tr><td>optimal wage</td><td>w_1^*</td><td>w_1^*</td><td>w_2^*</td></tr>
<tr><td rowspan="3">Riders</td><td>male</td><td>$\lambda_m^* = \Lambda_m$</td><td>$\lambda_m^* = \Lambda_m$</td><td>$\lambda_m^* = \Lambda_m$</td></tr>
<tr><td>safety-unconcerned female</td><td>$\lambda_{f_\phi}^* = \Lambda_{f_\phi}$</td><td>$\lambda_{f_\phi}^* = \Lambda_{f_\phi}$</td><td>$\lambda_{f_\phi}^* = \Lambda_{f_\phi}$</td></tr>
<tr><td>safety-concerned female</td><td colspan="2">$\lambda_{f_c}^* = \frac{\mu^2 N^2 (R - p_1^* - \frac{N_m a}{N})}{c + \mu N (R - p_1^* - \frac{a N_m}{N})} - \Lambda_\phi$</td><td>$\lambda_{f_c}^* = \lambda_{f_c}^e(p_2^*)$</td></tr>
<tr><td rowspan="3">Drivers</td><td>male</td><td>$n_m^* = N_m$</td><td>$n_m^* = N_m$</td><td>$n_m^* = N_m$</td></tr>
<tr><td>safety-unconcerned female</td><td>$n_{f_\phi}^* = N_{f_\phi}$</td><td>$n_{f_\phi}^* = N_{f_\phi}$</td><td>$n_{f_\phi}^* = N_{f_\phi}$</td></tr>
<tr><td>safety-concerned female</td><td>$n_{f_c}^* = N_{f_c}$</td><td>$n_{f_c}^* = N_{f_c}$</td><td>$n_{f_c}^* = 0$</td></tr>
<tr><td colspan="5">Remarks: (p_1^*, w_1^*) is the price and wage that characterized by Proposition B.2 and equation (22), respectively. $p_2^* = R - \frac{N_m a}{N_\phi} + \frac{c}{\mu N_\phi} - \sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}$, $w_2^* = \frac{r N_\phi}{\mu N_\phi - c \Big/ \sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)}}$, $\lambda_{f_c}^e(p_2^*) = \mu N_\phi - c \Big/ \sqrt{\left(\frac{c}{\mu N_\phi}\right)^2 + \frac{c}{\mu N_\phi}\left(R - \frac{N_m a}{N_\phi}\right)} - \Lambda_\phi$.</td></tr>
</table>

Appendix C The Hybrid System: Detailed Analyses

In a hybrid system, the male drivers and the male riders are only allowed to join male-driver subsystem while the female drivers are allowed to join female-driver subsystem but female riders can join both subsystems. Similar to that in a pooling system, here we also conduct the analysis over the hybrid system by considering two exhaustive and exclusive scenarios, $N_m \mu \leq \Lambda_\phi$ and $N_m \mu > \Lambda_\phi$.

C.1 When the number of male drivers is low: $N_m\mu \le \Lambda_\phi$

In a hybrid system, we consider that the platform adopts the subsystem-based pricing and wage policy. That is, the price and wage in the male-driver subsystem can be different from that in the female-driver subsystem. First, given these two price and wage pairs in the two subsystems, we analyze the equilibrium joining and participating behaviors of riders and drivers. Again, we focus on the equilibrium outcome in which riders join the two subsystems at non-zero rates. We now develop the conditions under which such equilibrium will exist in the following proposition.

Proposition C.1. *In a hybrid system, when $N_m\mu \le \Lambda_\phi$, if the platform sets prices and wages satisfying $p_j < R$, $j = F, M$, $w_M \ge \underline{w}_M := \frac{r(c+N_m\mu(R-p_M))}{\mu^2 N_m(R-p_M)}$ and $w_F \ge \underline{w}_F := \frac{r(c+N_f\mu(R-p_F))}{\mu^2 N_f(R-p_F)}$, then in equilibrium,*
(a). All male drivers join the male-driver subsystem, i.e., $n_m^e = N_m$. All female drivers join the female-driver subsystem, i.e., $n_f^e = N_f$.
(b). Male riders join the male-driver subsystem with rate $\lambda_m^e \in (0, \Lambda_m)$. Safety-concerned female riders join the female-driver subsystem with rate $\lambda_{f_c,F}^e \in (0, \Lambda_{f_c})$. As to safety-unconcerned female riders, they join both subsystems with rates $\lambda_{f_\phi,j}^e \in (0, \Lambda_{f_\phi}), j = F, M$, respectively. Moreover, those equilibrium effective joining rates satisfy

$$\begin{cases} \lambda_m^e + \lambda_{f_\phi,M}^e = \frac{N_m^2\mu^2(R-p_M)}{c+N_m\mu(R-p_M)}, \\ \lambda_{f_c,F}^e + \lambda_{f_\phi,F}^e = \frac{N_f^2\mu^2(R-p_F)}{c+N_f\mu(R-p_F)}. \end{cases} \tag{28}$$

Proof of Proposition C.1. We first prove that in equilibrium, no safety-concerned female riders join the male-driver subsystem, that is, $\lambda_{f_c,M}^e = 0$. We show this by contradiction. Assume that $\lambda_{f_c,M}^e > 0$. In the male-driver subsystem, by comparing the safety-unconcerned riders' joining utility stated in (10) with that of safety-concerned female riders stated in (11), we obtain that $U_{m,M} = U_{f_\phi,M} \ge U_{f_c,M}$. That is, once the safety-concerned female riders join the male-driver subsystem at a non-zero rate, it must be the case that all the safety-unconcerned riders have joined the male-driver subsystem so that $\lambda_m^e = \Lambda_m$ and $\lambda_{f_\phi,M}^e = \Lambda_{f_\phi}$. In this situation, even though all the N_m male drives participate to work in the male-driver subsystem, we have

$$N_m\mu \le \Lambda_\phi < \Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c,M}^e.$$

That is, the male-driver subsystem is not steady when $\lambda_{f_c,M}^e > 0$. Thus, it must be that $\lambda_{f_c,M}^e = 0$. Next, we analyze the joining and participating behaviors of riders and drivers in the two subsystems.

We begin with the male-driver subsystem. To ensure that at least one safety-unconcerned rider is willing to join the male-driver subsystem, we should have $p_M < R$. Since $N_m\mu \le \Lambda_\phi$, some safety-unconcerned riders must balk the system. Given that there are n_m male drivers participating in the male-driver subsystem, where $n_m \leq N_m$, the effective joining rates of male riders and safety-unconcerned female riders, λ_m^e and $\lambda_{f_\phi,M}^e$ can be obtained by solving

$$U_{i,M}(0, \lambda_{f_\phi,M}, \lambda_m; n_m) = R - p_M - \frac{c\left(\lambda_{f_\phi,M} + \lambda_m\right)}{n_m\mu(n_m\mu - \lambda_{f_\phi,M} - \lambda_m)} = 0, i = m, f_\phi.$$

It can be shown that there exist multiple solutions as long as in the male-driver subsystem,

$$\lambda_M^e(p_M, n_m) := \lambda_m^e(p_M, n_m) + \lambda_{f_\phi,M}^e(p_M, n_m) = \frac{n_m^2\mu^2(R - p_M)}{c + n_m\mu(R - p_M)}.$$

Applying the same logic used in the proof of Proposition B.1, we can show that the average demand allocated to a single driver in this subsystem is

$$\frac{\lambda_M^e(p_M, n_m)}{n_m} = \frac{n_m\mu^2(R - p_M)}{c + n_m\mu(R - p_M)},$$

which can be proved increasing in n_m as we have $\frac{d\left(\frac{\lambda_M^e(p_M,n_m)}{n_m}\right)}{dn_m} = \frac{c\mu^2(R-p_M)}{(c+n_m\mu(R-p_M))^2} > 0$. Recall that a male driver in the male-driver subsystem is willing to participate if and only if his net utility given in (12),

$$S_{m,M}(0, \lambda_{f_\phi,M}^e, \lambda_m^e; n_m) = \frac{\lambda_{f_\phi,M}^e + \lambda_m^e}{n_m} w_M - r = \frac{\lambda_M^e(p_M, n_m)}{n_m} w_M - r \geq 0.$$

As $\frac{\lambda_M^e(p_M,n_m)}{n_q}$ is increasing in N_m, we get the following result:

$$n_m^e = \begin{cases} N_m, & \text{if and only if} \quad w_M \geq \frac{rN_m}{\lambda_M^e(p_M,N_m)}, \\ 0, & \text{otherwise.} \end{cases}$$

When $n_m^e = 0$, no drivers participate in the male-driver subsystem, and the hybrid system degenerates to a female-driver subsystem. This equilibrium outcome is trivial and uninteresting. Thus, below, we restrict our attention to the equilibrium outcome where $n_m^e = N_m$, under which all the male drivers participate to work in the male-driver subsystem. The corresponding total effective joining rate of safety-unconcerned riders is $\lambda_M^e(p_M, N_m) = \frac{N_m^2\mu^2(R-p_M)}{c+N_m\mu(R-p_M)}$, and the wage needs to satisfy $w_M \geq \underline{w}_M := \frac{rN_m}{\lambda_M^e(p_M,N_m)} = \frac{r(c+N_m\mu(R-p_M))}{\mu^2N_m(R-p_M)}$.

As for the female-driver subsystem, similarly, we can show that in equilibrium, all the registered female drives participate in the system. The price p_F should satisfy $p_F < R$ to ensure that there is at least one female rider joining the female-driver subsystem. Correspondingly, the effective joining rate in the female-driver subsystem is $\lambda_F^e(p_F, N_f) := \lambda_{f_\phi,F}^e(p_F, N_f) + \lambda_{f_c,F}^e(p_F, N_f) = \frac{N_f^2\mu^2(R-p_F)}{c+N_f\mu(R-p_F)}$, and the wage is required to be $w_F \geq \underline{w}_F := \frac{r(c+N_f\mu(R-p_F))}{\mu^2N_f(R-p_F)}$. □

Next, we consider the platform's pricing and wage decisions. For the sake of notation simplicity, hereafter we suppress $\lambda^e_M(p_M, N_m)$ and $\lambda^e_F(p_F, N_f)$ as $\lambda^e_j(p_j)$, $j = M, F$. Note that in each subsystem j, $j = F, M$, for any given p_j, the platform has no incentive to offer a wage above $\underline{w}_j$. Hence, it is optimal for the platform to set

$$w_j(p_j) = \underline{w}_j(p_j), j = F, M. \tag{29}$$

Thus, in a hybrid system, the platform's optimization problem becomes

$$\begin{aligned}\widetilde{\Pi}^*_0 &= \max_{\underline{w}_j(p_j)<p_j, j\in\{F,M\}} \sum_{j\in\{F,M\}} (p_j - \underline{w}_j(p_j))\lambda^e_j(p_j) \\ &= (p_M - \underline{w}_M(p))\frac{N_m^2\mu^2(R-p_M)}{c+N_m\mu(R-p_M)} + (p_F - \underline{w}_F(p))\frac{N_f^2\mu^2(R-p_F)}{c+N_f\mu(R-p_F)} \\ &= \max_{\underline{w}_M(p_M)<p_M} \Pi_M(p_M) + \max_{\underline{w}_F(p_F)<p_F} \Pi_F(p_F),\end{aligned}$$

where $\Pi_j(p_j)$ is the subsystem j's profit function, $j = M, F$. This indicates that optimizing the total system profit can be derived by optimizing each subsystem's profit individually. Let $\widetilde{p}^*_{j0}$, $j = F, M$, be the optimal price in the subsystem j. Then, in the hybrid system the platform's optimal profit is

$$\widetilde{\Pi}^*_0 = \Pi_M(\widetilde{p}^*_{M_0}) + \Pi_F(\widetilde{p}^*_{F_0}).$$

Proposition C.2. *In a hybrid system, when $N_m\mu \le \Lambda_\phi$, the platform sets the optimal prices and wages as follows:*

$$\begin{cases} (\widetilde{p}^*_{M_0}, \widetilde{w}^*_{M_0}) = \left(R + \frac{c}{N_m\mu} - \sqrt{\frac{cR}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}, \frac{rN_m}{\mu N_m - c\Big/\sqrt{\frac{cR}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2}}\right), \\ (\widetilde{p}^*_{F_0}, \widetilde{w}^*_{F_0}) = \left(R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}, \frac{rN_f}{\mu N_f - c\Big/\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}}\right). \end{cases}$$

Moreover, $\widetilde{p}^*_{F_0} < \widetilde{p}^*_{M_0}$.

Proof of Proposition C.2. Recall that we can derive the optimal price for each subsystem individually. First, in the male-driver subsystem, the platform sets the price p_M to maximize its profit as follows:

$$\begin{aligned}\max_{\underline{w}_M(p_M)<p_M} \Pi_M(p_M) &= (p_M - \underline{w}_M(p_M))\frac{N_m^2\mu^2(R-p_M)}{c+N_m\mu(R-p_M)} \\ &= p_M\frac{N_m^2\mu^2(R-p_M)}{c+N_m\mu(R-p_M)} - rN_m.\end{aligned}$$

It can be easily shown that $\Pi_M(p_M)$ is concave in p_M as $\frac{d^2\Pi_M(p_M)}{dp_M^2} = \frac{-2(\mu N_m)^2(cp_M\mu N_m + c(c+\mu N_m(R-p_M)))}{(c+N_m\mu(R-p_M))^3} < 0$. Then, based on the first-order condition

$$\frac{d\Pi_M(p_M)}{dp_M} = (\mu N_m)^2 \frac{c(R-2p_M) + N_m\mu(R-p_M)^2}{(c+N_m\mu(R-p_M))^2} = 0,$$

we obtain the optimal price $\widetilde{p}^*_{M_0} = R + \frac{c}{N_m\mu} - \sqrt{\frac{cR}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}$, which is obviously smaller than R. Correspondingly, the effective arrival rate $\lambda^e_M(\widetilde{p}^*_M) = \mu N_m - \frac{c}{\sqrt{\frac{cR}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2}}$ and the optimal wage $\widetilde{w}^*_{M_0} = \frac{rN_m}{\lambda^e_M(\widetilde{p}^*_M)} = \frac{rN_m}{\mu N_m - c\Big/\sqrt{\frac{cR}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2}}$. The optimal profit of the male-driver subsystem is thus

$$\widetilde{\Pi}^*_{M_0} = \Pi_M(\widetilde{p}^*_{M_0}) = R\mu N_m + 2c - (R + \frac{c}{\mu N_m})c - \sqrt{c^2 + c\mu N_m R} - rN_m.$$

In the female-driver subsystem, the optimal price $\widetilde{p}^*_{F_0}$, the optimal wage $\widetilde{w}^*_{F_0}$ and the corresponding optimal profit $\widetilde{\Pi}^*_{F_0}$ can be derived similarly. We omit the detail here.

Lastly, we compare $\widetilde{p}^*_{F_0}$ and $\widetilde{p}^*_{M_0}$. Define $f(x) = R + \frac{c}{x\mu} - \sqrt{\frac{cR}{x\mu} + \left(\frac{c}{x\mu}\right)^2}$. It can be easily shown that

$$\frac{df(x)}{dx} = -\frac{c}{\mu x^2} - \frac{-\frac{cR}{\mu}\cdot\frac{1}{x^2} + \frac{2c}{x\mu}\cdot\frac{c}{\mu}\cdot\frac{-1}{x^2}}{2\sqrt{\frac{cR}{x\mu}+\left(\frac{c}{x\mu}\right)^2}} = \frac{c}{\mu x^2}\cdot\frac{R + \frac{2c}{\mu x} - 2\sqrt{\frac{cR}{x\mu}+\left(\frac{c}{x\mu}\right)^2}}{2\sqrt{\frac{cR}{x\mu}+\left(\frac{c}{x\mu}\right)^2}} > 0$$

due to $\left(R + \frac{2c}{\mu x}\right)^2 - 4\left(\frac{cR}{x\mu} + \left(\frac{c}{x\mu}\right)^2\right) = R^2 > 0$. Since $N_m > N_f$, the property of $f(x)$ implies that $\widetilde{p}^*_{F_0} < \widetilde{p}^*_{M_0}$. □

C.2 When the number of male drivers is large: $N_m\mu > \Lambda_\phi$

When the number of male drivers is sufficiently large ($N_m\mu > \Lambda_\phi$), the joining and participating behaviors of riders and drivers are much more complicated. We still focus on the equilibrium outcome in which riders join the two subsystems at non-zero rates. We now develop the conditions under which such equilibrium will exist in the following proposition.

Proposition C.3. *In a hybrid system, when $N_m\mu \le \Lambda_\phi$, depending on the magnitude of prices and wages, we further have that*

1. ***(Case $\mathcal{H}1$)*** *when $p_M \in \Omega_1 := \left[R - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu-\Lambda_\phi)}, R\right)$, $w_M \ge \underline{w}_M(p)$, $p_F < R$ and $w_F \ge \underline{w}_F(p)$, in equilibrium, the joining and participating behaviors of riders and drivers are exactly the same as those stated in Proposition C.1.*

2. ***(Case $\mathcal{H}2$)*** *when* $p_M \in \Omega_2 := \left(0, R - a - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu - \Lambda_\phi)}\right]$, $w_M \geq \frac{r(c+\mu N_m(R-p_M-a))}{\mu^2 N_m(R-p_M-a)}$, $p_F < R$ *and* $w_F \geq \underline{w}_F(p)$,
   - (a) *the drivers' equilibrium participating behaviors are exactly the same as those stated in Proposition C.1, that is,* $n_m^e = N_m$, $n_f^e = N_f$.
   - (b) Λ_m *male drivers and* Λ_{f_ϕ} *safety-unconcerned female riders all join the male-driver subsystem, i.e.,* $\lambda_m^e = \Lambda_m$ *and* $\lambda_{f_\phi,M}^e = \Lambda_{f_\phi}$. *Safety-concerned female riders join the male-driver subsystem with rate* $\lambda_{f_c,M}^e(p_M) = \frac{(\mu N_m)^2(R-p_M-a)}{c+\mu N_m(R-p_M-a)} - \Lambda_\phi$ *and join the female-driver subsystem with rate* $\lambda_{f_c,F}^e(p_F) = \frac{(\mu N_f)^2(R-p_F)}{c+\mu N_f(R-p_F)}$.

3. ***(Case $\mathcal{H}3$)*** *for any given* $p_j \geq w_j$, $j = F, M$, *the participating numbers and joining rates* $(\lambda_m^e = \Lambda_m, \lambda_{f_\phi,M}^e = \Lambda_{f_\phi}, \lambda_{f_c,M}^e = 0, \lambda_{f_c,F}^e; n_m^e, n_f^e = N_f)$ *are an equilibrium outcome if they satisfy the following set of conditions:*

$$\mathcal{H}3 \text{ Conditions}: \begin{cases} S_m = \frac{\Lambda_\phi}{n_m^e} w_M - r \geq 0, \\ S_f = \frac{\lambda_{f_c,F}^e}{N_f} w_F - r \geq 0, \\ U_m = U_{f_\phi,M} = R - p_M - \frac{c\Lambda_\phi}{\mu n_m^e(\mu n_m^e - \Lambda_\phi)} > 0, \\ U_{f_c,M} = R - a - p_M - \frac{c\Lambda_\phi}{\mu n_m^e(\mu n_m^e - \Lambda_\phi)} \leq 0, \\ U_{f_c,F} = R - p_F - \frac{c\lambda_{f_c,F}^e}{N_f\mu(N_f\mu - \lambda_{f_c,F}^e)} = 0. \end{cases}$$

Proof of Proposition C.3. As we focus on the cases where riders join the two subsystems at nonzero rates, we can further classify those cases according to the joining behaviors of female riders. Then, we have the following three cases.

One: safety-concerned female riders join both subsystems at non-zero rates. When safety-concerned female riders join the male-driver subsystem at a non-zero rate, by the same logic used in the proof of Proposition B.1, we know that it must be the case that all the safety-unconcerned riders have joined the male-driver subsystem. That is, $\lambda_m^e = \Lambda_m$ and $\lambda_{f_\phi,M}^e = \Lambda_{f_\phi}$. To ensure that at least one safety-concerned female rider is willing to join the male-driver subsystem, we need to require that her utility of joining is non-negative when all the N_m possible drivers have participated in the service, i.e.,

$$U_{f_c,M}(0, \Lambda_{f_\phi}, \Lambda_m; N_m) = R - p_M - cW(0, \Lambda_{f_\phi}, \Lambda_m; N_m) - a \geq 0.$$

This requires that $p_M \leq R - a - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu - \Lambda_\phi)}$.

Due to the limited supply of drives in the male-driver subsystem ($\mu N_m < \Lambda_\phi < \Lambda_\phi + \Lambda_{f_c}$), it is impossible that all the safety-concerned female riders join the male-driver subsystem in the steady state. Given the number of participating drivers n_m in the male-driver subsystem, where $n_m \le N_m$, the equilibrium effective joining rate of safety-concerned female riders can be obtained by solving

$$U_{f_c,M}(\lambda^e_{f_c,M}, \Lambda_{f_\phi}, \Lambda_m; n_m) = R - a - p_M - c\frac{\Lambda_\phi + \lambda^e_{f_c,M}}{n_m\mu\left(n_m\mu - \Lambda_\phi - \lambda^e_{f_c,M}\right)} = 0.$$

It can be shown that in the male-driver subsystem, the equilibrium total effective joining rate from all riders is

$$\lambda^e_M(p_M, n_m) = \Lambda_\phi + \lambda^e_{f_c,M}(p_M, n_m) = \frac{(\mu n_m)^2(R - p_M - a)}{c + \mu n_m(R - p_M - a)}.$$

Then, we can show that the average demand allocated to a single driver in the male-driver subsystem,

$$\frac{\lambda^e_M(p_M, n_m)}{n_m} = \frac{n_m\mu^2(R - p_M - a)}{c + \mu n_m(R - p_M - a)}.$$

is increasing n_m as we can show that $\frac{d\left(\frac{\lambda^e_M(p_M,n_m)}{n_m}\right)}{dn_m} = \frac{c\mu^2(R-p_M-a)}{(c+\mu n_m(R-p_M-a))^2} > 0$. Similar to the proof used in Proposition C.1, we can conclude that when $w_M \ge \frac{rN_m}{\lambda^e_M(p_M,N_m)} = \frac{r(c+\mu N_m(R-p_M-a))}{\mu^2 N_m(R-p_M-a)}$, all the N_m drivers participate to work in the male-driver subsystem. Correspondingly, $\lambda^e_{f_c,M}(p_M) = \frac{(\mu N_m)^2(R-p_M-a)}{c+\mu N_m(R-p_M-a)} - \Lambda_\phi$. Due to the constrained supply and overwhelming demand, all the related analyses regarding the female-driver system in the proof of Proposition C.1 can be applied here and it can be shown that $\lambda^e_{f_c,F}(p_F) = \frac{(\mu N_f)^2(R-p_F)}{c+\mu N_m(R-p_F)}$. This leads to the result stated in case $\mathcal{H}2$.

<u>Two: safety-concerned female riders only join the female-driver subsystem, and safety-unconcerned female riders join both subsystems.</u> To ensure that safety-unconcerned female riders join both subsystems, we should have

$$U_{f_\phi,M}(0, \Lambda_{f_\phi}, \Lambda_m; N_m) = R - p_M - cW(0, \Lambda_{f_\phi}, \Lambda_m; N_m) \le 0,$$

implying that if all the safety-unconcerned riders join the male-driver subsystem only, they receive a non-positive utility. Under this situation, $p_M \ge R - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu-\Lambda_\phi)}$ is required. As female riders' joining behaviors are exactly the same as those stated in Proposition C.1, the analyses in the proof of Proposition C.1 all hold here. This leads to the result summarized in case $\mathcal{H}1$.

<u>Three: safety-concerned female riders only join the female-driver subsystem, and safety-unconcerned female riders only join the male-driver subsystem.</u> Note that when safety-unconcerned female riders only join the male-driver subsystem, they shall receive a positive joining utility. The reason is that if in equilibrium, they receive a utility of zero, then some safety-unconcerned female riders is indifferent between joining the male-driver subsystem and balking. Also, note that due to the limited supply of female drivers, the female riders' joining

utility in the female-driver subsystem is also zero. Under such a case, the safety-unconcerned female riders shall be also indifferent between joining the male-driver subsystem and joining the female-driver subsystem. This indicates that if safety-unconcerned female riders only join the male-driver subsystem, their joining utility must be positive. Thus, safety-unconcerned female riders all join the male-driver subsystem, i.e., $\lambda^e_{f_\phi,M} = \Lambda_\phi$. Then, male riders shall all join the pooling system as well as they behave the same as the safety-unconcerned female riders, i.e., $\lambda^e_m = \Lambda_m$. Recall that under this case, $\lambda^e_{f_c,M} = 0$.

Next, we derive the conditions under which such an equilibrium exists. First, it requires the equilibrium participating number of drivers in the male-driver subsystem n^e_m satisfy

$$U_m = U_{f_\phi,M}(0, \Lambda_{f_\phi}, \Lambda_m; n^e_m) = R - p_M - \frac{c\Lambda_\phi}{\mu n^e_m(\mu n^e_m - \Lambda_\phi)} > 0;$$
$$U_{f_c,M}(0, \Lambda_{f_\phi}, \Lambda_m; n^e_m) = R - a - p_M - \frac{c\Lambda_\phi}{\mu n^e_m(\mu n^e_m - \Lambda_\phi)} \leq 0,$$

where the first inequality ensures the joining utility of safety-unconcerned riders is positive and the second utility guarantees that no safety-concerned female rider has incentive to join the male-driver subsystem. As to the driver side, it is required that

$$S_m = \frac{\Lambda_\phi}{n^e_m} w_M - r \geq 0.$$

Note that the demand rate per each driver, $\frac{\Lambda_\phi}{n^e_m}$, now decreases as the participating number of drivers increases. Put it differently, the participation of an additional driver hurts all the existing drivers in the system. In this situation, there is a one-to-one mapping between w_M and n^e_m. The higher the wage, the larger the participating number of drivers in the male-driver subsystem.

Regarding the female-driver subsystem, there are at most N_f female drivers participating in the female-driver subsystem. Due to the limited supply of female drivers and abundant female riders, in equilibrium, all the N_f drivers shall participate in the service; that is, $n^e_f = N_f$. As to the safety-concerned female riders, below we prove that not all of them join the female-driver subsystem. Suppose that all safety-concerned female riders join the female-driver subsystem, that is, $\lambda^e_{f_c,F} = \Lambda_{f_c}$. In the steady state, to ensure the stability of the queueing system, we must have $\mu N_f > \Lambda_{f_c}$ (the female-driver subsystem) and that $\mu n^e_m > \Lambda_m + \Lambda_{f_\phi}$ (the male-driver subsystem). This implies that $\mu(N_f + n^e_m) > \Lambda_m + \Lambda_{f_\phi} + \Lambda_{f_c} = \Lambda$, which contradicts our assumption that $\mu N < \Lambda$. Hence, it is impossible that all the safety-concerned female riders join the female-driver subsystem. Furthermore, the equilibrium effective joining rate of safety-concerned female riders $\lambda^e_{f_c,F}$ shall satisfy

$$U_{f_c,F} = R - p_F - \frac{c\lambda^e_{f_c,F}}{N_f\mu(N_f\mu - \lambda^e_{f_c,F})} = 0.$$

In summary, for any given $p_j \geq w_j$, $j = F, M$, the following participating numbers and joining rates of users, $(\lambda^e_m = \Lambda_m, \lambda^e_{f_\phi,M} = \Lambda_{f_\phi}, \lambda^e_{f_c,M} = 0, \lambda^e_{f_c,F}; n^e_m, n^e_f = N_f)$, are an equilibrium outcome if they satisfy the following set of conditions:

$$\begin{cases} S_m = \frac{\Lambda_\phi}{n^e_m} w_M - r \geq 0, \\ S_f = \frac{\lambda^e_{f_c,F}}{N_f} w_F - r \geq 0, \\ U_m = U_{f_\phi,M} = R - p_M - \frac{c\Lambda_\phi}{\mu n^e_m(\mu n^e_m - \Lambda_\phi)} > 0, \\ U_{f_c,M} = R - a - p_M - \frac{c\Lambda_\phi}{\mu n^e_m(\mu n^e_m - \Lambda_\phi)} \leq 0, \\ U_{f_c,F} = R - p_F - \frac{c\lambda^e_{f_c,F}}{N_f \mu(N_f \mu - \lambda^e_{f_c,F})} = 0. \end{cases}$$

□

We now consider the platform's pricing and wage decisions. First, we show that when the platform maximizes its profit, case $\mathcal{H}3$ will be dominated by case $\mathcal{H}1$ under optimization. Note that under case $\mathcal{H}3$, the rider's joining utility in the male-driver subsystem shall be positive. However, when maximizing its profit, the platform can always increase its price p_M to reduce the riders' joining utility to zero, under which case $\mathcal{H}3$ degenerates to case $\mathcal{H}1$. In this way, we can focus on the platform's optimal price and wage decisions under cases $\mathcal{H}1$ and $\mathcal{H}2$.

Under case $\mathcal{H}1$, as the joining and participating behaviors of riders and drivers are exactly the same as those stated in Proposition C.1, the platform's pricing and wage optimization problem is also similar to that presented in the online Appendix §C.1, which can be written as follows:

$$\begin{aligned} \widetilde{\Pi}^*_1 &= \max \widetilde{\Pi}_1 = \max_{p_M \in \Omega_1} \Pi_M(p_M) + \max_{\underline{w}_F(p_F) < p_F} \Pi_F(p_F), \\ &= \max_{p_M \in \Omega_1} (p_M - \underline{w}_M(p)) \frac{(\mu N_m)^2 (R - p_M)}{c + \mu N_m (R - p_M)} + \max_{\underline{w}_F(p_F) < p_F} (p_F - \underline{w}_F(p)) \frac{(\mu N_f)^2 (R - p_F)}{c + \mu N_f (R - p_F)}. \end{aligned}$$

Let $(\widetilde{p}^*_{j_1}, \widetilde{w}^*_{j_1})$ be the optimal price and wage of the subsystem j, $j = M, F$ under case $\mathcal{H}1$. Similarly, under case $\mathcal{H}2$, the platform's pricing and wage optimization problem can be derived as

$$\begin{aligned} \widetilde{\Pi}^*_2 &= \max \widetilde{\Pi}_2 = \max_{p_M \in \Omega_2} \Pi_M(p_M) + \max_{\underline{w}_F(p_F) < p_F} \Pi_F(p_F), \\ &= \max_{p_M \in \Omega_2} \left(p_M - \frac{r\,(c + \mu N_m (R - p_M - a))}{\mu^2 N_m (R - p_M - a)} \right) \frac{(\mu N_m)^2 (R - p_M - a)}{c + \mu N_m (R - p_M - a)} \\ &\quad + \max_{\underline{w}_F(p_F) < p_F} (p_F - \underline{w}_F(p)) \frac{(\mu N_f)^2 (R - p_F)}{c + \mu N_f (R - p_F)}. \end{aligned}$$

Denote $(\widetilde{p}^*_{j_2}, \widetilde{w}^*_{j_2})$ as the optimal price and wage of the subsystem j, $j = M, F$ under case $\mathcal{H}2$. The platform compared the optimal profits under the two cases and choose the one that leads to a higher profit. Thus, the platform's profit $\widetilde{\Pi}^* = \max\{\widetilde{\Pi}_1^*, \widetilde{\Pi}_2^*\}$. Analogous to the derivation process shown in the proof of Proposition C.2, we can show that the optimal prices and wages are

$$\begin{cases} (\widetilde{p}_{F_k}^*, \widetilde{w}_{F_k}^*) = \left(R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}, \frac{rN_f}{\mu N_f - c\Big/\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}} \right), k = 1, 2. \\ (\widetilde{p}_{M_1}^*, \widetilde{w}_{M_1}^*) = \left(\max\left\{ R - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu - \Lambda_\phi)}, R + \frac{c}{N_m\mu} - \sqrt{\frac{cR}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2} \right\}, \frac{r(c + N_m\mu(R - \widetilde{p}_{M_1}^*))}{\mu^2 N_m (R - \widetilde{p}_{M_1}^*)} \right). \\ \widetilde{p}_{M_2}^* = \min\left\{ R - a - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu - \Lambda_\phi)}, R - a + \frac{c}{N_m\mu} - \sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2} \right\}. \\ \widetilde{w}_{M_2}^* = \frac{r(c + N_m\mu(R - a - \widetilde{p}_{M_2}^*))}{\mu^2 N_m(R - a - \widetilde{p}_{M_2}^*)}. \end{cases} \tag{30}$$

Based on (30) and Proposition C.3, we can obtain the equilibrium joining rates of all types of users and the corresponding optimal profits under cases $\mathcal{H}1$ and $\mathcal{H}2$. Then, we have the following result.

Proposition C.4. *In a hybrid system, when $N_m\mu > \Lambda_\phi$, there exists a threshold $\widehat{a}$ such that if the safety-concerned female riders' mismatch cost $a > \widehat{a}$, the platform sets the optimal price and wage $(\widetilde{p}_M^*, \widetilde{w}_M^*; \widetilde{p}_F^*, \widetilde{w}_F^*) = (\widetilde{p}_{M_1}^*, \widetilde{w}_{M_1}^*; \widetilde{p}_{F_1}^*, \widetilde{w}_{F_1}^*)$. Otherwise, the platform sets $(\widetilde{p}_M^*, \widetilde{w}_M^*; \widetilde{p}_F^*, \widetilde{w}_F^*) = (\widetilde{p}_{M_2}^*, \widetilde{w}_{M_2}^*; \widetilde{p}_{F_2}^*, \widetilde{w}_{F_2}^*)$.*

Proof of Proposition C.4. It is easy to check that $\widetilde{\Pi}_1^*$ is independent of a. We next prove that $\widetilde{\Pi}_2^*$ decreases in a. It can be shown that

$$\begin{aligned} \widetilde{\Pi}_2^* &= (\widetilde{p}_{F_2}^* - \widetilde{w}_{F_2}^*) \cdot \lambda_{F_2}^e(\widetilde{p}_{F_2}^*) + (\widetilde{p}_{M_2}^* - \widetilde{w}_{M_2}^*) \cdot \lambda_{M_2}^e(\widetilde{p}_{M_2}^*), \\ &= \widetilde{\Pi}_{F_2}^* + \begin{cases} \left(R - a - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu - \Lambda_\phi)}\right)\Lambda_\phi - rN_m \ \text{ if } \widetilde{p}_{M_2}^* = R - a - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu-\Lambda_\phi)}, \\ \left(R - a + \frac{c}{N_m\mu} - \sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}\right)\left(\mu N_m - \frac{c}{\sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}}\right) - rN_m \\ \quad = (\sqrt{(R-a)\mu N_m + c} - \sqrt{c})^2 - rN_m, \ \text{if } \widetilde{p}_{M_2}^* = R - a + \frac{c}{N_m\mu} - \sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}. \end{cases} \end{aligned}$$

where $\widetilde{\Pi}_{F_2}^*$ is independent of a. (1) When $\widetilde{p}_{M_2}^* = R - a - \frac{c\Lambda_\phi}{N_m\mu(N_m\mu - \Lambda_\phi)}$, it is easy to show $\frac{d\widetilde{\Pi}_2^*}{da} = -\Lambda_\phi < 0$. (2) When $\widetilde{p}_{M_2}^* = R - a + \frac{c}{N_m\mu} - \sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}$, $\lambda_{M_2}^e(\widetilde{p}_{M_2}^*) = \mu N_m - \frac{c}{\sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}}$ is obviously decreasing in a. It can be shown that $\frac{\widetilde{p}_{M_2}^*}{da} = \frac{\frac{c}{N_m\mu} - 2\sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}}{2\sqrt{\frac{c(R-a)}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}} < 0$. Therefore, $(\widetilde{p}_{M_2}^* - \widetilde{w}_{M_2}^*) \cdot \lambda_{M_2}^e(\widetilde{p}_{M_2}^*) = \widetilde{p}_{M_2}^* \cdot \lambda_{M_2}^e(\widetilde{p}_{M_2}^*) - rN_m$ decreases in a. Consequently, $\widetilde{\Pi}_2^*$ decreases in a.

Let $\widehat{a}$ is the unique solution of

$$\left(\widetilde{\Pi}_2^* - \widetilde{\Pi}_1^*\right)\Big|_{a=\widehat{a}} = 0, \tag{31}$$

if it exists. If $(\widetilde{\Pi}_2^* - \widetilde{\Pi}_1^*)\ |_{a\to 0} < 0$, we let $\widehat{a} = 0$. Then, if $a \leq \widehat{a}$, $\widetilde{\Pi}_2^* \geq \widetilde{\Pi}_1^*$; otherwise, $\widetilde{\Pi}_2^* < \widetilde{\Pi}_1^*$. □

For ease of reference, we now summarize the equilibrium outcome under the hybrid system in Table C.1 based on the above discussions. Note that in Table C.1, $\widetilde{\lambda}_{m_0}^* + \widetilde{\lambda}_{f_\phi,M_0}^* = \frac{N_m^2\mu^2(R-\widetilde{p}_{M_0}^*)}{c+N_m\mu(R-\widetilde{p}_{M_0}^*)}$, $\widetilde{\lambda}_{f_c,F_0}^* + \widetilde{\lambda}_{f_\phi,F_0}^* = \frac{N_f^2\mu^2(R-\widetilde{p}_{F_0}^*)}{c+N_f\mu(R-\widetilde{p}_{F_0}^*)}$, $\widetilde{\lambda}_{f_\phi,F_0}^* + \widetilde{\lambda}_{f_\phi,M_0}^* \leq \Lambda_{f_\phi}$; $\widetilde{\lambda}_{m_1}^* + \widetilde{\lambda}_{f_\phi,M_1}^* = \frac{N_m^2\mu^2(R-\widetilde{p}_{M_1}^*)}{c+N_m\mu(R-\widetilde{p}_{M_1}^*)}$, $\widetilde{\lambda}_{f_c,F_1}^* + \widetilde{\lambda}_{f_\phi,F_1}^* = \frac{N_f^2\mu^2(R-\widetilde{p}_{F_1}^*)}{c+N_f\mu(R-\widetilde{p}_{F_1}^*)}$, $\widetilde{\lambda}_{f_\phi,F_1}^* + \widetilde{\lambda}_{f_\phi,M_1}^* \leq \Lambda_{f_\phi}$; and $\widetilde{\lambda}_{f_c,M_2}^* = \frac{N_m^2\mu^2(R-a-\widetilde{p}_{M_2}^*)}{c+N_m\mu(R-a-\widetilde{p}_{M_2}^*)} - \Lambda_\phi$, $\widetilde{\lambda}_{f_c,F_2}^* = \frac{N_f^2\mu^2(R-\widetilde{p}_{F_2}^*)}{c+N_f\mu(R-\widetilde{p}_{F_2}^*)}$.

Table C.1: Equilibrium Price, Wage and User Joining Behaviors in a Hybrid System

Player		If $N_m\mu \leq \Lambda_\phi$
Platform	decision in F subsystem	$(\widetilde{p}^*_{F_0}, \widetilde{w}^*_{F_0}) = (R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}, \frac{rN_f}{\mu N_f - c/\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}})$
	decision in M subsystem	$(\widetilde{p}^*_{M_0}, \widetilde{w}^*_{M_0}) = \left(R + \frac{c}{N_m\mu} - \sqrt{\frac{cR}{N_m\mu} + \left(\frac{c}{N_m\mu}\right)^2}, \frac{rN_m}{\mu N_m - c/\sqrt{\frac{cR}{N_m\mu}+\left(\frac{c}{N_m\mu}\right)^2}}\right)$
	profit in F subsystem	$\widetilde{\Pi}^*_{F_0} = \left(\sqrt{RN_f\mu} - \sqrt{c}\right)^2 - rN_f$
	profit in M subsystem	$\widetilde{\Pi}^*_{M_0} = \left(\sqrt{RN_m\mu} - \sqrt{c}\right)^2 - rN_m.$
	profit in hybrid system	$\widetilde{\Pi}^*_0 = \widetilde{\Pi}^*_{M_0} + \widetilde{\Pi}^*_{F_0}$
Riders	male	join male-driver subsystem at rate $\widetilde{\lambda}^*_{m_0}$
	type-f_ϕ female	join female-driver subsystem at rate $\widetilde{\lambda}^*_{f_\phi,F_0}$ join male-driver subsystem at rate $\widetilde{\lambda}^*_{f_\phi,M_0}$
	type-f_c female	join female-driver subsystem at rate $\widetilde{\lambda}^*_{f_c,F_0}$
Drivers	male	All N_m male drivers join male-driver subsystem
	female	All N_f female drivers join female-driver subsystem
Player		If $N_m\mu > \Lambda_\phi$: case $\mathcal{H}1$
Platform	decision in F subsystem	$(\widetilde{p}^*_{F_1}, \widetilde{w}^*_{F_1}) = \left(R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}, \frac{rN_f}{\mu N_f - c/\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}}\right)$
	decision in M subsystem	$(\widetilde{p}^*_{M_1}, \widetilde{w}^*_{M_1})$: equation (30)
	profit in F subsystem	$\widetilde{\Pi}^*_{F_1} = \left(\sqrt{RN_f\mu} - \sqrt{c}\right)^2 - rN_f.$
	profit in M subsystem	$\widetilde{\Pi}^*_{M_1} = \left(\widetilde{p}^*_{M_1} - \widetilde{w}^*_{M_1}\right) \frac{N_m^2\mu^2(R-\widetilde{p}^*_{M_0})}{c+N_m\mu(R-\widetilde{p}^*_{M_0})}$
	profit in hybrid system	$\widetilde{\Pi}^*_1 = \widetilde{\Pi}^*_{F_1} + \widetilde{\Pi}^*_{M_1}$
Riders	male	join male-driver subsystem at rate $\widetilde{\lambda}^*_{m_1}$
	type-f_ϕ female	join female-driver subsystem at rate $\widetilde{\lambda}^*_{f_\phi,F_1}$ join male-driver subsystem at rate $\widetilde{\lambda}^*_{f_\phi,M_1}$
	type-f_c female	join female-driver subsystem at rate $\widetilde{\lambda}^*_{f_c,F_1}$
Drivers	both types	same as that in the case $N_m\mu \leq \Lambda_\phi$
Player		If $N_m\mu > \Lambda_\phi$: case $\mathcal{H}2$
Platform	decision in F subsystem	$(\widetilde{p}^*_{F_2}, \widetilde{w}^*_{F_2}) = \left(R + \frac{c}{N_f\mu} - \sqrt{\frac{cR}{N_f\mu} + \left(\frac{c}{N_f\mu}\right)^2}, \frac{rN_f}{\mu N_f - c/\sqrt{\frac{cR}{N_f\mu}+\left(\frac{c}{N_f\mu}\right)^2}}\right)$
	decision in M subsystem	$(\widetilde{p}^*_{M_2}, \widetilde{w}^*_{M_2})$: equation (30)
	profit in F subsystem	$\widetilde{\Pi}^*_{F_2} = \left(\sqrt{RN_f\mu} - \sqrt{c}\right)^2 - rN_f.$
	profit in M subsystem	$\widetilde{\Pi}^*_{M_2} = \left(\widetilde{p}^*_{M_2} - \widetilde{w}^*_{M_2}\right) \frac{N_m^2\mu^2(R-a-\widetilde{p}^*_{M_2})}{c+N_m\mu(R-a-\widetilde{p}^*_{M_2})}$
	profit in hybrid system	$\widetilde{\Pi}^*_2 = \widetilde{\Pi}^*_{F_2} + \widetilde{\Pi}^*_{M_2}$
Riders	male	all join male-driver subsystem (i.e., at rate Λ_m)
	type-f_ϕ female	all join male-driver subsystem(i.e., at rate Λ_{f_ϕ})
	type-f_c female	join female-driver subsystem at rate $\widetilde{\lambda}^*_{f_c,F_2}$ join male-driver subsystem at rate $\widetilde{\lambda}^*_{f_c,M_2}$