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A two-stage stochastic nonlinear integer-programming model for slot allocation of a liner container shipping service

Abstract

In this study, we propose a container slot allocation problem for a liner shipping service. A liner containership provides a regular shipping service with a fixed itinerary and schedule. In practice, the liner containership may not be fully loaded, which results in a loss of revenue. We therefore segment shippers into two classes: contract shippers and spot shippers. A contract shipper has a contract with the shipping company and negotiates a fixed minimum quantity, so that the shipping company can secure a steady revenue. The remaining containership slots are open to spot shippers, allowing the shipping company to obtain ad hoc revenue. The container slot allocation problem is investigated in this study using a two-stage stochastic mixed-integer nonlinear programming model. We use the sample average approximation based on Lagrangian relaxation and dual decomposition techniques to effectively solve the model. Finally, we conduct a case study to evaluate the applicability and effectiveness of the proposed model and the solution algorithm.

Keywords: container slot allocation; two-stage stochastic mixed-integer nonlinear programming; sample average approximation; Lagrangian relaxation and dual decomposition.

1. Introduction

A liner container shipping company provides regular shipping services according to fixed itineraries and schedules. While such services benefit the shipping company in canvassing shipping demands for cargo, the reliance on a fixed schedule means that containerships may not be fully loaded when they depart from a port. Therefore, liner container shipping companies need an effective container slot allocation procedure for shippers contracting containerships. Such a procedure requires completing shippers' shipping demand with the simultaneous aim of maximizing the profit of shipping the cargo. This issue is a major concern for the liner container shipping industry.

Most liner container shipping companies manage container slots by relying on the judgment of experienced employees or on a simple "first come first serve" (FCFS) principle. Such a container slot management approach makes little or no use of decision support systems, and it is far from comprehensive, dynamic, computerized, or integrated (Ting and Tzeng, 2004). Hence, making good decisions regarding slot allocation for shippers' requests is crucial for a liner container shipping company to improve its revenue. The characteristics of slot allocation in the liner shipping industry, such as fixed capacities, advanced bookings, and demand segmentation, mean that revenue management (RM) is a high priority in slot allocation (Zurheide and Fischer, 2010; Meng et al., 2019).

RM has been widely applied and studied for more than 50 years in the airline industry (McGill and van Ryzin, 1999), and the studies have mainly focused on seat allocation and inventory control (Belobaba, 1987; Brumelle and McGill, 1990), pricing and overbooking control (Kunnumkal and Topaloglu, 2011), and air cargo transportation (Huang and Chang, 2010; Levin et al., 2012; Moussawi-Haidar, 2014; He et al., 2019). Although studies on RM applications in the airline industry are valuable to the liner shipping industry, there are significant differences between the two industries. In terms of seat inventory control, airlines commonly use advance bookings and cancellation penalties to differentiate discount fare classes (Huang and Liang, 2011), whereas in liner shipping the use of different classes is uncommon (Acciaro, 2011). As for pricing, bookings by negotiated contracts are rare in the airline industry, whereas the vast majority of liner trade is fulfilled through service contracts in which the freight rates are confidential and negotiated on a one-to-one basis (Marlow and Nair, 2008). In terms of overbooking, airlines may compensate passengers if more passengers show up at the time of a flight than the seats available on that flight; in contrast, shipping companies rarely compensate shippers even if containers have to wait in the yard for the next liner containership. As for air cargo transportation, there are two main differences. First, the space available for air cargo depends on the size of passengers' baggage. Second, air cargo transportation is almost point-to-point as part of a hub-and-spoke network; the number of airports on a flight itinerary is very small (usually less than two stops), and the cargo will be discharged to empty the aircraft at the final airport of the flight. In container liner shipping, the network structure may not be hub-and-spoke and each liner shipping route consists of a number of ports (usually more than three ports), and the containership may not be emptied at the final port of a liner voyage. because it may carry cargo to be unloaded at the destination port of the next voyage (Zurheide and Fischer, 2012). All of these differences mean that the RM models proposed for the airline industry cannot be directly transferred to the liner shipping industry. The slot allocation problem for liner container shipping services is thus still an interesting issue that deserves study. Accordingly, this paper makes the following contributions to the literature:

(i) It proposes a new slot allocation problem integrating the issues of shipping demand uncertainty, empty container repositioning, and freight rate pricing.

(ii) The proposed slot allocation problem for a liner container shipping service is formulated as a two-stage stochastic mixed-integer nonlinear programming (2SSMINP) model. As this model is intractable by using the solution methods proposed in the literature, this paper develops a solution algorithm to solve the proposed 2SSMINP model, and its convergence is proved mathematically. The methodology used in the solution algorithm is the most significant contribution of this paper.

(iii) A number of experiments are implemented to test the proposed model and solution algorithm. The computational results verify the applicability of the proposed model and the efficiency of the solution algorithm, and evaluate the effect of the proposed model on profit growth.

The remainder of this paper is organized as follows. Section 2 reviews the relevant studies and Section 3 discusses the container slot allocation problem. Section 4 develops the model and Section 5 presents the solution algorithm. Section 6 conducts a numerical experiment to evaluate the model and the solution algorithm. Finally, Section 7 concludes the study and provides recommendations for future research.

2. Literature review

The slot allocation problem in container shipping has been studied for decades. Maragos (1994) took the first step in studying the problem of slot allocation and pricing in the context of both single-segment and multi-segment container shipping. However, his study did not consider the associated problem of repositioning empty containers, although this is common practice in shipping operations and management and can increase a shipping company’s profitability. Ting and Tzeng (2004) proposed different models to determine the optimal number of containers to accept for each port pair. Lee et al. (2007, 2009) proposed a heuristic to solve a RM problem for sea cargo on a single journey leg. Feng and Chang (2008) studied the optimal slot allocation problem for ocean carriers serving a specific shipping route, considering slot allocation for empty and laden containers (Feng and Chang, 2009). Zurheide and Fischer (2010) applied RM to propose a slot allocation model with prioritization for the liner shipping industry. In their work, the freight rate of ad hoc containers is treated as a predetermined parameter, but no solution algorithm is presented. Brouer et al. (2011) revisited the slot allocation problem alongside the issue of empty containers repositioning and presented two container allocation models based on arc flow and path flow. Bell et al. (2011) proposed a frequency-based container assignment model to minimize the sailing and dwell time of containers. Bell et al. (2013) further studied a container-route assignment problem for a shipping network to minimize the sum of container handling costs, laden container inventory cost, and laden and empty container leasing costs. Wang et al. (2015) investigated a liner container seasonal shipping RM problem. Based on the transit-time-sensitive demand of the shipping context, Wang et al. (2016) proposed a tactical-level container assignment model for a liner shipping network to maximize total profit. However, in all of these studies, shipping demand is assumed to be known and deterministic, when in reality shipping demand is generally uncertain (e.g., Wang and Meng 2019).

Considering the uncertainty of shipping demand, Bu et al. (2005) developed two stochastic programming models to address the slot allocation problem with and without empty container transportation. Lu et al. (2010) considered the fluctuations in container demand and proposed a seasonal slot plan based on path flow for a liner container shipping service. Wang et al. (2015) presented a slot allocation problem in which container shipping demand depended on the freight rate and formulated the problem as a profit-based container assignment model. Fu et al. (2016) addressed the slot allocation problem with minimum quantity commitment (MQC) under uncertain demand, formulating it as a robust optimization model. Recently, Zurheide and Fischer (2015) developed a bid-price strategy to handle container booking acceptance and slot allocation. Ting and Tzeng (2016) formulated the slot allocation problem as a bi-objective model to deal with two conflicting objectives: a carrier’s freight contribution and an agent’s level of satisfaction. However, in all of these studies, the freight rate of for shipping containers is treated as a given parameter, when in reality it is more reasonable to consider it as a decision variable.

To summarize, none of the above studies integrates the issues of shipping demand uncertainty, empty container repositioning, and freight rate pricing in the context of the slot allocation problem for a liner container shipping service. Therefore, we aim to fill this gap by considering these issues. It should be noted that in this paper, the slot allocation problem for a liner container shipping service is seen as an operational issue, in which the ship routes are known; therefore, the loading/unloading operations of containers at ports are beyond the research scope. In terms of tactical level issues, such as ship routing problems, interested readers can refer to Pang and Liu (2014). Regarding operational issues such as container operations, readers can refer to Bierwirth and Meisel (2010, 2015) and Gharehgozli and Zaerpour (2018), among others.

3. Problem Description

This section describes the container slot allocation problem. In practice, the liner containerships deployed on a shipping route are not fully loaded, for two main reasons. The first reason is the low reliability of shippers. Reports indicates that the no-show rate of shippers can be as high as 30% in the liner shipping industry (Leach, 2011), and the cancellation rate of container slot bookings is high among shippers (Zhao et al., 2019 and 2020). The main reason is that there are currently no penalties for no-shows and booking cancellation in the liner container shipping industry. The second reason concerns the essential characteristics of liner shipping services. As stated earlier, a liner container shipping company provides regular shipping services with fixed itineraries and schedules; containerships should depart on time to respect the fixed schedules (Zurheide and Fischer, 2010). It should be noted that in practice, schedules are not so rigid, and sailing times may deviate from the schedule. These two reasons can make the liner container shipping company's revenue unstable and not maximized. For the sake of presentation, "containers" refer to twenty-foot equivalent units (TEUs), where all cargo is stored, and a shipping service refers to a shipping voyage comprising a number of port calls. Therefore, we consider the slot allocation problem for the voyage of a containership deployed on a specific shipping route.

To maintain stable and maximum revenue, the liner container shipping company can use a strategy commonly used in the industry: segmenting shippers into two classes, contract shippers and spot shippers (Lee et al., 2007). The liner container shipping company gives preferential freight rates to contract shippers; to attract contract shippers and obtain a steady revenue, it offers shippers an agreement stipulating a specific number of containers to be shipped over a given period, called “contract containers”. The remaining slots of the containership are left open to spot shippers, who are charged higher freight rates; thus, the liner container shipping company can accept shipping orders based on the number of available slots, called “ad hoc containers”. Accordingly, contract shippers hope that the stipulated number of containers signed in the agreement is as high as possible. However, accepting all potential containers from contract shippers is not the best choice for the liner container shipping company, because leaving slots for ad hoc shippers will bring more revenue. Therefore, the best choice for the liner container shipping company is to sign an agreement stipulating a fixed minimum quantity for contract shippers, called MQC. Then, the liner container shipping company reserves certain slots for these contract shippers.

Ad hoc containers are temporary, but bring extra revenue for the liner container shipping company. Recall that the freight rates of contract containers are predetermined by negotiation between contract shippers and the liner container shipping company, whereas the freight rates of ad hoc containers are dynamic and are unilaterally determined by the liner container shipping company. Usually, spot shippers at each port will book slots within a certain period before the containership departs from the port. Let T be the length of the booking period, and define $\mathcal{T} \coloneqq \{1, \dots, t, \dots, T\}$ as the set of booking times. In other words, spot shippers at each port can book slots at time t. The higher the value of t, the closer the booking time is to final booking deadline.

Due to trade imbalances between ports, some ports may need empty containers, resulting in empty container repositioning. Certain slots must be reserved for empty container repositioning, generating costs for the shipping company. Hence, empty container repositioning needs to be considered in the slot allocation problem. It should be noted that the laden container flow is driven externally by customer demands, whereas the empty container flow is driven by the laden container flow and determined internally by shipping companies themselves; so, the empty containers must either be accumulated in advance to meet demand or be repositioned to the depots where they are most urgently needed (Song and Dong, 2015).

Therefore, the framework of slot allocation can be described as follows. First, the slots for contract containers and empty containers are reserved; this can be seen as the first-stage decision problem. If there are still vacant slots beyond the slots determined in the first stage, they are available for ad hoc containers from spot shippers, which is considered the second-stage decision problem. Spot shippers at each port will book slots within a certain period before the containership departs from th2 port. Accordingly, this paper adopts a two-stage decision method based on shipper segmentation to deal with the proposed slot allocation problem, as illustrated in Figure 1. Our objective is to maximize the total expected profit from these two stages over a round-trip journey. Note that our problem concerns a liner shipping service; in a future study, we plan to extend our method to a shipping network consisting of multiple shipping routes.

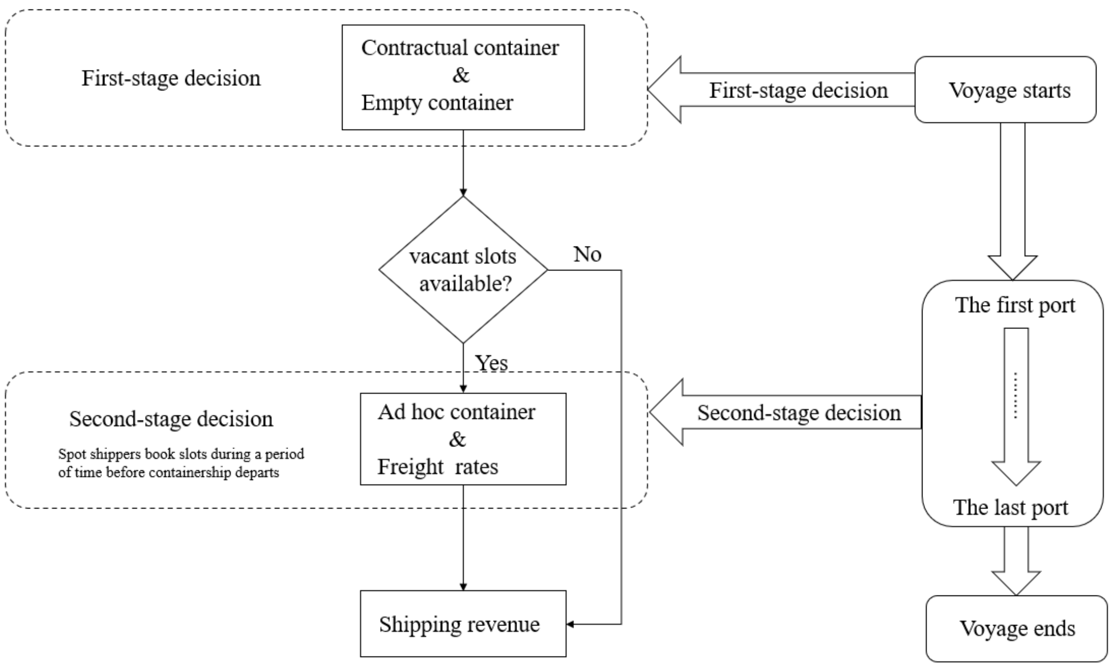

Figure 1. Illustration of two-stage slot allocation

4. Model Development

Before we turn to the model formulation, we introduce the notation used in this paper:

Sets

$\mathcal{P}$	Set of ports indexed by i, $\mathcal{P} = \{1, \dots, i, \dots, P\}$
$\mathcal{P}_s^{out}$	Set of outgoing ports for repositioning empty containers of type s
$\mathcal{P}_s^{in}$	Set of incoming ports for repositioning empty containers of type s
$\mathcal{W}$	Set of port pairs, $\mathcal{W} = \{(i,j) \mid i \in \mathcal{P}, j \in \mathcal{P}\}$

$\mathcal{L}$	Set of legs indexed by l, $\mathcal{L} = \{1, \dots, l, \dots, L\}$
$\mathcal{E}$	Set of types for empty containers
$\mathcal{F}$	Set of types for laden containers
$\mathcal{R}$	Set of types for laden reefer containers
$\mathcal{T}$	Set of booking times for spot shippers indexed by t, $\mathcal{T} = \{1, \dots, t, \dots, T\}$

Parameters

$\hat{p}_s^{(i,j)}$	Freight rate for a contract container of type s with O-D pair (i, j)
$\hat{c}_s^{(i,j)}$	Cost for a laden container of type s with O-D pair (i, j)
$c_s^{(i,j)}$	Cost for an empty container of type s with O-D pair (i, j)
d_s	Dimension in TEU of a container of type s
$\widehat{w}_s$	Average weight in tons of a laden container of type s
w_s	Average weight in tons of an empty container of type s
CAP	Capacity in TEU of the deployed containership
$\bar{U}_s^{(i,j)}$	Maximum potential demand for contract containers of type s with O-D pair (i, j)
DWT_l	Deadweight in tons of the deployed containership on leg l
RP	Number of reefer plugs of the deployed containership
$MQC_s^{(i,j)}$	Minimum quantity of contract containers of type s with O-D pair (i, j)
N_s^i	Number of empty containers of type s $(s \in \mathcal{E})$ that can be repositioned from port i
$\tilde{N}_s^j$	Number of empty containers of type s $(s \in \mathcal{E})$ to be repositioned to port j
α_t	Decreasing rate of ad hoc containers' shipment at time t

Decision variables

$\hat{x}_s^{(i,j)}$	Number of contract containers of type s with O-D pair (i, j)
$x_s^{(i,j)}$	Number of empty containers of type s with O-D pair (i, j)
$\tilde{x}_{st}^{(i,j)}$	Number of ad hoc containers of type s with O-D pair (i, j) at time t
$\tilde{p}_{st}^{(i,j)}$	Freight rate for an ad hoc container of type s with O-D pair (i, j) at time t

Auxiliary decision variables

$RCAP_l^t$	Remaining capacity in TEU of the containership on leg l at time t
$RDWT_l^t$	Remaining deadweight in tons of the containership on leg l at time t

RRP_l^t	Remaining number of reefer plugs of the containership on leg l at time t

According to the problem description in Section 3, the profit of the first stage in the round-trip journey can be given by:

$$\text{Profit}_1 = \sum_{s\in\mathcal{F}\cup\mathcal{R}}\sum_{(i,j)\in\mathcal{W}}\left(\hat{p}_s^{(i,j)} - \hat{c}_s^{(i,j)}\right)\hat{x}_s^{(i,j)} - \sum_{s\in\mathcal{E}}\sum_{(i,j)\in\mathcal{W}} c_s^{(i,j)} x_s^{(i,j)} \tag{1}$$

Eq. (1) can be written in the following vector form:

$$\text{Profit}_1 = \mathbf{c}'\mathbf{v} \tag{2}$$

where vector $\mathbf{v} \coloneqq \left(\ldots, \hat{x}_s^{(i,j)}, \ldots, x_s^{(i,j)}, \ldots\right)$ represents all of the first-stage decision variables and $\mathbf{c} \coloneqq \left(\ldots, \left(\hat{p}_s^{(i,j)} - \hat{c}_s^{(i,j)}\right), \ldots, c_s^{(i,j)}, \ldots\right)$ denotes all of the coefficients associated with the first-stage decision variables.

In general, demand decreases as the price increases. Thus, it is rational to assume that the shipping demand for ad hoc containers will reach a maximum when the freight rates for ad hoc containers fall to the freight rates for contract containers. Because the decision variables $\tilde{x}_{st}^{(i,j)}$ refer to the number of ad hoc containers accepted from spot shippers, it is reasonable to treat the maximum potential ad hoc shipping demand as a random variable. Therefore, let $\overline{\mathbf{D}} \coloneqq \left\{\overline{D}_s^{(i,j)}: (i,j) \in \mathcal{W}, s \in \mathcal{F} \cup \mathcal{R}\right\}$ be the vector of the maximum potential ad hoc shipping demand defined over a probability space $(\Omega, \mathbb{F}, \mathbb{P})$, with elementary outcomes $\Omega = \left\{\overline{D}_s^{(i,j)}(\omega): (i,j) \in \mathcal{W}, s \in \mathcal{F} \cup \mathcal{R}\right\}$, where $\overline{D}_s^{(i,j)}(\omega)$ is an elementary outcome, $\mathbb{F}$ is the event space, and $\mathbb{P}$ is the probability measure. Therefore, the slot allocation problem in this paper can be formulated by the 2SSMINP model：

$$[\text{2SSMINP}] \qquad \max Z(\mathbf{v}) = \mathbf{c}'\mathbf{v} + \mathbb{E}\left[\!\left[Q_{\overline{\mathbf{D}}}\left(\mathbf{v}, \overline{\mathbf{D}}\right)\right]\!\right] \tag{3}$$

subject to

$$\sum_{(i,j)\in\mathcal{W}} \rho_l^{(i,j)}\left(\sum_{s\in\mathcal{F}\cup\mathcal{R}} d_s \hat{x}_s^{(i,j)} + \sum_{s\in\mathcal{E}} d_s x_s^{(i,j)}\right) \le CAP \qquad \forall l \in \mathcal{L} \tag{4}$$

$$\sum_{(i,j)\in\mathcal{W}} \rho_l^{(i,j)}\left(\sum_{s\in\mathcal{F}\cup\mathcal{R}} \hat{w}_s \hat{x}_s^{(i,j)} + \sum_{s\in\mathcal{E}} w_s x_s^{(i,j)}\right) \le DWT_l \qquad \forall l \in \mathcal{L} \tag{5}$$

$$\sum_{s\in\mathcal{R}}\sum_{(i,j)\in\mathcal{W}} \rho_l^{(i,j)} \hat{x}_s^{(i,j)} \le RP \qquad \forall l \in \mathcal{L} \tag{6}$$

$$\bar{U}_s^{(i,j)} \ge \hat{x}_s^{(i,j)} \ge MQC_s^{(i,j)} \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R} \tag{7}$$

$$\sum_{j\in\mathcal{P}_s^{in}} x_s^{(i,j)} \le N_s^i \qquad \forall i\in\mathcal{P}_s^{out}, s\in\mathcal{E} \tag{8}$$

$$\sum_{i\in\mathcal{P}_s^{out}} x_s^{(i,j)} \ge \tilde{N}_s^j \qquad \forall j\in\mathcal{P}_s^{in}, s\in\mathcal{E} \tag{9}$$

$$\hat{x}_s^{(i,j)}, x_s^{(i,j)} \in \mathbb{Z}^+ \bigcup \{0\} \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}\cup\mathcal{E} \tag{10}$$

where $\mathbb{E}[\![\boldsymbol{Q}_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}})]\!]$ is the expected recourse function in which $\boldsymbol{Q}_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}})$ is the optimal objective function value for the second-stage optimization problem, with a given vector $\mathbf{v}$ and the random maximum potential ad hoc shipping demands denoted by $\bar{\mathbf{D}}$. $\rho_l^{(i,j)}$ is a binary parameter that equals 1 if the shipping journey of containers with an O-D pair (i,j) contains leg l, and 0 otherwise. For a particular realization of the maximum potential ad hoc shipping demand $\bar{\mathbf{D}}(\omega)$, we let $Q_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}}(\omega))$ be the value of the second-stage optimization model, defined as follows:

$$Q_{\bar{\mathbf{D}}}\left(\mathbf{v},\bar{\mathbf{D}}(\omega)\right) = \max \sum_{t\in\mathcal{T}}\sum_{(i,j)\in\mathcal{W}}\sum_{s\in\mathcal{F}\cup\mathcal{R}} \left(\tilde{p}_{st}^{(i,j)} - \hat{c}_s^{(i,j)}\right)\times \tilde{x}_{st}^{(i,j)} \tag{11}$$

subject to

$$\sum_{(i,j)\in\mathcal{W}} \rho_l^{(i,j)} \left(\sum_{s\in\mathcal{F}\cup\mathcal{R}} d_s \hat{x}_s^{(i,j)} + \sum_{s\in\mathcal{E}} d_s x_s^{(i,j)} + \sum_{s\in\mathcal{F}\cup\mathcal{R}}\sum_{k=1}^{t} d_s \tilde{x}_{sk}^{(i,j)} \right) \le CAP \quad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{12}$$

$$\sum_{(i,j)\in\mathcal{W}} \rho_l^{(i,j)} \left(\sum_{s\in\mathcal{F}\cup\mathcal{R}} \hat{w}_s \hat{x}_s^{(i,j)} + \sum_{s\in\mathcal{E}} w_s x_s^{(i,j)} + \sum_{s\in\mathcal{F}\cup\mathcal{R}}\sum_{k=1}^{t} \hat{w}_s \tilde{x}_{sk}^{(i,j)} \right) \le DWT_l \quad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{13}$$

$$\sum_{(i,j)\in\mathcal{W}} \rho_l^{(i,j)} \left(\sum_{s\in\mathcal{R}} \hat{x}_s^{(i,j)} + \sum_{s\in\mathcal{R}}\sum_{k=1}^{t} \tilde{x}_{sk}^{(i,j)} \right) \le RP \quad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{14}$$

$$\tilde{x}_{st}^{(i,j)} \le \bar{D}_s^{(i,j)}(\omega) - \alpha_t \left(\tilde{p}_{st}^{(i,j)} - \hat{p}_s^{(i,j)}\right) \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}, t\in\mathcal{T} \tag{15}$$

$$\tilde{p}_{st}^{(i,j)} \ge \hat{p}_s^{(i,j)} \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}, t\in\mathcal{T} \tag{16}$$

$$\tilde{p}_{st}^{(i,j)} \le \hat{p}_s^{(i,j)} + \bar{D}_s^{(i,j)}(\omega)/\alpha_t \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}, t\in\mathcal{T} \tag{17}$$

$$\tilde{x}_{st}^{(i,j)} \in \mathbb{Z}^+ \bigcup \{0\} \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}, t\in\mathcal{T} \tag{18}$$

where $\bar{D}_s^{(i,j)}(\omega)$ is the realization of the maximum potential shipping demand for ad hoc containers of type s with an O-D pair (i,j).

Eq. (3) is the objective function of the 2SSMINP model, which is equivalent to maximizing the expected total profit obtained from the two stages. Constraints (4)-(6) specify that the containers carried on each leg of the voyage cannot exceed the capacity, deadweight, and available reefer plugs of the containership deployed on the voyage, respectively. The number of accepted contract containers is restricted to the range given in Constraints (7). Constraints (8) ensure that the total number of empty containers to be repositioned into other ports cannot exceed the maximum number of empty containers from its outgoing port. Constraints (9) require the total number of empty containers from outgoing ports to meet the need of the port where they will be repositioned in. Constraint (10) defines the ranges of the first-stage decision variables.

Eq. (11) is the objective function of the second-stage optimization model, given the first-stage decision variables of vector $\mathbf{v}$ and the realization of the random maximum potential ad hoc shipping demand $\bar{\mathbf{D}}$, which aims to maximize the profit from ad hoc containers. Constraints (12) -(14) specify that the total number of containers on board on each leg until booking time t cannot exceed the capacity, deadweight, and available reefer plugs of the containership, respectively. The right sides of Constraints (15) compute the shipping demand for ad hoc containers with a given realization of its random maximum potential quantity, which reveal the relationship between shipping demand and the freight rate of ad hoc containers and indicate that as the freight rates of ad hoc containers increases, shipping demand decreases. The left sides of Constraints (15) represent the number of accepted ad hoc containers. Therefore, Constraints (15) specify that the number of ad hoc containers accepted cannot exceed the shipping demands. Constraints (16) and (17) give the lower bound and upper bound of the freight rates of ad hoc containers, respectively. Note that the freight rates for ad hoc containers and the number of accepted ad hoc containers obtained by solving the second-stage optimization model are related to the realization of $\bar{\mathbf{D}}$. In addition, at different booking times, the freight rates of ad hoc containers are usually different, which results in different spot demands. Therefore, we introduce a time index for the decision variables presented in Constraints (15) and (17). Constraints (18) define the ranges of the second-stage decision variables.

5. Solution Algorithm

It is found that the 2SSMINP model cannot be solved by traditional optimization algorithms or commercial solvers, due to the following four characteristics: (i) the expected value function $\mathbb{E}[\![Q_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}})]\!]$ does not have a closed form and incorporates an optimization model in the second-stage decision problem; (ii) the objective function value of the second-stage optimization problem, $Q_{\bar{\mathbf{D}}}\big(\mathbf{v},\bar{\mathbf{D}}(\omega)\big)$, can be obtained only when the first-stage decision variables and realizations of the random shipping demand for ad hoc containers are provided; (iii) the 2SSMINP model involves the MQC constraints shown in Eq. (7), and Lim et al. (2006) demonstrated that a transportation problem with MQC constraints is NP-hard; and (iv) the bilinear terms $\tilde{p}_{st}^{(i,j)} \times \tilde{x}_{st}^{(i,j)}$ in Eq. (11) make $Q_{\bar{\mathbf{D}}}\big(\mathbf{v},\bar{\mathbf{D}}(\omega)\big)$ a nonlinear integer programming model, which increases the difficulty. The first two characteristics motivate us to use the sample average approximation (SAA) method proposed by Kleywegt et al. (2001), and the last two characteristics prompt us design a branch-and-bound (B&B) approach combined with a heuristic method to solve the 2SSMINP model.

The overall procedure of the solution algorithm goes as follows. First, we use SAA to approximate the expected value function $\mathbb{E}[\![Q_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}})]\!]$ with the sample average function denoted by $N^{-1}\sum_{n=1}^{N} Q_{\bar{\mathbf{D}}}\big(\mathbf{v},\bar{\mathbf{D}}_{\boldsymbol{n}}(\omega)\big)$. Second, the SAA problem with a sample average function is decomposed into a series of sub-problems using the dual decomposition method, and these sub-problems are then relaxed by applying the Lagrangian relaxation technique, resulting in a Lagrangian dual problem. Third, the Lagrangian dual problem is solved by the surrogate sub-gradient method, in which the surrogate sub-gradient is obtained using the B&B algorithm and a heuristic algorithm. The following sections elaborate these procedures.

5.1 SAA to approximate the expected value function $\mathbb{E}[\![Q_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}})]\!]$

The SAA method proposed by Kleywegt et al. (2001) uses a Monte Carlo simulation-based approach and its basic idea is to approximate the expected value $\mathbb{E}[\![Q_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}})]\!]$ by the sample mean. The key procedures of the SAA method are as follows. First, a sample $\bar{\mathbf{D}}_{\mathbf{1}},\ldots,\bar{\mathbf{D}}_{N}$ of *N* realizations of the random shipping demand vector of ad hoc containers $\bar{\mathbf{D}}$ is generated; then, the expected value function $\mathbb{E}[\![Q_{\bar{\mathbf{D}}}(\mathbf{v},\bar{\mathbf{D}})]\!]$ is approximated by the sample average function $N^{-1}\sum_{n=1}^{N} Q_{\bar{D}}\big(\mathbf{v},\bar{\mathbf{D}}_{\boldsymbol{n}}(\omega)\big)$. The 2SSMINP model is thus approximated by the following SAA model:

[SAA] $$\max Z_N(\mathbf{v}) = \mathbf{c}'\mathbf{v} + N^{-1}\sum_{n=1}^{N} Q_{\bar{\mathbf{D}}}\left(\mathbf{v},\bar{\mathbf{D}}_n(\omega)\right) \tag{19}$$

$$\text{s.t. constraints (4)-(10).}$$

where $Q_{\bar{\mathbf{D}}}(\mathbf{v}, \bar{\mathbf{D}}_n(\omega))$ $(n = 1, \dots, N)$ is the optimal objective function value for the n^{th} second-stage optimization problem with a given vector $\mathbf{v}$ and a given realization $\bar{\mathbf{D}}_n(\omega)$. $Q_{\bar{\mathbf{D}}}(\mathbf{v}, \bar{\mathbf{D}}_n(\omega))$ is expressed as follows:

$$Q_{\bar{\mathbf{D}}}\left(\mathbf{v}, \bar{\mathbf{D}}_n(\omega)\right) = \max \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{W}} \sum_{s\in\mathcal{F}\cup\mathcal{R}} \left(\tilde{p}_{stn}^{(i,j)} - \hat{c}_s^{(i,j)}\right) \times \tilde{x}_{stn}^{(i,j)} \tag{20}$$

subject to constraints (12)-(18), duplicated for each realization of container shipping demand.

It should be noted that by using the SAA method, we are able to obtain the unbiased estimators for the lower and upper bounds of the objective function value of the 2SSMINP model, denoted by LB and UB, respectively. Therefore, we can calculate the gap between LB and UB. Readers can refer to Kleywegt et al. (2001) for more details on SAA.

5.2 Dual decomposition and Lagrangian relaxation to decompose the SAA problem

It can be seen that the SAA problem involves N optimization models corresponding to N realizations. This motivates us to decompose the SAA problem into N sub-problems based on the realizations, to reduce the difficulty of solving this SAA problem (Carøe and Schultz, 1999; Ahmed, 2013). To carry out the decomposition, the first-stage decision variables are duplicated for each realization, denoted by $\mathbf{v}_n \coloneqq \left(\dots, \hat{x}_{sn}^{(i,j)}, \dots, x_{sn}^{(i,j)}, \dots\right)$, $n = 1, \dots, N$. The SAA problem can thus be rewritten as follows:

$$\max \frac{1}{N} \sum_{n=1}^{N} \left(\mathbf{c}'\mathbf{v}_n + \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{W}} \sum_{s\in\mathcal{F}\cup\mathcal{R}} \left(\tilde{p}_{stn}^{(i,j)} - \hat{c}_s^{(i,j)}\right) \times \tilde{x}_{stn}^{(i,j)} \right) \tag{21}$$

subject to constraints (4)-(10), constraints (12)-(18), and the non-anticipativity constraints

$$\mathbf{v}_1 = \mathbf{v}_2 = \dots = \mathbf{v}_N \tag{22}$$

The non-anticipativity constraints (22) can be equivalently written as follows:

$$\sum_{n=1}^{N} \mathbf{H}_n \mathbf{v}_n = \mathbf{0} \tag{23}$$

where $\mathbf{0}$ is a zero vector with a dimension of $\tilde{R} \coloneqq \tilde{N}(N-1)$, and $\mathbf{H}_n (n = 1, \dots, N)$ is a matrix with $\tilde{R}$ rows and $\tilde{N}$ columns ($\tilde{N}$ is the cardinality of vector $\mathbf{v}_n$), defined as follows:

$$\begin{aligned} &\mathbf{H}_1 = (\mathbf{I}, \mathbf{0}, \cdots, \mathbf{0})', \mathbf{H}_2 = (-\mathbf{I}, \mathbf{I}, \mathbf{0}, \cdots \mathbf{0})', \mathbf{H}_3 = (\mathbf{0}, -\mathbf{I}, \mathbf{I}, \cdots \mathbf{0})', \cdots, \\ &\mathbf{H}_{N-1} = (\mathbf{0}, \cdots, -\mathbf{I}, \mathbf{I})', \mathbf{H}_N = (\mathbf{0}, \cdots \mathbf{0}, -\mathbf{I})' \end{aligned} \tag{24}$$

in which $\mathbf{I}$ and $\mathbf{0}$ are the square unity matrix and the zero matrix with size $\widetilde{N}$, respectively. If $\boldsymbol{\lambda}$ denotes an $\tilde{R}$-dimensional vector of Lagrangian multipliers associated with Constraints (23), we then have a corresponding Lagrangian relaxation problem to that of the SAA, as shown below:

$$\text{[LR]} \qquad LR(\boldsymbol{\lambda}) = \max \sum_{n=1}^{N} \left[\frac{1}{N} \left(\mathbf{c}'\mathbf{v}_n + \sum_{t \in \mathcal{T}} \sum_{(i,j) \in \mathcal{W}} \sum_{s \in \mathcal{F} \cup \mathcal{R}} \left(\tilde{p}_{stn}^{(i,j)} - \hat{c}_s^{(i,j)} \right) \times \tilde{x}_{stn}^{(i,j)} \right) + \boldsymbol{\lambda}'\mathbf{H}_n \mathbf{v}_n \right] \tag{25}$$

subject to Constraints (4)-(10), with Constraints (12)-(18) duplicated for each realization of container shipping demand. Furthermore, the LR model (25) can be split into N separate mixed-integer programming problems corresponding to the N realizations of shipping demand for ad hoc containers, namely:

$$LR(\boldsymbol{\lambda}) = \sum_{n=1}^{N} LR_n(\boldsymbol{\lambda}) \tag{26}$$

where

$$LR_n(\boldsymbol{\lambda}) = \max \frac{1}{N} \left(\mathbf{c}'\mathbf{v}_n + \sum_{t \in \mathcal{T}} \sum_{(i,j) \in \mathcal{W}} \sum_{s \in \mathcal{F} \cup \mathcal{R}} \left(\tilde{p}_{stn}^{(i,j)} - \hat{c}_s^{(i,j)} \right) \times \tilde{x}_{stn}^{(i,j)} \right) + \boldsymbol{\lambda}'\mathbf{H}_n \mathbf{v}_n \tag{27}$$

subject to Constraints (4)-(10), with constraints (12)-(18) associated with the n^{th} realization of container shipping demand.

Finally, we can obtain the best or tightest upper bound by solving the Lagrangian dual model:

$$\text{[LD]} \qquad LD = \min_{\boldsymbol{\lambda}} LR(\boldsymbol{\lambda}) \tag{28}$$

5.3 Surrogate sub-gradient method for solving the Lagrangian dual model

As the sub-gradient method is easy to implement and works well when applied to numerous practical problems, it has become a popular method for solving the Lagrangian dual model (Fisher, 2004). According to the results of Nemhauser and Wolsey (1998), $\mathbf{g} \coloneqq \sum_{n=1}^{N} \mathbf{H}_n \mathbf{v}_n^*$ is a sub-gradient for $LR(\boldsymbol{\lambda})$, where $\mathbf{v}_n^*$ is the optimal solution to the n^{th} subproblem $LR_n(\boldsymbol{\lambda})$. Therefore, this method needs to solve the optimization models for all sub-problems to obtain the direction of the sub-gradient, which makes it cumbersome for large-scale problems. Zhao et al. (1999) proposed a surrogate sub-gradient method to replace the sub-gradient by the surrogate sub-gradient which is denoted by $\tilde{\mathbf{g}} \coloneqq \sum_{n=1}^{N} \mathbf{H}_n \mathbf{v}_n$ (here, $\mathbf{v}_n$ satisfies the surrogate optimality condition). The surrogate sub-gradient method does not require solving all of the sub-problems, implying that it takes considerably less effort than the sub-gradient method. Therefore, we adopt the surrogate sub-gradient method proposed by Zhao et al. (1999) to solve the LD model shown by Eq. (28), as follows:

Step 0: (Initialize) Take an initial Lagrangian multiplier vector $\boldsymbol{\lambda}^0$ (usually set $\boldsymbol{\lambda}^0 = \mathbf{0}$) and solve all of the sub-problems $LR_n(\boldsymbol{\lambda})$ shown in Eq. (27) to obtain the initial solution, $\mathbf{v}_n^{0*}$ (the algorithm used to solve these sub-problems, $LR_n(\boldsymbol{\lambda})$, is described in Section 5.4). Next, set the initial surrogate dual as the dual (the surrogate dual is denoted by $\tilde{L}(\boldsymbol{\lambda}, \mathbf{v}) \coloneqq \sum_{n=1}^{N} \frac{1}{N}\left(\mathbf{c}'\mathbf{v}_n + \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{W}} \sum_{s\in\mathcal{F}\cup\mathcal{R}} \left(\tilde{p}_{stn}^{(i,j)} - \hat{c}_s^{(i,j)}\right) \times \tilde{x}_{stn}^{(i,j)}\right) + \boldsymbol{\lambda}'\mathbf{H}_n\mathbf{v}_n$), and set the initial surrogate sub-gradient as the sub-gradient, i.e. $\tilde{L}(\boldsymbol{\lambda}^0, \mathbf{v}^0) = LR(\boldsymbol{\lambda}^0)$, $\tilde{\mathbf{g}}^0 \coloneqq \sum_{n=1}^{N} \mathbf{H}_n\mathbf{v}_n^{0*}$. Estimate the lower bound of LD denoted by $\underline{Z}$, which can be obtained by applying a heuristic to the SAA problem (21) (see Section 5.5). Set the initial step size as $\tau^0 = \left(\tilde{L}^0 - \underline{Z}\right)/\|\tilde{\mathbf{g}}^0\|^2$.

Step 1: (Update the Lagrangian multiplier vector) Let $\boldsymbol{\lambda}^{\ell+1} = \boldsymbol{\lambda}^{\ell} + \tau^{\ell}\tilde{\mathbf{g}}^{\ell}$, where $\tilde{\mathbf{g}}^{\ell}$ and τ^{ℓ} denote the surrogate sub-gradient and step size in iteration ℓ, respectively, and are given by $\tilde{\mathbf{g}}^{\ell} = \sum_{n=1}^{N} \mathbf{H}_n\mathbf{v}_n^{\ell}$ and $\tau^{\ell} = \frac{\beta^{\ell}\tau^{\ell-1}\|\tilde{\mathbf{g}}^{\ell-1}\|}{\|\tilde{\mathbf{g}}^{\ell}\|}$. β^{ℓ} is a scalar satisfying $0 < \beta^{\ell} < 1$ and is set as $\beta^{\ell} = 1 - \frac{1}{R\ell^p}, p = 1 - \frac{1}{\ell^r}, \ell = 1,2,\ldots,$ where R and r are given positive constants satisfying $R \geq 1, 0 < r < 1$. The Lagrangian multiplier vector, $\boldsymbol{\lambda}^{\ell}$, converges to a unique fixed point $\boldsymbol{\lambda}^*$ (see Theorem 2.1 in Bragin et al. (2015)).

Step 2: (Update) Obtain $\mathbf{v}^{\ell+1}$ by setting it as $\mathbf{v}^{\ell+1} \coloneqq \left(\mathbf{v}_1^{\ell}, \ldots, \mathbf{v}_{i-1}^{\ell}, \mathbf{v}_i^{\ell+1^*}, \mathbf{v}_{i+1}^{\ell}, \ldots, \mathbf{v}_N^{\ell}\right)$, where $\mathbf{v}_i^{\ell+1^*}$ is the optimal solution to the i^{th} sub-problem $LR_i\left(\boldsymbol{\lambda}^{\ell+1}\right)$.

Step 3: (Check the stopping criteria) For a given tolerance $\delta > 0$, if $\left|\frac{LR(\boldsymbol{\lambda}^{\ell+1}) - LR(\boldsymbol{\lambda}^{\ell})}{LR(\boldsymbol{\lambda}^{\ell})}\right| \leq \delta$, then the algorithm is terminated. Otherwise, let $\ell = \ell + 1$ and go to Step 1.

5.4 A branch and bound algorithm to solve $LR_n(\boldsymbol{\lambda})$

5.4.1 Non-convexity of $LR_n(\boldsymbol{\lambda})$

$LR_n(\boldsymbol{\lambda})$, shown in Eq. (27) with a given Lagrangian multiplier vector $\boldsymbol{\lambda}$, can be considered as a nonlinear integer programming problem, given the bilinear terms $\tilde{p}_{stn}^{(i,j)} \times \tilde{x}_{stn}^{(i,j)}$ in the objective function. For the sake of presentation, we define vector $\mathbf{u}_n \coloneqq (\mathbf{v}_\mathbf{n}, \tilde{\mathbf{p}}_n, \tilde{\mathbf{x}}_n)$ where $\tilde{\mathbf{p}}_n \coloneqq$

$\left(\ldots,\tilde{p}_{stn}^{(i,j)},\ldots\right)$ and $\tilde{\mathbf{x}}_n := \left(\ldots,\tilde{x}_{stn}^{(i,j)},\ldots\right)$. Subsequently, the second-order derivative of the objective function of $LR_n(\boldsymbol{\lambda})$ with respect to $\mathbf{u}_n$, i.e. the Hessian matrix denoted by $\nabla^2\left(LR_n(\boldsymbol{\lambda})\right)$, can be depicted as follows:

$$\nabla^2\left(LR_n(\boldsymbol{\lambda})\right)=\begin{pmatrix}\mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3\\ \mathbf{A}_4 & \mathbf{A}_5 & \mathbf{A}_6\\ \mathbf{A}_7 & \mathbf{A}_8 & \mathbf{A}_9\end{pmatrix}_{\left(\tilde{N}+2\tilde{\tilde{N}}\right)\times\left(\tilde{N}+2\tilde{\tilde{N}}\right)} \tag{29}$$

where $\tilde{\tilde{N}}$ is the number of the elements in vector $\tilde{\mathbf{p}}_n$, $\mathbf{A}_1 = (\mathbf{0})_{\tilde{N}\times\tilde{N}}, \mathbf{A}_2 = \mathbf{A}_3 = (\mathbf{0})_{\tilde{N}\times\tilde{\tilde{N}}}, \mathbf{A}_4 = \mathbf{A}_7 = (\mathbf{0})_{\tilde{\tilde{N}}\times\tilde{N}}, \mathbf{A}_5 = \mathbf{A}_9 = (\mathbf{0})_{\tilde{\tilde{N}}\times\tilde{\tilde{N}}}, \mathbf{A}_6 = \mathbf{A}_8 = \frac{1}{N}(\mathbf{I})_{\tilde{\tilde{N}}\times\tilde{\tilde{N}}}$. It is clear that the Hessian matrix is not a positive semidefinite matrix, which indicates that $LR_n(\boldsymbol{\lambda})$ is a non-convex, nonlinear mixed-integer programming model. Therefore, the existing convex optimization techniques embedded in computerized solvers cannot be applied to solve $LR_n(\boldsymbol{\lambda})$.

5.4.2 Lower and upper bounds on $LR_n(\boldsymbol{\lambda})$

A non-convex mixed-integer nonlinear programming model can be relaxed to a mixed-integer linear programming model by replacing the non-convex terms by convex under- and over-estimators (Al-Khayyal and Falk, 1983). The bilinear term xy in the domain $[x^L, x^U] \times [y^L, y^U]$ can be relaxed by applying the following linear over-estimators:

$$xy \le \min\left\{x^U y + xy^L - x^U y^L, x^L y + xy^U - x^L y^U\right\} \tag{30}$$

Accordingly, we obtain the following linear over-estimators for the bilinear terms $\tilde{p}_{stn}^{(i,j)} \times \tilde{x}_{stn}^{(i,j)}$ in the objective function of $LR_n(\boldsymbol{\lambda})$:

$$\tilde{p}_{stn}^{(i,j)} \times \tilde{x}_{stn}^{(i,j)} \le \min\left\{\overline{\tilde{p}}_{stn}^{(i,j)}\tilde{x}_{stn}^{(i,j)} + \tilde{p}_{stn}^{(i,j)}\underline{\tilde{x}}_{stn}^{(i,j)} - \overline{\tilde{p}}_{stn}^{(i,j)}\underline{\tilde{x}}_{stn}^{(i,j)}, \underline{\tilde{p}}_{stn}^{(i,j)}\tilde{x}_{stn}^{(i,j)} + \tilde{p}_{stn}^{(i,j)}\overline{\tilde{x}}_{stn}^{(i,j)} - \underline{\tilde{p}}_{stn}^{(i,j)}\overline{\tilde{x}}_{stn}^{(i,j)}\right\} \tag{31}$$

where $\underline{\tilde{p}}_{stn}^{(i,j)}$ and $\overline{\tilde{p}}_{stn}^{(i,j)}$ are the lower and upper bounds of $\tilde{p}_{stn}^{(i,j)}$, respectively. They are given as $\underline{\tilde{p}}_{stn}^{(i,j)} = \hat{p}_s^{(i,j)}$ and $\overline{\tilde{p}}_{stn}^{(i,j)} = \overline{D}_{sn}^{(i,j)}(\omega)/\alpha_t + \hat{p}_s^{(i,j)}$, based on constraints (16) and (17). $\underline{\tilde{x}}_{stn}^{(i,j)}$ and $\overline{\tilde{x}}_{stn}^{(i,j)}$ are the lower and upper bounds of $\tilde{x}_{stn}^{(i,j)}$, respectively, and they are given as $\underline{\tilde{x}}_{stn}^{(i,j)} = 0$ and $\overline{\tilde{x}}_{stn}^{(i,j)} = \overline{D}_{sn}^{(i,j)}(\omega)$.

We can thus introduce the auxiliary decision variables $\tilde{y}_{stn}^{(i,j)}$ to replace the bilinear terms $\tilde{p}_{stn}^{(i,j)} \times \tilde{x}_{stn}^{(i,j)}$ and relax $LR_n(\boldsymbol{\lambda})$ as a mixed-integer linear model by using the over-estimators, as follows:

$$MILPLR_n(\boldsymbol{\lambda}) = \max \frac{1}{N}\left(\mathbf{c}'\mathbf{v}_n + \sum_{t\in\mathcal{T}}\sum_{(i,j)\in\mathcal{W}}\sum_{s\in\mathcal{F}\cup\mathcal{R}}\left(\tilde{y}_{stn}^{(i,j)} - \hat{c}_s^{(i,j)}\tilde{x}_{stn}^{(i,j)}\right)\right) + \boldsymbol{\lambda}'\mathbf{H}_n\mathbf{v}_n \tag{32}$$

subject to constraints (4)-(10) and constraints (12)-(18), associated with the n^{th} container shipment demand realization, whereby:

$$\tilde{y}_{stn}^{(i,j)} \le \overline{\tilde{p}}_{stn}^{(i,j)}\tilde{x}_{stn}^{(i,j)} + \tilde{p}_{stn}^{(i,j)}\underline{\tilde{x}}_{stn}^{(i,j)} - \overline{\tilde{p}}_{stn}^{(i,j)}\underline{\tilde{x}}_{stn}^{(i,j)} \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}, t\in\mathcal{T} \tag{33}$$

$$\tilde{y}_{stn}^{(i,j)} \le \underline{\tilde{p}}_{stn}^{(i,j)}\tilde{x}_{stn}^{(i,j)} + \tilde{p}_{stn}^{(i,j)}\overline{\tilde{x}}_{stn}^{(i,j)} - \underline{\tilde{p}}_{stn}^{(i,j)}\overline{\tilde{x}}_{stn}^{(i,j)} \qquad \forall (i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}, t\in\mathcal{T} \tag{34}$$

Let $\tilde{\mathbf{u}}_n \coloneqq (\mathbf{v}_n, \tilde{\mathbf{p}}_n, \tilde{\mathbf{x}}_n, \tilde{\mathbf{y}}_n)$ be the decision variable vector of $MILPLR_n(\boldsymbol{\lambda})$; then the following proposition is straightforward:

Proposition 1: Let $\tilde{\mathbf{u}}_n^* \coloneqq (\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*, \tilde{\mathbf{y}}_n^*)$ be the optimal solution of $MILPLR_n(\boldsymbol{\lambda})$, and $LR_n^*(\boldsymbol{\lambda})$ the optimal objective function value of $LR_n(\boldsymbol{\lambda})$. We thus obtain the following relationship:

$$LR_n(\boldsymbol{\lambda})\Big|_{\mathbf{u}_n^* := (\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*)} \le LR_n^*(\boldsymbol{\lambda}) \le MILPLR_n^*(\boldsymbol{\lambda})\Big|_{\tilde{\mathbf{u}}_n^* := (\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*, \tilde{\mathbf{y}}_n^*)} \tag{35}$$

Proposition 1 reveals that $MILPLR_n(\boldsymbol{\lambda})$ approximates $LR_n(\boldsymbol{\lambda})$ with a relative error given as follows:

$$relative\ error = \frac{MILPLR_n^*(\boldsymbol{\lambda})\Big|_{\tilde{\mathbf{u}}_n^*} - LR_n(\boldsymbol{\lambda})\Big|_{\mathbf{u}_n^*}}{MILPLR_n^*(\boldsymbol{\lambda})\Big|_{\tilde{\mathbf{u}}_n^*}} \tag{36}$$

5.4.3 A tailored branch-and-bound algorithm and its convergence

Proposition 1 gives us the upper and lower bounds of the optimal objective function value of $LR_n(\boldsymbol{\lambda})$, which enables us to propose a tailored B&B method to solve $LR_n(\boldsymbol{\lambda})$. This is elaborated below:

Step 0: (Initialize) Let ε be the maximum tolerance; set the empty active problem as $\mathbb{P} = \emptyset$ and the incumbent problem as $P_c = NULL$, the current solution as $CS = NULL$, the lower bound as $LB_{LR_n(\boldsymbol{\lambda})} = -\infty$, and the upper bound as $UB_{LR_n(\boldsymbol{\lambda})} = +\infty$.

Step 1: (Solve the root problem) Set the root problem, $MILPLR_n^0(\boldsymbol{\lambda})$, defined in Eq. (32), and set the incumbent problem, $P_c = MILPLR_n^0(\boldsymbol{\lambda})$. Solve the incumbent problem using CPLEX to obtain the optimal solution $(\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*, \tilde{\mathbf{y}}_n^*)$ and the corresponding optimal objective function value $P_c^{obj^*}$. Set $UB_{LR_n(\boldsymbol{\lambda})} = P_c^{obj^*}$ and substitute the optimal solution to $LR_n(\boldsymbol{\lambda})$, then set $LB_{LR_n(\boldsymbol{\lambda})} = LR_n(\boldsymbol{\lambda})|_{(\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*)}$. Set $CS = (\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*)$.

Step 2: (Stop for criteria check) If $\frac{UB_{LR_n(\boldsymbol{\lambda})} - LB_{LR_n(\boldsymbol{\lambda})}}{UB_{LR_n(\boldsymbol{\lambda})}} < \varepsilon$, stop and output the current solution *CS* and lower bound $LB_{LR_n(\boldsymbol{\lambda})}$. Otherwise, go to Step 3.

Step 3: (Branch the problem) We adopt the branching strategy for the port pair with the largest gap between the lower and upper bounds of the profit of shipping ad hoc containers. The rationale for the branching strategy is that the upper bound of $LR_n(\boldsymbol{\lambda})$ is obtained by solving the mixed-integer linear model $MILPLR_n(\boldsymbol{\lambda})$ in which the over-estimators are used to replace the bilinear terms $\tilde{p}_{stn}^{(i,j)} \times \tilde{x}_{stn}^{(i,j)}$ in the objective function of $LR_n(\boldsymbol{\lambda})$. It should be noted that we branch the domain of ad hoc freight rate $\tilde{p}_{stn}^{(i,j)}$, but not the number of ad hoc containers, $\tilde{x}_{stn}^{(i,j)}$. Let (i^*, j^*), s^*, and t^* be the port pair, container type, and booking time, respectively, with the largest contribution to the gap between the lower and upper bounds of the profit of shipping ad hoc containers, namely,

$$\left((i^*, j^*), s^*, t^*\right) \in \underset{(i,j)\in\mathcal{W}, s\in\mathcal{F}\cup\mathcal{R}, t\in\mathcal{T}}{\operatorname{argmax}} \left\{\tilde{y}_{stn}^{(i,j)^*} - \tilde{p}_{stn}^{(i,j)^*} \times \tilde{x}_{stn}^{(i,j)^*}\right\} \tag{37}$$

where $\tilde{p}_{stn}^{(i,j)^*}, \tilde{x}_{stn}^{(i,j)^*} \tilde{y}_{stn}^{(i,j)^*}$ represents the corresponding optimal solution for P_c. Next, P_c is branched into two sub-problems by dividing the domain of $\tilde{p}_{stn}^{(i,j)}$ into two sub-intervals, such that

P_{sub1}:

$$\underline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)} = \underline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)}, \overline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)} = \tilde{p}_{s^*t^*n}^{(i^*,j^*)^*}$$

s.t.:

$$\tilde{p}_{stn}^{(i,j)} \in \left[\underline{\tilde{p}}_{stn}^{(i,j)}, \overline{\tilde{p}}_{stn}^{(i,j)}\right], \forall (i,j) \in \mathcal{W}, s \in \mathcal{F} \cup \mathcal{R}, t \in \mathcal{T} \tag{38}$$

P_{sub2}:

$$\underline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)} = \tilde{p}_{s^*t^*n}^{(i^*,j^*)^*}, \overline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)} = \overline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)}$$

s.t.:

$$\tilde{p}_{stn}^{(i,j)} \in \left[\underline{\tilde{p}}_{stn}^{(i,j)}, \bar{\tilde{p}}_{stn}^{(i,j)} \right], \forall (i,j) \in \mathcal{W}, s \in \mathcal{F} \cup \mathcal{R}, t \in \mathcal{T} \quad (39)$$

Step 4: Solve each sub-problems P_{subq} to obtain the optimal solution $(\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*, \tilde{\mathbf{y}}_n^*)_{P_{subq}}$, and the optimal objective function value, $P_{subq}^{obj^*}$, for $q = 1,2$. Substitute $(\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*, \tilde{\mathbf{y}}_n^*)_{P_{subq}}$ into $LR_n(\boldsymbol{\lambda})$ to obtain the objective function value, $LR_n(\boldsymbol{\lambda})|_{(\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*, \tilde{\mathbf{y}}_n^*)_{P_{subq}}}, q = 1,2$.

Step 5: (Update the lower bound) For each sub-problem P_{subq}, if $LR_n(\boldsymbol{\lambda})|_{(\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*)_{P_{subq}}} > LB_{LR_n(\boldsymbol{\lambda})}$, then set $LB_{LR_n(\boldsymbol{\lambda})} = LR_n(\boldsymbol{\lambda})|_{(\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*)_{P_{subq}}}$ and $CS = (\mathbf{v}_n^*, \tilde{\mathbf{p}}_n^*, \tilde{\mathbf{x}}_n^*)_{P_{subq}}$.

Step 6: (Fathom the problem) For each subproblem P_{subi}, if $P_{subq}^{obj^*} > \frac{LB_{LR_n(\boldsymbol{\lambda})}}{1-\varepsilon}$, add this to set $\mathbb{P}$ and go to Step 3; if $LB_{LR_n(\boldsymbol{\lambda})} < P_{subq}^{obj^*} \leq \frac{LB_{LR_n(\boldsymbol{\lambda})}}{1-\varepsilon}$, then the sub-problem is fathomed and go to Step 2; if $P_{subq}^{obj^*} \leq LB_{LR_n(\boldsymbol{\lambda})}$, then discard the sub-problem.

Step 7: (Select the incumbent problem) Select the problem from set $\mathbb{P}$ with the largest optimal objective function value $P_c^{obj^{\max}}$. Set $UB_{LR_n(\boldsymbol{\lambda})} = P_c^{obj^{\max}}$, and set this problem as the incumbent problem P_c. Remove this question from set $\mathbb{P}$. Go to Step 2.

Proposition 2: The tailored B&B algorithm gives an optimal solution within the relative tolerance ε in a finite number of iterations for any $\varepsilon > 0$ (see the proof in the appendix).

5.5 Heuristic algorithm to estimate lower bound $\underline{Z}$

Note that any feasible solution to the SAA problem (21) yields a lower bound of LD denoted by $\underline{Z}$. Hence, we can split the SAA problem into two sub-problems. We first solve the first-stage optimization problem subject to Constraints (4)-(10) to obtain the optimal solutions of the first-stage decision variables, denoted by $\mathbf{v}^*$. We then substitute $\mathbf{v}^*$ into the second-stage optimization problem to define a new second-stage optimization problem, in which $\tilde{p}_{stn}^{(i,j)} = \hat{p}_s^{(i,j)}$, subject to Constraints (12)-(15) and (18). We solve the new second-stage optimization problem using the given $\mathbf{v}^*$, denoted by $Q'_{\overline{\mathbf{D}}}(\mathbf{v}^*, \overline{\mathbf{D}}_n(\omega))$. Finally, $\underline{Z}$ is computed as $\underline{Z} = \mathbf{c}'\mathbf{v}^* + \frac{1}{N}\sum_{n=1}^{N} Q'_{\overline{\mathbf{D}}}(\mathbf{v}^*, \overline{\mathbf{D}}_n(\omega))$.

6. Computational Experiments

6.1 Case study description

In this section, we take the trans-Pacific route from Asia to Pacific North West (APNW) shown in Figure 2 as a case study, which is operated by OOCL—a global liner container shipping company with headquarters in Hong Kong (www.oocl.com). We report the process and results of the case study to assess the applicability of the developed model and solution algorithm. Route APNW calls at 10 ports, Qingdao → Hong Kong → Yantian → Kaohsiung → Shanghai → Ningbo → Tacoma → Vancouver → Tokyo → Osaka → Qingdao, and provides a container shipping service for these ports. A round-trip voyage on this route takes 28 days, and a fleet of four full-container ships is deployed on the route for the weekly shipping service, with each containership holding a capacity of 13,208 TEUs and a maximum available deadweight of 144,131 tons. In our case study, we assume that each containership has 1,000 reefer plugs and that there are eight types of containers with different weights and volumes (see Table 1).

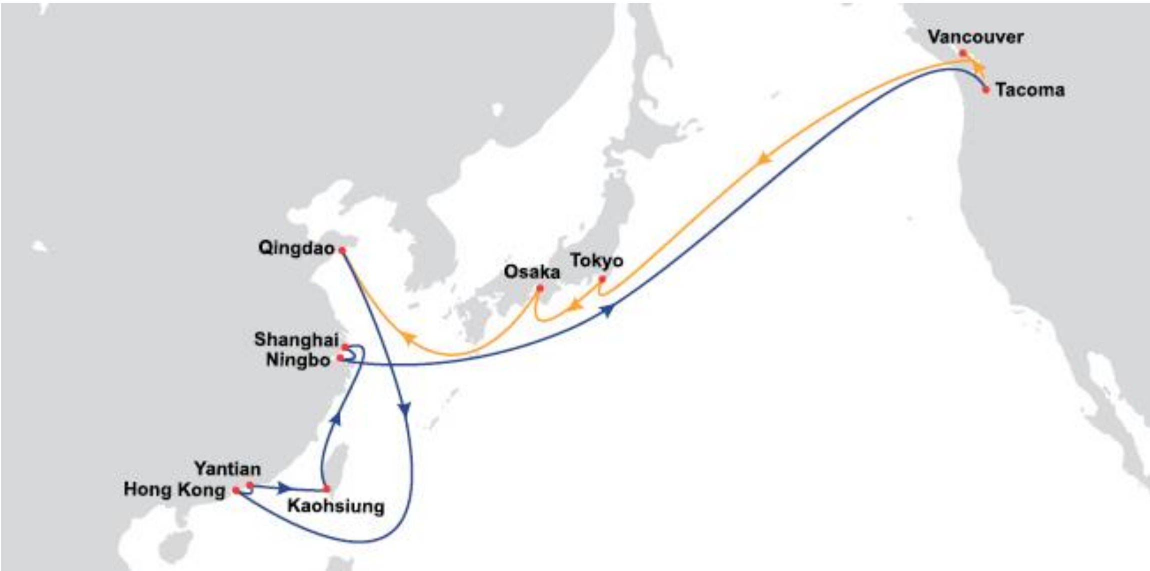

Figure 2. Port rotation of the APNW service

Table 1 Data of container weight and volume

Type code	20’D	20’R	40’D	40’R	40’HC	40’HCR	20’E	40’E
Container type	20’ dry	20’ reefer	40’ dry	40’ reefer	40’ high cube	40’ high cube reefer	20’empty	40’empty
Weight (ton)	17	17	23	23	23	23	2	4
Volume (TEU)	1	1	2	2	2.25	2.25	1	2

Real freight rates and costs are unavailable because they are confidential. For the purposes of this paper, we consider the freight rate of a 20’D contract container for each port pair as the benchmark, assuming that its freight rate is charged to shippers by US$ 0.30/nautical mile. The freight rate of a 20’D contract container for each port pair in route APNW is shown in Table 2. The freight rates of 40’D contract containers and 40’HC contract containers are assumed to be 1.5 times and 2 times those of 20’D contract containers, respectively. The cost of an empty container of each type is assumed to be 50% of that of a corresponding laden contract container, given that an empty container has no insurance, commission, or weighting fees. We also assume that the freight rate of a reefer contract container is 5% higher than that of a corresponding dry contract container, due to the refrigeration requirement.

Table 2 Freight rate of a 20’D container for each port pair in Route APNW

	Qingdao	Hong Kong	Yantian	Kaohsiung	Shanghai	Ningbo	Tacoma	Vancouver	Tokyo	Osaka
Qingdao	0	324	323	260	98	121	1537	1520	341	255
Hong Kong	324	0	12	104	241	218	1722	1702	486	422
Yantian	323	12	0	102	239	217	1720	1700	485	421
Kaohsiung	260	104	102	0	176	154	1652	1628	412	350
Shanghai	98	241	239	176	0	38	1517	1500	309	234
Ningbo	121	218	217	154	38	0	1533	1513	314	246
Tacoma	1537	1722	1720	1652	1517	1533	0	47	1283	1360
Vancouver	1520	1702	1700	1628	1500	1513	47	0	1259	1343
Tokyo	341	486	485	412	309	314	1283	1259	0	110
Osaka	255	422	421	350	234	246	1360	1343	110	0

Note: Freight rates are obtained by rounding up the resulting figures from the calculation 0.3$/nautical mile×distance. The distance of each port pair is obtained from the website: https://www.searates.com/reference/portdistance.

In terms of uncertain spot demand, we assume that it follows a log-normal distribution, which is suitable for modeling uncertain demand (Kamath and Pakkala, 2002). Specifically, $\ln \overline{D}_s^{(i,j)} \sim N\left(\mu_s^{(i,j)}, \sigma_s^{(i,j)^2}\right)\left((i,j) \in \mathcal{W}, s \in \mathcal{F} \cup \mathcal{R}\right)$, where $\mu_s^{(i,j)}$ and $\sigma_s^{(i,j)}$ are the mean and standard deviation of a normal distribution, respectively (note that the mean and variance for $\overline{D}_s^{(i,j)}$ are $e^{\mu_s^{(i,j)}+\sigma_s^{(i,j)^2}/2}$ and $\left(e^{\sigma_s^{(i,j)^2}}-1\right)e^{2\mu_s^{(i,j)}+\sigma_s^{(i,j)^2}}$, respectively). To simplify our presentation, we consider the values of $\mu_s^{(i,j)}$ for the 20’D container type between each port pair as the benchmark, which are randomly generated by a uniform distribution on the interval $[3,5]$. We also assume that the coefficients of variations, i.e. the ratios $\sigma_s^{(i,j)}/\mu_s^{(i,j)}$, all follow a uniform distribution over interval $[0,0.05]$. Once the values of $\mu_s^{(i,j)}$ and $\sigma_s^{(i,j)}$ are given, we can use them to randomly generate a data-set that follows a normal distribution. Finally, we calculate the exponents with respect to these generated values to obtain the corresponding values of $\overline{D}_s^{(i,j)}$ for the 20'D container type between each port pair. In addition, we assume that the values of $\overline{D}_s^{(i,j)}$ for 20'R, 40'D, 40'R, 40'HC, and 40'HCR containers are 50%, 40%, 30%, 20%, and 10% of the corresponding values for the 20'D container, respectively. Note that the values of $\overline{D}_s^{(i,j)}$ may not be integers; hence we round these values to the nearest integer. In general, early bookings of slots are more frequent than bookings at a later time. Therefore, we assume that the value of the decreasing rate α_t at $t = 1,\dots,T$ follows a uniform distribution over the interval $[t,T]$. Here, we set the booking period as 7 days $(T = 7)$, and spot shippers can book slots on each of the 7 days.

6.2 Computational performance analysis

We set the stop tolerance as $\delta = 10^{-3}$ in the surrogate sub-gradient method and as $\varepsilon = 10^{-6}$ in the B&B algorithm, and the number of samples is $M = 20$, with a sample size N ($N \in \{20,30,40,50,60\}$) and $\widehat{N} = 1000$ in the SAA method. All programs are coded in the programming language Lua calling CPLEX 12.6, to solve linear optimization models and mixed-integer linear optimization models, on a PC with an Intel (R) Core TM2 T9600 @ 2.8 GHz processor and 4.0 GB of RAM.

We first examine the performance of the surrogate sub-gradient method for each sample size $N \in \{20,30,40,50,60\}$, as shown in Figure 3. Here, it can be seen that for each sample size $N \in \{20,30,40,50,60\}$, the surrogate sub-gradient method fulfills the stopping criteria with a given tolerance of $\delta = 10^{-3}$ after 60 iterations. The computation time for each iteration is about 24 seconds, meaning that with the given tolerance $\delta = 10^{-3}$, the surrogate sub-gradient method stops within 24 minutes.

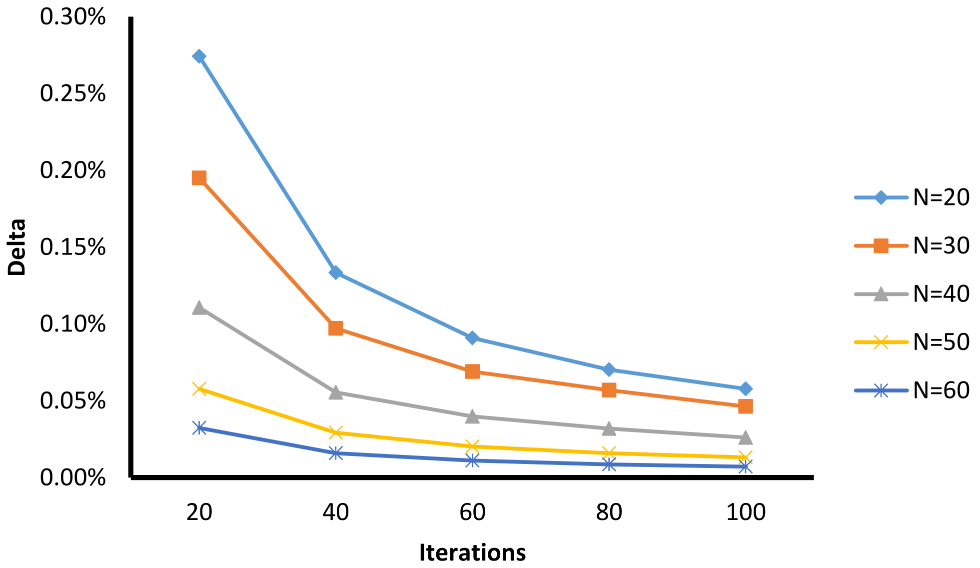

Figure 3. Convergence rates of the surrogate sub-gradient method with different sample sizes

We now present our investigation of the sensitivity of the sample size on the lower bound, upper bound, gap, and confidence interval of the gap obtained from SAA. Table 3 provides the lower bound, upper bound, gap, and 90% confidence interval of the gap, for each sample size. As can be seen, the confidence interval of the optimality gap generally becomes narrower as the sample size increases. We thus use $N = 60$ as the sample size in the subsequent analysis based on the acceptable confidence interval obtained with this sample size.

Table 3 Statistics for SAA with $M = 20$ and $\hat{N} = 1000$ (unit: million US dollars)

N	LB	σ_{LB}	UB	σ_{UB}	Gap	σ_{gap}	90% Confidence interval of gap
20	45.4622	2.4764	55.3548	4.9732	9.8926	5.5557	(0.7536,19.0316)
30	46.4876	2.2315	51.4713	3.8145	4.9837	4.4193	(−2.286,12.2534)
40	48.3286	1.7641	50.2516	2.5516	1.923	3.1021	(−3.1799,7.0259)
50	49.0521	1.1832	52.8851	1.5162	3.833	1.9232	(0.6693,6.9967)
60	48.6623	0.6148	50.6053	0.7069	1.943	0.9369	(0.4019,3.4841)

6.3 Model analysis

We now present our analysis of the effect of demand uncertainty for ad hoc containers. First, we replace the uncertain parameters in the 2SSMINP model with their mean value, to obtain the *expected value problem*, and then solve this problem to obtain its optimal first-stage solutions, called *expected value* solutions (EV solutions). Next, we compute the expected result by implementing the EV solutions for a large number of realizations of shipping demand for ad hoc containers, and denote the expected result by EEV (see Birge and Louveaux, 1997). Then, we compare the average profit obtained from the proposed 2SSMINP model with that obtained from the EEV, to explore the superiority of the 2SSMINP model over the EEV in terms of average profit.

To compare the 2SSMINP model and the EEV model, three levels (low, medium, and high) of the standard deviation of random demand variables are considered, where “low” and “high” indicate 20% below and above the standard deviations of demand at the medium level, respectively. Figure 4 depicts the EEV values and the expected profits from the 2SSMINP model associated with low, medium and high standard deviations. From this, it can be seen that the 2SSMINP models corresponding to the three levels of variance all yield a higher profit than the EEV model, indicating the superiority of the 2SSMINP model over the EEV model.

Figure 4 also illustrates that the ratios between the values of the 2SSMINP and EEV models increase as the variance level increases (as expected). However, we have to acknowledge that the average profit resulting from the 2SSMINP model is moderate because we can only set *appropriate*, but not precise, values of the SAA parameters M, N, and $\widehat{N}$. In addition, although $LR(\boldsymbol{\lambda}) \rightarrow LD$ in the dual decomposition method was theoretically proved by Shore (1985), it is quite difficult to reach the convergence point in practice. We can only set the tolerance as δ to find a relatively good solution with an acceptable level of precision (here, we set $\delta = 10^{-3}$).

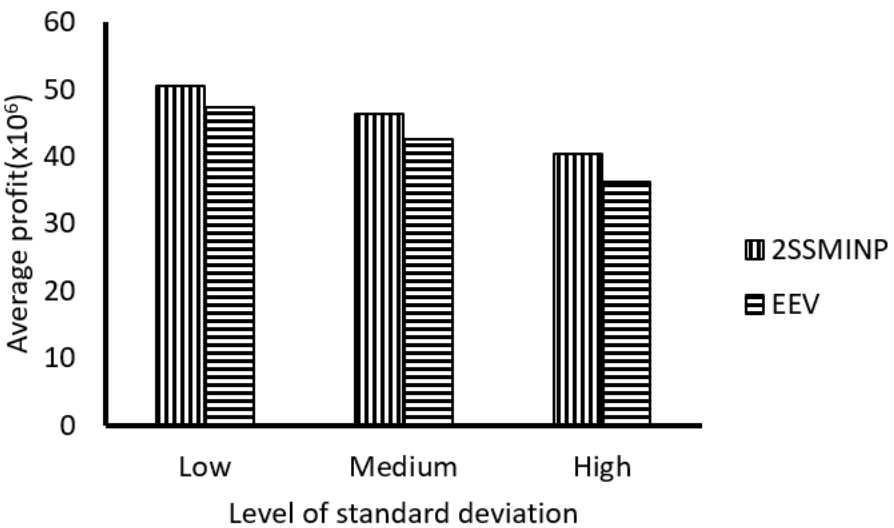

Figure 4. Average profits of 2SSMINP model and EEV for different levels of variance

6.4 Sensitivity analysis

We now perform a sensitivity analysis of the parameters of contract shippers

$\left(MQC_s^{(i,j)} \text{ and } \overline{U}_s^{(i,j)}\right)$, empty containers $\left(N_s^i \text{ and } \widetilde{N}_s^j\right)$, and spot shippers $\left(\overline{D}_s^{(i,j)}\right)$, separately, as they affect slot allocation, further affecting the total expected profit of shipping containers. For the sake of presentation, we follow the methodology used in Section 6.1 to generate the values of these parameters. We first let the value of each parameter for the 20'D container type between each port pair be the benchmark, and then, set three levels (low, medium, and high) for the benchmark values. Once the values of all parameters are obtained, we use the factorial design to explore their interaction.

To be specific, the benchmark values of $MQC_s^{(i,j)}$ at the low, medium, and high levels are randomly generated from the three intervals $[0,50], [50,100], \text{and } [100,150]$, respectively. Similarly, the values of $\overline{U}_s^{(i,j)}$ are randomly generated from the three intervals $[50,100], [100,150], \text{and } [150,200]$, respectively. Regarding the values of N_s^i and $\widetilde{N}_s^j$ at the three levels, they are randomly generated from the three intervals $[0,50], [50,150], \text{and } [150,400]$, respectively. For the values of $\overline{D}_s^{(i,j)}$, we first set three intervals $[1,3], [3,5], \text{and } [5,7]$ for the means $\mu_s^{(i,j)}$, and then use the same method described in Section 6.1 to generate the values of $\overline{D}_s^{(i,j)}$. As mentioned earlier, we set three levels for each parameter, and thus have 27 combinations of different levels in the factorial design. The corresponding results of each combination are shown in Table 4.

Table 4 Results in the factorial design for the sensitivity analysis (unit: million US dollars)

Levels for different types of containers			Profit/cost of different containers			
Contractual container	Empty container	Ad hoc container	Profit of contractual container	Cost of empty container	Expected profit of ad hoc container	Total Profit
L	L	L	21.6352	0.0706	3.2343	24.7989
L	L	M	30.3662	0.0823	20.2356	50.5195
L	L	H	25.7832	0.0941	28.1604	53.8495
L	M	L	24.7217	0.2352	4.2185	28.7050
L	M	M	28.9421	0.2940	21.9792	50.6273
L	M	H	27.8325	0.1764	22.6975	50.3536
L	H	L	22.4382	0.4704	3.4875	25.4553

L	H	M	18.3267	0.7644	26.4582	44.0205
L	H	H	20.7264	0.5880	30.3665	50.5049
M	L	L	26.4328	0.0621	3.9872	30.3579
M	L	M	29.9327	0.0761	18.4517	48.3083
M	L	H	32.3347	0.0893	20.6543	52.8997
M	M	L	39.1036	0.2214	4.0213	42.9035
M	M	M	37.8821	0.2517	14.6892	52.3196
M	M	H	36.4231	0.1932	16.8825	53.1124
M	H	L	28.9357	0.5321	3.5267	31.9303
M	H	M	35.5512	0.7572	15.8845	50.6785
M	H	H	34.7365	0.6887	18.3428	52.3906
H	L	L	45.6781	0.0598	3.8762	49.4945
H	L	M	43.2316	0.0625	7.7348	50.9039
H	L	H	44.2836	0.0777	6.8935	51.0994
H	M	L	46.8215	0.1983	3.6764	50.2996
H	M	M	47.1025	0.2432	3.4704	50.3297
H	M	H	44.2374	0.2337	6.7509	50.7546
H	H	L	48.5691	0.6712	2.5212	50.4191
H	H	M	42.7423	0.7323	8.4563	50.4663
H	H	H	43.6426	0.6012	7.7845	50.8259

Note: “L” “M” and “H” denote the low, medium and high level, respectively.

Based on Table 4, we have the following observations: 1) With the increase in levels from low to high, the contract and ad hoc containers generally generate more profits and the empty containers incur more costs, which is obvious and reasonable. The mean and variance of the profit of contract and ad hoc containers and the cost of empty containers are calculated and shown in Table 5. 2) However, as can be seen in Table 4, the values of the expected profit of ad hoc containers generally decrease with the increase in the levels of contract and empty containers. This can be easily explained by our modeling, because the numbers of accepted contract and repositioned empty containers are the decision variables in the first stage, while the numbers of accepted ad hoc containers are the decision variables in the second stage, which indicates that the numbers of accepted ad hoc containers are determined after the determination of the first-stage decision variables. Therefore, with the levels of contract and empty containers increasing from low to high, the remaining vacant slots for ad hoc containers become increasingly limited, and so are for the accepted ad hoc containers even when the spot demand is high. 3) The expected profits of ad hoc containers fluctuate differently under different levels of contract and empty containers. For example, when the contract and empty containers are at a low/medium/high level, the maximum fluctuations in the expected profit of ad hoc containers range from 3.4875 to 30.3665, from 3.9827 to 20.6543, and from 2.5212 to 8.4563, respectively. The fluctuations also show a tendency to become relatively calm. This is rational because the number of accepted ad hoc containers not only depends on the number of remaining vacant slots but also on the demand for ad hoc containers. When the levels of contract and empty containers are low, and the number of remaining vacant slots is sufficiently large, the number of accepted ad hoc containers mainly depends on the demand for ad hoc containers. Therefore, if the demand for ad hoc containers at different levels fluctuates significantly, the expected profit of ad hoc containers fluctuates intensely (e.g. from 3.4875 to 30.3665). When the levels of contract and empty containers increase, the number of remaining vacant slots decreases, but the impact on the number of accepted ad hoc containers increases, which means that the expected profit of ad hoc containers at different levels of demand fluctuates less intensely. Simultaneously, for different levels of ad hoc containers, contract containers have different effects on the expected profit of ad hoc containers. For example, when ad hoc containers are at a low level, although their expected profit of ad hoc containers varies under different levels of contract and empty containers, their values always fluctuate around 3.6166, and therefore the variance is also small (see Table 5). However, when ad hoc containers are at a medium/high level, the impacts of the contract and empty containers on the expected profit of ad hoc containers are much larger, resulting in fluctuations from 3.4704 to 26.4582 and from 6.7509 to 28.1604, respectively. Similarly, when the demand for ad hoc containers is low, or even lower than the number of remaining vacant slots, the number of ad hoc containers mainly depends on demand, and consequently, the expected profit of ad hoc containers shows slight fluctuations regardless of the levels of contract and empty containers. However, when the demand for ad hoc containers is medium/high, the number of ad hoc containers is constrained by the number of remaining vacant slots and their demand, and consequently, the expected profits of ad hoc containers show intense fluctuations. 4) Although an ad hoc container has a higher freight rate than a contract container, this does not mean that leaving more slots for ad hoc containers is always the best choice for the container shipping company. As can be seen from Table 4 that the maximum total profit of the container shipping company is 53.8495, when the levels of contract, empty, and ad hoc containers are low, low, and high, respectively. However, the variance in the total profit at low levels of contract and empty containers is quite large (see Table 6). Therefore, this slot allocation strategy is not the best for a risk averse container shipping company.

Table 5 Average profit/cost and variance in each level of different containers (unit: million US dollars)

	Contractual container		Empty container		Ad hoc container	
	Mean	Variance	Mean	Variance	Mean	Variance
Low	24.5302	16.4682	0.0658	0.0007	3.6166	0.2631
Medium	33.4814	18.7178	0.2275	0.0011	15.2622	56.0151
High	45.1454	3.9885	0.6451	0.0095	17.6148	79.9449

Table 6 Mean and variance of total profit at different levels of different types of containers

Types	Contractual container			Empty container			Ad hoc container		
Levels	Low	Medium	High	Low	Medium	High	Low	Medium	High
Mean	42.0927	46.1001	50.5103	45.8035	47.7117	45.1879	37.1516	49.7971	51.7546
Variance	147.4883	82.2895	0.2225	111.3816	59.1331	95.4765	121.0602	5.7366	1.7206

6.5 Demand Information Analysis

The uncertainty of spot market demand is the basis for us to formulate the proposed slot allocation problem as a two-stage stochastic programming model. That is, the proposed slot allocation model is lacks perfect information on spot demand, which indicates that spot demand information has a significant impact on modeling. Therefore, we now analyze the value of spot demand information and compare the expected shipping profits with and without perfect spot demand information. We first compute the expected shipping profit without perfect spot demand information. Consequently, we still assume that spot demand follows a log-normal distribution.

Then, we obtain the corresponding expected profit by implementing the solution algorithm, denoted by Z^{NPI}.

As for the expected shipping profit with perfect spot demand information, we first set three levels for spot demand: high, medium, and low, where spot demand at medium level is equal to the mean of the log-normal distribution, and "low" and "high" indicate that spot demand is one standard deviation below and above the mean of the log-normal distribution, respectively. The spot demand at each level is known. Then, we can solve the slot allocation model with the known spot demand at each level to obtain the corresponding shipping profits. Finally, we can obtain the expected profit by averaging these shipping profits, denoted by Z^{PI}. Consequently, the expected value of perfect information, denoted by $EVPI$, is given by $EVPI = Z^{PI} - Z^{NPI}$, as depicted in Figure 5.

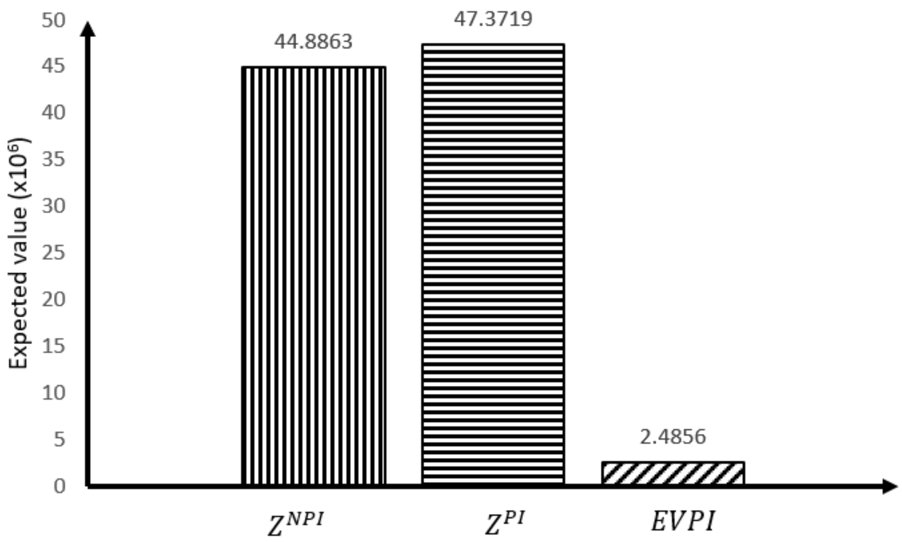

Figure 5. Expected value of depot demand information

Perfect spot demand information is expected to have a positive impact on shipping profit. Figure 5 specifically validates this expectation, and we can see that perfect spot demand information can increase shipping profit at a rate of 5.5%, which reflects the value of perfect information. If ad hoc demand is known and certain, the container shipping company will obtain maximum profit. Therefore, information about ad hoc containers can significantly help the container shipping company to increase its shipping profit.

7. Conclusions

This paper proposes an interesting container slot allocation problem arising from liner container shipping services. The proposed problem is formulated as the 2SSMINP model. The greatest difficulty in solving the 2SSMINP model lies in dealing with the expected recourse function, which is only implicitly defined and depends on the first-stage decisions, typically involving optimization problems embedded in expectations. To solve the model efficiently, we develop a solution algorithm integrating the SAA method, the Lagrangian relaxation and dual decomposition techniques, the surrogate sub-gradient, and the B&B algorithm. The devleoped model and solution algorithm are assessed using a case study, and a series of numerical experiments with different parameter values are implemented to analyze their sensitivity. The gaps between the lower and upper bounds emerge as small, indicating that these methods are effective. It is also found that the variability of the uncertain parameters has a significant effect on the solutions. In particular, the value of demand information is tested and the results show that knowledge of the spot market can increase the profit of the shipping company.

We intend to extend the container slot allocation problem from a specific route to a network, in addition to considering container transshipment. The models and solution algorithms involved are expected to be much more complex when considering container transshipment operations. Another possible extension is considering other RM strategies in the context of the slot allocation problem, such as overbooking control strategies to manage container booking acceptance and slot allocation.

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APPENDIX

Proof of Proposition 2. For any problem P in set $\mathbb{P}$ defined in the B&B algorithm, let $\vartheta_P = \min\left\{\bar{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)} - \tilde{p}_{s^*t^*n}^{(i^*,j^*)^*}, \tilde{p}_{s^*t^*n}^{(i^*,j^*)^*} - \underline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)}\right\}$, where (i^*,j^*), s^*, and t^* satisfy Eq. (37), $\tilde{p}_{s^*t^*n}^{(i^*,j^*)^*}$ belongs to the optimal solution of problem P, and $\bar{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)}$ and $\underline{\tilde{p}}_{s^*t^*n}^{(i^*,j^*)}$ are the upper and lower bounds of $\tilde{p}_{s^*t^*n}^{(i^*,j^*)}$, respectively. Then, according to Eq. (33) and Eq. (34), we have the following inequalities:

$$
\begin{aligned}
\tilde{y}_{s^*t^*n}^{\left(i^*,j^*\right)^*} - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} &\le \left(\bar{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} + \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\underline{\tilde{x}}_{s^*t^*n}^{\left(i^*,j^*\right)} - \bar{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)}\underline{\tilde{x}}_{s^*t^*n}^{\left(i^*,j^*\right)}\right) - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} \\
&= \left(\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} - \underline{\tilde{x}}_{s^*t^*n}^{\left(i^*,j^*\right)}\right)\left(\bar{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)} - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\right) \\
&\le CAP\left(\bar{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)} - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\right)
\end{aligned}
\tag{40}
$$

and

$$
\begin{aligned}
\tilde{y}_{s^*t^*n}^{\left(i^*,j^*\right)^*} - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} &\le \left(\underline{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} + \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\bar{\tilde{x}}_{s^*t^*n}^{\left(i^*,j^*\right)} - \underline{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)}\bar{\tilde{x}}_{s^*t^*n}^{\left(i^*,j^*\right)}\right) - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} \\
&= \left(\bar{\tilde{x}}_{s^*t^*n}^{\left(i^*,j^*\right)} - \tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\right)\left(\tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*} - \underline{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)}\right) \\
&\le CAP\left(\tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*} - \underline{\tilde{p}}_{s^*t^*n}^{\left(i^*,j^*\right)}\right)
\end{aligned}
\tag{41}
$$

Therefore, it follows that

$$
\tilde{y}_{s^*t^*n}^{\left(i^*,j^*\right)^*} - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} \le CAP \times \vartheta_P \tag{42}
$$

From Eq. (37), we know that

$$
\tilde{y}_{stn}^{(i,j)^*} - \tilde{p}_{stn}^{(i,j)^*}\tilde{x}_{stn}^{(i,j)^*} \le \tilde{y}_{s^*t^*n}^{\left(i^*,j^*\right)^*} - \tilde{p}_{s^*t^*n}^{\left(i^*,j^*\right)^*}\tilde{x}_{s^*t^*n}^{\left(i^*,j^*\right)^*} \quad \forall (i,j) \in \mathcal{W}, s \in \mathcal{F} \cup \mathcal{R}, t \in \mathcal{T} \tag{43}
$$

Consequently, for problem P, we have

$$
MILPLR_n^*(\boldsymbol{\lambda})\big|_{P^*} - LR_n(\boldsymbol{\lambda})\big|_{P^*} = \sum_{t\in\mathcal{T}}\sum_{(i,j)\in\mathcal{W}}\sum_{s\in\mathcal{F}\cup\mathcal{R}}\left(\tilde{y}_{stn}^{(i,j)^*} - \tilde{p}_{stn}^{(i,j)^*}\tilde{x}_{stn}^{(i,j)^*}\right) \le \tilde{\tilde{N}} \times CAP \times \vartheta_P \tag{44}
$$

For the root problem P_0 in problem set $\mathfrak{P}$, the B&B algorithm ensures that

$$
MILPLR_n^*(\boldsymbol{\lambda})\big|_{P_0^*} - LR_n(\boldsymbol{\lambda})\big|_{P_0^*} \ge MILPLR_n^*(\boldsymbol{\lambda})\big|_{P_0^*} \times \varepsilon \tag{45}
$$

Combining Eq. (44) and Eq. (45), we have

$$
\vartheta_P \ge \frac{MILPLR_n^*(\boldsymbol{\lambda})\big|_{P_0^*} \times \varepsilon}{\tilde{\tilde{N}} \times CAP} \tag{46}
$$

Therefore, the longest times for problem branching can be estimated as

$$
N = \sum_{t\in\mathcal{T}}\sum_{(i,j)\in\mathcal{W}}\sum_{s\in\mathcal{F}\cup\mathcal{R}}\left\lceil \frac{\bar{\tilde{p}}_{stn}^{(i,j)} - \underline{\tilde{p}}_{stn}^{(i,j)}}{\vartheta_P} \right\rceil \le \sum_{t\in\mathcal{T}}\sum_{(i,j)\in\mathcal{W}}\sum_{s\in\mathcal{F}\cup\mathcal{R}}\left\lceil \frac{\left(\bar{\tilde{p}}_{stn}^{(i,j)} - \underline{\tilde{p}}_{stn}^{(i,j)}\right) \times \tilde{\tilde{N}} \times CAP}{MILPLR_n^*(\boldsymbol{\lambda})\big|_{P_0^*} \times \varepsilon} \right\rceil < +\infty \tag{47}
$$

As the branching times are finite, the B&B algorithm must terminate after a finite number of iterations. □

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School of Economics and Management
Department of Management Science and Engineering
Wuhan University

Statement of Contribution/Potential Impact

This paper studies a slot allocation problem for a liner shipping service. A liner containership provides a regular shipping service on a fixed itinerary and at a fixed schedule for shippers. The liner containership may be not fully loaded, in which case it loses the revenue of potential shippers. This paper thus divides shippers into two classes: contractual shippers and spot shippers. Contractual shippers sign contracts with a liner container shipping company and promise Minimum Quantity Commitments, so that the liner container shipping company can obtain steady revenue. The remaining slots of the containership are open to spot shippers, so that the liner container shipping company can obtain ad hoc revenue. We formulate the slot allocation problem proposed in this paper as a two-stage stochastic nonlinear integer-programming model. We then use the sample average approximation based on Lagrangian relaxation and dual decomposition to solve the model. Finally, we use a case study to evaluate the applicability of the proposed model and the performance of the proposed solution algorithm.

The contributions of this study are summarized as follows:

1) This paper contributes to the literature by proposing a new slot allocation problem integrating the issues of uncertainty of shipping demand, empty container repositioning, and pricing of freight rate.

2) The proposed slot allocation problem for a liner container shipping service is formulated as a two-stage stochastic nonlinear integer-programming (2SSNIP) model. As this model is intractable by the solution methods proposed in the existing literature, this paper designs a solution algorithm to solve the proposed 2SSNIP model, and its convergence has been mathematically proved. The methodology used in the solution algorithm is the most significant contribution of this paper.

3) A number of experiments have been implemented on the proposed model and solution algorithm. The computational results verify the applicability of the proposed model and the efficiency of the solution algorithm, and evaluate the effect on profit increase of the proposed model.