TRANSACTIONS ON EMERGING TELECOMMUNICATIONS TECHNOLOGIES
Trans. Emerging Tel. Tech. 2014; **00**:1–7
DOI: 10.1002/ett

SHORT RESEARCH LETTER

Energy efficiency optimization in full-duplex relay systems

Yunxiang Jiang[1], Francis C.M. Lau[1,2], Hongbin Chen[3], Feng Zhao[3]

1 Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong
2 The Hong Kong Polytechnic University Shenzhen Research Institute, Shenzhen, China
3 Key Laboratory of Cognitive Radio and Information Processing (Guilin University of Electronic Technology), Ministry of Education, Guilin 541004, China

ABSTRACT

Energy efficiency of relay communications has attracted much interest recently. Most research efforts have focused on half-duplex systems. As there has been significant progress in practical implementation of self-interference cancellation, full-duplex systems will have a promising potential in the near future. In this letter, energy efficiency of a full-duplex relay system under the total power constraint and fixed circuitry power consumption is studied. An optimization problem is formulated towards maximizing the system energy efficiency. Unfortunately, this problem is non-trivial and cannot be solved by conventional fractional programming methods, such as the Dinbelbach's method. To resolve this issue, an algorithm called sequential parametric convex approximation-Dinbelbach is proposed in this letter. Simulation results show that the proposed algorithm can converge to the global optimum very quickly. Copyright © 2014 John Wiley & Sons, Ltd.

*Correspondence
Hongbin Chen, Key Laboratory of Cognitive Radio and Information Processing (Guilin University of Electronic Technology), Ministry of Education, Guilin 541004, China

E-mail: chbscut@guet.edu.cn

1. INTRODUCTION

It is well known that full-duplex (FD) transmission can achieve higher throughput than half-duplex transmission theoretically [1]. However, its applications in wireless communications were rather limited due to a variety of practical implementation challenges. The major problem is the so called self-interference. It stems from simultaneous transmission and reception in the same frequency band, and causes the strong transmitted signal to couple directly with the receiving path. Recently, some practical self-interference cancellation schemes have been developed to effectively cancel self-interference in FD transmission [2–6]. These progresses inspire theories and applications of FD transmission, especially in FD relay systems.

Meanwhile, energy efficiency (EE) has also attracted much interest in the telecommunications community [7–17]. Increasing EE has become an important and urgent task. In relay communications, recent research efforts have focused on EE in half-duplex relay systems [8, 14–16, 18–20]. In [8, 15, 16], the main algorithms used to optimize EE are based on the Dinbelbach's method [21], which has been commonly used to solve fractional programming problems. In [14, 17], the authors have proposed a dual method that maximizes EE in amplify-and-forward relay systems. To the best of our knowledge, there is no open literature focusing on EE of FD relay systems.

In this letter, a FD relay system, in which a relay helps information delivery from the source to the destination in FD manner, is considered. The decode-and-forward relaying protocol is adopted since the relay has to decode the signals in order to perform self-interference cancellation. First, an optimization problem is formulated to maximize the EE in the FD relay system. The optimization problem is non-trivial and cannot be solved by conventional fractional programming methods, such as the Dinbelbach's method [21]. Then, the optimization problem is converted into an equivalent problem that can be further decomposed into two subproblems. The first subproblem can be solved by the Dinbelbach's method directly. The second subproblem is not quasi-concave because of the non-convex constraint, which cannot be solved by the Dinbelbach's method directly. An effective algorithm called sequential parametric convex approximation (SPCA) [22] is utilized to iteratively approach the optimum value at each iteration of the Dinbelbach's method. The joint algorithm is called SPCA-Dinbelbach.

Prepared using ettauth.cls [Version: 2012/06/19 v2.10]

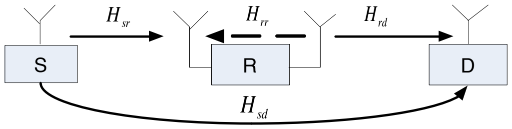

Figure 1. A full-duplex relay system. Solid lines denote information transmission and the dashed line denotes self-interference.

2. SYSTEM MODEL

We consider a three-node cooperative communication system in which the source S communicates with the destination D via a relay R, as shown in Fig. 1. The source and destination are equipped with a single antenna, while the relay is equipped with one receiving antenna and one transmitting antenna (can receive and transmit signals simultaneously). Let $x_s(t)$ and $x_r(t)$ denote the signals transmitted from S and from the transmitting antenna of R at the time instant t, respectively, where the average powers of transmitted symbols equal to 1, i.e., $E[x_s(t)'x_s(t)] = E[x_r(t)'x_r(t)] = 1$. Let H_{sr} denote the channel coefficient of the link between the source S and the relay R, H_{sd} denote the channel coefficient of the link between the source S and the destination D, and H_{rd} denote the channel coefficient of the link between the relay R and the destination D. The transmission powers of the source S and the relay R are denoted as P_s and P_r, respectively. Then, the received signals at the receiving antenna of R and D, denoted by $y_r(t)$ and $y_d(t)$ respectively, can be written as

$$y_r(t) = \sqrt{P_s} H_{sr} x_s(t) + \sqrt{P_r} H_{rr} x_r(t) + n_r(t), \quad (1)$$
$$y_d(t) = \sqrt{P_r} H_{rd} x_r(t) + \sqrt{P_s} H_{sd} x_s(t) + n_d(t), \quad (2)$$

where $n_r(t)$ and $n_d(t)$ are the additive white Gaussian noises at the relay R and the destination D, respectively, and follow $\mathcal{CN}(0, \sigma_z^2)$. It is assumed that the channel coefficient between node i and node j is $H_{ij} \sim \mathcal{CN}(0, \Omega_{i,j})$. Here $\Omega_{i,j}$ is determined by the path-loss, i.e., $\Omega_{i,j} = (d_0/d_{ij})^m$, where m is the path-loss exponent, d_{ij} is the distance between node i and node j, and d_0 is the reference distance. The self-interference channel coefficient H_{rr} is modeled as $\sqrt{\beta} H_{SI}$, where β is the self-interference attenuation and $H_{SI} \sim \mathcal{CN}(0, 1)$*. The effective channel gains are then defined as $G_{ij} = |H_{ij}|^2/\sigma_z^2$, $i \in \{s, r\}$, and $j \in \{r, d\}$.

*Before analog domain cancellation, the self-interference channel has a strong line-of-sight component. So it can be modeled as a Ricean distribution with a large K-factor. It is shown experimentally in [23] that after applying a sufficiently large analog domain cancellation, the strong line-of-sight component is attenuated, resulting in a Ricean distribution with a small K-factor or a Rayleigh distribution.

The decode-and-forward relaying protocol is adopted at the relay. Then, the achievable rate is given by [24, eq.7]

$$R_{\mathrm{DF}}(P_s, P_r) = \min\{R_{\mathrm{DF},1}(P_s, P_r), R_{\mathrm{DF},2}(P_s, P_r)\} \quad (3)$$

where $R_{\mathrm{DF},1}(P_s, P_r)$ and $R_{\mathrm{DF},2}(P_s, P_r)$ are defined as

$$R_{\mathrm{DF},1}(P_s, P_r) = \log_2\left(1 + \frac{P_s G_{sr}}{1 + \beta P_r G_{rr}}\right), \quad (4)$$
$$R_{\mathrm{DF},2}(P_s, P_r) = \log_2(1 + P_s G_{sd} + P_r G_{rd}). \quad (5)$$

However, if $G_{sd} \geq G_{sr}$, the achievable rate in (3) boils down to the rate of the direct transmission between the source and the destination, which is given by

$$R_{\mathrm{D}}(P_s) = \log_2(1 + P_s G_{sd}). \quad (6)$$

Hence, the overall achievable rate can be re-written as:

$$R(P_s, P_r) = \max\{R_{\mathrm{DF}}(P_s, P_r), R_{\mathrm{D}}(P_s)\}. \quad (7)$$

The EE of the FD relay system is studied, and is defined as

$$U_{\mathrm{eff}} = R(P_s, P_r)/P_T(P_s, P_r) \quad [\text{bits/Joule}], \quad (8)$$

where the total power consumption P_T is calculated by

$$P_T(P_s, P_r) = P_s + P_r + P_c. \quad (9)$$

In (9), P_c denotes the energy consumed by the circuitry of the whole relay system, and is assumed to be constant. Assuming that the total power constraint of the source and relay is given by $P_s + P_r \leq P_{max}$, the optimal transmission powers P_s and P_r will be obtained by solving

Optimization Problem (P1)

$$\begin{aligned} \max_{P_s, P_r} \quad & U_{\mathrm{eff}}(P_s, P_r) \\ \text{s.t.} \quad & C1: P_s + P_r \leq P_{max}, \\ & C2: P_s, P_r \geq 0. \end{aligned} \quad (10)$$

3. ALGORITHM

In this section, firstly the optimization problem **P1** will be transformed into an equivalent problem which can be further decomposed into two subproblems. Then, the subproblems will be solved one by one. In the optimization problem **P1**, the variables to be optimized are P_s and P_r. The objective function is a nonlinear fractional function, in which the numerator is a max-min function. Thus, this problem is very difficult to solve directly. Then, the optimization problem **P1** will be transformed into a simpler and equivalent problem, which can be solved. Assuming all the channel state information is perfectly known at the source, the source performs the optimization procedure in a centralized way, and then transmits the value of the optimized power to the relay.

Trans. Emerging Tel. Tech. 2014; **00**:1–7 © 2014 John Wiley & Sons, Ltd.
DOI: 10.1002/ett
Prepared using ettauth.cls

Algorithm 1 Dinkelbach's Method

1: Set $I_{\max}$ (maximum number of iterations),
   $\epsilon_o > 0$ (convergence tolerance),
2: $q_1 = 0$ and $q_0 = 1$.
3: $i \leftarrow 1$.
4: **while** $q_i - q_{i-1} > \epsilon_o$ and $i < I_{\max}$ **do**
5: Solve $\max\limits_{P_s}\{F(q_i) = R(P_s) - q_i P_T(P_s)\}$ subject to $0 \le P_s \le P_{max}$ to obtain the optimal solution P_s.
6: $q_i \leftarrow R(P_s)/P_T(P_s)$.
7: $i \leftarrow i + 1$.
8: **end while**
9: **return**

Proposition 1: The solution to the optimization problem **P1** is equivalent to the solution to the optimization problem **P2**, as described bellow.

Equivalent Optimization Problem (P2)

$$\mathbf{P2.1}: \max_{P_s} \quad R_{\mathrm{D}}(P_s)/P_T(P_s, 0) \tag{11}$$
$$\text{s.t.} \quad 0 \le P_s \le P_{max};$$
$$\mathbf{P2.2}: \max_{P_s, P_r} \quad R_{\mathrm{DF},2}(P_s, P_r)/P_T(P_s, P_r) \tag{12}$$
$$\text{s.t.} \quad P_s + P_r \le P_{max}, \tag{13}$$
$$P_s, P_r \ge 0, \tag{14}$$
$$R_{\mathrm{DF},1}(P_s, P_r) \ge R_{\mathrm{DF},2}(P_s, P_r), \tag{15}$$

where $R_{\mathrm{DF},1}(P_s, P_r)$, $R_{\mathrm{DF},2}(P_s, P_r)$, and $R_{\mathrm{D}}(P_s)$ are defined in (4), (5), and (6), respectively. If $G_{sd} \ge G_{sr}$, solve the subproblem **P2.1**; otherwise, solve the subproblem **P2.2**.

Proof: Please refer to the Appendix 1.

3.1. Solution to Problem P2.1

It is not hard to prove that the objective function of the problem **P2.1** is a quasi-concave function with respect to P_s, and the constraint is affine. Thus, we can directly use the traditional Dinkelbach's method to solve the problem seen in **Algorithm 1** . The detail of the Dinkelbach's method is given in the Appendix 2. It is worth to mention that at step 5 of **Algorithm 1**, we can derive a closed-form solution in each iteration. As follows, by applying the Karush-Kuhn-Tucker (KKT) conditions [25], it can be obtained that

$$P_s = \left[\frac{1}{q_i} - \frac{1}{G_{sd}}\right]_0^{P_{max}} \tag{16}$$

where $[*]_0^{P_{max}} = \min(P_{max}, \max(0, *))$ is the box constraint.

3.2. Solution to Problem P2.2

Due to the non-convex constraint (15) , the problem **P2.2** is not a quasi-concave problem. It is not possible to use the traditional Dinkelbach's method because that convex optimization algorithm is not valid at step 5 of the Dinkelbach's Method in **Algorithm 1**. Specifically, the constraint (15) in the problem **P2.2** is extended as

$$\alpha_1 P_s + \alpha_2 P_r + \alpha_3 P_r P_s + \alpha_4 P_r^2 \le 0, \tag{17}$$

where $\alpha_1 = G_{sd} - G_{sr}$, $\alpha_2 = G_{rd}$, $\alpha_3 = \beta G_{sd} G_{rr}$, and $\alpha_4 = \beta G_{sd} G_{rr}$.

Note that (17) is obviously a non-convex function. To deal with the non-convex constraint, a SPCA method is utilized to iteratively solve the problem. Herein, we give the key lemma for the SPCA method.

Lemma 1: Considering an optimization problem with non-convex constraint $g(\boldsymbol{x})$. If the function $G(\boldsymbol{x}, \lambda)$ have the following properties: i) for any $\boldsymbol{x}$, $g(\boldsymbol{x}) \le G(\boldsymbol{x}, \lambda), \lambda > 0$; ii) for a given feasible point $\boldsymbol{x_0}$, there exists a $\lambda = \psi(\boldsymbol{x_0})$ satisfying $g(\boldsymbol{x}) = G(\boldsymbol{x}, \lambda)$ and $\nabla g(\boldsymbol{x}) = \nabla G(\boldsymbol{x}, \lambda)$, then $G(\boldsymbol{x}, \lambda)$ can replace $\lambda^{(l)}$ by $\psi(\boldsymbol{x}^{(l-1)})$ such that the relaxed problem with convex constraint $G(\boldsymbol{x}, \lambda)$ is solved iteratively until convergence. The iterative solution would finally converge to a KKT point.

Proof: see [22].

It is observed in the constraint (15) that the unique effective part for non-convexity is $P_s P_r$. Thus, one only need to find a convex upper-bound to approach $P_s P_r$ iteratively. To do this, the following function is defined:

$$G([P_s, P_r], \lambda) = \frac{1}{2\lambda} P_s^2 + \frac{\lambda}{2} P_r^2, \tag{18}$$

which is a convex function used to over-estimate $P_r P_s$. Additionally, $\lambda^{(l+1)}$ is updated by $P_s^{(l)}/P_r^{(l)}$ iteratively. It is very easy to verify that the function $G([P_s, P_r], \lambda)$ satisfies **Lemma 3** (see Page 5).

Replacing $P_r P_s$ in (17) by $G([P_s, P_r], \lambda)$, the relaxed constraint is expressed as

$$\alpha_1 P_s + \alpha_2 P_r + \frac{\alpha_3}{2\lambda} P_s^2 + \frac{\alpha_3 \lambda}{2} P_r^2 + \alpha_4 P_r^2 \le 0, \tag{19}$$

which can be proved as a convex constraint by the Hessian function [25].

The proposed SPCA-Dinkelbach algorithm is depicted as **Algorithm 2**. The algorithm is a dual iterative algorithm, in which the outer iteration is based on the Dinkelbach method and the inner iteration is based on SPCA. Although SPCA is converged to a solution satisfying the KKT conditions (i.e., local optimum is achieved), we conclude through extensive numerical simulations that the solution is in fact the global optimal.

4. SIMULATION RESULTS

This section presents the results of applying the proposed SPCA-Dinkelbach algorithm to the FD relay system. The reference distance is $D_0 = 1$ m. The distance

Algorithm 2 SPCA-Dinkelbach Algorithm

1: Set $I^0_{\max}$ (maximum number of outer iterations), $\epsilon_o > 0$ (convergence tolerance of outer iterations),
2: Set $I^l_{\max}$ (maximum number of inner iterations), $\epsilon_l > 0$ (convergence tolerance of inner iterations),
3: $q_1 = 0$ and $q_0 = 1$,
4: $\lambda^{(1)} = 0$ and $\lambda^{(0)} = 1$.
5: $i \leftarrow 1$.
6: **while** $q_i - q_{i-1} > \epsilon_o$ and $i < I_{\max}$ **do**
7:   **while** $|\lambda^{(l)} - \lambda^{(l-1)}| > \epsilon_l$ and $l < I^l_{\max}$ **do**
8:     Using the standard convex optimization (e.g., interior-point method) to solve the problem $\max\limits_{P_s, P_r} F(q_i) = R_{\mathrm{DF},2}(P_s, P_r) - q_i P_T(P_s, P_r)$, s.t. (13), (14), and (19). Obtain the optimal transmission powers $P_s^{(l)}$ and $P_r^{(l)}$.
9:     $\lambda^{(l+1)} \leftarrow P_s^{(l)}/P_r^{(l)}$.
10:     $l \leftarrow l + 1$.
11:   **end while**
12:   $q_i \leftarrow R(P_s^{(l-1)}, P_r^{(l-1)})/P_T(P_s^{(l-1)}, P_r^{(l-1)})$.
13:   $i \leftarrow i + 1$.
14: **end while**
15: **return**

between the source and the destination is 10 m, and the relay is at the mid-point of the line connecting the source and the destination. The path-loss exponent is $m = 3$ and the noise power is $\sigma_z^2 = 10^{-6}$. The power consumption of circuitry is P_c = 20 dBm [26]. The convergence tolerance ϵ_o and ϵ_l are set as $\epsilon = 10^{-5}$. The results are retrieved by averaging over 1000 different channel realizations. We assume that the optimization process would be completed within one channel realization such that adaptive power can be optimally assigned from the source to the relay.

4.1. Convergence of the Proposed Algorithm

Fig. 2 illustrates the convergence behavior of the proposed SPCA-Dinkelbach algorithm. As seen in Fig. 2, the proposed algorithm converges to the optimal value within five outer iterations. We also study the number of inner iterations required during the second outer iteration. We find that the SPCA method converges within five inner iterations. The result demonstrates that the proposed algorithm indeed obtains the global optimal solution, even though the SPCA method only reaches the KKT conditions theoretically. We also find that the convergence speed is not highly related to β (the self-interference attenuation factor) in both inner and outer iterations. It is trivial to obtain that the computational complexity of the exhaustive search algorithm is proportional to $1/\epsilon = 10^5$. It is hard to derive the computational complexity of the SPCA-Dinkelbach method directly. However, based on the simulation results seen in Fig. 2, we can see that the maximal number of iterations of the outer iteration and the inner iteration in the SPCA-Dinkelbach method are both fixed as 4. In addition, the computational complexity of interior-point method to solve the convex problem in step 8 of algorithm 2 is $n^{3.5}\log(1/\epsilon)$ [25], in which n represents the number of optimized variables. To sum up, in this our optimization problem, the approximate computational complexity is $4 * 4 * 2^{3.5}\log(1/\epsilon) = 905$ which is much lower than that of exhaustive search algorithm.

4.2. Effects of P_{max} and β on Average EE

Fig. 3 illustrates the average EE against P_{max} for EE-maximization and rate-maximization schemes with different β. The EE-maximization scheme is implemented by our proposed SPCA-Dinkelbach algorithm. It can be observed in Fig. 3 that the average EE increases upon increasing P_{max}, and remains unchanged when P_{max} reaches a certain threshold. Specifically, when P_{max} is larger than 25 dBm, the average EE no longer increases under our simulation settings. It can be also obtained that the average EE increases as β decreases. When β equals -70 dBm and -90 dBm, the average EE under the settings are nearly the same. It implies that when β is very low and the total power is high (in our simulation $\beta = -70$ dB and $P_{max} = 25$ dBm), the optimal average EE does not change. In order to further exploit the performance improvement of EE-maximization scheme, we consider the rate-maximization scheme as a comparison. The rate-maximisation scheme is achieved by the similar procedure in Algorithm 2 except for choosing $q_i = 0$, which means that the outer iteration of **Algorithm 2** is not needed. In the rate-maximization scheme, when P_{max} increases, the average EE first increases and then decreases. As the metric is average EE, the EE-maximization scheme is always better than or equal to the rate-maximization scheme. Interestingly, at low P_{max}, the results are the same except for $\beta = -10$ dB. The reason is that at low P_{max}, the total power constraint is always satisfied with equality, i.e., the source and relay would utilize the total power. Then, according to the expression of energy consumption in (9), the consumed power is always constant, i.e., $P_{max} + P_c$, which is also verified by our simulations. Due to the limited space, the corresponding simulation results are not given. Thus, we can conclude that the average EE of the EE-maximization scheme is equivalent to that of rate-maximization scheme.

4.3. Effects of P_{max} and β on Average Achievable Rate

We then compare the EE-maximization scheme with the rate-maximization scheme for the performance metrics of EE and achievable rate. In Fig. 4, it is obvious that the rate-maximization scheme can have better achievable rate than the EE-maximization scheme. As a sequel, The EE-maximization scheme has a performance degradation in terms of rate. Besides, it can be obtain that the EE-maximization scheme keeps constant achievable rate in

Trans. Emerging Tel. Tech. 2014; **00**:1–7
DOI: 10.1002/ett
Prepared using ettauth.cls

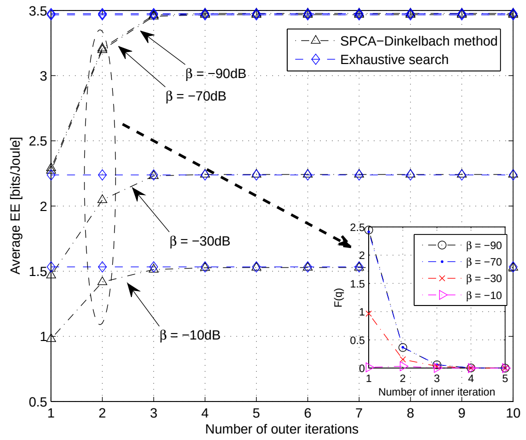

Figure 2. Convergence of the SPCA-Dinkelbach algorith (P_{max} = 30 dBm).

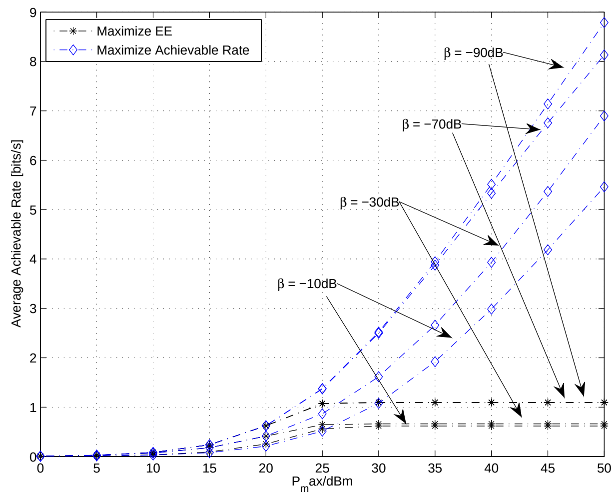

Figure 4. Average achievable rate against P_{max} for the scheme of maximizing EE and maximizing achievable rate under different settings of β.

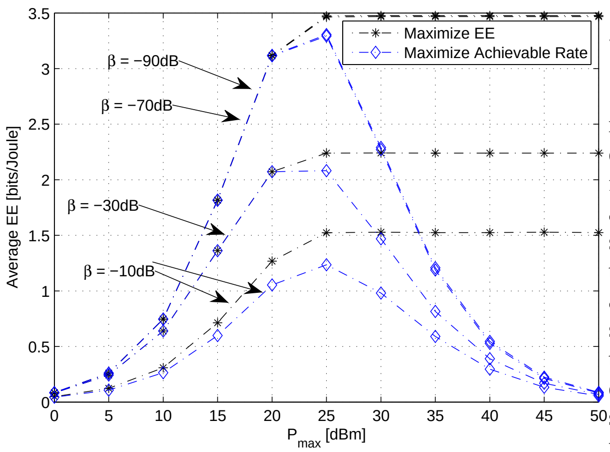

Figure 3. Average EE against P_{max} for the scheme of maximizing EE and maximizing achievable rate under different settings of β.

high SNR region, specifically when the SNR is larger than 25dB. Combining with the results in Fig. 3, we can conclude that when the maximal total transmit power is increased, the actual total transmit power will be firstly increased and then fixed, which is similar as the performance metric of EE. Interestingly, we find that the achievable rate in $\beta = -10dB$ and $\beta = -30dB$ are almost the same while the EE in $\beta = -30dB$ is larger than that in $\beta = -10dB$. This fact explains that the higher the self-interference is, the more power would be consumed. However, when the self-interference is very low, i.e., $\beta = -70dB$ and $\beta = -90dB$, the performance of achievable rate and EE are nearly the same.

5. CONCLUSION

In this paper, energy efficiency of a full-duplex relay system under the total power constraint and fixed circuitry power consumption has been studied. The formulated optimization problem is transformed into an equivalent problem which is decomposed into two subproblems. The first subproblem is solved by the traditional Dinbelbach's method. Then, a dual iterative algorithm called SPCA-Dinbelbach method is used to solve the second subproblem. Simulation results show that the proposed algorithm can converge to the global optimum at different levels of self-interference. They also show that when the total power constraint of source and relay is large, optimization the energy efficiency will reduce the maximum achievable rate.

APPENDIX

1. Proof of Proposition 1

It is straightforward that if $G_{sd} \geq G_{sr}$, $R(P_s, P_r) = R_{\text{D}}(P_s)$ according to (7). Thus, the problem can be simplified to

$$U_{\text{eff}} = \frac{R_{\text{D}}(P_s)}{P_T(P_s, P_r)} = \frac{R_{\text{D}}(P_s)}{P_T(P_s, 0)}. \tag{20}$$

As a result, we need to solve P2.1 when $G_{sd} \geq G_{sr}$.

When $G_{sd} < G_{sr}$, we have

$$\begin{aligned} U_{\text{eff}} &= \frac{R_{\text{DF}}(P_s)}{P_T(P_s, P_r)} \\ &= \frac{\min\{R_{\text{DF},1}(P_s, P_r), R_{\text{DF},2}(P_s, P_r)\}}{P_T(P_s, P_r)}. \end{aligned} \tag{21}$$

We firstly give some lemmas before proving P2.2 which are obvious.

Lemma 2 $R_{\mathrm{DF},1}(P_s, P_r)$ is an increasing function with respect to (w.r.t.) P_s, and a decreasing function w.r.t. P_r.

Lemma 3: i) $P_T(P_s, P_r)$ is an increasing function w.r.t. P_s and P_r; ii) $P_T(P_s, P_r)$ is also a linear function w.r.t. P_s, P_r.

We are now ready to prove P2.2.

Proof of P2.2: We prove it by self-contradiction. For the sake of notational simplicity, we define the constraint set as $\mathcal{F}$. We assume that $\{P_s^*, P_r^*\} \in \mathcal{F}$ as the optimal policy with the following constraint:

$$R_{\mathrm{DF},1}(P_s^*, P_r^*) < R_{\mathrm{DF},2}(P_s^*, P_r^*). \tag{22}$$

The optimal energy efficiency U_{eff}^* is therefore expressed as

$$\begin{aligned} U_{\mathrm{eff}}^* &= \frac{\min\{R_{\mathrm{DF},1}(P_s^*, P_r^*), R_{\mathrm{DF},2}(P_s^*, P_r^*)\}}{P_T(P_s^*, P_r^*)} \\ &= \frac{R_{\mathrm{DF},1}(P_s^*, P_r^*)}{P_T(P_s^*, P_r^*)}. \end{aligned} \tag{23}$$

According to Lemma 2 and 3, $R_{\mathrm{DF},1}(P_s^*, P_r^*)$ is a decreasing function of P_r^* and $P_T(P_s^*, P_r^*)$ is an increasing function of P_r^*. Consequently, based on (23), U_{eff}^* should be optimal when $P_r^* = 0$. Substituting $P_r^* = 0$ into (4) and (5) gives

$$R_{\mathrm{DF},1}(P_s^*, P_r^*) = \log_2(1 + P_s^* G_{sr}), \tag{24}$$

$$R_{\mathrm{DF},2}(P_s^*, P_r^*) = \log_2(1 + P_s^* G_{sd}). \tag{25}$$

Since it is given that $G_{sd} < G_{sr}$, the above results indicate $R_{\mathrm{DF},1}(P_s^*, P_r^*) > R_{\mathrm{DF},2}(P_s^*, P_r^*)$, which is contradictory to our assumption in (22). In conclusion, under any P_r^* in the optimal solution, $R_{\mathrm{DF},1}$ is larger than $R_{\mathrm{DF},2}$. Therefore, we prove that P2 is equivalent to P1.

2. Dinkelbach's Method

Consider a fractional programming problem

$$\begin{aligned} \max_{\boldsymbol{x} \in \mathcal{S}_0} \quad & R(\boldsymbol{x})/P_T(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{g}(\boldsymbol{x}) \leq 0. \end{aligned} \tag{26}$$

This problem is formed by denoting the objective function value as q so that a subtractive form of the objective function can be written as

$$F(q) = \max\{R(\boldsymbol{x}) - qP_T(\boldsymbol{x})) | \boldsymbol{g}(\boldsymbol{x}) \leq 0\}, q \in \mathbb{R}. \tag{27}$$

Additionally, it requires that $P_T(\boldsymbol{x}) > 0$ for all $\boldsymbol{x} \in \mathcal{S}$ where $\mathcal{S}$ is the feasible set of $\boldsymbol{x}$. Then, the function $F(q)$ has a series of important properties which are given in [21]. Explicitly, the solution to $F(q^*)$ is equivalent to the solution to the fractional programming problem (26). Dinkelbach has proposed an iterative method to find increasing q values, which are feasible, by solving the parameterized problem of $\max_{\boldsymbol{x}}\{R(\boldsymbol{x} - q_{i-1}P_T(\boldsymbol{x}))\}$ at each iteration. Hence, it can be shown that the method produces an increasing sequence of q values, which converges to the optimal value $F(q^*) = 0$. Each iteration corresponds to solving $\max_{\boldsymbol{x}}\{R(\boldsymbol{x}) - q_{i-1}P_T(\boldsymbol{x})\}$, where q_{i-1} is a given value of the parameter q, to obtain the optimum value $\boldsymbol{x}^*$ at the ith iteration of the Dinkelbach's method. For more details and the proof of convergence, please refer to [21].

ACKNOWLEDGEMENT

The work described in this paper was supported by a grant from the RGC of the Hong Kong SAR, China (Project No. PolyU 519011), the National Natural Science Foundation of China (Grant numbers 61372095 and 61162008), and the Guangxi Natural Science Foundation under Grant 2013GXNSFGA019004.

REFERENCES

[1] Cover TM, Thomas JA. *Elements of Information Theory*. John Wiley and Sons, 2012.

[2] Choi JI, Jain M, Srinivasan K, Levis P, Katti S. Achieving single channel, full duplex wireless communication. *Proceedings of the 16th annual International Conference on Mobile Computing and Networking (MobiCom'10)*, New York, USA, 2010; 1–12.

[3] Jain M, Choi JI, Kim T, Bharadia D, Seth S, Srinivasan K, Levis P, Katti S, Sinha P. Practical, real-time, full duplex wireless. *Proceedings of the 17th annual International Conference on Mobile Computing and Networking (MobiCom'11)*, New York,USA, 2011; 301–312.

[4] Duarte M, Sabharwal A. Full-duplex wireless communications using off-the-shelf radios: feasibility and first results. *Conference Record of the Forty Fourth Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, USA, 2010; 1558–1562.

[5] Bharadia D, McMilin E, Katti S. Full duplex radios. *Proceedings of the ACM SIGCOMM'13*, Hong Kong, China, 2013; 375–386.

[6] Korpi D, Anttila L, Syrjälä V, Valkama M. Widely-linear digital self-interference cancellation in direct-conversion full-duplex transceiver. *IEEE Journal on Selected Areas in Communications* June 2014; **PP**(99):1.

[7] Isheden C, Chong Z, Jorswieck E, Fettweis G. Framework for link-level energy efficiency optimization with informed transmitter. *IEEE Transactions on Wireless Communications* August 2012; **11**(8):2946–2957.

[8] Cheung KTK, Yang S, Hanzo L. Achieving maximum energy-efficiency in multi-relay ofdma cellular networks: a fractional programming approach. *IEEE Transactions on Communications* July 2013; **61**(7):2746–2757.

[9] Karmokar AK, Anpalagan A. Energy-efficient cross-layer design of dynamic rate and power allocation techniques for cognitive green radio networks. *Transactions on Emerging Telecommunications Technologies* October 2013; **24**(7-8):762–776.

[10] Tang X, Pu J, Gao Y, Xiong Z, Weng Y. Energy-efficient multicast routing scheme for wireless sensor networks. *Transactions on Emerging Telecommunications Technologies* May 2013; **PP**(99):DOI: 10.1002/ett.2661.

[11] Ashraf M, Sohaib S. Energy-efficient delay tolerant space time codes for asynchronous cooperative communications. *Transactions on Emerging Telecommunications Technologies* January 2014; **PP**(99):DOI: 10.1002/ett.2781.

[12] Reviriego P, Maestro JA, Hernández JA, Larrabeiti D. Study of the potential energy savings in ethernet by combining energy efficient ethernet and adaptive link rate. *Transactions on Emerging Telecommunications Technologies* October 2012; **23**(3):227–233.

[13] Ejaz W, Shah GA, ul Hasan N, Kim HS. Energy and throughput efficient cooperative spectrum sensing in cognitive radio sensor networks. *Transactions on Emerging Telecommunications Technologies* March 2014; **PP**(99):DOI: 10.1002/ett.2803.

[14] Li C, Cioffi JM, Yang L. Optimal energy efficient joint power allocation for two-hop single-antenna relaying systems. *Transactions on Emerging Telecommunications Technologies* June 2014; **25**(7):745–751.

[15] Naeem M, Illanko K, Karmokar AK, Anpalagan A, Jaseemuddin M. Power allocation in decode and forward relaying for green cooperative cognitive radio systems. *Proceedings of the Wireless Communications and Networking Conference (WCNC'13)*, Istanbul, Turkey, 2013; 3806–3810.

[16] Cheung KTK, Yang S, Hanzo L. Maximizing energy-efficiency in multi-relay ofdma cellular networks. *arXiv preprint 1401.6083* 2013; **PP**(99):1.

[17] Li C, Sun F, Cioffi J, Yang L. Energy efficient mimo relay transmissions via joint power allocations. *IEEE Transactions on Circuits and Systems II: Express Briefs* July 2011; **61**(7):531–535.

[18] Zafar A, Radaydeh RM, Chen Y, Alouini MS. Energy-efficient power allocation for fixed-gain amplify-and-forward relay networks with partial channel state information. *IEEE Wireless Communications Letters* December 2012; **1**(6):553–556.

[19] Jing Y, Jafarkhani H. Single and multiple relay selection schemes and their achievable diversity orders. *IEEE Transactions on Wireless Communications* March 2009; **8**(3):1414–1423.

[20] Atapattu S, Jing Y, Jiang H, Tellambura C. Relay selection and performance analysis in multiple-user networks. *IEEE Journal on Selected Areas in Communications* August 2013; **31**(8):1517–1529.

[21] Dinkelbach W. On nonlinear fractional programming. *Management Science* March 1967; **13**(7):492–498.

[22] Beck A, Ben-Tal A, Tetruashvili L. A sequential parametric convex approximation method with applications to nonconvex truss topology design problems. *Journal of Global Optimization* May 2010; **47**(1):29–51.

[23] Duarte M, Dick C, Sabharwal A. Experiment-driven characterization of full-duplex wireless systems. *IEEE Transactions on Wireless Communications* December 2011; **11**(12):4296–4307.

[24] Host-Madsen A, Zhang J. Capacity bounds and power allocation for wireless relay channels. *IEEE Transactions on Information Theory* June 2005; **51**(6):2020–2040.

[25] Boyd SP, Vandenberghe L. *Convex Optimization*. Cambridge University Press, 2004.

[26] Auer G, Giannini V, Desset C, Godor I, Skillermark P, Olsson M, Imran MA, Sabella D, Gonzalez MJ, Blume O, *et al.*. How much energy is needed to run a wireless network? *IEEE Wireless Communications* Oct 2011; **18**(5):40–49.