Managing Navigation Channel Traffic and Anchorage Area Utilization of a Container Port

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Abstract

Navigation channels are fairways for vessels to travel in and out of the terminal basin of a container port. The capacity of a navigation channel is restricted by the number of traffic lanes and safety clearance of vessels, and the availability of a navigation channel is usually affected by tides. The limited capacity and availability of a navigation channel could lead to congestion in the terminal basin. When the navigation channels run out of capacity, the anchorage areas in the terminal basin could serve as a buffer. This paper aims to develop a mathematical model which simultaneously optimizes the navigation channel traffic and anchorage area utilization. We provide a mixed integer programming formulation of the problem, analyze its complexity, and propose a Lagrangian relaxation heuristic in which the relaxed problem is decomposed into two asymmetric assignment problems. Computational performance of the Lagrangian relaxation heuristic is tested on problem instances generated based on the operational data of a port in Shanghai. Computational results show that the proposed heuristic is able to achieve satisfactory performance within reasonable computation time.

Keywords: port operations; vessel traffic service; navigation channels; anchorage areas; Lagrangian relaxation

1 Introduction

Navigation channels are fairways where vessels receive official pilotage services when traveling in and out of the terminal basin of a container port. The vessel traffic in navigation channels is regulated by vessel traffic service (VTS) operators (International Maritime Organization 1997). For tidal ports such as the Port of Shanghai where the availability of the navigation channel is limited due to tidal effect, the management of vessel traffic by VTS operators plays a crucial role in congestion mitigation. In these tidal ports, terminal operators determine the berthing and unberthing times of calling vessels based on their knowledge about the availability of the berths and navigation channels. After receiving the berth plans proposed by the terminal operators, the VTS operator of the port needs to schedule the vessels for traveling through the navigation channels such that the time windows for the vessels to utilize the channels coordinate with the vessels' planned berthing and unberthing times at the terminals. Because of the limited number of traffic lanes and the safety clearance requirement in navigation channels, the number of vessels that can sail in the channels is limited. When the channels run out of capacity, the anchorage area in the terminal basin could serve as a buffer. However, the poor planning of navigation channel and anchorage area utilization often leads to congestion, which lowers terminal operation's efficiency and incurs more vessel emissions. If the VTS operator fails to arrange a schedule for some vessels to utilize the navigation channel, the berth plans will be rejected by the VTS operator, and the terminal operators will have to revise their plans.

Figure 1 provides a schematic layout of a container port. During each planning horizon, some calling vessels need to be served at a container port according to the berth plans proposed by the terminal operators. Each vessel arrives at the anchorage ground of the port at a given time with a planned berthing time at which it is expected to arrive at its designated berth. After the vessel completes its service at the berth, it will leave the terminal basin, and is expected to arrive at the anchorage ground by a certain expected departure time. Vessels that enter or exit the terminal basin should pass through a navigation channel. The navigation channel consists of two traffic lanes: one for incoming vessels (i.e., vessels that enter the terminal basin for service) and the other for outgoing vessels (i.e., vessels that exit the terminal basin after service). Since each traffic lane

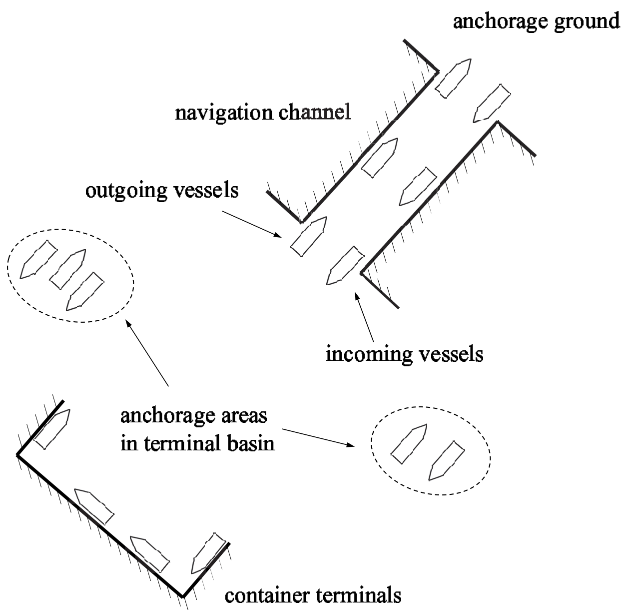

Figure 1: Schematic layout of a container port.

bears one-way traffic, incoming or outgoing vessels need to enter a lane one by one and sail in a single line when getting through the channel. When sailing in the navigation channel, vessels in the same traffic lane should keep a safety clearance among them. Because of tidal effect, the water depth in the navigation channel varies over time. As a result, a vessel can only pass through the navigation channel when the water level becomes deep enough (Du et al. 2015, Ding et al. 2016). This tidal constraint, together with the limited capacity of the navigation channel, imposes a serious restriction on the number of vessels that can enter and leave the terminal basin. Vessels in the terminal basin should either moor at berth, or park at some small anchorage areas, which we refer to as *staging anchorages*. The staging anchorages are utilized in the following ways: (i) An incoming vessel that enters the terminal basin earlier than expected should wait at a staging anchorage until it can moor as planned. (ii) After an outgoing vessel finishes service, the vessel should wait at a staging anchorage if it cannot enter the navigation channel due to either inadequate water level or limited channel capacity. To ensure that vessels can berth and depart as planned, the VTS operator should manage the traffic by scheduling the utilization of the navigation channel and the staging anchorages.

In this paper, we consider a container port with a single navigation channel. We develop and analyze an optimization model that simultaneously allocates space in the anchorage area to vessels and determines the time for each vessel to utilize the navigation channel. The objective of this model is to minimize vessels' berthing and departure tardiness, as well as unsatisfied service requests. The model can be used for constructing weekly "rough-cut" plans that determine the usage of pilots and the number of tugboats needed for guiding the vessels, as well as shorter term plans (e.g., 1- or 2-day plans) that determine the actual schedule of the vessels.

Seaside operations planning problems in container ports have been extensively studied by logistics and operations researchers, and various mathematical models have been developed for different applications. One common feature of the existing models is that problems are formulated from the perspective of a terminal operator. These models include berth allocation models with different spatial, temporal, and handling time attributes, quay crane scheduling models with different task, crane, and interference attributes, as well as various models that integrate berth allocation and quay crane scheduling decisions. Bierwirth and Meisel (2010, 2015) provide comprehensive reviews on various berth allocation and quay crane scheduling problems. For other recent reviews on mathematical models for container terminal operations, see Carlo, Vis, and Roodbergen (2015), Kim and Lee (2015), and Gharehgozli, Roy, and de Koster (2016). In addition to the limited availability of terminal resources, low water level caused by tidal effect in navigation channels and limited space in the anchorage area are other issues faced by many container ports. Yet, most existing seaside operations planning models in the literature have excluded these factors. The tidal condition in navigation channels often leads to tight time windows for vessels with deep drafts to enter and exit the terminal basin. As a result, deep-draft vessels may need to enter and leave their allocated berths at some specific time points, occupy the berth space for a longer time period, or wait at the anchorage area in the terminal basin for berthing and departure.

A number of seaside operations planning models reported in the literature have considered tidal conditions of a port. Barros et al. (2011), Xu, Li, and Leung (2012), Lalla-Ruiz, Melián Batista, and Moreno Vega (2013), Lalla-Ruiz et al. (2016, 2017), Qin, Du, and Sha (2016), and Lalla-Ruiz (2017) develop and analyze different seaside operations planning models with tidal effects at the berths. Ilati, Sheikholeslami, and Hassannayebi (2014), Sheikholeslami, Ilati, and Kobari (2014), Du et al. (2015), Ding et al. (2016), Dadashi et al. (2017), Yu, Wang, and Zhen (2017), and Zhen et al. (2017) develop and analyze different seaside operations planning models by taking the tidal impact on navigation channels into consideration. Existing works that study the tidal impact on a navigation channel typically focus on examining how the tidal windows in the navigation channel affect berth planning decisions. However, the limited traffic capacity of the navigation channel is rarely considered in these works. Since the traffic capacity of the navigation channel is restricted by the number of traffic lanes and the safety clearance of vessels, the number of incoming vessels and the number of outgoing vessel that can enter the navigation channel at a time are limited. This restriction would have a direct impact on the berthing and unberthing times of vessels.

Different from the abovementioned seaside operations planning models which allocate terminal resources to calling vessels, our model takes vessel berthing information as input and focuses on scheduling vessel traffic in a busy container port. Vessel traffic scheduling problems in restricted water areas have been studied by some researchers. Petersen and Taylor (1988), Nauss (2008), Verstichel et al. (2014), and Passchyn et al. (2016) model and solve lock scheduling problems for inland waterways. Ulusçu et al. (2009) and Sluiman (2017) schedule the traffic of transit vessels in straits. There also exist a few works on the scheduling of navigation channels of seaports that are more related to our work. Kelareva et al. (2012) and Kelareva, Tierney, and Kilby (2013) consider the scheduling of incoming and outgoing vessels of a navigation channel with predetermined vessel berthing positions, tidal constraints in the channel, and constraints on tug availability, so as to maximize the cargo throughput of the port. Zhang et al. (2016) also study the scheduling of incoming and outgoing vessels of a navigation channel with predetermined vessel berthing positions and tidal constraints in the channel, where the objective is to minimize the vessels' total waiting time. Tang et al. (2016) study the master planning of a new container terminal by taking the dimensions of a navigation channel into account to avoid possible bottlenecks for the port's future performance. Lalla-Ruiz, Shi, and Voß (2018) consider a problem that schedules vessels for traveling through a set of navigation channels with tidal constraints. They model the problem as a mixed integer linear program (MILP) and propose a metaheuristic for minimizing vessels' waiting time at the anchorage ground plus the travel time in the navigation channels. Hill et al. (2018) reformulate

Lalla-Ruiz, Shi, and Voß's problem as a variant of the multi-mode resource-constrained project scheduling problem and develop a compact MILP that can be solved efficiently by a mathematical programming solver. However, existing works on scheduling vessel traffic in navigation channels of seaports either ignore the utilization of staging anchorages or assume that the capacity of staging anchorages are infinite. The issue of how the utilization of the limited anchorage capacity can mitigate congestion in navigation channels during low-tide periods has yet to be addressed. This paper therefore aims to scientifically model, solve, and analyze the anchorage area and navigation channel planning problem of a container port. Similar to the model of Lalla-Ruiz, Shi, and Voß (2018), scheduling the navigation channel traffic of a port is a key component of our model. However, our model considers the assignment of the staging anchorages to vessels, takes into account of the safety clearance of vessels and the pre-determined berth plans when scheduling the vessels, and has an objective function different from that of Lalla-Ruiz, Shi, and Voß's model.

Our main contributions are twofold. First, we develop a mathematical model for scheduling the vessel traffic at a container port. Our model captures important aspects of practical operations such as vessels' tidal windows for traveling through a navigation channel, safety clearance of consecutive vessels, the vessels' designated berthing positions, and most importantly, the capacity of the anchorage area. The objective is to minimize the total tardiness cost of the vessels. It imposes a large penalty on each unsatisfied service request, so that the first priority is to satisfy all service requests. Any unsatisfied service request in the solution indicates that the berth plans proposed by the terminal operators need to be revised. Second, we analyze the computational complexity of the problem and develop a Lagrangian relaxation heuristic for solving the problem. We evaluate the performance of the heuristic computationally using extensive test data with the parameter setting developed based on the characteristics of the Yangshan Deep-water Port of Shanghai.

The rest of the paper is organized as follows. In Section 2, we define our problem mathematically, present a MILP formulation of the problem, and discuss its computational complexity. In Section 3, we present the Lagrangian relaxation heuristic and analyze the properties of the relaxation problem. In Section 4, we conduct computational experiments to test the effectiveness and efficiency of the Lagrangian relaxation heuristic and to assess the benefits of taking the anchorage area's capacity into consideration when planning navigation channel traffic. Section 5 concludes the study and proposes some future research directions. The proofs of the lemmas and theorem, pseudo-codes, statistics and operational data of the Yangshan Deep-water Port used in Section 4.1, and some detailed computational results are provided in online appendices. The data set used in our computational experiments is provided as online supplement, which is available at the journal's website and at http://www.mypolyuweb.hk/∼lgtzx/nav/result.htm.

2 Problem Definition and Complexity

Let $T > 0$ be the length of the planning horizon. During the planning horizon, $n_1 \geq 0$ incoming vessels and $n_2 \geq 0$ outgoing vessels are scheduled for service at the container terminals. Note that the number of incoming vessels may be different from the number of outgoing vessels, as a vessel that enters the terminal basin during a planning cycle may leave in another cycle. Each incoming vessel i has an arrival time $A_i \geq 0$. Thus, vessel i may enter the navigation channel at or after time A_i. Incoming vessel i also has a planned berthing time $B_i \geq 0$ as specified by the berth plans. Each outgoing vessel i has a planned service completion time $E_i \geq 0$ at the berth, and a target departure time $D_i \geq 0$ (i.e., time for the vessel to arrive at the anchorage ground after service).

The water level in the navigation channel is tide-dependent, and a vessel can only sail in the channel when the water becomes deep enough to hold the draft. Because the water level rises and drops with the fluctuation of tide, each vessel i is subject to a number of tidal windows, during which it can sail through the channel with a satisfactory water level. The vessels' tidal windows are "nested," i.e., the tidal windows for vessels with a deeper draft are subsets of the tidal windows for vessels with a shallower draft. Figure 2 illustrates the variation of the water level over time, and the nested tidal windows for vessels with different drafts. We let $u_i > 0$ denote the number of tidal windows that vessel i has during the planning horizon, and $[\underline{w}_{il}, \bar{w}_{il}]$ denote the lth tidal window for vessel i.

Each vessel i has been assigned to a berthing position $b(i)$ where the vessel should moor and be handled. Here, $b(i)$ represents a berth if the cargo terminal has a discrete berth layout, and a specific position along the quay if the cargo terminal has a continuous berth layout. As mentioned

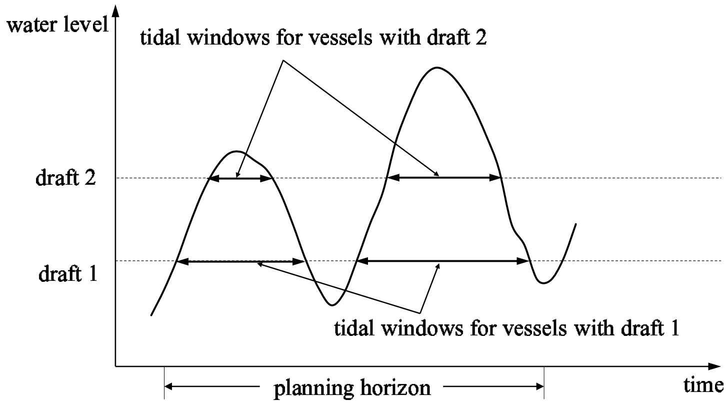

Figure 2: Nested tidal windows of vessels with different drafts.

in Section 1, incoming vessels may utilize the staging anchorages if they arrive at the terminal basin too early, and outgoing vessels may utilize the staging anchorages if they need to wait for the availability of the navigation channel. Denote $\bar{\tau}$ as the navigation time in the navigation channel. When traveling through the navigation channel, vessels in the same traffic lane must keep a safety clearance σ, in terms of the length of travel time between two successive vessels. There are m separate staging anchorages in the terminal basin, which may be located at different locations. We denote $\tau_{0,b(i)}$, τ'_{0k}, and $\tau''_{k,b(i)}$ as the travel time between the end of the navigation channel and berthing position of vessel i, the travel time between the end of the navigation channel and staging anchorage k, and the travel time between berthing position of vessel i and staging anchorage k, respectively (see Figure 3).

To comply with the predetermined berth plans, an incoming vessel should not berth earlier than its planned berthing time B_i in order to avoid interference on the berthing activities of other vessels. In practice, some flexibility is allowed for the berthing of each incoming vessel, but a deadline $\bar{B}_i$ ($\bar{B}_i \geq B_i$) on the actual berthing time of each incoming vessel i is imposed by the cargo terminal. Therefore, vessel i should berth within the feasible time interval $[B_i, \bar{B}_i]$ so as to complete service on time. If the actual berthing time is later than B_i, berthing tardiness is incurred, resulting in a penalty cost. The tardiness penalty for late berthing is $C_{1i} > 0$ per time unit for each incoming vessel i. If the actual berthing time is later than $\bar{B}_i$, service completion will be delayed, which may affect the actual berthing time of subsequent vessels, causing uncontrolled execution of the predetermined berth plans. Each outgoing vessel i must unberth at time E_i. After unberthing,

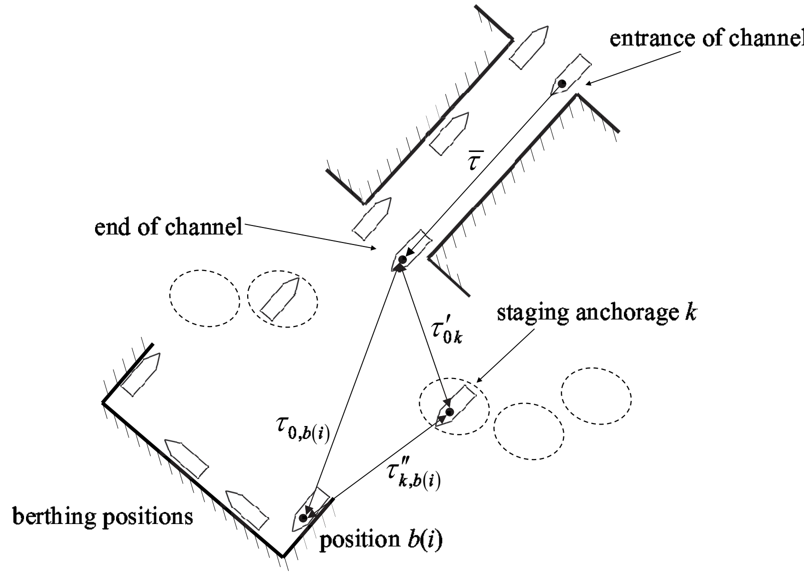

Figure 3: Possible movements of vessels at the seaport and their corresponding travel times.

the outgoing vessel will either sail directly through the navigation channel to the anchorage ground for departure, or wait at a staging anchorage before sailing through the channel. On-time or early departure is preferred, but late departure, i.e., departure later than D_i, is allowed. The tardiness penalty for late departure is $C_{2i} > 0$ per unit time for each outgoing vessel i. If a vessel cannot be served at its minimum requirement (i.e., an incoming vessel i cannot berth before its berthing deadline $\bar{B}_i$ or an outgoing vessel i cannot unberth at time E_i), the berth plan for the vessel will need to be revised by the terminal operator. We let $C_{0i} > 0$ be a large lump-sum penalty on such incidence for each vessel i.

We assume that all time-related parameters in our model are multiples of the safety clearance σ. Because σ is relatively small compared to other travel time parameters, the inaccuracy of the solution caused by this assumption is relatively insignificant. Thus, by setting $\sigma = 1$, all time-related parameters in our model become integer-valued. The problem involves decisions for determining the actual berthing time of incoming vessels, the actual departure time of outgoing vessels, and the time points for the incoming and outgoing vessels to enter the channel. In addition, the utilization of the staging anchorages (i.e., whether a vessel should occupy a staging anchorage or not, which staging anchorage is assigned to a vessel, and the time points for a vessel to enter and leave the staging anchorage) must also be determined. The objective of the problem is to minimize the total cost incurred by berthing tardiness of incoming vessels and departure tardiness of outgoing vessels, as well as the cost incurred by the unsatisfied service requests. We denote this problem as **P**. Additional assumptions of problem **P** are as follows:

(i) Each staging anchorage can accommodate exactly one vessel. In addition, a vessel cannot stay partially at one staging anchorage and partially at another staging anchorage.

(ii) All the staging anchorages are available for the vessels throughout the planning horizon.

(iii) The travel speed is constant and identical for all vessels in the navigation channel; the travel speed is also constant and identical for all vessels in the terminal basin. Thus, the time for traveling through the navigation channel is identical for all vessels, and the time for traveling between any two locations in the terminal basin is symmetric and identical for all vessels.

For notational convenience, we number the incoming vessels from 1 to n_1, and the outgoing vessels from $n_1 + 1$ to $n_1 + n_2$. Let M denote a large number. Problem **P** can be formulated as an MILP as follows.

Input parameters:

T: Length of planning horizon

n_1: Number of incoming vessels

n_2: Number of outgoing vessels

m: Number of staging anchorages

$b(i)$: Designated berthing position for vessel i

u_i: Number of tidal windows for vessel i

$[\underline{w}_{il}, \bar{w}_{il}]$: lth tidal window for vessel i

A_i: Arrival time of incoming vessel i (i.e., the earliest possible time to enter the navigation channel)

B_i: Planned berthing time of incoming vessel i

$\bar{B}_i$: Latest allowed berthing time of incoming vessel i

E_i: The time that outgoing vessel i must leave berthing position $b(i)$

D_i: Expected departure time of outgoing vessel i

$\bar{\tau}$: Amount of time for a vessel to travel through the navigation channel

$\tau_{0,b(i)}$: Amount of time for vessel i to travel directly between the end of the navigation channel and berthing position $b(i)$

τ'_{0k}: Amount of time for a vessel to travel between the end of the navigation channel and staging anchorage k ($\tau'_{00} = 0$)

$\tau''_{k,b(i)}$: Amount of time for vessel i to travel between staging anchorage k and berthing position $b(i)$ ($\tau''_{0,b(i)} = \tau_{0,b(i)}$)

C_{1i}: Unit cost of berthing tardiness for incoming vessel i

C_{2i}: Unit cost of departure tardiness for outgoing vessel i

C_{0i}: A (large) lump-sum cost if the service request of vessel i cannot be satisfied.

Decision variables:

x_{it}: =1 if vessel i enters the navigation channel at time t; 0 otherwise

y_{ik}: =1 if vessel i occupies staging anchorage k; 0 otherwise

y_{i0}: =1 if vessel i travels directly between the end of the channel and berthing position $b(i)$ without occupying any staging anchorage; 0 otherwise

z_{ikt}: =1 if vessel i occupies staging anchorage k at time t; 0 otherwise

e_i: Time point for vessel i to enter a staging anchorage

f_i: Time point for vessel i to leave a staging anchorage

g_i: Berthing time of incoming vessel i

L_{1i}: Berthing tardiness of incoming vessel i

L_{2i}: Departure tardiness of outgoing vessel i

U_i: =1 if the service request of vessel i cannot be satisfied; 0 otherwise.

MILP formulation:

$$\mathbf{P}: \text{ minimize } \quad \sum_{i=1}^{n_1} C_{1i}L_{1i} + \sum_{i=n_1+1}^{n_1+n_2} C_{2i}L_{2i} + \sum_{i=1}^{n_1+n_2} C_{0i}U_i \tag{1}$$

$$\text{subject to } \quad \sum_{i=1}^{n_1} x_{it} \le 1 \quad (t = 0, 1, \ldots, T) \tag{2}$$

$$\sum_{i=n_1+1}^{n_1+n_2} x_{it} \le 1 \quad (t = 0, 1, \ldots, T) \tag{3}$$

$$\sum_{t=0}^{T} x_{it} = 1 - U_i \quad (i = 1, \ldots, n_1 + n_2) \tag{4}$$

$$x_{it} = 0 \quad (i = 1, \ldots, n_1;\ t = 0, 1, \ldots, A_i - 1) \tag{5}$$

$$x_{it} = 0 \quad (i = 1, \ldots, n_1 + n_2;\ t \in \{0, 1, \ldots, T\} \setminus \textstyle\bigcup_{l=1}^{u_i} [\underline{w}_{il}, \bar{w}_{il} - \bar{\tau}]) \tag{6}$$

$$\textstyle\sum_{k=0}^{m} y_{ik} = 1 - U_i \quad (i = 1, \ldots, n_1 + n_2) \tag{7}$$

$$e_i = \textstyle\sum_{t=0}^{T} (t + \bar{\tau}) x_{it} + \sum_{k=0}^{m} \tau'_{0k} y_{ik} \quad (i = 1, \ldots, n_1) \tag{8}$$

$$f_i = g_i - \textstyle\sum_{k=0}^{m} \tau''_{k,b(i)} y_{ik} \quad (i = 1, \ldots, n_1) \tag{9}$$

$$e_i = \textstyle\sum_{k=0}^{m} (E_i + \tau''_{k,b(i)}) y_{ik} \quad (i = n_1 + 1, \ldots, n_1 + n_2) \tag{10}$$

$$f_i = \textstyle\sum_{t=0}^{T} t x_{it} - \sum_{k=0}^{m} \tau'_{0k} y_{ik} \quad (i = n_1 + 1, \ldots, n_1 + n_2) \tag{11}$$

$$f_i - M(1 - y_{i0}) \le e_i \le f_i \quad (i = 1, \ldots, n_1 + n_2) \tag{12}$$

$$\textstyle\sum_{i=1}^{n_1+n_2} z_{ikt} \le 1 \quad (k = 1, \ldots, m;\ t = 0, 1, \ldots, T) \tag{13}$$

$$e_i - M(1 - z_{ikt}) \le t \le f_i + M(1 - z_{ikt})$$

$$(i = 1, \ldots, n_1 + n_2;\ k = 1, \ldots, m;\ t = 0, 1, \ldots, T) \tag{14}$$

$$f_i - e_i + 1 - M(1 - y_{ik}) \le \textstyle\sum_{t=0}^{T} z_{ikt} \le M y_{ik}$$

$$(i = 1, \ldots, n_1 + n_2;\ k = 1, \ldots, m) \tag{15}$$

$$B_i(1 - U_i) \le g_i \le \bar{B}_i(1 - U_i) \quad (i = 1, \ldots, n_1) \tag{16}$$

$$L_{1i} \ge g_i - B_i \quad (i = 1, \ldots, n_1) \tag{17}$$

$$L_{2i} \ge \textstyle\sum_{t=0}^{T} (t + \bar{\tau}) x_{it} - D_i(1 - U_i) \quad (i = n_1 + 1, \ldots, n_1 + n_2) \tag{18}$$

$$x_{it} \in \{0, 1\} \quad (i = 1, \ldots, n_1 + n_2;\ t = 0, 1, \ldots, T) \tag{19}$$

$$y_{ik} \in \{0, 1\} \quad (i = 1, \ldots, n_1 + n_2;\ k = 0, 1, \ldots, m) \tag{20}$$

$$z_{ikt} \in \{0, 1\} \quad (i, j = 1, \ldots, n_1 + n_2;\ k = 1, \ldots, m;\ t = 0, 1, \ldots, T) \tag{21}$$

$$U_i \in \{0, 1\} \quad (i = 1, \ldots, n_1 + n_2) \tag{22}$$

$$e_i, f_i \ge 0 \quad (i = 1, \ldots, n_1 + n_2) \tag{23}$$

$$g_i, L_{1i} \ge 0 \quad (i = 1, \ldots, n_1) \tag{24}$$

$$L_{2i} \ge 0 \quad (i = n_1 + 1, \ldots, n_1 + n_2) \tag{25}$$

Objective function (1) minimizes the total penalty, which includes the total berthing tardiness penalty of the incoming vessels, the total departure tardiness penalty of the outgoing vessels, and the penalty for unsatisfied service requests of vessels. Constraint (2) ensures that at most one incoming vessel can enter the navigation channel from the anchorage ground at each time point. Constraint (3) ensures that at most one outgoing vessel can enter the channel from the terminal basin at each time point. Constraint (4) requires each accepted vessel to enter the channel once. Constraint (5) specifies that an incoming vessel i cannot enter the channel before A_i. Constraint (6) requires each accepted vessel i to enter the channel during time interval $[\underline{w}_{il}, \bar{w}_{il} - \bar{\tau}]$ for some $l = 1, \ldots, u_i$, so that the vessel can pass through the channel within tidal window $[\underline{w}_{il}, \bar{w}_{il}]$. Constraint (7) ensures that each accepted vessel either travels directly between the end of the navigation channel and its designated berthing position, or makes use of exactly one staging anchorage. Constraints (8) and (9) define the time points when incoming vessel i enters and leaves a staging anchorage (if $y_{i0} = 1$, then e_i and f_i are both set equal to the time point when incoming vessel i arrives at the end of the navigation channel). Similarly, constraints (10) and (11) define the time points when outgoing vessel i enters and leaves a staging anchorage. Constraint (12) specifies a relationship between e_i and f_i. It also implies that $f_i = e_i$ if vessel i does not make use of any staging anchorage. Constraint (13) ensures that at most one vessel dwells at staging anchorage k at each time point. We refer to this constraint as the capacity constraint of staging anchorage k. Constraint (14) ensures that z_{ikt} can be set equal to 1 only when $t \in [e_i, f_i]$, i.e., vessel i dwells at a staging anchorage only within time window $[e_i, f_i]$. Constraint (15) states that if vessel i does not make use of staging anchorage k, then it cannot dwell at staging anchorage k at any time point; otherwise it must occupy staging anchorage k for $f_i - e_i + 1$ time points (i.e., time points $e_i, e_i + 1, \ldots, f_i$). Constraint (16) ensures that each incoming vessel i berths within the feasible time interval $[B_i, \bar{B}_i]$ to satisfy its service request. Constraint (17) determines the berthing tardiness L_{1i} for each incoming vessel i. Constraint (18) determines the departure tardiness L_{2i} for each outgoing vessel i. Finally, constraints (19)–(25) specify the binary and nonnegativity requirements of the decision variables.

A simple example that illustrates problem **P** is given as follows:

- Length of planning horizon: $T = 12$
- Number of incoming vessels: $n_1 = 2$
- Number of outgoing vessels: $n_2 = 2$

- Number of staging anchorages: $m = 1$
- Number of tidal window for vessel i: $u_i = 1$ for $i = 1, 2, 3, 4$
- Tidal windows for incoming vessels: $[\underline{w}_{11}, \bar{w}_{11}] = [3, 8]$; $[\underline{w}_{21}, \bar{w}_{21}] = [0, 12]$
- Tidal windows for outgoing vessels: $[\underline{w}_{31}, \bar{w}_{31}] = [3, 8]$; $[\underline{w}_{41}, \bar{w}_{41}] = [0, 12]$
- Arrival times of incoming vessels: $A_1 = 2$; $A_2 = 3$
- Berthing time windows of incoming vessels: $[B_1, \bar{B}_1] = [11, 12]$; $[B_2, \bar{B}_2] = [9, 10]$
- Time points that outgoing vessels must leave their berthing positions: $E_3 = 0$; $E_4 = 2$
- Expected departure times of outgoing vessels: $D_3 = 7$; $D_4 = 10$
- Amount of time for a vessel to travel through the navigation channel: $\bar{\tau} = 5$
- Amount of time for vessel i to travel directly between the end of the navigation channel and berthing position $b(i)$: $\tau_{0,b(i)} = 1$ for $i = 1, 2, 3, 4$
- Amount of time for a vessel to travel between the end of the navigation channel and the staging anchorage: $\tau'_{01} = 1$
- Amount of time for vessel i to travel between the staging anchorage and berthing position $b(i)$: $\tau''_{1,b(i)} = 1$ for $i = 1, 2, 3, 4$
- Unit costs of berthing tardiness for incoming vessels: $C_{11} = 2$; $C_{12} = 3$
- Unit costs of departure tardiness for outgoing vessels: $C_{23} = 2$; $C_{24} = 3$
- Lump-sum cost if the service request of vessel i cannot be satisfied: $C_{0i} = 100$ for $i = 1, 2, 3, 4$

In this example, there is only one staging anchorage, and each vessel has only one tidal window. A solution of this example is depicted in Figure 4. In this solution, incoming vessel 1 travels through the navigation channel during the time interval $[3, 8]$, travels to the staging anchorage, and then travels to berthing position $b(1)$. Vessel 1 arrives at the staging anchorage at $e_1 = 8 + \tau'_{01} = 9$ and leaves the staging anchorage at $f_1 = 10$. Thus, the berthing tardiness of vessel 1 is $L_{11} = f_1 + \tau''_{1,b(1)} - B_1 = 0$, and the tardiness cost of the vessel is $C_{11}L_{11} = 0$. Incoming vessel 2 travels through the navigation channel during the time interval $[4, 9]$ and then travels directly to berthing position $b(2)$ without making use of the staging anchorage. The berthing tardiness of vessel 2 is $L_{12} = 9 + \tau_{0,b(2)} - B_2 = 1$, and the tardiness cost of the vessel is $C_{12}L_{12} = 3$. Outgoing vessel 3 leaves berthing position $b(3)$ at time $E_3 = 0$, travels to the staging anchorage, and then travels to

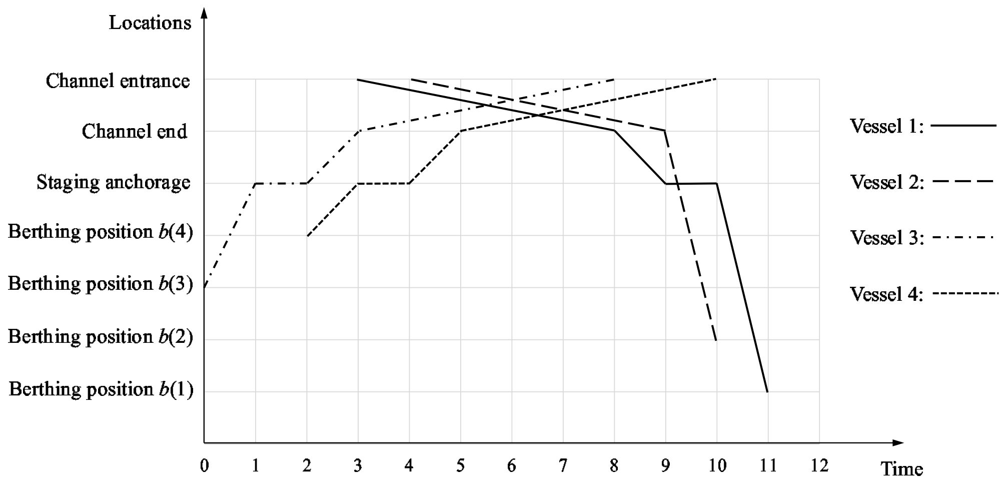

Figure 4: Routes of vessels in a feasible solution of the example.

the navigation channel. Vessel 3 arrives at the staging anchorage at $e_3 = 0 + \tau''_{1,b(3)} = 1$, and leaves the staging anchorage at $f_3 = 2$. Thus, the departure tardiness of vessel 3 is $L_{23} = \max\{0,\ f_3 + \tau'_{01} + \bar{\tau} - D_3\} = 1$, and the tardiness cost of the vessel is $C_{23}L_{23} = 2$. Outgoing vessel 4 leaves berthing position $b(4)$ at time $E_4 = 2$, travels to the staging anchorage, and then travels to the navigation channel. Vessel 4 arrives at the staging anchorage at $e_4 = 2 + \tau''_{1,b(4)} = 3$, and leaves the staging anchorage at $f_4 = 4$. Thus, the departure tardiness of vessel 4 is $L_{24} = \max\{0,\ f_4 + \tau'_{01} + \bar{\tau} - D_4\} = 0$, and the tardiness cost of the vessel is $C_{24}L_{24} = 0$. Since the service requests of all vessels are satisfied, the total cost of the solution is $C_{11}L_{11} + C_{22}L_{22} + C_{23}L_{23} + C_{24}L_{24} = 5$. Note that in this solution, (i) each outgoing vessel i unberths at E_i; (ii) each incoming vessel i berths within its berthing time window $[B_i, \bar{B}_i]$; (iii) each incoming vessel enters the navigation channel no earlier than its arrival time; (iv) each vessel passes through the channel during its tidal window; (v) at any time point, there is at most one incoming vessel entering the channel and at most one outgoing vessel entering the channel; and (vi) at any time point, there is at most one vessel occupying the staging anchorage. Hence, this solution is feasible.

The following theorem states the computational complexity of problem $\mathbf{P}$.

Theorem 1 *Problem* $\mathbf{P}$ *is NP-hard in the strong sense.*

Remark 1 *In the proof of Theorem 1, we have shown that the answer to the 3-Dimensional Matching problem is "yes" if and only if the constructed instance of problem* $\mathbf{P}$ *has a zero total cost. Thus,* *unless $P = NP$, there exists no approximation algorithm for problem* **P** *with a constant bound on the relative error.*

Remark 2 *In the proof of Theorem 1, the constructed instance of problem* **P** *has no outgoing vessel, and all travel times satisfy the triangle inequality. Thus, problem* **P** *is NP-hard in the strong sense even when there are only incoming vessels and all travel times satisfy the triangle inequality.*

3 Lagrangian Relaxation Heuristic

In this section, we present a Lagrangian relaxation heuristic for problem **P**. We first present the Lagrangian dual problem, and show that the subproblems obtained by relaxing the capacity constraints of the staging anchorages can be solved in pseudo-polynomial time. We then present a heuristic for constructing a feasible primal solution from the solution to the Lagrangian dual problem, as well as the subgradient optimization procedure for searching the Lagrangian multipliers.

3.1 The Lagrangian Relaxation Problem and Its Subproblems

Relaxing the staging anchorage capacity constraint (13) and placing it in the objective function of problem **P** with Lagrangian multipliers $\lambda_{kt} \geq 0$ ($k = 1, \ldots, m$; $t = 0, 1, \ldots, T$), we obtain the following relaxed problem:

$$\mathbf{P}(\lambda): \ \text{minimize} \quad \sum_{i=1}^{n_1} C_{1i}L_{1i} + \sum_{i=n_1+1}^{n_1+n_2} C_{2i}L_{2i} + \sum_{i=1}^{n_1+n_2} C_{0i}U_i + \sum_{k=1}^{m}\sum_{t=0}^{T} \lambda_{kt}\Big(\sum_{i=1}^{n_1+n_2} z_{ikt} - 1\Big)$$

$$\text{subject to} \quad (2)\text{–}(12) \text{ and } (14)\text{–}(25)$$

where λ denotes the vector of the λ_{kt} values. Let $L(\lambda)$ denote the optimal objective value of problem $\mathbf{P}(\lambda)$. The goal of the Lagrangian relaxation heuristic is to solve the following Lagrangian dual problem:

$$\mathbf{P}_{\text{LD}}: \quad \max_{\lambda} L(\lambda).$$

We search the optimal Lagrangian multipliers by applying the subgradient optimization method, which will be described in the next section. In the following, we will show that given the values of the Lagrangian multipliers, problem $\mathbf{P}(\lambda)$ can be solved in pseudo-polynomial time.

After dropping the constant term $-\sum_{k=1}^{m}\sum_{t=0}^{T}\lambda_{kt}$ from the objective function, problem $\mathbf{P}(\lambda)$ can be decomposed into the following two independent subproblems:

$$\begin{aligned}\mathbf{P}_{\text{in}}(\lambda): \text{ minimize } & \textstyle\sum_{i=1}^{n_1}\left\{C_{1i}L_{1i}+C_{0i}U_i+\sum_{k=1}^{m}\sum_{t=0}^{T}\lambda_{kt}z_{ikt}\right\}\\ \text{subject to } & (2), (4)\text{–}(9), (12), (14)\text{–}(17), \text{and } (19)\text{–}(24)\\ & (\text{for } i=1,\ldots,n_1 \text{ only})\end{aligned}$$

and

$$\begin{aligned}\mathbf{P}_{\text{out}}(\lambda): \text{ minimize } & \textstyle\sum_{i=n_1+1}^{n_1+n_2}\left\{C_{2i}L_{2i}+C_{0i}U_i+\sum_{k=1}^{m}\sum_{t=0}^{T}\lambda_{kt}z_{ikt}\right\}\\ \text{subject to } & (3)\text{–}(4), (6)\text{–}(7), (10)\text{–}(12), (14)\text{–}(15), (18)\text{–}(23), \text{and } (25)\\ & (\text{for } i=n_1+1,\ldots,n_1+n_2 \text{ only})\end{aligned}$$

Subproblem $\mathbf{P}_{\text{in}}(\lambda)$ involves only decision variables of the incoming vessels, while subproblem $\mathbf{P}_{\text{out}}(\lambda)$ involves only decision variables of the outgoing vessels. We will show that these subproblems can both be transformed into asymmetric assignment problems.

We first consider subproblem $\mathbf{P}_{\text{in}}(\lambda)$. For ease of presentation, we introduce a dummy time point $T+i$ and denote $x_{i,T+i}=U_i$ for each incoming vessel $i=1,\ldots,n_1$. For $i=1,\ldots,n_1$ and $t=0,1,\ldots,T+n_1$, define

$$F_{it}(\lambda)=\begin{cases}\min_{k=0,1,\ldots,m}\left\{F_{it}^{(k)}(\lambda)\right\}, & \text{if } A_i\le t\le T \text{ and } t\in\bigcup_{l=1}^{u_i}[\underline{w}_{il},\bar{w}_{il}-\bar{\tau}];\\ C_{0i}, & \text{if } t=T+i;\\ +\infty, & \text{otherwise;}\end{cases}$$

where

$$F_{it}^{(k)}(\lambda)=\begin{cases}C_{1i}(t+\bar{\tau}+\tau_{0,b(i)}-B_i), & \text{if } k=0 \text{ and } B_i\le t+\bar{\tau}+\tau_{0,b(i)}\le\bar{B}_i;\\ C_{1i}(t+\bar{\tau}+\tau'_{0k}+\tau''_{k,b(i)}-B_i)+\lambda_{k,t+\bar{\tau}+\tau'_{0k}}, & \text{if } k>0 \text{ and } B_i\le t+\bar{\tau}+\tau'_{0k}+\tau''_{k,b(i)}\le\bar{B}_i;\\ \sum_{t'=t+\bar{\tau}+\tau'_{0k}}^{B_i-\tau''_{k,b(i)}}\lambda_{kt'}, & \text{if } k>0 \text{ and } t+\bar{\tau}+\tau'_{0k}+\tau''_{k,b(i)}<B_i;\\ +\infty, & \text{otherwise.}\end{cases}$$

Here, $F_{it}(\lambda)$ represents the cost of letting incoming vessel i enter the navigation channel at time t in the Lagrangian subproblem, $F_{it}^{(k)}(\lambda)$ represents this cost when the vessel is assigned staging anchorage k (if $k > 0$), and $F_{it}^{(0)}(\lambda)$ represents this cost when the vessel is not assigned any staging anchorage. Define

$$\begin{aligned} \mathbf{P}'_{\text{in}}(\lambda): \text{ minimize } \quad & \textstyle\sum_{i=1}^{n_1} \sum_{t=0}^{T+n_1} F_{it}(\lambda) x_{it} \\ \text{subject to } \quad & \textstyle\sum_{i=1}^{n_1} x_{it} \le 1 \quad (t = 0, 1, \dots, T + n_1) \\ & \textstyle\sum_{t=0}^{T+n_1} x_{it} = 1 \quad (i = 1, \dots, n_1) \\ & x_{it} \in \{0, 1\} \quad (i = 1, \dots, n_1;\, t = 0, 1, \dots, T + n_1) \end{aligned}$$

Lemma 1 *Solving problem $\mathbf{P}'_{\text{in}}(\lambda)$ yields the optimal objective value of problem $\mathbf{P}_{\text{in}}(\lambda)$.*

Next, we consider subproblem $\mathbf{P}_{\text{out}}(\lambda)$. Again, we introduce a dummy time point $T + i$ and denote $x_{i,T+i} = U_i$ for each outgoing vessel $i = n_1 + 1, \dots, n_1 + n_2$. Similar to subproblem $\mathbf{P}'_{\text{in}}(\lambda)$, for $i = n_1 + 1, \dots, n_1 + n_2$ and $t = 0, 1, \dots, T + n_2$, we define

$$G_{it}(\lambda) = \begin{cases} \min_{k=0,1,\dots,m} \left\{G_{it}^{(k)}(\lambda)\right\}, & \text{if } t \in \bigcup_{l=1}^{u_i} [\underline{w}_{il}, \bar{w}_{il} - \bar{\tau}]; \\ C_{0i}, & \text{if } t = T + i - n_1; \\ +\infty, & \text{otherwise;} \end{cases}$$

where

$$G_{it}^{(k)}(\lambda) = \begin{cases} C_{2i} \max\{t + \bar{\tau} - D_i,\, 0\}, & \text{if } k = 0 \text{ and } t = E_i + \tau_{0,b(i)}; \\ C_{2i} \max\{t + \bar{\tau} - D_i,\, 0\} + \sum_{t'=E_i+\tau''_{k,b(i)}}^{t-\tau'_{0k}} \lambda_{kt'}, & \text{if } k > 0 \text{ and } t \ge E_i + \tau''_{k,b(i)} + \tau'_{0k}; \\ +\infty, & \text{otherwise.} \end{cases}$$

Define

$$\begin{aligned} \mathbf{P}'_{\text{out}}(\lambda): \text{ minimize } \quad & \textstyle\sum_{i=n_1+1}^{n_1+n_2} \sum_{t=0}^{T+n_2} G_{it}(\lambda) x_{it} \\ \text{subject to } \quad & \textstyle\sum_{i=n_1+1}^{n_1+n_2} x_{it} \le 1 \quad (t = 0, 1, \dots, T + n_2) \\ & \textstyle\sum_{t=0}^{T+n_2} x_{it} = 1 \quad (i = n_1 + 1, \dots, n_1 + n_2) \\ & x_{it} \in \{0, 1\} \quad (i = n_1 + 1, \dots, n_1 + n_2;\, t = 0, 1, \dots, T + n_2) \end{aligned}$$

Lemma 2 *Solving problem $\mathbf{P}'_{\text{out}}(\lambda)$ yields the optimal objective value for problem $\mathbf{P}_{\text{out}}(\lambda)$.*

Problems $\mathbf{P}'_{\text{in}}(\lambda)$ and $\mathbf{P}'_{\text{out}}(\lambda)$ are $n_1 \times \big(T + n_1 + 1\big)$ and $n_2 \times \big(T + n_2 + 1\big)$ asymmetric assignment problems, respectively, and can be solved by the Hungarian method whose running time is polynomial in the problem size. Since the sizes of these asymmetric assignment problems are linear functions of T, these two problems can be solved in pseudo-polynomial time.

3.2 Upper Bound Heuristic and the Subgradient Method

Given any vector λ of λ_{kt} values, the optimal objective value of problem $\mathbf{P}(\lambda)$, i.e., $L(\lambda)$, is a lower bound on the optimal objective value of problem $\mathbf{P}$. We now develop a heuristic method for constructing a feasible solution of problem $\mathbf{P}$ based on the optimal solution of problem $\mathbf{P}(\lambda)$. The objective value of this feasible solution serves as an upper bound on the optimal objective value of problem $\mathbf{P}$, and this upper bound is used for updating the Lagrangian multipliers in a subgradient optimization framework, which will be described later. In the optimal solution of problem $\mathbf{P}(\lambda)$, each vessel i either cannot be served successfully (i.e., $\sum_{t=0}^{T} x_{it} = 0$) or is assigned a time point for entering the navigation channel (i.e., $\sum_{t=0}^{T} x_{it} = 1$). A feasible solution of problem $\mathbf{P}$ can be constructed by fixing these x_{it} values and then solving the MILP of problem $\mathbf{P}$. After fixing the x_{it} values, the MILP of problem $\mathbf{P}$ can be simplified as follows. Denote $\Omega = \{i \mid \sum_{t=0}^{T} x_{it} = 1; i = 1, \ldots, n_1 + n_2\}$. For $i, j \in \Omega$ such that $i \neq j$, define new binary variables $\delta_{ij} = 1$ if vessel i arrives at its staging anchorage earlier than vessel i, and $\delta_{ij} = 0$ otherwise. Then, a feasible solution to problem $\mathbf{P}$ can be obtained by solving the following problem:

$$
\begin{aligned}
\mathbf{P}_{\text{ub}}: \quad & \text{minimize} \quad \textstyle\sum_{i=1}^{n_1} C_{1i} L_{1i} + \sum_{i=n_1+1}^{n_1+n_2} C_{2i} L_{2i} + \sum_{i=1}^{n_1+n_2} C_{0i} U_i \\
& \text{subject to} \quad (7)\text{–}(12),\ (16)\text{–}(18),\ (20),\ (22)\text{–}(25) \\
& \qquad e_j \geq f_i + 1 - M(1 - \delta_{ij}) \quad (i, j \in \Omega;\ i \neq j) && (26) \\
& \qquad \delta_{ij} + \delta_{ji} \geq y_{ik} + y_{jk} - 1 \quad (i, j \in \Omega;\ i \neq j;\ k = 1, \ldots, m) && (27) \\
& \qquad \delta_{ij} \in \{0, 1\} \quad (i, j \in \Omega;\ i \neq j) && (28)
\end{aligned}
$$

Problem $\mathbf{P}_{\text{ub}}$ differs from problem $\mathbf{P}$ in that (i) x_{it} has become an input parameter, (ii) variable z_{ikt} and constraints (2)–(6) and (13)–(15) have been removed, and (iii) variable δ_{ij} and constraints (26)–(27) have been introduced to ensure that $[e_i, f_i]$ and $[e_j, f_j]$ do not overlap if vessels i and j make use of the same staging anchorage. After solving problem $\mathbf{P}_{\text{ub}}$, we let $z_{ikt} = 0$ if $y_{ik} = 0$ or $t \notin [e_i, f_i]$, and let $z_{ikt} = 1$ if $y_{ik} = 1$ and $t \in [e_i, f_i]$. It is easy to see that these z_{ikt} values satisfy constraints (13)–(15). Hence, solving problem $\mathbf{P}_{\text{ub}}$ yields a feasible solution of problem $\mathbf{P}$. Note that the number of variables in problem $\mathbf{P}_{\text{ub}}$ is independent of T; therefore, the number of decision variables and the number of constraints of problem $\mathbf{P}_{\text{ub}}$ are polynomial in the input size.

We apply the standard subgradient optimization method (see Held, Wolfe, and Crowder 1974) to update the Lagrangian multipliers. The subgradient for constraint (13) at the ℓ-th iteration of the subgradient algorithm is the vector Δ^ℓ with components

$$\Delta_{kt}^\ell = \textstyle\sum_{i=1}^{n_1+n_2} z_{ikt}^\ell - 1,$$

for $k = 1, \ldots, m$ and $t = 0, 1, \ldots, T$. Denote λ_{kt}^ℓ as the value of λ_{kt} at the ℓ-th iteration of the subgradient algorithm. The value of λ_{kt} is updated as follows:

$$\lambda_{kt}^\ell = \begin{cases} 0, & \text{if } \ell = 1; \\ \max\{0, \lambda_{kt}^{\ell-1} + \zeta^{\ell-1}\Delta_{kt}^{\ell-1}\}, & \text{if } \ell \geq 2; \end{cases} \tag{29}$$

where ζ^ℓ is the step size at the ℓ-th iteration.

Let Z_{LD} be the optimal objective value of the Lagrangian dual problem $\mathbf{P}_{\text{LD}}$, and $\underline{Z}(\lambda^\ell)$ be the optimal objective value of problem $\mathbf{P}(\lambda^\ell)$. According to Fisher (2004), the most commonly used step size rule is in the following form:

$$\zeta^\ell = \varepsilon \frac{Z^* - \underline{Z}(\lambda^\ell)}{\|\Delta^\ell\|^2}, \tag{30}$$

where Z^* is an estimate of Z_{LD}, and ε is a step size control parameter. Held, Wolfe, and Crowder (1974) showed that if Z^* is a lower bound of Z_{LD} and $0 < \varepsilon \leq 2$, then $\underline{Z}(\lambda^\ell)$ converges to either Z^* or a value between Z^* and Z_{LD}. Since a good lower bound of Z_{LD} is typically not known, Z^* is usually set to be an upper bound of the optimal objective value of the primal problem $\mathbf{P}$. Meanwhile, to ensure convergence of $\underline{Z}(\lambda^\ell)$, ε is often decreased iteratively so that ζ^ℓ converges to 0.

In our implementation, we determine the step size according to (30). We can make use of an upper bound $\bar{Z}(\lambda^\ell)$ obtained by solving problem $\mathbf{P}_{\text{ub}}$ to estimate Z_{LD}. However, as mentioned earlier, the cost coefficients of unsatisfied service requests (i.e., the C_{0i} values) are often significantly larger than the cost coefficients of berthing and departure tardiness (i.e., the C_{1i} and C_{2i} values). Hence, $\bar{Z}(\lambda^\ell)$ can be much larger than Z_{LD} if $U_i = 1$ for some i in the upper bound solution but $U_i = 0$ for all i in an optimal solution of problem **P**. To overcome this pitfall, we use $Z^* = \min\{\bar{Z}(\lambda^\ell), 2\underline{Z}(\lambda^\ell)\}$ as a better estimate of Z_{LD}.

We start the subgradient algorithm with ε initially set to 1. Let $\bar{Z}$ and $\underline{Z}$ denote the best upper bound and the best lower bound found so far, respectively. When the value of $\underline{Z}$ is not improved for 5 consecutive iterations, we reduce ε by 20%. The subgradient algorithm is terminated when (i) we reach 100 iterations, or (ii) the optimality gap, measured by $\big(\bar{Z} - \underline{Z}\big)/\underline{Z} \times 100\%$, is below 1%. A pseudo-code of our solution method is provided in Appendix B.

4 Computational Experiments

The goal of the computational experiments is threefold. First, we would like to evaluate the computational performance of the Lagrangian relaxation heuristic for problems of different sizes. For this purpose, we generate problem instances with planning horizons of different lengths and compute the optimality gaps of the solutions obtained by the heuristic. We also compare the computational performance of Lagrangian relaxation heuristic with those of benchmark solutions. Two benchmark solution methods are used for comparison. One method is to solve the MILP of problem **P** directly using CPLEX, a well-known mathematical programming solver. Another method is to adopt a rule-based approach which mimics the current practice of the VTS operator at the Yangshan Deep-water Port of Shanghai. Since in practice some parameter values may be different under different situations, our second goal is to investigate how the performance of the Lagrangian relaxation heuristic is affected as these parameters vary. In particular, since weather conditions may affect the availability of staging anchorages in the terminal basin, we examine how the results are affected as the number of staging anchorages varies. We also analyze the sensitivity of the Lagrangian heuristic's performance to different settings of tardiness penalties (i.e., the C_{1i} and C_{2i} values). Because it is not easy to quantify the tardiness penalties, a solution with lower sensitivity to the values of the tardiness penalties is considered more robust, and thus of higher practicality. Our third goal is to evaluate the benefits of taking the anchorage area's capacity into consideration when planning navigation channel traffic. To achieve this, we analyze the solutions obtained by several vessel sequencing policies when the anchorage area's capacity is ignored.

All algorithms were implemented in C#.Net and ran on a computer with a 64-bit Intel i7-6700 3.40GHz CPU and 32GB RAM. In the Lagrangian relaxation heuristic, the upper bound was obtained by solving the MILP of problem $\mathbf{P}_{\text{ub}}$. All MILPs were solved by CPLEX 12.5 with default configurations.

4.1 Generation of Problem Instances

In this subsection, we describe the test instances used in our computational experiments. These test instances are randomly generated with the parameter setting selected based on the physical layout and the characteristics of the operational data of the Yangshan Deep-water Port of Shanghai. The port has one navigation channel with two traffic lanes. One traffic lane is for incoming vessels, and the other is for outgoing vessels. Details of the operational data, the layout of the Yangshan Deep-water Port, and justifications of the parameter setting of the test instances are presented in Appendix C. We let $U\{\alpha, .., \beta\}$ denote the random number generator which returns a uniformly distributed random integer from $\{\alpha, \alpha + 1, \ldots, \beta\}$, and we say that $R \sim U\{\alpha, .., \beta\}$ if R is a random number generated by $U\{\alpha, .., \beta\}$. We let $U[\alpha, \beta]$ denote the random number generator which returns a uniformly distributed random real number lying within the interval $[\alpha, \beta]$, and we say that $R \sim U[\alpha, \beta]$ if R is a random number generated by $U[\alpha, \beta]$.

The safety clearance between vessels traveling through the navigation channel is set equal to 10 minutes. Thus, each time unit represents 10 minutes. The amount of time for a vessel to travel through the navigation channel $\bar{\tau}$ is set equal to 12 time units. We generate test instances with planning horizons varying from 1 day to 7 days (i.e., T is set equal to $144, 288, \ldots, 1008$ time units) in three categories, namely the low-traffic case, the medium-traffic case, and the heavy-traffic case, and develop 21 problem sets. For each problem set, we generate 5 random test instances, and thus, there are 105 test instances in total. For a test instance with a planning horizon of d days, the number of incoming vessels is obtained by setting $n_1 \sim U\{10d, .., 12d\}$ for the low-traffic case, $n_1 \sim U\{12d, .., 14d\}$ for the medium-traffic case, and $n_1 \sim U\{14d, .., 16d\}$ for the heavy-traffic case. The number of outgoing vessels n_2 is set equal to n_1. See Table 1 for a summary of the problem sets.

Table 1: Problem sets used in the computational study.

	Low-traffic		Medium-traffic		Heavy-traffic	
T	Problem set	n_1 and n_2	Problem set	n_1 and n_2	Problem set	n_1 and n_2
144	L-1	$U\{10,..,12\}$	M-1	$U\{12,..,14\}$	H-1	$U\{14,..,16\}$
288	L-2	$U\{20,..,24\}$	M-2	$U\{24,..,28\}$	H-2	$U\{28,..,32\}$
432	L-3	$U\{30,..,36\}$	M-3	$U\{36,..,42\}$	H-3	$U\{42,..,48\}$
576	L-4	$U\{40,..,48\}$	M-4	$U\{48,..,56\}$	H-4	$U\{56,..,64\}$
720	L-5	$U\{50,..,60\}$	M-5	$U\{60,..,70\}$	H-5	$U\{70,..,80\}$
864	L-6	$U\{60,..,72\}$	M-6	$U\{72,..,84\}$	H-6	$U\{84,..,96\}$
1008	L-7	$U\{70,..,84\}$	M-7	$U\{84,..,98\}$	H-7	$U\{98,..,102\}$

In each test instance, there are 16 berthing positions, numbered from 1 to 16. For example, if vessel i is designated to berth at the 10th position, then $b(i) = 10$. The number of staging anchorages is set to $m = 3$. The travel speed of all vessels in the terminal basin is set equal to 1000 meters per time unit. The coordinates of the berthing positions, staging anchorages, and end of navigation channel, measured in meters, are provided in Table 2. The travel times $\tau_{0,b(i)}$, τ'_{0k}, and $\tau''_{k,b(i)}$ are obtained by dividing the Euclidean distances between the two locations concerned by the travel speed of the vessel and rounding the results to the nearest integers. For example, suppose vessel i is assigned to staging anchorage 1 and $b(i) = 10$. Since the Euclidean distance between staging anchorage 1 and berthing position 10 is $\sqrt{(3500-1800)^2+(0-2000)^2} \approx 2625$ meters, we have $\tau''_{1,b(i)} = 2625/1000 \approx 3$.

The vessel-related parameters are generated randomly as follows. For each incoming vessel i, $b(i) \sim U\{1,..,16\}$, $B_i \sim U\{20,..,T\}$, $A_i = \max\{0, B_i - R\}$ with $R \sim U\{100,..,250\}$, and

Table 2: Coordinates of different locations.

Berthing positions and their coordinates		Staging anchorages and their coordinates		Coordinates of the end of navigation channel
1	(350, 0)	1	(1800, 2000)	(0, 600)
2	(700, 0)	2	(2800, 2000)	
3	(1050, 0)	3	(3800, 2000)	
4	(1400, 0)			
5	(1750, 0)			
6	(2100, 0)			
7	(2450, 0)			
8	(2800, 0)			
9	(3150, 0)			
10	(3500, 0)			
11	(3850, 0)			
12	(4200, 0)			
13	(4550, 0)			
14	(4900, 0)			
15	(5250, 0)			
16	(5600, 0)			

$\bar{B}_i = \min\{B_i + R, T\}$ with $R \sim U\{150, .., 180\}$. For each outgoing vessel i, $b(i) \sim U\{1, .., 16\}$, $E_i \sim U\{0, .., T-20\}$, and $D_i = \max\{0, E_i + R\}$ with $R \sim U\{-40, .., 80\}$. Vessels are classified into "deep-draft vessels" (i.e., vessels with draft greater than 12.5) and "non-deep-draft vessels." Non-deep-draft vessels are unaffected by tides and have a single tidal window $[0, T]$, while the utilization of the navigation channel of deep-draft vessels may be affected by tides. Among the $n_1 + n_2$ vessels, 24% of them are selected to be deep-draft vessels. The draft of each deep-draft vessel is generated by $U[12.5, 15.2]$, and the tidal window of each deep-draft vessel is derived from the water level. We simulate the fluctuation of water level by a sine wave $16 + 1.5 \sin \frac{\pi t}{36}$, which represents an average water level of 16 meters, an average highest/lowest tide of ± 1.5 meters, and an average tidal cycle of 72 time units. When deriving the tidal windows of vessels, a seabed clearance of 2 meters is taken into account. For example, when $T = 144$, a deep-draft vessel with draft 12.6 has tidal windows $\left\{t \;\middle|\; 16 + 1.5 \sin \frac{\pi t}{36} \geq 14.6;\, 0 \leq t \leq T\right\} = [0, 49.7] \cup [58.3, 121.7] \cup [130.3, 144]$. In practice, deep-draft vessels have a higher service priority than non-deep-draft vessels. To capture the service priorities of vessels, we set the tardiness penalties C_{1i} and C_{2i} to 2 if vessel i is a deep-draft vessel, and set C_{1i} and C_{2i} to 1 if vessel i is a non-deep-draft vessel. Since satisfying the vessel service requests is the most important goal of the VTS operator, we impose a sufficiently large penalty cost on each unsatisfied service request. In our test instances, the penalty C_{0i} is set equal to 10000 for all i. This large penalty, if incurred, reflects the situation where the solution fails to schedule all the vessels existing in the given berth plan.

4.2 Benchmark Methods

To evaluate the computational performance of the Lagrangian relaxation heuristic, we introduce two benchmark methods for solving problem **P** and compare the performance of the Lagrangian relaxation heuristic with the performances of these benchmark methods. The first benchmark method is to solve the MILP formulation of problem **P** presented in Section 2 directly by CPLEX. The second benchmark method is a rule-based heuristic, which simulates the decision of a VTS operator. In the following, we provide a description on the manual decision process.

The VTS officers at the Yangshan Deep-water Port currently adopt a rule-based approach to manage the traffic in the navigation channel and the utilization of the staging anchorages. This rule-based heuristic gives priority to outgoing vessels to ensure on-time unberthing, so as to provide berth vacancy for subsequent vessels. It first considers the outgoing vessels one by one according to their unberthing times. In each step, the heuristic allocates a time slot of the navigation channel to the outgoing vessel being considered. It also allocates some time slots of a staging anchorage to this vessel if the vessel cannot travel to the channel and exit the port directly. If two outgoing vessels have the same unberthing time, then the one with the higher service priority (i.e., larger C_{2i} value) is considered first. Next, the heuristic considers the incoming vessels one by one according to their planned berthing times. In each step, it allocates the next available time slot of the navigation channel to the incoming vessel being considered. It also allocates some time slots of a staging anchorage to this vessel if the vessel is too early to travel to its berth directly. If two incoming vessels have the same planned berthing time, then the one with the higher service priority (i.e., larger C_{1i} value) is considered first.

The pseudo-code of the rule-based heuristic used to simulate the manual decision process is presented in Appendix D.

4.3 Computational Results: Comparison with Benchmark Methods

We solve the problem sets presented in Table 1 in Section 4.1 using the Lagrangian relaxation heuristic and the two benchmark methods, namely solving the MILP directly via CPLEX (which we refer to as the "MILP/CPLEX method") and the rule-based heuristic. When solving each problem, the running time of the MILP/CPLEX method is restricted to no more than 3600 seconds. Table 3 reports the computational results obtained by the Lagrangian relaxation heuristic, the MILP/CPLEX method, and the rule-based heuristic, where each row summarizes the results of 5 random test instances. For each problem set and each solution method, the "#I" column reports the number of instances (out of 5 instances) in which some service requests cannot be satisfied, and the "#R" column reports the average number of unsatisfied service requests. Detailed computational results, including the upper bound and lower bound values of each test instance, are provided in Appendix E.

Table 3: Computational results for the tested problem sets.

Problem set	Lagrangian Relaxation					MILP/CPLEX					Rule-based Heuristic				
	#I	#R	G1 (%)	G2 (%)	Time	#I	#R	G1 (%)	G2 (%)	Time	#I	#R	G1 (%)	G2 (%)	Time
Low-traffic															
L-1	0	0	0.0	0.0	0.1	0	0	0.0	0.0	11.5	1	0.4	≥ 100	0.5	0.0
L-2	0	0	2.7	2.7	3.1	0	0	5.5	5.5	2414.1	1	0.2	≥ 100	12.1	0.0
L-3	0	0	0.7	0.7	3.1	1	0.4	≥ 100	11.4	3168.4	0	0	14.3	14.3	0.0
L-4	0	0	0.0	0.0	0.8	3	1.0	≥ 100	6.6	3580.6	1	0.2	≥ 100	16.4	0.0
L-5	1	0.2	≥ 100	1.6	21.1	5	7.2	≥ 100	–	3600	2	0.6	≥ 100	9.5	0.0
L-6	0	0	1.2	1.2	36.8	5	26.8	≥ 100	–	3600	2	0.4	≥ 100	21.0	0.0
L-7	2	0.4	≥ 100	1.4	75.1	5	82.8	≥ 100	–	3600	1	0.6	≥ 100	18.3	0.0
Medium-traffic															
M-1	0	0	0.0	0.0	0.1	0	0	0.0	0.0	16.1	0	0	4.4	4.4	0.0
M-2	0	0	1.9	1.9	1.7	0	0	8.4	8.4	2932.5	2	0.4	≥ 100	21.2	0.0
M-3	0	0	3.1	3.1	13.6	2	0.4	≥ 100	27.3	3600	0	0	14.5	14.5	0.0
M-4	0	0	3.2	3.2	34.4	5	4.8	≥ 100	–	3600	0	0	26.5	26.5	0.0
M-5	0	0	4.4	4.4	60.7	5	7.6	≥ 100	–	3600	1	0.2	≥ 100	21.8	0.0
M-6	0	0	1.5	1.5	98.7	5	28.4	≥ 100	–	3600	1	0.2	≥ 100	23.2	0.0
M-7	1	0.4	≥ 100	4.8	176.2	5	87.8	≥ 100	–	3600	1	0.4	≥ 100	32.2	0.0
Heavy-traffic															
H-1	0	0	0.1	0.1	0.2	0	0	0.0	0.0	54.3	1	0.4	≥ 100	3.2	0.0
H-2	0	0	0.5	0.5	1.6	1	0.4	≥ 100	18.4	2978.1	0	0	23.4	23.4	0.0
H-3	0	0	1.8	1.8	20.8	4	2.2	≥ 100	29.1	3600	2	0.4	≥ 100	23.7	0.0
H-4	1	0.2	≥ 100	4.7	39.5	5	6.0	≥ 100	–	3600	2	1.2	≥ 100	30.7	0.0
H-5	0	0	7.7	7.7	106.8	5	22.8	≥ 100	–	3600	1	0.2	≥ 100	35.6	0.0
H-6	1	0.4	≥ 100	4.4	133.5	5	91.4	≥ 100	–	3600	3	1.8	≥ 100	40.9	0.0
H-7	0	0	4.2	4.2	273.1	5	134.0	≥ 100	–	3600	3	0.6	≥ 100	33.6	0.0

The “G1” columns report the average optimality gaps of the objective values of the test instances in each problem set. The optimality gap of a test instance is given by $(UB - LB)/LB \times 100\%$, where UB is the objective value of the solution generated by the solution method concerned, and LB is the lower bound generated by the Lagrangian relaxation heuristic. Recall that the penalty on each unsatisfied service request, C_{0i}, is significantly larger than the tardiness penalties C_{1i} and C_{2i}. Hence, if there exists an instance for which all vessel service requests are satisfied in the lower bound solution but some are not satisfied in the upper bound solution, then the optimality gap of that instance, and thus the G1 value of the problem set, would become very large. As a supplementary to the G1 columns, we report in the “G2” columns the average optimality gaps of those test instances for which all vessel service requests are satisfied in the upper bound solutions. Hence, the G2 values represent the accuracy of the solution methods on determining the berthing and departure tardiness costs. For each problem set, if service requests are not satisfied in each of the 5 instances, then the G2 value for this problem set is unavailable. The running time (in seconds) of the solution methods is reported in the “Time” columns.

From Table 3, we observe that the values in the "#I" column of Lagrangian relaxation heuristic are small, showing that the Lagrangian relaxation heuristic could identify complete vessel schedules for most of the test instances. In particular, for those instances with planning horizons no more than 3 days (i.e., instances in L-1, L-2, L-3, M-1, M-2, M-3, H-1, H-2, and H-3), the Lagrangian relaxation heuristic successfully finds solutions with no unsatisfied service request. In contrast, both the MILP/CPLEX method and rule-based heuristic fail to identify complete vessel schedules frequently. Values in the "#R" columns show that the Lagrangian relaxation heuristic results in very few unsatisfied service requests. However, the MILP/CPLEX method results in far more unsatisfied service requests. For instances with planning horizons longer than 3 days, the number of unsatisfied service requests generated by the MILP/CPLEX method increases dramatically as the length of the planning horizon increases. The rule-based heuristic also results in more unsatisfied service requests than the Lagrangian relaxation heuristic. Thus, the Lagrangian relaxation heuristic outperforms the benchmark methods by far in minimizing the number of unsatisfied service requests.

To evaluate the performance of the solution methods on minimizing the berthing and departure tardiness, we compare the optimality gaps on the tardiness cost, i.e., the values in the G2 columns. The performance of the Lagrangian relaxation algorithm is as good as that of CPLEX for small-sized instances in L-1 and M-1. For larger problem instances, the G2 values of MILP/CPLEX are either unavailable or greater than those of the Lagrangian relaxation heuristic. The G2 values of rule-based heuristic are significantly worse than those of the Lagrangian relaxation heuristic. Hence, the Lagrangian relaxation heuristic is superior over the benchmark methods in minimizing the berthing and departure tardiness of vessels. Overall, the average optimality gap of the Lagrangian relaxation heuristic on the tardiness cost is within 8% for each of the tested problem sets. The Lagrangian relaxation heuristic reaches the stopping criteria within a reasonable amount of computer time. On the other hand, the MILP/CPLEX method finds optimal solutions within the running time limit only for small instances.

From Table 3, we observe that the #I, #R, G1, and G2 values of all three solution methods tend to increase as the length of the planning horizon increases, indicating that it is more difficult to schedule the vessel traffic for longer term planning. The performance of the three solution methods is also affected by the traffic density. As the traffic density increases, it is more difficult to ensure on-time berthing and departure, and thus the objective values of the solutions generated by the three solution methods, as well as the Lagrangian relaxation lower bound, tend to increase as the traffic density increases (see Tables A2–A4 in Appendix E). In addition, we observe that the G1 and G2 values of all three solution methods also tend to increase as the traffic becomes heavier. This shows the increase in difficulty in finding good solutions as the traffic density increases. However, the #I and #R values of Lagrangian relaxation heuristic are hardly affected by traffic density. This shows that the Lagrangian relaxation heuristic is quite capable in minimizing unsatisfied service requests under heavy traffic conditions.

As mentioned earlier, the rule-based heuristic mimics the current practice of the VTS operator at the Yangshan Deep-water Port of Shanghai. Hence, a comparison between the performance of the Lagrangian relaxation heuristic and that of the rule-based heuristic reflects the benefit of using the Lagrangian relaxation heuristic. Table 4 summarizes the overall improvement generated by the Lagrangian relaxation heuristic as opposed to the rule-based heuristic. The comparison shows that the Lagrangian relaxation heuristic results in a 17.9% reduction in vessel tardiness. The most significant benefit brought by the Lagrangian relaxation heuristic is that the Lagrangian relaxation solutions have substantially fewer unsatisfied service requests. Because the terminal operators will have to adjust the berth plans when vessels' service requests cannot be satisfied, the Lagrangian relaxation heuristic can help reduce the frequency of berth plan adjustment.

Table 4: Performance improvement of Lagrangian relaxation over rule-based heuristic.

	Number of instances with unsatisfied service requests	Average number of unsatisfied service requests per instance	Average tardiness cost per instance	Average total cost per instance
Lagrangian Relaxation	6	0.1	1183.4	1945.3
Rule-based Heuristic	25	0.3	1440.9	5345.7
Percentage Improvement	76.0%	82.9%	17.9%	63.6%

4.4 Computational Results: Varying Anchorage Capacity and Tardiness Penalties

In the computational study presented in Section 4.3, we have set the number of staging anchorages to 3. However, in practice, the number of available staging anchorages in the terminal basin is different under different situations. Thus, we investigate how the computational performance of the Lagrangian relaxation heuristic is affected as the anchorage capacity varies. For this purpose, we solve the problem sets with the longest planning horizon (i.e., L-7, M-7, and H-7) using the Lagrangian relaxation heuristic with different values of m.

We analyze the computational results with m set equal to 1, 2, 3, 4, and 5. When changing the value of m, we regenerate the locations of the staging anchorages by evenly dividing the anchorage area of the terminal basin (see Figure A1 in Appendix C) into m parts. Table 5 summarizes the computational results. From these results, we observe that the Lagrangian relaxation heuristic is successful in identifying complete vessel schedules when m equals 5, but often fails to do so when m equals 1 and 2. This is because when the number of staging anchorages decreases, the chance that the optimal solution contains no unsatisfied service request drops. We also observe that the optimality gap G2 improves as m increases. This is because the Lagrangian relaxation problem $\mathbf{P}(\lambda)$ is developed by dualizing constraint (13). Since the purpose of constraint (13) is to ensure that at most one vessel dwells at each staging anchorage at each time point, increasing the number of staging anchorages makes this constraint less restrictive, and, as a result, tightens the lower bound.

In the computational study presented in Section 4.3, we have set the tardiness penalties C_{1i} and C_{2i} to 1 or 2 depending on whether vessel i is a deep-draft vessel or not. In practice, there

Table 5: Computational results under different values of m.

Problem set	m	#I	#R	G1 (%)	G2 (%)	Time
L-7	1	5	4.4	$\geq$ 100	–	49.0
	2	2	2.8	$\geq$ 100	8.2	78.2
	3	2	0.4	$\geq$ 100	1.4	75.1
	4	0	0	0.5	0.4	25.9
	5	0	0	0.1	0.1	11.6
M-7	1	5	5	$\geq$ 100	–	96.9
	2	3	1	$\geq$ 100	11.4	112.7
	3	1	0.4	$\geq$ 100	4.8	176.2
	4	1	0.2	$\geq$ 100	1.7	89.6
	5	0	0	0.2	0.2	20.1
H-7	1	5	3.2	$\geq$ 100	–	150.2
	2	2	0.4	$\geq$ 100	17.8	201.2
	3	0	0	4.2	4.2	273.1
	4	0	0	1.2	1.2	172.5
	5	0	0	0.2	0.2	52.8

may be other factors affecting a vessel's service priority. Thus, we investigate the sensitivity of the Lagrangian relaxation heuristic performance to a change in the tardiness penalties' distribution. For this purpose, we generate new instances from the problem sets L-7, M-7, and H-7 by randomly perturbing the tardiness penalties of vessels. Let $\nu \in [0, 1)$ be a parameter, and let C'_{1i} and C'_{2i} be the tardiness penalties of vessels after perturbation. For each instance in the original data set, we generate new instances by setting $C'_{1i} = \rho C_{1i}$ with $\rho \sim U[1-\nu, 1+\nu]$ for each $i = 1, \ldots, n_1$, and setting $C'_{2i} = \rho C_{2i}$ with $\rho \sim U[1-\nu, 1+\nu]$ for each $i = n_1 + 1, \ldots, n_1 + n_2$. For each instance in problem sets L-10, M-10, and H-10, we generate four new instances with ν set equal to 0.1, 0.2, 0.3, 0.4. We solve these new instances using the Lagrangian relaxation heuristic.

Table 6 summarizes the computational results under different values of ν. From these results, we observe that the #I and #R values are insensitive to the change in ν value, showing the number of unsatisfied service requests is insensitive to the change of tardiness penalties. We also observe that the G1 and G2 values are also insensitive to the change in ν value. This implies that the performance of the Lagrangian relaxation heuristic is insensitive to the C_{1i} and C_{2i} values.

Table 6: Computational results under different values of ν.

Problem set	ν	#I	#R	G1 (%)	G2 (%)	Time
L-7	0	2	0.4	$\geq$ 100	1.4	75.1
	0.1	2	0.4	$\geq$ 100	1.3	76.1
	0.2	2	0.4	$\geq$ 100	1.4	93.4
	0.3	2	0.4	$\geq$ 100	1.2	86.2
	0.4	2	0.4	$\geq$ 100	1.2	80.2
M-7	0	1	0.4	$\geq$ 100	4.8	176.2
	0.1	1	0.4	$\geq$ 100	5.4	170.7
	0.2	1	0.4	$\geq$ 100	5.8	172.3
	0.3	1	0.4	$\geq$ 100	5.3	185.1
	0.4	1	0.4	$\geq$ 100	6.0	182.2
H-7	0	0	0	4.2	4.2	273.1
	0.1	0	0	4.8	4.8	291.4
	0.2	0	0	4.9	4.9	282.6
	0.3	0	0	5.2	5.2	292.5
	0.4	0	0	4.7	4.7	269.1

4.5 Computational Results: Sequencing Vessels by Ignoring Anchorage Capacity

One key feature of our optimization model is that the capacity of the anchorage area and the capacity of the navigation channel are considered simultaneously. If the capacity of the anchorage area is ignored, then the problem becomes a pure vessel-sequencing problem that can be solved efficiently via some heuristic rules. However, the effectiveness of the solution is affected if the capacity of the anchorage area is not considered when the traffic of the navigation channel is scheduled. In this subsection, we computationally assess the benefits of taking the anchorage area's capacity into consideration when planning navigation channel traffic.

Suppose the capacity of the anchorage area is ignored. Then, the main problem is to determine the sequence of incoming vessels that enters the navigation channel from the anchorage ground, as well as the sequence of outgoing vessels that enters the navigation channel from the port basin. We consider several vessel sequencing policies that other researchers have considered for port management, including the first-come first-served (FCFS) policy, the shortest time windows length (STW) policy, and the random sequencing (RS) policy; see Cordeau et al. (2005) and Lalla-Ruiz, Shi, and Voß (2018).

We computationally evaluate the performances of the FCFS, STW, and RS policies for sequencing incoming and outgoing vessels when these policies are applied to our model. When applying the FCFS policy to our problem, we let the incoming vessels enter the navigation channel in ascending order of their port arrival times, and let the outgoing vessels enter the navigation channel in ascending order of their unberthing time, with ties broken arbitrarily. When applying the STW policy to our problem, we let the incoming vessels enter the navigation channel in ascending order of their tidal window lengths, and let the outgoing vessels enter the navigation channel in ascending order of their tidal window lengths, with ties broken arbitrarily. When applying the RS policy to our problem, a random sequence of incoming vessels is generated by selecting vessels one by one with equal probability, and a random sequence of outgoing vessels is generated by selecting vessels one by one with equal probability.

Because we have ignored the anchorage capacity when scheduling the navigation channel traffic, the anchorage area may run out of capacity during execution. Thus, we need a mechanism to allocate the anchorage space. In our computational experiments, after obtaining the sequence of incoming vessels and the sequence of outgoing vessels, we allocate the staging anchorages to the vessels in a greedy manner as follows:

Step 1: Set $t \leftarrow 0$.

Step 2: Consider the next vessel i in the given outgoing vessel sequence.

- Step 2.1: Determine the earliest time point $t' \geq t+1$ at which vessel i can enter the navigation channel.
- Step 2.2: If vessel i cannot travel directly to the navigation channel and reach the end of the channel at time t', then search for an available staging anchorage for vessel i (with ties broken arbitrarily), and let i be an unsatisfied service request if no staging anchorage is available.
- Step 2.3: Set $t \leftarrow t'$. If vessel i is not the last vessel in the outgoing vessel sequence, then repeat Step 2.

Step 3: Set $t \leftarrow 0$.

Step 4: Consider the next vessel i in the given incoming vessel sequence.

- Step 4.1: Determine the earliest time point $t' \geq t+1$ at which vessel i can enter the navigation channel.
- Step 4.2: If vessel i cannot travel directly to its berthing position after entering the channel at time point t', then search for an available staging anchorage for vessel i so that vessel i can reach $b(i)$ during the time window $[B_i, \bar{B}_i]$ (with ties broken arbitrarily), and let i be an unsatisfied service request if no such staging anchorage is available.
- Step 4.3: Set $t \leftarrow t'$. If vessel i is not the last vessel in the incoming vessel sequence, then repeat Step 4.

Note that in the above steps, priority is given to outgoing vessels in order to ensure on-time unberthing and to provide berth vacancy for subsequent vessels. As mentioned by Lalla-Ruiz, Shi, and Voß (2018), the RS policy has the advantage that different vessel sequences can be obtained by repeating the policy multiple times and that each run requires very little time effort. Hence, in our computational study, if the RS policy fails to generate a solution with no unsatisfied vessel service, then we repeat the RS policy with different vessel sequences until either (i) a solution with no unsatisfied vessel service is generated, or (ii) the RS policy has been repeated 100 times.

We test the FCFS, STW, and RS policies using the same test instances as in Section 4.3. Table 7 reports the results, where each row summarizes the results of 5 random test instances. Since the execution of these policies requires very little computational time, the running times are

Table 7: Comparison with different vessel sequencing policies.

Problem set	Lagrangian Relaxation				FCFS				STW				RS			
	#I	#R	G1 (%)	G2 (%)	#I	#R	G1 (%)	G2 (%)	#I	#R	G1 (%)	G2 (%)	#I	#R	G1 (%)	G2 (%)
Low-traffic																
L-1	0	0	0.0	0.0	0	0	≥ 100	≥ 100	0	0	≥ 100	≥ 100	0	0	97.1	97.1
L-2	0	0	2.7	2.7	0	0	39.4	39.4	5	9.2	≥ 100	–	5	16.2	≥ 100	–
L-3	0	0	0.7	0.7	0	0	44.7	44.7	5	20.2	≥ 100	–	5	25.2	≥ 100	–
L-4	0	0	0.0	0.0	0	0	43.3	43.3	5	35.4	≥ 100	–	5	37.8	≥ 100	–
L-5	1	0.2	≥ 100	1.6	0	0	52.1	52.1	5	46.2	≥ 100	–	5	51	≥ 100	–
L-6	0	0	1.2	1.2	0	0	65.8	65.8	5	59	≥ 100	–	5	63.6	≥ 100	–
L-7	2	0.4	≥ 100	1.4	2	0.6	≥ 100	50.7	5	72.8	≥ 100	–	5	74.8	≥ 100	–
Medium-traffic																
M-1	0	0	0.0	0.0	0	0	33.1	33.1	0	0	41.8	41.8	0	0	≥ 100	≥ 100
M-2	0	0	1.9	1.9	0	0	69.7	69.7	4	12	≥ 100	≥ 100	4	16.8	≥ 100	≥ 100
M-3	0	0	3.1	3.1	0	0	54.7	54.7	5	26.2	≥ 100	–	5	34.6	≥ 100	–
M-4	0	0	3.2	3.2	0	0	80.7	80.7	5	46.6	≥ 100	–	5	50.6	≥ 100	–
M-5	0	0	4.4	4.4	0	0	70.3	70.3	5	60.4	≥ 100	–	5	62.6	≥ 100	–
M-6	0	0	1.5	1.5	1	0.2	≥ 100	87.9	5	74.2	≥ 100	–	5	76	≥ 100	–
M-7	1	0.4	≥ 100	4.8	1	0.4	≥ 100	58.0	5	88	≥ 100	–	5	91	≥ 100	–
Heavy-traffic																
H-1	0	0	0.1	0.1	0	0	≥ 100	≥ 100	0	0	≥ 100	≥ 100	0	0	≥ 100	≥ 100
H-2	0	0	0.5	0.5	0	0	≥ 100	≥ 100	5	15.2	≥ 100	–	5	23.4	≥ 100	–
H-3	0	0	1.8	1.8	1	0.2	≥ 100	78.0	5	37	≥ 100	–	5	35.6	≥ 100	–
H-4	1	0.2	≥ 100	4.7	1	0.2	≥ 100	95.1	5	55.2	≥ 100	–	5	56.4	≥ 100	–
H-5	0	0	7.7	7.7	0	0	≥ 100	≥ 100	5	70	≥ 100	–	5	71.8	≥ 100	–
H-6	1	0.4	≥ 100	4.4	1	0.4	≥ 100	87.3	5	84	≥ 100	–	5	88.6	≥ 100	–
H-7	0	0	4.2	4.2	1	0.2	≥ 100	88.1	5	98.4	≥ 100	–	5	≥ 100	≥ 100	–

not included in this table. Detailed computational results for each test instance are provided in Appendix E.

From Table 7, we observe that the #I and #R values of the FCFS policy are comparable to those of the Lagrangian relaxation heuristic, and are significantly smaller than those of the STW and RS policies. This is because berth planners typically plan the incoming vessels' berthing times based on their arrival times, and plan the outgoing vessels' unberthing times based on their expected departure times. Thus, in the given data, the latest allowed berthing times of the incoming vessels are positively correlated with their arrival times, and the unberthing times of the outgoing vessels are positively correlated with their expected departure times. Since the FCFS policy sequences the incoming and outgoing vessels based on the their arrival and unberthing times, respectively, it requires less anchorage capacity for the vessels than the other policies. Hence, it is relatively easy for the FCFS policy to satisfy all service requests. However, since these vessel sequencing policies ignore the utilization of the staging anchorages when they sequence the vessels, the staging anchorages tend to be utilized ineffectively in the solutions generated by these policies. Therefore, as indicated by the G2 values in Table 7, the performances of the FCFS, STW, and RS policies in minimizing the berthing and departure tardiness of vessels are worse than that of the Lagrangian relaxation heuristic. Overall, the computational results indicate that integrating the management of the anchorage area utilization into the decision for channel traffic control can significantly improve the vessel service.

5 Conclusions

In this research, we model and solve the vessel traffic scheduling problem of a container port. The problem involves the management of navigation channel traffic and the utilization of staging anchorages. We develop an MILP formulation for the problem and show that the problem is strongly NP-hard. We also show that after relaxing the anchorage capacity constraint, the resulting subproblems can be transformed into two asymmetric assignment problems that are solvable in pseudo-polynomial time. Based on this observation, we develop a Lagrangian relaxation heuristic for the problem.

We generate test instances based on the layout and operational data of the Yangshan Deep-Water Port of Shanghai, and compare the computational performance of the Lagrangian relaxation heuristic with those of CPLEX and a rule-based heuristic that mimics the current practice of a VTS operator. The computational results indicate that the manual decision process for scheduling vessel traffic often results in large tardiness penalties and a large number of unsatisfied service requests, while CPLEX finds optimal solutions within reasonable computation times only for the smallest instances. On the other hand, the Lagrangian relaxation heuristic provides high-quality solutions in relatively short computation times for instances with planning horizons varying from one day to one week. Therefore, the Lagrangian relaxation heuristic achieves a good balance between effectiveness and efficiency, and thus can serve as a decision support tool for congestion mitigation and improving vessel service levels at container ports. Furthermore, we computationally test the performances of various vessel sequencing policies by ignoring the anchorage capacity, and find that the solutions obtained by ignoring the anchorage capacity incur significantly larger penalty costs than the solutions obtained by the Lagrangian relaxation heuristic. This indicates the importance of considering the anchorage capacity when scheduling the navigation channel traffic.

Future research could focus on developing new models and solution approaches for addressing a variety of problem extensions. For instance, vessels are guided by tugboats when sailing in or out of a seaport (see, e.g., Ilati, Sheikholeslami, and Hassannayebi 2014). When the availability of tugboats is limited, decisions for scheduling the tugboats should be integrated into the navigation channel and staging anchorage management problem. Some seaports have navigation channels with single traffic lane. Those seaports are often highly congested as incoming and outgoing vessels share the same traffic lane when traveling through a navigation channel. Modeling and solving a problem with incoming and outgoing vessels sharing a single traffic lane could be a future research topic. Another interesting research direction could be to develop and analyze a model that integrates the decisions for berth allocation, navigation channel traffic, and staging anchorage utilization, since such integration could significantly improve port performance in terms of throughput enhancement and congestion mitigation. However, as mentioned in Section 1, berth allocation decisions are made by terminal operators whereas vessel traffic is regulated by VTS operators. Thus, a model that integrates berth allocation and vessel traffic management is applicable only if there is an authority that could centralize the decision making for both terminal operations and vessel traffic regulations. In the current study, all the input parameters of the problem are assumed to be given in advance. However, in practice, due to uncertainties caused by weather conditions and unexpected events, the actual values of some parameters may change as time evolves. In this case, our model and solution method can be applied by following a rolling-horizon approach, where the navigation and anchorage utilization plan needs to be re-optimized whenever the values of some input parameters are changed. An alternative approach to tackling the problem with uncertainty in input parameters is to formulate the problem into stochastic programming models or robust optimization models, which we leave to future research. Finally, our current solution method is not an exact approach. Developing an exact solution method and comparing its computational performance with the performance of our current solution method could be another research direction.

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