Financing an Agricultural Supply Chain with a Capital-Constrained Smallholder Farmer in Developing Economies

Zelong Yi

Department of Transportation Economics and Logistics Management, College of Economics, Shenzhen University, Shenzhen, Guangdong, China, yizl@szu.edu.cn

Yulan Wang*[1]

Faculty of Business, The Hong Kong Polytechnic University, Kowloon, Hong Kong, yulan.wang@polyu.edu.hk

Ying-Ju Chen

Department of ISOM, School of Business and Management & Department of IELM, School of Engineering, Hong Kong University of Science and Technology, imchen@ust.hk

---

We consider an agricultural supply chain consisting of a capital-constrained smallholder farmer and an intermediary platform. The smallholder farmer sells agricultural products through the intermediary platform but lacks financial resources for planting. In addition to traditional *bank financing* (provided by a bank), the creditworthy intermediary platform can provide loans directly to the farmer (known as *direct financing*) or serve as a guarantor if the farmer's creditworthiness is insufficient to access bank loans (known as *guarantor financing*). We show that under guarantor and direct financing, the smallholder farmer's production level can be even higher than that in a centralized system. The farmer prefers direct financing when the production cost is low but the unit commission fee is sufficiently high. Otherwise, he prefers guarantor financing. The intermediary platform will encourage the farmer to resort to bank financing when the farmer's production cost is sufficiently high and the commission fee is low. Otherwise, it will provide direct financing. Guarantor financing makes the platform weakly worse off than direct financing and will be adopted only when the platform is also capital-constrained. The involvement of the intermediary platform significantly improves the welfare of the farmer and the total profit of the supply chain. Moreover, the increased concern for social responsibility of the intermediary platform can lead to a *win-win-win* outcome for the farmer, the platform, and the whole supply chain.

Keywords: Smallholder Farmer; Bank Financing; Guarantor Financing; Direct Financing; Agricultural Supply Chain; Social Responsibility Concern.

---

[1] Corresponding author.

1. Introduction

As the backbone of social development in most developing countries, agriculture plays a pivotal role in economic stability. Traditionally, smallholder farmers in developing countries sell agricultural products locally due to limitations such as logistics, transportation, finance, and information resources. Thanks to recently developed electronic platforms, farmers can now access a larger market and trade with customers far from their locality. As a result, these intermediary platforms have become increasingly popular among farmers to sell their products.[1] For instance, Jingdong (JD.com; JD hereafter), a China-based e-commerce platform, provides an intermediary service for farmers.[2] However, a hard fact in the agricultural industry is that smallholder farmers in developing countries often lack financial resources to procure inputs such as seeds, fertilizers, pesticides and farm equipment. This financial insufficiency may significantly affect the supply of agricultural products or even the stability of the whole supply chain. The World Bank reports that the need to invest in agricultural products has increased due to a growing world population and changing dietary preferences for high-value food products among the middle class in emerging markets.[3] Therefore, the importance of financing agricultural products can never be overestimated. A variety of financing approaches, including commercial banks, non-profit organizations, and government-led farmer support programs, have been launched to finance smallholder farmers in Asia, North Africa, and Latin America (Thierry et al. 2011; IFC 2014; Daniela 2015).

A traditional method of raising funds is through the banking market. However, the lack of creditworthiness and valuable collateral means that the bank treats the smallholder farmer as a high-risk candidate for a loan. Thus, bank financing, if government intervention is not considered, is often an unavailable or inefficient channel for the smallholder farmer to alleviate his capital stress. Meanwhile, an intermediary platform can be a key player in the agricultural product supply chain, and its profitability is undoubtedly negatively affected by the farmer's lack of access to finance. In extreme situations, the business relationship between the platform and the smallholder farmer may break down if the farmer does not have the financial resources to start production. Intermediary platforms with very high creditworthiness, such as JD, are willing to provide a *guarantor financing* service by assuming the role of guarantor. This service guarantees the safety of the bank's investment as the intermediary platform will fill the financial gap when the farmer's realized revenue cannot fully cover the repayment to the bank. With the provision of a guarantee

[1] For the news report on China's recent agricultural e-commerce boom, please see the following website: `http://technode.com/2015/08/21/e-commerce-agriculture/`. An example is Jingdong (JD) Mall, which is dedicated to farm products.

[2] Several other e-commerce platforms, including Alibaba and Yihaodian, now provide similar intermediary services in China. Throughout this paper, we use only JD as our motivating example for simplicity, but our analysis can be applied to other platforms.

[3] For more details about these investments, please visit the following website: `http://www.worldbank.org/en/topic/financialsector/brief/agriculture-finance`.

by the platform, the farmer with low or no creditworthiness can then more easily obtain a bank loan to start the planting process. This financing format is widely acknowledged in practice (Shi 2013; Zhou et al. 2020).[4] Note that besides platforms, other parties in the agricultural supply chain have contributed to the development of guarantor financing. For example, New Hope Liuhe Co., Ltd, a Chinese company specializing in agricultural and animal husbandry products, established its agriculture/livestock *guarantee* company.[5] It also cooperates with JD to issue loans to farmers to increase their profitability by providing a financial guarantee service.[6]

The intermediary platform can also *directly finance* the farmer's planting activity by lending money to cover his planting costs and solve the pain point problem. One example is the micro loan scheme *Jing Nong Dai* launched by JD in 2015 to raise funds for farmers to plant specific agricultural products in specific areas and improve productivity. For instance, this financial service is provided to farmers in Renshou, Sichuan Province, to plant a unique fruit, *loquats*. In this micro loan scheme, farmers can apply for input loans without any collateral to purchase seeds, fertilizers, pesticides, or farm equipment. Repayment is based on trade with JD, which sells loquats on its own platform. Similar schemes (e.g., *Jing Bao Bei*) are also provided by JD to ensure a sustainable supply chain for products in more diverse categories.[7] Other intermediary platforms such as Alibaba have similar agricultural finance programs. In essence, direct financing by an intermediary platform is similar to bank financing, in which the farmer has limited liability and simply needs to repay the loan (principal plus interest) up to his realized revenue in the sales horizon.

Both guarantor financing and direct financing are highly implementable from the viewpoint of the intermediary platform. On the one hand, as the smallholder farmer sells online through the intermediary platform, all transaction details, such as time, quantity, price, and logistical arrangements, are recorded and available to the farmer and the platform. The farmer's realized revenue is also collected through the platform's advanced electronic payment system. This close connection facilitates cooperation between the two parties, both operationally and financially. On the other hand, the intermediary platforms often have strong incentives to join the government-led farmer support programs. Platforms like JD are concerned with both firm profitability and social responsibility.[8] That is, they take into account not only their own profitability, but also the welfare

[4] For a more detailed introduction, see `http://www.360doc.com/content/16/0411/08/32327246_549630561.shtml`.

[5] As stated in its official website, "The fund always becomes the bottleneck for farmers to develop the modern livestock/poultry farming. As a result, we have proposed the industrial service plan of providing farmers with financial guarantee. After years of hard work, we have developed it very well and we are disseminating the plan widely ..." For more details, please see the following link: `http://www.newhopeagri.com/en/ServiceDBInfo.aspx?type=256`

[6] For more details, please see the following link: `http://nj.jd.com/`

[7] See more details on the following web page: `https://thenextweb.com/asia/2013/12/06/chinese-online-retailer-jingdong-wants-loan-money-suppliers-help-grow-faster/`

[8] Please see the following reports for more detailed information about both JD's and Alibaba's social responsibility concerns: https://corporate.jd.com/static/pdf/Corporate%20Social%20Responsibility%20Report.pdf https://www.centennialcollege.hku.hk/f/upload/2136/Alibaba%20CSR_15_009C.pdf.

of the farmer in the agricultural supply chain. They can actively do so by using their cash on hand and/or creditworthiness to help farmers and gain long-term goodwill.

Undoubtedly, the contractual relationship between the farmer and the intermediary platform and the performance of the whole agricultural supply chain are affected by the farmer's financing approach. However, the effect of different financing formats–bank financing, guarantor financing, or direct financing–on the performance of the farmer, the platform, and the whole supply chain remains underexplored and requires further investigation. To this end, we aim to answer the following research questions in this paper:

1. How do different financing formats affect the smallholder farmer's production decision?
2. Under what conditions does each financing format most benefit the smallholder farmer and the whole agricultural supply chain?
3. Under what conditions should the intermediary platform provide guarantor financing or directly finance the smallholder farmer in equilibrium?
4. How does the intermediary platform's social responsibility concern affect the smallholder farmer and the whole agricultural supply chain?

To facilitate our analysis, we use a Stackelberg game to capture the interplay between a capital-constrained smallholder farmer and the intermediary platform (possibly with the bank involved). We assume that the farmer plants a specific agricultural product and sells exclusively through the intermediary platform. The platform charges the farmer a unit-based commission fee. To alleviate his financial constraints, the farmer can raise funds from the commercial bank alone or with the intermediary platform acting as a guarantor. The farmer can also obtain direct financing from the intermediary platform. We first obtain the decisions and corresponding payoffs under the three financing formats, then derive the equilibrium from the perspective of the platform.

We show that the farmer produces the most under guarantor financing or direct financing. Under guarantor and direct financing, the farmer's production quantity can be even higher than that in a centralized supply chain with/without capital constraints. In terms of payoffs, when the three financing formats are provided exclusively, bank financing never benefits the farmer. The farmer's preference between guarantor and direct financing depends heavily on the production cost and the unit commission fee. Specifically, he prefers direct financing when the production cost is low and the unit commission fee is sufficiently high. Otherwise, guarantor financing benefits him. However, the whole supply chain may prefer any of the three financing formats. Specifically, when the unit commission fee is low, bank financing benefits the whole supply chain if the production cost is either extremely high or relatively low, while guarantor financing benefits the whole supply chain if the production cost is extremely low. The whole supply chain prefers direct financing only when the production cost is relatively high. For the intermediary platform, when making its own financing decision, it will not provide any financing service when the commission fee is low and the farmer's production cost is sufficiently high. Instead, it will encourage the farmer to resort to bank financing. Otherwise, the platform will provide direct financing because this type of financing makes the platform weakly better off compared with guarantor financing. Guarantor financing will be adopted by the platform only when it is also capital constrained. These results are quite robust regardless of whether the intermediary platform exhibits social responsibility concern with respect to farmer welfare. We further show that a higher level of such social responsibility concern makes the whole agricultural supply chain more coordinated, leading to a lower interest rate and a higher level of production. As a result, the social responsibility concern of the intermediary platform benefits both the farmer and the whole supply chain.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. We present the model setup in Section 3. In Section 4, we analyze the equilibrium decisions of the smallholder farmer and the intermediary platform under three financing formats. We then examine the effect of these financing formats on the supply chain participants in Section 5. We also derive the equilibrium financing format when the intermediary platform acts as the game leader and discuss the partial credit guarantee. In Section 6, we consider that the intermediary platform is concerned with social responsibility and cares about the farmer's welfare. We then examine how this concern affects the performance of the agricultural supply chain and the equilibrium outcome. Concluding remarks are provided in Section 7. All of the proofs are relegated to the online Appendix.

2. Literature Review

This study belongs to the growing line of research on supply chain finance that connects the fields of operations management and finance (Buzacott and Zhang 2004; Kouvelis and Zhao 2011; Li et al. 2013; Yang and Birge 2013; Cai et al. 2014; Yang et al. 2015; Kouvelis and Zhao 2016; Kouvelis and Zhao 2018). Traditionally, operations management research focuses on the operational issues of the supply chain and ignores the financial constraints of individual firms. In their influential work, Lariviere and Porteus (2001) examine a Stackelberg game between a supplier and a retailer in a newsvendor setting without considering liquidity constraints. Most subsequent studies on "*financing the newsvendor*" build on this model by adding liquidity constraints and exploring the effectiveness of various financing schemes in alleviating them. For example, Dada and Hu (2008) consider bank financing of a retailer and characterize the Stackelberg equilibrium. Lai et al. (2009) examine short-term bank financing issues in a capital-constrained one-supplier-one-retailer setting when both the supplier and the retailer face bankruptcy costs. Lai et al. (2012) investigate how a downstream buyer's short-term interest in capital market valuation influences the behavior and performance of supply chain participants. Yang et al. (2015) demonstrate how a firm's financial distress and ease of bankruptcy reorganization alter market competition and supplier-buyer interactions.

Our study is also closely related to research on trade credit, advance payment, and guarantee service (Chen and Cai 2011; Jing et al. 2012; Chod 2017; Deng et al. 2018; Tang et al. 2018; Tunca and Zhu 2018; Zhen et al. 2020; Zhou et al. 2020). Rajan and Zingales (1996) provide empirical reasons why trade credit is so popular and imply that trade credit loans may be more expensive than bank loans. Gupta and Wang (2009) argue that the structure of the base-stock inventory control policy is not affected by common credit terms under a discrete-time model with random demand. Jing et al. (2012) and Jing and Seidmann (2014) identify the conditions for the upstream supplier to provide trade credit to the financially limited downstream retailer. Chen and Cai (2011) suggest that when a third-party logistics (3PL) firm provides logistics and trade credit, it generates higher profits not only for the 3PL firm, but also for the supplier and the retailer. Kouvelis and Zhao (2012) and Yang and Birge (2013) also attempt to provide a better understanding of trade credit from an operational perspective. They model trade credit in detail, including early payment discount, debt structure, contract structure, and financial distress cost. Chen and Gupta (2014) study trade finance contracts for small business suppliers and characterize the effects of third-party bank financing. Kouvelis and Zhao (2018) investigate the effect of the credit ratings of supply chain members to determine who should take on the financing role. Tang et al. (2018) compare buyer direct financing (i.e., advance payment) and purchase order financing in a supply chain consisting of one capital-constrained supplier and one manufacturer, and investigate the effect of information asymmetry. Furthermore, Deng et al. (2018) show that in an assembly system with suppliers with heterogeneous capital, the assembler may offer buyer finance (i.e., advance payment) even if its own opportunity cost of capital is higher than the bank's risk-free interest rate. Zhen et al. (2020) investigate the financing strategy of a capital-constrained manufacturer by comparing bank financing, platform financing, and retailer financing in a dual-channel supply chain. Zhou et al. (2020) consider guarantor financing in a four-party supply chain with a variety of guarantee scenarios. They focus in particular on the effect of supply chain leadership position. Tunca and Zhu (2018) theoretically demonstrate that guarantor financing (called buyer intermediation) can not only lower both the loan interest rate and wholesale price, but also increase order quantity and supply chain efficiency. Unlike the aforementioned studies, we investigate the problem of financing a platform-based agricultural supply chain by *simultaneously* considering three financing formats: bank financing, guarantor financing, and direct financing. The intermediary platform, as the downstream supply chain partner, can provide both operational and financing services to its supplier of agricultural products, the capital-constrained smallholder farmer. We are the first to conduct a game-theoretical analysis and equilibrium comparison considering these three financing schemes, which helps explain the coexistence of guarantor and direct financing in reality. Moreover, we investigate the effect of the platform's social responsibility concern on the welfare of the farmer and the profitability of the whole agricultural supply chain, which was overlooked in the supply chain financing literature.

Our work is also related to studies on social responsibility. For instance, Hsu et al. (2019) examine an innovative agricultural partnership in the dairy industry consisting of a group of farmers and a firm. When the firm shows its social responsibility concern, it will enter into contracts with more farmers and produce more dairy products. Sodhi and Tang (2011), Sodhi and Tang (2014), and Lee and Tang (2017) also discuss research opportunities in social responsibility. They elaborate how supply chains can mobilize the poor in rural areas as their producers to improve their earnings and run more efficient operations. In this paper, we also consider that the intermediary platform is concerned with social responsibility and cares about the welfare of the farmer. We show that the platform's social responsibility concern can benefit both the farmer and the whole supply chain.

3. Model Setup

Consider a capital-constrained smallholder farmer (labeled f) who plants and sells one type of agricultural product. He has no direct access to the end market and relies exclusively on an intermediary electronic platform (labeled i) such as JD to distribute his products. The farmer must therefore pay a fraction of his realized revenue as a commission fee to the platform.[9]

The demand for this agricultural product is stochastic and denoted by a continuous random variable ξ. It follows a distribution with a probability density function $f(\cdot)$, a cumulative density function (CDF) $F(\cdot)$, and a complementary CDF $\overline{F}(\cdot)$. We also define its failure rate function and generalized failure rate function as

$$g(\xi) = \frac{f(\xi)}{\overline{F}(\xi)} \text{ and } G(\xi) = \frac{\xi f(\xi)}{\overline{F}(\xi)},$$

respectively. We assume that the demand distribution has an increasing failure rate, i.e., $0 \leq g(\xi_1) < g(\xi_2)$ for $0 \leq \xi_1 < \xi_2 < +\infty$. This is a relatively weak requirement satisfied by many common distribution types, including uniform, exponential, truncated normal, and Weibull (Lariviere 2006).

The smallholder farmer has little power and is therefore unable to affect the market price. Following the operations-finance interface literature, we assume that the unit price that customers are willing to pay is fixed and normalize it to 1. This is consistent with our motivating example. Note that the price of the agricultural product, which is essential to society as a whole, is subject to strict government regulation and cannot be fully controlled by the farmer (Cummings et al. 2006). The market for a product in this category is also very competitive, so the price is often the equilibrium market price, and the farmer behaves as a price taker.[10]

[9] In our model, the supplier of agricultural products can also be an agricultural social enterprise or cooperative, which serves and makes decisions on behalf of a group of farmers.

[10] The agricultural production market is frequently cited as a real example of perfect competition, in which no individual person or business can control prices. For more information, please see the following link: https://www.ag.ndsu.edu/aglawandmanagement/agmgmt/coursematerials/competition

The intermediary platform charges the smallholder farmer a *unit commission fee* t, which is also the revenue sharing rate received by the platform per transaction. We assume that t is exogenously given for the following reasons.[11] In practice, a platform like JD deals with many farmers simultaneously, so it may not be practical for the platform to customize its commission fee for a specific farmer. A quick survey of the websites of e-commerce platforms, such as JD, Tmall, and Yihaodian, indicates that the commission rate is around 2% for agricultural products and is almost the same across all platforms.[12] In addition, most third-party platforms, such as Amazon and Taobao, apply fee policies before offering lending services (Zhen et al. 2020).

The cost of planting one unit of product is $c \in (0,1)$. The *net marginal revenue* and *net marginal profit* of the farmer are then $1-t$ and $1-t-c$, respectively. Basically, $t \leq 1-c$ is necessary to ensure the participation of the smallholder farmer. We assume that the production cost is public information, which may otherwise lead to an agency problem. When the farmer's production cost is private information, the farmer may intentionally increase his production cost to obtain more financial support. However, in practice, a farmer does not have too much room to artificially increase the cost. Take *Jing Nong Dai* in Renshou, Sichuan Province as an example. JD actively participates in the entire production process and serves farmers through various means far beyond finance, such as logistics, sales, or training. Farmers' farming activities are closely guided by the government, social enterprises, and JD together. The agency problem can then be solved by adopting advanced information technology, which can record in detail the cost of materials purchased by farmers from recommended sellers. With these detailed records, cost information becomes transparent.

The smallholder farmer must bear the cost before the start of the sales horizon for inputs, such as seeds, farm equipment, fertilizers, and labor. However, the smallholder farmer is constrained by his capital resources and does not have sufficient endowment to cover his production cost. We assume that his initial capital is zero. As the smallholder farmer has few individual assets, he is unlikely to have valuable collateral for a loan application. The agricultural product is perishable and cannot be used as collateral, so its salvage value is zero. However, the farmer can raise funds via a commercial bank (traditional *bank financing*) or an intermediary platform (innovative *direct financing*). Note that the farmer has only limited liability. He pays back both principal and interest if he can generate enough revenue during the sales period, but if he cannot, he only returns the amount he receives from the sales of his product and declares bankruptcy afterwards. The loan provider (either the bank or the platform) then embraces a certain level of default risk. We also

[11] We have also extended our analysis to a situation with an endogenous unit commission fee for theoretical completeness, available on request.

[12] For more information on the commission rates charged by e-commerce platforms in China, please see the following links: `https://site.douban.com/289763/widget/notes/193732525/note/690453689/`; `http://www.china-briefing.com/news/tmall-yihaodian-and-jd-a-comparison-of-chinas-top-e-commerce-platforms-for-foreign-enterprises/`; `https://img.alicdn.com/tps/i1/T1Srg.FjXXXXaR1ePy-953-8196.jpg`

consider a third financing format, *guarantor financing*, in which the intermediary platform provides the guarantee for the farmer to raise funds from the bank and will have to repay the debt if the farmer defaults on the agreed repayment (Zhou et al. 2020). Therefore, guarantor financing solves the problem of the lack of creditworthiness issue of the capital-constrained farmer and ensures the stable production of his agricultural product.

In this paper, we consider that the capital market is perfect (no taxes, transaction fees, and bankruptcy costs) and that all bank loans are competitively priced (with a perfectly competitive banking sector). In other words, the bank does not make a positive profit and the interest rate r is set at the break-even level (Chen and Cai 2011; Jing et al. 2012; Tang et al. 2018). We also normalize the risk-free interest rate and the opportunity cost of capital (i.e., the time value of cash) to zero and assume that the farmer and the intermediary platform are both risk-neutral.

4. Individual Analysis of the Three Financing Formats

One main purpose is to investigate how to finance an agricultural supply chain with a capital-limited smallholder farmer. In this section, we start by separately analyzing the equilibrium outcomes under the three financing formats, bank financing, guarantor financing, and direct financing.

4.1. Bank Financing

Here, we consider a scenario in which the farmer lacks financial resources but bank financing is viable. This financing format is very practical in developing countries like China, where state-owned banks are encouraged by the central government to offer commercial loans to farmers to support agricultural development.[13] Knowing the unit commission fee t, the bank sets the interest rate r at the break-even level, and the farmer decides his production quantity q_b, raises funds from the bank, and plants his product accordingly. After that, demand is realized and the sales revenue is collected. Finally, the liability-limited farmer pays back the total loan amount up to his maximal wealth level. See Figure 1 for an illustration.

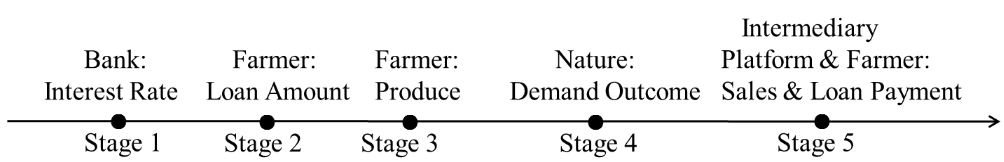


Figure 1 The sequence of events under bank financing

As a result, the bank determines the interest rate to satisfy the following break-even condition:

$$cq_b = \mathsf{E}[\min\{c(1+r)q_b, \min\{\xi, q_b\}(1-t)\}], \tag{1}$$

[13] For more information, please see the following link: `http://assets.fsnforum.fao.org.s3-eu-west-1.amazonaws.com/public/files/120_FSN_APEC/Sep_9a.pdf`

where $\mathsf{E}[\cdot]$ is the expectation operator. As the farmer's liability is limited, the bank bears some default risk if the realized demand is not sufficiently high.

The liability-limited farmer decides his production quantity q_b to maximize his expected profit subject to the bank's constraint (1):

$$\Pi_f^b(q_b) = \mathsf{E}[\min\{\xi, q_b\}(1-t) - c(1+r)q_b]^+, \tag{2}$$

where $[x]^+ = \max\{x, 0\}$. The following lemma shows the farmer's quantity decision.

LEMMA 1. *Under bank financing, the capital-constrained farmer determines q_b^** as if *he is not limited by capital resources, that is,*

$$\overline{F}(q_b^*) = \frac{c}{1-t}.$$

Moreover, q_b^* decreases with the unit commission fee t and is smaller than the optimal production quantity in a centralized capital-unconstrained supply chain denoted by q^c, where $\overline{F}(q^c) = c$.[14]

Lemma 1 shows that the production decision of the capital-constrained farmer is the same as that without capital constraints. In addition, when the bank provides financing, the capital-constrained supply chain behaves *as if* there is no capital constraint, and the farmer can freely leverage the bank's financial resources to plant. The underlying reason is that when the loan is competitively priced, the bank only claims the time value of its investment (which is zero in our setting). It charges interest that can exactly cover default risk in loan repayment. Thus, in equilibrium, the bank makes no profit. It simply serves as a capital buffer for the farmer. For the farmer, although he has limited liability, he is exposed to *default* risk and must pay the interest rate premium. That is, he must balance the *underage* and *overage* costs when making his production decision. This result differs from the intuition that the farmer's financial constraints lead him to produce more aggressively than he would with sufficient capital. Similar results are observed in Xu and Birge (2004) and Jing et al. (2012) but under a different context. However, the equilibrium production quantity is less than a centralized production quantity due to the double marginalization effect caused by the introduction of an intermediary platform in the agricultural supply chain. Moreover, the quantity decreases with the unit commission fee t, as an increase in t reduces the farmer's profit margin and decreases his production incentive.

Anticipating the best responses from the farmer and the bank, the expected profit of the intermediary platform can be obtained by

$$\Pi_i^b = \mathsf{E}[\min\{\xi, q_b^*\}t] = \left[\int_0^{q_b^*} \overline{F}(\xi)d\xi\right] t.$$

The following proposition presents how the platform's profit changes with t.

[14] In a centralized chain, the profit function is $\Pi_c = \mathsf{E}[\min\{\xi, q^c\}] - cq^c$. The well-known *critical fractile* solution can be immediately obtained, i.e., $\overline{F}(q^c) = c$.

PROPOSITION 1. *Under bank financing, the profit of the platform first increases and then decreases with the unit commission fee t while the profits of the smallholder farmer and the whole supply chain decrease with t.*

The unit commission fee has two effects on the profitability of the intermediary platform: on one hand, a higher unit commission fee implies a higher profit margin for the intermediary platform; on the other hand, it leads to a decrease in the production quantity. The balance between these two driving forces indicates that a moderate unit commission fee is the most advantageous for the intermediary platform under bank financing, as shown in Proposition 1. For the smallholder farmer, an increase in the unit commission fee leads to a lower production level and a smaller profit margin, naturally reducing his welfare. For the whole supply chain, given the fixed selling price, a lower production level implies lower profitability.

4.2. Guarantor Financing

Guarantor financing has grown in popularity in recent years, especially in developing economies where small firms often lack creditworthiness and a loan guarantee is required by the bank to control risk. To ensure that the smallholder farmer can operate sustainably, the intermediary platform, as the downstream supply chain partner, is undoubtedly a natural candidate to provide this guarantor service. It is worth noting that a guarantee from the intermediary platform is usually considered reliable and acceptable by the bank, as the platform is often large and has sufficient working capital and fixed assets, and a solid cooperation record. Under guarantor financing, the smallholder farmer and the intermediary platform cooperate not only through traditional product distribution, but also through innovative supply chain financing. The sequence of events is similar to that under bank financing except that the intermediary platform agrees in advance to provide the guarantor service to the farmer. In the last stage, if the farmer eventually defaults, the intermediary platform repays the farmer's debt. See Figure 2 for an illustration.

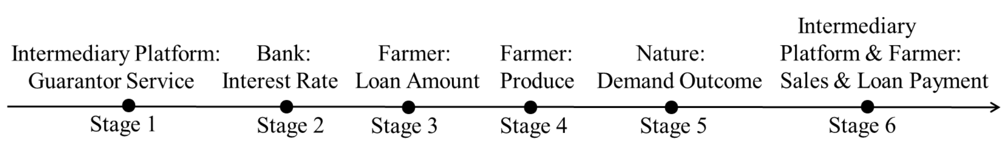


Figure 2 **The sequence of events under guarantor financing**

As the intermediary platform promises to repay the farmer's debt if the farmer files for bankruptcy, it bears all downside risks. The bank is completely free of default risk in this financing format. Therefore, the interest rate charged by the bank is $r = 0$ in this situation.[15] That is,

[15] We refer the readers to Zhou et al. (2020) for a similar conclusion that the commercial bank does not take the debtor's default risk and the interest rate should equal the risk-free rate, which is normalized to zero in our setting.

compared with bank financing, the bank is more aggressive in issuing a loan to the farmer because of the platform's promise.

As the interest rate $r=0$, the liability-limited farmer makes the following production decision q_g to maximize his profit:

$$\Pi_f^g(q_g) = \mathsf{E}[\min\{\xi, q_g\}(1-t) - c(1+r)q_g]^+ = \mathsf{E}[\min\{\xi, q_g\}(1-t) - cq_g]^+. \tag{3}$$

To ensure a non-negative production incentive for the farmer, $c \leq 1-t$ is necessary. Define

$$\widetilde{c} = \frac{c}{1-t}.$$

We then have the following results.

LEMMA 2. *Under guarantor financing, when $c \leq 1-t$, i.e., $\widetilde{c} \leq 1$, the optimal production quantity of the capital-constrained smallholder farmer q_g^* is uniquely determined by $\overline{F}(q_g^*) = \widetilde{c}\overline{F}(\widetilde{c}q_g^*)$. q_g^* decreases with $\widetilde{c}$ and the unit commission fee t, while both $\widetilde{c}q_g^*$ and $q_g^*\overline{F}(q_g^*)$ increase with $\widetilde{c}$ and t.*

By Lemma 2, the production quantity under guarantor financing decreases with the commission fee t. Indeed, as t increases, the farmer obtains a lower profit margin and shares a smaller portion of the total profit of the supply chain. Consequently, the farmer has less incentive to produce.

Anticipating the farmer's optimal production decision q_g^*, the expected profit Π_i^g of the intermediary platform can be written as

$$\Pi_i^g = \underbrace{\mathsf{E}[\min\{\xi, q_g^*\}t]}_{\text{Revenues}} - \underbrace{\mathsf{E}[cq_g^* - \min\{\xi, q_g^*\}(1-t)]^+}_{\text{Loss by Guarantor Service}}. \tag{4}$$

That is, under guarantor financing, the profit of the intermediary platform is equal to the revenue obtained by serving as a sales channel minus the loss incurred by serving as a guarantor. Hereinafter, to facilitate our analysis, we make the following technical assumption.

Assumption 1: The cost elasticity of the production quantity, that is, the ratio of the percentage change in the production quantity q_g^* to the percentage change in the effective cost $\widetilde{c}$,

$$z(\widetilde{c}) := \frac{dq_g^*/q_g^*}{d\widetilde{c}/\widetilde{c}} = \frac{1-G(\widetilde{c}q_g^*)}{G(\widetilde{c}q_g^*) - G(q_g^*)}$$

decreases with $\widetilde{c}$.

This assumption only depends on the demand distribution, and we have the following lemma.

LEMMA 3. *Assumption 1 is satisfied by distributions such as uniform over the interval $[0,a]$ and exponential with a parameter τ over the interval $[0,+\infty)$.*

We then examine how the unit commission fee t affects the profitability of the supply chain participants.

Proposition 2. *Under guarantor financing, the profits of the platform and the whole supply chain first increase and then decrease with t, while the farmer's profit decreases with t.*

Similar to bank financing, a higher unit commission fee under guarantor financing indicates a higher profit margin for the intermediary platform but a lower production incentive for the farmer. Proposition 2 shows that even with the provision of the guarantee service, the profit of the intermediary platform (and that of the whole supply chain) can still either increase or decrease with the unit commission fee. Again, the underlying driving force is the tradeoff between the benefit brought by the increased profit margin and the loss brought by the reduction in the production quantity of the smallholder farmer. In contrast, an increase in the unit commission fee always harms the welfare of the farmer, as it reduces both his production incentive and his profit margin.

4.3. Direct Financing

In this section, we consider direct financing, in which the capital-constrained smallholder farmer borrows money directly from the intermediary platform. That is, the intermediary platform serves as both a distribution channel and a loan provider. The sequence of events is as follows. First, the intermediary platform determines the interest rate r at its discretion. Next, the farmer decides his production quantity q_d, borrows money from the platform, and cultivates his agricultural product accordingly. Demand is then realized and the revenue is collected. The farmer finally realizes his profit after the intermediary platform deducts the loan (principal plus interest) to his maximal wealth, or if he defaults, after he declares bankruptcy.

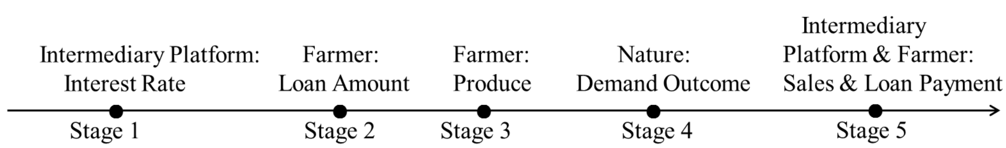


Figure 3 The sequence of events under direct financing

Given the interest rate r, the smallholder farmer makes the following production decision to maximize his expected profit:

$$\Pi_f^d(q_d) = \mathsf{E}[\min\{\xi, q_d)\}(1-t) - c(1+r)q_d\}]^+. \tag{5}$$

To ensure that the farmer has a non-negative production incentive, $c(1+r) \leq 1-t$ is necessary. Define

$$\widehat{c} = \frac{c(1+r)}{1-t}.$$

We then have the following results.

Lemma 4. *Under direct financing, the farmer's production quantity q_d^* is uniquely determined by $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$. Moreover, q_d^* decreases with $\widehat{c}$, while $\widehat{c}q_d^*$ and $q_d^*\overline{F}(q_d^*)$ increase with $\widehat{c}$.*

By anticipating the farmer's production decision, the intermediary platform decides the interest rate of its financing service to maximize its expected profit

$$\Pi_i^d = \underbrace{\mathsf{E}[\min\{\xi, q_d^*(r)\}t]}_{\text{Revenues}} - \left(\underbrace{cq_d^*(r)}_{\text{Loan}} - \underbrace{\mathsf{E}[\min\{c(1+r)q_d^*(r), \min\{\xi, q_d^*(r)\}(1-t)\}]}_{\text{Repayment}}\right), \tag{6}$$

subject to $c(1+r) \le 1-t$. That is, under direct financing, the profit of the intermediary platform is equal to the revenue obtained by serving as a sales channel minus the loss incurred by serving as a loan provider. Note that if the intermediary platform sets the interest rate $r = 0$, its profit function is exactly the same as in (4) under guarantor financing. In this situation, the platform carries the same level of farmer default risk under both guarantor and direct financing.

Let q^0 satisfy

$$G(q^0) = \frac{q^0 f(q^0)}{\overline{F}(q^0)} = 1.$$

Then, we have the following results for the interest rate decision of the intermediary platform.

Lemma 5. *Under direct financing, the optimal interest rate r^* charged by the intermediary platform or, equivalently, $\widehat{c}^* = \frac{c(1+r^*)}{1-t}$, is as follows:*
(i). $\widehat{c}^* = 1$ when

$$t < \frac{c}{2\overline{F}(q^0)} + \frac{1}{2};$$

(ii). otherwise, $\widehat{c}^*$ satisfies

$$\frac{\overline{F}(q_d^*)}{\overline{F}(q_d^*) - c}(1-t) + \frac{1 - G(\widehat{c}^* q_d^*)}{G(\widehat{c}^* q_d^*) - G(q_d^*)} = 0.$$

Moreover, $\widehat{c}^*$ weakly decreases with the commission fee t, while q_d^* weakly increases with t.

Lemma 5 shows that under direct financing, the production quantity weakly increases with the unit commission fee t, which is in stark contrast to that under bank and guarantor financing, in which the production quantity decreases with t (see Lemmas 1 and 2). This result is due to the interaction between the platform's operational and financing aspects: when the unit commission fee is significantly high, the platform has the incentive to encourage the farmer to produce more by lowering its interest rate, which may even be negative, i.e., falling below its opportunity cost of capital. That is, the platform can even financially subsidize the farmer's production. The operational gain resulting from collecting more commission fees can offset the corresponding financial loss by charging a lower interest rate, which actually benefits the platform.

Based on the optimal interest rate decision, we next investigate how the profit of the intermediary platform changes with the unit commission fee t.

Proposition 3. *Under direct financing, the profits of the platform and the whole supply chain weakly increase with t.*

Proposition 3 shows that unlike bank financing and guarantor financing, a higher commission fee always improves the profitability of the intermediary platform and that of the whole supply chain under direct financing. Note that a higher commission fee reduces the farmer's production incentive. However, the intermediary platform has the flexibility to adjust its interest rate, which can reverse the farmer's production incentive. The net effect of these two forces leads to a higher production level. We also note that this involves a weakly higher level of investment risk for the platform.

5. Comparison of the Three Financing Formats and Equilibrium Analysis

So far, we have derived all of the equilibrium outcomes associated with the three financing formats when only one of them is available in the market. An in-depth comparison can help us examine the effect of these financing formats on the performance of the whole supply chain and identify the conditions under which one format is preferred over the other two by supply chain participants.

We note that guarantor financing usually exists due to the supplier's lack of creditworthiness. For this reason, bank financing (with a creditworthy farmer) and guarantor financing (with a non-creditworthy farmer) do not coexist in one setting in many cases. However, we also note that the availability of frictionless bank financing for the smallholder farmer is to a large extent policy-driven and influenced by the government. This is related not only to the behavior of the financial market, but also to a macroeconomic policy to support the development of the agricultural sector. For example, the People's Bank of China, China's central bank, pushes banks in China to increase their financing support to the agricultural sector and to issue loans to farmers to boost the economy. It is clearly stated in its official documents that "*the banking industry should enhance the availability of financing for the agricultural sector at an affordable cost*." As a result, bank loans to the agricultural sector in China increased at an annual average of 21.7% between 2007 and 2014.[16] Therefore, with the support of governmental policy, all three financing formats can be available.

In this section, based on the results in Sections §4.1, §4.2, and §4.3, we first investigate the financing preferences of supply chain participants when all three financing formats are provided. Next, we consider a scenario in which the platform is the leader and makes its financing format decision first and derive the corresponding equilibrium outcome. We also discuss a situation in which the platform offers a partial credit guarantee or may be capital constrained. For convenience and tractability, below we limit our attention to a uniform demand distribution between 0 and 1.[17]

[16] For a more detailed overview of China's banking policies for agricultural development, please see `https://www.nytimes.com/2015/03/26/business/international/chinas-farms-need-more-loans-central-bank-says.html`

[17] Extensive numerical experiments reveal that the key insights regarding the optimal production quantity and the financing selection of supply chain participants with uniform distribution remain intact when demand is exponentially or normally distributed.

5.1. Financing Format Preferences of Supply Chain Participants

We first compare the smallholder farmer's optimal production quantity under the three financing formats. Note that the availability of agricultural products is essential to people's lives and thus affects the stability of society and its sustainable development.

PROPOSITION 4. *The optimal planting quantity of the smallholder farmer under bank financing (q_b^*), guarantor financing (q_g^*), and direct financing (q_d^*) has the following relationship:*

(i). q_b^* is always less than q_g^*, that is, $q_b^* < q_g^*$.

(ii). There exists a critical threshold $\bar{t}$ such that $q_d^* < q_g^*$ only when $t < \bar{t}$, where

$$\bar{t} = \begin{cases} c + 1 - \sqrt{c}, & \text{if } c < \frac{1}{4}, \\ 1 - c, & \text{otherwise.} \end{cases}$$

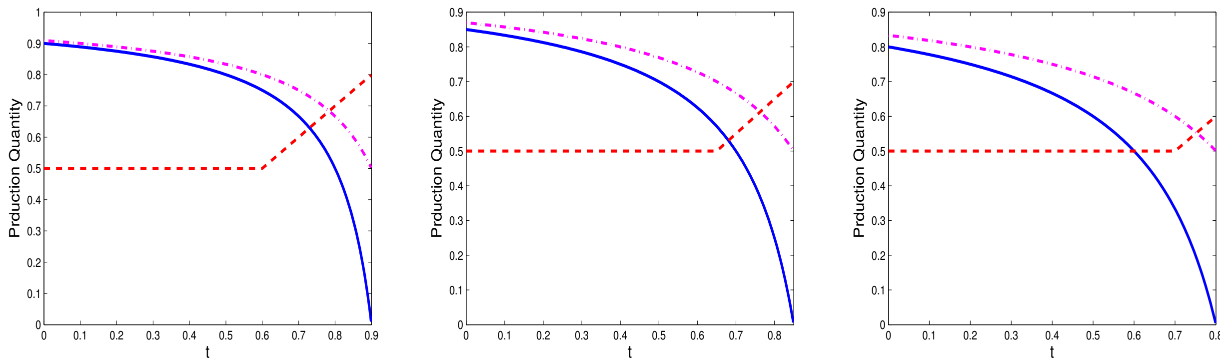
(iii). When the production cost $c > \frac{1}{2}$, q_b^* is always less than q_d^* (i.e., $q_b^* < q_d^*$). Otherwise, there exists a critical threshold $\bar{\bar{t}}$ such that $q_b^* > q_d^*$ only when the unit commission fee $t < \bar{\bar{t}}$, where

$$\bar{\bar{t}} = \begin{cases} \frac{c}{2} + 1 - \frac{\sqrt{c^2+4c}}{2}, & \text{if } c < \frac{1}{6}, \\ 1 - 2c, & \text{otherwise.} \end{cases}$$

(iv). Compared with the optimal production quantity in a centralized capital-unconstrained supply chain q^c, $q_b^* < q^c$ always holds; $q_g^* > q^c$ only when the unit commission fee $t < c$; and $q_d^* > q^c$ only when the production cost $c > \frac{1}{2}$.

Proposition 4 shows that the production quantity under bank financing q_b^* is less than that under guarantor financing q_g^*. This is because under bank financing, the loan is costly ($r^* > 0$), and the farmer himself is exposed to default risk. The farmer must pay the interest rate premium. Therefore, the farmer is relatively conservative in production. In contrast, under guarantor financing, the farmer's default risk is borne by the platform. The bank therefore bears no risk and issues the loan with a zero interest rate ($r^* = 0$). Consequently, the farmer becomes more aggressive in production, leading to $q_b^* < q_g^*$.

Under direct financing, the intermediary platform provides financial resources to the farmer and bears his default risk. It can control this risk by fine-tuning the interest rate to change the farmer's production incentive. Specifically, when the unit commission fee t is low, the farmer has a strong incentive to produce due to a relatively high profit margin; thus, the platform can charge a high interest rate to extract the surplus from the farmer. In contrast, when t is high, the farmer is reluctant to produce more because of a low profit margin. Then, the platform will reduce its interest rate, which may be even lower than its capital opportunity cost, to encourage the farmer to produce more. Overall, the equilibrium production quantity weakly increases with t. Recall from Lemmas 1 and 2 that the equilibrium production quantity under bank and guarantor financing decreases with t. As a result, there exists a unit commission fee threshold below/above which the



(a) $c=0.1$ (b) $c=0.15$ (c) $c=0.2$

Figure 4 Optimal Production quantity as a function of the commission fee t under the three financing formats: $\xi \sim U[0,1]$. Solid line: bank financing; dash-dotted line: guarantor financing; dashed ine: direct financing.

equilibrium production quantity under direct financing is less/greater than that under bank (or guarantor) financing, as shown in Figure 4.

Interestingly, Proposition 4 shows that the farmer's production quantity under both guarantor and direct financing can be even higher than that in a centralized supply chain with sufficient capital. The underlying reason is that under guarantor financing, the interest rate charged by the bank is zero; thus, the farmer can freely take advantage of the bank's capital resources to actively plant. More specifically, compared with a centralized capital-unconstrained supply chain, two effects counteract each other in our decentralized system under guarantor financing. One is the *repayment guarantee effect* induced by the platform's guarantor service, which benefits the farmer as he no longer bears default risk. This increases his production incentive. The other is the *double marginalization effect* caused by the decentralization of the supply chain due to the involvement of the platform, which reduces the farmer's profit margin from $1-c$ to $1-t-c$ and decreases his production incentive. When the unit commission fee t is low, the first effect outweighs the second, leading to overproduction. However, as t increases, the second effect becomes dominant and the problem of overproduction disappears. That being so, the platform's guarantee promise combined with a low unit commission fee leads to overproduction by the farmer.

Similar to guarantor financing, under direct financing, the liability-limited farmer does not bear any financial risk and can therefore produce his agricultural product aggressively with the financial support of the intermediary platform. In particular, when the production cost is extremely high ($c > \frac{1}{2}$), the production incentive of the centralized capital-unconstrained supply chain is low. However, under such a circumstance, in this decentralized capital-constrained supply chain, the platform can increase the farmer's production incentive by setting a low interest rate, which may even be lower than the opportunity cost of capital, resulting in overproduction. Therefore, the platform's financing flexibility combined with a high production cost can also lead to the farmer's overproduction.

It is worth noting that agricultural overproduction is widespread in practice and has long been a popular topic. One widely discussed driving force is commercial loans or government subsidies.[18] A promising way to solve this problem is to use a credit line. That is, the intermediary platform, which is concerned about the performance of the whole supply chain, can *actively* control the farmer's production quantity by setting a loan limit. With this limit, the farmer cannot obtain as much financial support as he wants, which can mitigate or even eliminate the problem of overproduction.

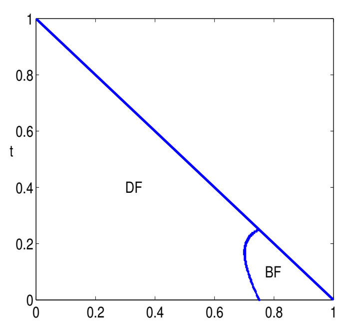
Below, we examine the financing format preferences of the three parties when the three financing formats are provided exclusively.

PROPOSITION 5. *Between bank financing, guarantor financing, and direct financing,*

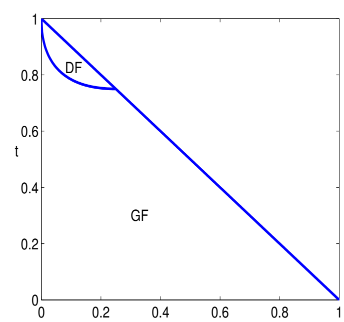
(i). The intermediary platform prefers bank financing when the unit commission fee $t < \frac{1}{4}$ *and the production cost* $c > \frac{1-t-\sqrt{5t^2-5t+1}}{2t}(1-t)$*; otherwise, it prefers direct financing.*

(ii). The farmer prefers direct financing when $c < \frac{1}{4}$ *and* $t > c + 1 - \sqrt{c}$*; otherwise, he prefers guarantor financing.*

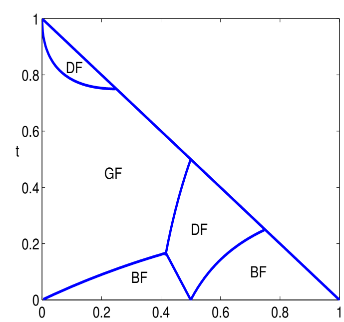
(iii). The whole supply chain prefers bank financing when (1) $c < \frac{1}{2}$ *and* $t < \min\left\{\frac{c+1-\sqrt{c^2+1}}{2}, 1-2c\right\}$*, or (2)* $c > \frac{1}{2}$ *and* $t < \min\left\{\frac{2c-1}{4c-1}, 1-c\right\}$*; and prefers guarantor financing when* $\widetilde{t}(c) < t < \widehat{t}(c)$ *and* $c < \frac{1}{2}$*, where the detailed expressions of* $\widetilde{t}(c)$ *and* $\widehat{t}(c)$ *are presented in* (15) *of the online Appendix. Otherwise, it prefers direct financing.*



(a) Intermediary platform



(b) Smallholder farmer



(c) Whole supply chain

Figure 5 Financing format preferences of supply chain participants: $\xi \sim U[0,1]$**. BF: bank financing; GF: guarantor financing; DF: direct financing.**

Proposition 5 shows that direct financing always weakly dominates guarantor financing from the viewpoint of the intermediary platform because the platform plays the role of a bank (financing

[18] For a more detailed discussion of the overproduction problem in the United States and India, respectively, see the following links: https://www.nytimes.com/2005/11/09/business/mountains-of-corn-and-a-sea-of-farm-subsidies.html and https://medium.com/@AmritHallan/if-overproduction-is-a-problem-for-indian-farmers-how-can-underproduction-solve-it-ce529742df6e

the farmer's production cost and bearing his default risk) and a sales channel. A closer look at the arguments in §4.2 and §4.3 indicates that in the extreme situation in which the platform charges a zero interest rate, the performance of the whole supply chain under direct financing is exactly the same as that under guarantor financing. As the intermediary platform has the flexibility to adjust the interest rate under direct financing, its optimal interest rate decision will result in a higher profit than under guarantor financing. Between bank and direct financing, the platform prefers bank financing only when the farmer's production cost is high but the unit commission fee is low, as shown in Figure 5a. The underlying reasons are as follows. Under the competitive pricing of bank loans, the bank makes no profit when granting the loan. As a result, the farmer and the platform enjoy the full profit of the supply chain, regardless of the financing format chosen. However, the platform has to invest heavily upfront when providing the direct financing service and thus bears a high level of default risk due to the farmer's limited liability. In this situation, when the unit commission fee is low, the revenue obtained by the platform is insufficient to cover its default risk from direct financing.

Proposition 5 indicates that bank financing is never preferred by the farmer because guarantor financing benefits the farmer more than bank financing. With the creditworthy intermediary platform as a guarantor, the bank bears no financial risk and lends more to the liability-limited farmer, which leads to more aggressive production. In addition, under guarantor financing, the intermediary platform cannot fully extract the farmer's surplus given the fixed unit commission fee. This implies that the availability of platform-backed financing is essential for the welfare of the farmer. Moreover, the farmer prefers direct financing over guarantor financing when the unit commission fee is sufficiently high (see Figure 5b). In this situation, from an operational perspective, a high commission fee reduces the farmer's profit margin and exacerbates double marginalization. However, from a financial perspective, a high commission fee incentivizes the platform to duly cut its interest rate, which alleviates the farmer's repayment stress and benefits him.

Proposition 5 also shows that the preference of the whole supply chain for the three financing formats strongly depends on the production cost c and the unit commission fee t, as depicted in Figure 5c. Note that due to the competitive pricing of bank loans, the bank makes no profit and thus does not intervene in the profit of the whole supply chain. For a sufficiently low production cost c, if the unit commission fee t is low, bank financing is preferred by the supply chain; if t is moderate, guarantor financing is preferred; and if t is high, direct financing is preferred, as the platform can duly cut the interest rate to effectively reduce double marginalization. In a similar vein, when the unit commission fee is sufficiently low, any of these three financing formats can be preferred by the supply chain, depending on the production cost. Recall that the unit commission fee for agricultural products is around 2%. This result is therefore very insightful, as it helps explain why the three financing formats are observed in real business practices. A closer look at Figures

5a and 5c reveals that when the unit commission fee is sufficiently low (which fits the agricultural product), the preferences of the intermediary platform and the whole supply chain can be aligned if the farmer's production cost is relatively high. This implies that the involvement of the platform can help improve the performance of the whole supply chain.

Proposition 5 also implies that when the production cost is low but the commission fee is extremely high, direct financing benefits both the smallholder farmer and the intermediary platform and therefore the whole supply chain. That is, even if the unit commission fee is extremely high, the intermediary platform can adjust its interest rate *downward*, inducing the liability-limited smallholder farmer to produce more. The duly reduced interest rate then mitigates the double marginalization effect caused by the high commission fee. Thus, the high commission fee favors the platform, while the much lower interest rate benefits the farmer. As a result, the whole supply chain becomes better off, leading to a *win-win-win* outcome for all the parties. Accordingly, their financing format preferences are aligned. Note that without providing the financial service (either direct financing or guarantor financing), the intermediary platform serves only as a sales channel, and the farmer can only resort to bank financing. Proposition 5 then indicates that by providing the financial service in addition to its operational function, the involvement of the intermediary platform can significantly improve the welfare of the farmer and the total profit of the supply chain. As such, our analysis suggests that the government should encourage platforms to provide both operational and financing services to benefit the entire agricultural supply chain.

For tractability and ease of exposition, we have assumed that the farmer's initial endowment is zero. We now relax this assumption. However, due to the lack of exact expressions of the optimal solutions, we have to rely on extensive numerical studies to examine the effect of the farmer's initial endowment. Interestingly, our numerical experiments show that all of our main results remain qualitatively unchanged. The farmer's initial endowment only affects the amount of financial support that the bank/platform must provide upfront. The underlying tradeoffs for selecting the optimal financing format remain the same as those discussed above.

So far, we have examined the preferences of supply chain participants over the three financing formats if they are provided exclusively. Recall from Proposition 5 that in a theoretical sense, it is never in the best interest of the intermediary platform to offer guarantor financing. Now, let us consider a scenario in which the smallholder farmer has to decide in advance which financing format to adopt. Proposition 5 then indicates that guarantor financing can be the equilibrium financing format when the farmer has the power to decide which financing format to adopt. In this *sequential* setting, bank financing is available as a benchmark and the platform can offer both guarantor and direct financing. We examine the effect of the commission fee on equilibrium performance.

Proposition 6. *When all three financing formats are available and the smallholder farmer determines the financing format to adopt, in equilibrium,* *(i). the farmer's profit increases with t when $\frac{3}{2}-\sqrt{2}<c<\frac{1}{4}$ and $c+1-\sqrt{c}<t<\frac{c}{2}+\frac{3}{4}$, and decreases with t otherwise.*

(ii). the profit of the intermediary platform always increases with t.

(iii). the total supply chain profit increases with t when (1) $c<\frac{1}{4}$ and $t\in[0,c)\cup(c+1-\sqrt{c},1)$ or (2) $c>\frac{1}{4}$ and $t<\min\{c,1-c\}$; otherwise, it decreases with t.

Proposition 6 implies that an increase in the unit commission fee can generate a higher profit for the farmer. In this case, direct financing is preferred by the smallholder farmer (see Proposition 5), as the intermediary platform has the incentive to cut the interest rate to encourage the farmer to produce more, which in turn benefits the farmer. Proposition 6 further shows that when the farmer decides on the financing format to adopt, the increase in the unit commission fee always benefits the intermediary platform. When the supply chain participants face no financial constraints, common knowledge suggests that an increase in the commission fee t *exacerbates* the double marginalization effect and thus affects the efficiency of the supply chain. However, with financial constraints, Proposition 6 suggests that an increase in the unit commission fee may actually benefit the whole supply chain. Again, this is due to the following financial reason: a higher unit commission fee can result in a lower interest rate for the farmer, leading to a higher production quantity, mitigating the double marginalization effect and thus benefiting the whole supply chain. This implies that in our capital-constrained supply chain, an increase in the unit commission fee may unexpectedly improve the performance of the entire agricultural supply chain in terms of farmer welfare and total supply chain profit.

5.2. When the Intermediary Platform Determines the Financing Format

In the foregoing analysis, we have compared the equilibrium outcomes associated with three financing formats assuming that they are all provided. However, both direct financing and guarantor financing are provided by the intermediary platform, who should be able to decide whether and when to offer either (or neither) of these formats. In this subsection, we consider the scenario in which the intermediary platform acts as the game leader and makes its financing format decision in advance. Then, depending on the financing formats available, the farmer selects the one that maximizes his profit. In this setting, the platform's financing format competes directly with bank financing, especially under direct financing in which the platform also decides the interest rate. That is, the platform can influence the farmer's selection by fine-tuning the decision variables, such as the interest rate.

In addition, note that when the smallholder farmer is not creditworthy (and without the support of government policies), bank financing is not available. Then, the farmer must rely on either direct or guarantor financing to obtain a loan. In contrast, when the farmer is creditworthy (or has the support of government policies), he can obtain a loan either from the bank or from the intermediary platform. The following proposition summarizes the financing format decisions of the intermediary platform when dealing with a non-creditworthy farmer and a creditworthy farmer, respectively.

PROPOSITION 7. *When the farmer is not creditworthy, between guarantor financing and direct financing, the intermediary platform adopts direct financing. When the farmer is creditworthy, the intermediary platform does not provide any financing service and encourages the farmer to resort to bank financing when the unit commission fee* $t < \frac{1}{4}$ *and the production cost* $c > 4t(1-t)$*, and provides direct financing otherwise.*

When faced with a non-creditworthy capital-constrained smallholder farmer, the platform, as his supply chain partner, must provide the financing service to ensure the sustainability of his agricultural production. Proposition 7 shows that the platform always intends to provide direct financing rather than guarantor financing, because in this situation, the intermediary platform can extract all of his surplus if direct financing is adopted. When the farmer is creditworthy (possibly with the support of government policies), bank financing is also viable in the market, so there is no need for the platform to provide financing. In this situation, Proposition 7 shows that the platform will choose not to provide any financial service when the farmer's production cost is high but the unit commission fee is low. The underlying reasons are as follows. Recall from Proposition 5 that the platform weakly prefers bank financing over guarantor financing. When the production cost is high, the platform must invest heavily upfront if it chooses direct financing. Moreover, to entice the farmer to choose direct financing, the platform must set a very competitive interest rate, which negatively affects its default risk. Then, the platform prefers not to offer any financing service.

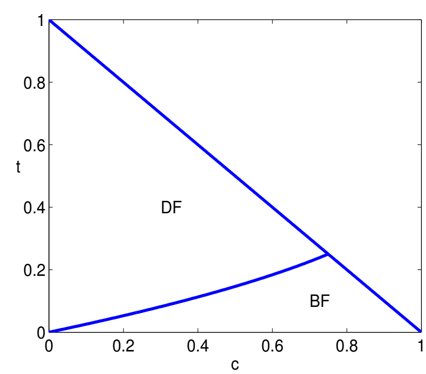


Figure 6 Equilibrium outcomes when the intermediary platform selects the financing format

5.3. Discussion of Partial Credit Guarantee and Internal Financial Constraints

Until now, we have implicitly assumed that the intermediary platform provides a full credit guarantee for the smallholder farmer. However, a partial credit guarantee is also observed in practice from time to time (Yan et al. 2016). Here, we consider such a partial credit guarantee and characterize how it affects the equilibrium outcome and the performance of the whole supply chain.

Whether the platform promises only a partial or full credit guarantee has no impact on the equilibrium outcome, including the optimal interest rate and production quantity, under both bank and direct financing. Accordingly, the system performance of the whole supply chain remains the same as that described in §4.1 and §4.3. However, under guarantor financing, as the intermediary platform promises to only *partially* repay the farmer's debt if the farmer files for bankruptcy, it only bears partial downside risk. Thus, the bank also bears some default risk and has to charge a positive interest rate r, which is zero when the platform provides the full credit guarantee.

We now analyze guarantor financing under the partial credit guarantee. Given the interest rate r, the liability-limited farmer makes the following production decision q_g to maximize his profit:

$$\Pi_f^g(q_g) = \mathsf{E}[\min\{\xi, q_g\}(1-t) - c(1+r)q_g]^+. \tag{7}$$

To ensure a non-negative production incentive for the farmer, $c(1+r) \le 1-t$ is necessary. Equation (7) is exactly the same as that under direct financing except that the interest rate here is determined by the bank. In a similar way, recalling $\widehat{c} = \frac{c(1+r)}{1-t}$, we can obtain the following results.

Lemma 6. *Under guarantor financing with the partial credit guarantee, when $c(1+r) \le 1-t$, i.e., $\widehat{c} \le 1$, the optimal production quantity of the capital-constrained farmer q_g^* is uniquely determined by $\overline{F}(q_g^*) = \widehat{c}\overline{F}(\widehat{c}q_g^*)$. Moreover, q_g^* decreases with $\widehat{c}$, while $\widehat{c}q_g^*$ and $q_g^*\overline{F}(q_g^*)$ increase with $\widehat{c}$.*

For the intermediary platform, its expected profit when providing the partial credit guarantee Π_i^g can be written as

$$\Pi_i^g = \underbrace{\mathsf{E}[\min\{\xi, q_g^*\}t]}_{\text{Revenues}} - \underbrace{\phi\mathsf{E}[c(1+r)q_g^* - \min\{\xi, q_g^*\}(1-t)]^+}_{\text{Loss by Guarantor Service}},$$

where $\phi \in (0,1)$ is the partial credit guarantee parameter. Anticipating the farmer's production decision, the bank determines interest rate $r^*(\phi)$ that satisfies the following break-even condition:

$$cq_g^*(r) = \underbrace{\mathsf{E}[\min\{c(1+r)q_g^*(r), \min\{\xi, q_g^*(r)\}(1-t)\}]}_{\text{Repayment by Farmer}} + \underbrace{\phi\mathsf{E}[c(1+r)q_g^*(r) - \min\{\xi, q_g^*(r)\}(1-t)]^+}_{\text{Repayment by Platform}}.$$

Then, we can show that for a uniformly-distributed demand between 0 and 1, $r^*(\phi) = \frac{1}{c}\left(\frac{\sqrt{\widetilde{c}^2+2\phi\widetilde{c}+1}-1+\widetilde{c}}{1+\phi}\right)(1-t)-1$. The corresponding equilibrium profits can then be obtained.

In §5.1, we show that guarantor financing is a special case of direct financing under a full credit guarantee. Here, under direct financing, the platform can also mimic guarantor financing with a partial credit guarantee by setting $r = r^*(\phi)$ or $\widehat{c} = \widehat{c}^*(\phi)$. As a result, same as that stated in Proposition 7, when the farmer is not creditworthy and bank financing is not available, the intermediary platform always provides direct financing. We now consider that the farmer is creditworthy and/or bank financing is available. In this situation, the platform needs to fine-tune its interest rate to compete against bank financing if this is in its best interest. Note that the bank also charges an interest rate when the platform provides guarantor financing with a partial credit guarantee. The equilibrium comparison then becomes more complicated and there is no analytical solution. We have to rely on extensive numerical analysis instead. Our numerical experiments reveal that guarantor financing can be the platform's equilibrium financing choice in this case. This is in stark contrast to that when the platform offers a full credit guarantee, in which case guarantor financing is never chosen by the platform. This result is insightful, as it provides an explanation why guarantor financing is observed in business practices.

We have assumed in the above analysis that the intermediary platform has sufficient financial resources. However, in practice, the platform may be capital-constrained. Note that the platform often operates a variety of businesses and needs to allocate its financial resources between a wide range of business units. For instance, JD continues to invest heavily in logistics infrastructure and has continuously reported net financial losses for some time.[19] Thus, the intermediary platform may face internal budget constraints and may not fully satisfy the smallholder farmer's financial needs, especially when there are a huge number of capital-constrained farmers in the market. We now consider that the platform is also capital-constrained (i.e., with a budget limit) and examine how it affects its choice of financing format. As this analysis is very complex, we have to rely on extensive numerical studies. Through extensive numerical experiments, we find that when the platform faces financial constraints, guarantor financing is more likely to be provided by the platform. In addition, the more financially constrained the platform, the more likely the adoption of guarantor financing. Again, this helps explain why guarantor financing is observed in real business practices and so the coexistence of guarantor financing and direct financing in the market.

6. When the Platform Shows Concern for Social Responsibility

In the above analysis, we assume that the intermediary platform maximizes its own economic profit when cooperating with the smallholder farmer. However, in an agricultural supply chain, the platform may show its concern for social responsibility and care not only for its own profitability, but also for the welfare of the farmer (Hsu et al. 2019). Hereafter, we consider this possibility and investigate how the social responsibility concern affects the financing format preferences of supply chain participants and the equilibrium outcomes.

[19] For more information, see the following link: `http://www.chinadaily.com.cn/business/tech/2016-10/31/content_27221793.htm`

6.1. Direct Financing with the Platform Concerned with Social Responsibility

When the intermediary platform shows its social responsibility concern and cares about the welfare of the farmer, it aims to maximize the following objective function under each financing format k:

$$\Omega_i^k(\widehat{c}) = \Pi_i^k(\widehat{c}) + \lambda \Pi_f^k(\widehat{c}), \;\; k = b, g, d,$$

where the parameter $\lambda \in (0,1)$ measures the level of the platform's social responsibility concern. When $\lambda = 0$, the intermediary platform does not take into account the farmer's profit when making its decision, and the whole supply chain is completely decentralized in the sense that all players purely maximize their own profit. When $\lambda = 1$, the platform takes equally into account its own profit and that of the smallholder farmer and aims to maximize the total profit of the supply chain.

Although the intermediary platform exhibits social responsibility concern, it does not affect the equilibrium outcomes (including the optimal interest rate and production quantity) under both bank and guarantor financing. This is because under these two financing formats, the platform plays a relatively passive role. The performance of the whole supply chain remains the same as described in §4.1 and §4.2. However, when the platform actively provides the financing service, its social responsibility concern does play a role and affects the equilibrium outcome under direct financing. Specifically, anticipating the smallholder farmer's production decision, the intermediary platform decides the interest rate to maximize its objective function as follows:

$$\begin{aligned} \Omega_i^d(\widehat{c}) &= \Pi_i^d(\widehat{c}) + \lambda \Pi_f^d(\widehat{c}) = \Pi_{sc}^d(\widehat{c}) - (1-\lambda)\Pi_f^d(\widehat{c}) \\ &= \mathsf{E}[\min\{\xi, q_d^*(r)\}] - c q_d^*(r) - (1-\lambda)(1-t)\mathsf{E}[\min\{\xi, q_d^*(r)\} - \widehat{c} q_d^*(r)]^+, \end{aligned} \tag{8}$$

subject to the smallholder farmer's participation constraint, $c(1+r) \le 1-t$. We then have the following results for the platform's optimal interest rate decision.

Lemma 7. *When the intermediary platform is concerned about the smallholder farmer's profit, under direct financing, the optimal interest rate r^* charged by the platform, or equivalently $\widehat{c}^* = \frac{c(1+r^*)}{1-t}$, is as follows:*
(i). $\widehat{c}^* = 1$ when

$$t < \frac{c}{2(1-\lambda)\overline{F}(q^0)} + \frac{1-2\lambda}{2(1-\lambda)};$$

(ii). otherwise, $\widehat{c}^*$ satisfies

$$\frac{(1-\lambda)\overline{F}(q_d^*)}{\overline{F}(q_d^*) - c}(1-t) + \frac{1 - G(\widehat{c}^* q_d^*)}{G(\widehat{c}^* q_d^*) - G(q_d^*)} = 0.$$

Next, we examine the effect of social responsibility concern on the equilibrium outcomes, such as the production quantity and supply chain profitability. We then have the following results.

Corollary 1. *Assume that demand ξ is uniformly distributed between 0 and 1. Then, under direct financing,* *(i). the farmer's optimal production quantity q_d^* (weakly) increases with λ, the level of the intermediary platform's social responsibility concern while the optimal interest rate charged by the platform r^* (weakly) decreases with λ.*

(ii). the profit of the smallholder farmer Π_f^{d*}, the total supply chain profit Π_{sc}^{d*}, and the platform's overall social payoff Ω_i^{d*} (weakly) increase with λ.

Corollary 1 shows that under direct financing, when the intermediary platform is more concerned about the welfare of the farmer, it will charge the farmer a lower interest rate, leading to a lower *effective* production cost for the farmer. Subsequently, the farmer's production level will increase. As a result, both the farmer and the whole supply chain earn more. Note that as the social responsibility concern parameter λ increases, the platform becomes more concerned about the overall performance of the supply chain. Consequently, the optimal decision of the platform makes the whole supply chain more coordinated, leading to a higher total profit for the supply chain. Moreover, the overall social payoff of the intermediary platform also increases. This indicates that a *win-win-win* outcome is achievable when the platform shows more concern about the welfare of the farmer. Recall that the government often encourages intermediary platforms to increase their sense of social responsibility in business practices, such as launching and involving platforms in farmer support programs.[20] Corollary 1 implies that these policies can benefit all parties and improve the performance of the entire agricultural supply chain.

6.2. Comparison of the Three Financing Formats

Similar to that in §5, in which the platform shows no social responsibility concern, here we first compare the optimal production quantity under the three financing formats.

LEMMA 8. *When the platform exhibits social responsibility concern, the optimal planting quantity of the smallholder farmer under bank financing (q_b^*), guarantor financing (q_g^*), and direct financing (q_d^*) has the following relationship:*

(i). q_b^* is always less than q_g^*, that is, $q_b^* < q_g^*$.

(ii). There exists a critical threshold $\bar{t}'$ such that $q_d^* < q_g^*$ only if $t < \bar{t}'$, where

$$\bar{t}' = \begin{cases} \frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)}, & \text{if } c < \frac{1}{2(2-\lambda)}, \\ 1-c, & \text{otherwise.} \end{cases}$$

(iii). When the production cost $c > \frac{1}{2}$, $q_b^* < q_d^*$ always holds. However, when $c \le \frac{1}{2}$, there exists a critical threshold $\bar{\bar{t}}'$ such that $q_b^* > q_d^*$ only if $t < \bar{\bar{t}}'$, where

$$\bar{\bar{t}}' = \begin{cases} \frac{2+c-2\lambda-\sqrt{c(4+c-4\lambda)}}{2(1-\lambda)}, & \text{if } c < \frac{1}{2(3-2\lambda)}, \\ 1-2c, & \text{otherwise.} \end{cases}$$

[20] For example, see JD's social responsibility concern at https://corporate.jd.com/static/pdf/Corporate%20Social%20Responsibility%20Report.pdf

(iv). Compared with the optimal production quantity in the centralized capital-unconstrained supply chain q^c, $q_b^* < q^c$ *always holds;* $q_g^* > q^c$ *only when* $t < c$*; and* $q_d^* > q^c$ *only when* $c > \frac{1}{2}$.

A closer look at Proposition 4 and Lemma 8 indicates that the relationship between the optimal production quantities under the three financing formats remains qualitatively the same, regardless of whether the platform exhibits the social responsibility concern. Overproduction can still occur when the platform cares about the welfare of the farmer.

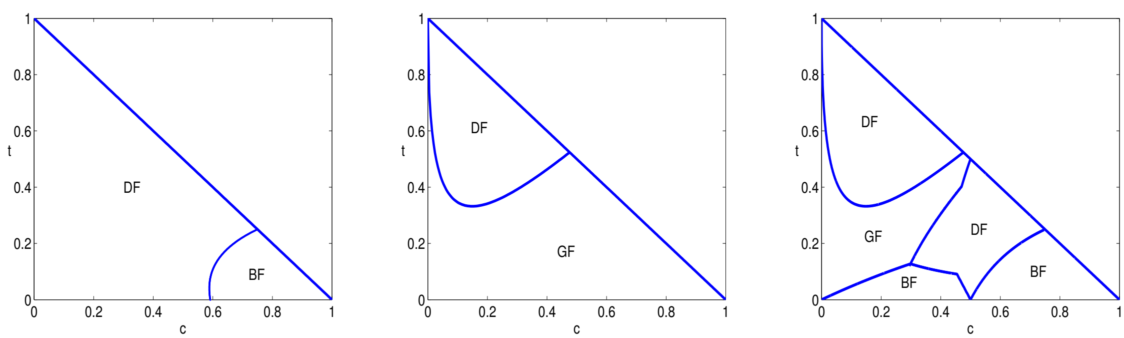
Next, we examine the preferences of supply chain participants for the three financing formats when they are provided separately and obtain the following results.

Proposition 8. *When the intermediary platform exhibits the social responsibility concern,*

(i). the platform prefers bank financing when the unit commission fee $t < \frac{1}{4}$ *and the production cost* $c > \frac{(1-t)(1-2\lambda)-\sqrt{5t^2-5t+5t\lambda-4t^2\lambda+1-\lambda}}{2(t-\lambda+t\lambda)}(1-t)$*; otherwise, it prefers direct financing.*

(ii). the farmer prefers direct financing when $c < \frac{1}{2(2-\lambda)}$ *and* $t > \frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)}$*; otherwise, he prefers guarantor financing.*

(iii). the whole supply chain prefers bank financing when (1) $c < \frac{1}{2}$ *and* $t < \min\left\{\frac{c+1-\sqrt{c^2+1}}{2}, \frac{c-2\lambda+2-\sqrt{c(c-4\lambda+4)}}{2(1-\lambda)}, 1-2c\right\}$*, or (2)* $c > \frac{1}{2}$ *and* $t < \min\left\{\frac{2c-1}{4c-1}, 1-c\right\}$*; and prefers guarantor financing when* $\widetilde{t}'(c) < t < \widehat{t}'(c)$ *and* $c < \frac{1}{2}$*, where the detailed expressions of* $\widetilde{t}'(c)$ *and* $\widehat{t}'(c)$ *can be found in* (16) *and* (17) *of the online Appendix. Otherwise, it prefers direct financing.*

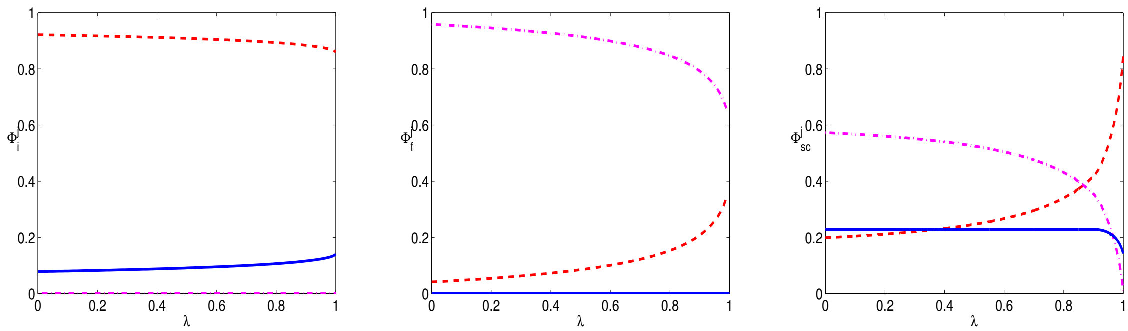


(a) Intermediary platform (b) Smallholder farmer (c) Whole supply chain

Figure 7 Financing format preferences of supply chain participants when the platform is concerned with farmer welfare: $\xi \sim U[0,1]$ **and** $\lambda = 0.95$**. BF: bank financing; GF: guarantor financing; DF: direct financing.**

Again, a closer look at Propositions 5 and 8 indicates that the preferences of the smallholder farmer, the platform, and the whole supply chain over the three financing formats remain qualitatively unchanged, regardless of whether the intermediary platform exhibits the social responsibility concern. See Figure 7 for an illustration. However, the level of the platform's social responsibility concern affects the adoption rate of each format, that is, the frequency/likelihood that each format will be adopted. To this end, let Φ_l^k be the adoption rate of the financing format $k \in \{b, g, d\}$ by the supply chain participant $l \in \{i, f, sc\}$. It can be obtained by integrating all possible combinations of the production cost c and the unit commission fee t given that $c \leq 1 - t$. Take guarantor financing as an illustration. Its adoption rate Φ_{sc}^g can be obtained by

$$\Phi_{sc}^g = \frac{\int_0^{\frac{1}{2}} (\widehat{t}'(c) - \widetilde{t}'(c))dc}{\int_0^1 \int_0^{1-t} dtdc} = 2\int_0^{\frac{1}{2}} (\widehat{t}'(c) - \widetilde{t}'(c))dc.$$



(a) Intermediary platform (b) Smallholder farmer (c) Entire supply chain

Figure 8 **The effect of λ on the adoption of each financial format: $\xi \sim U[0,1]$. Solid line: bank financing; dash-dotted line: guarantor financing; dashed line: direct financing.**

Then, we quantitatively investigate how the platform's social responsibility concern affects the financing format selection of supply chain participants using numerical experiments. See Figure 8 for an illustration. Figure 8 shows that the platform's concern for the welfare of the farmer λ increases, it is more preferable for the farmer to borrow money from the bank than directly from the platform. The underlying reason is that an increase in λ makes the platform further reduce its interest rate, making it riskier to directly finance the farmer. As a result, an increase in λ makes it more likely for the smallholder farmer and the whole supply chain to prefer direct financing. In other words, when the platform is more concerned with social responsibility, direct financing becomes more beneficial for the farmer and the whole supply chain. As a result, guarantor financing is less likely to be adopted.

6.3. When the Intermediary Platform Determines the Financing Format

In this section, similar to in §5.2, we consider a scenario in which a platform concerned with social responsibility acts as the leader and makes its financing format decision in advance. Then, depending on the financing formats available, the farmer selects the one that maximizes his profit. Again, we consider two types of farmer: a creditworthy farmer and a non-creditworthy farmer.

The following proposition summarizes the financing format decision of the intermediary platform when dealing with a non-creditworthy farmer and a creditworthy farmer, respectively.

PROPOSITION 9. *For a creditworthy or non-creditworthy farmer, the results provided in Proposition 7 still hold. That is, the financing format decision of the intermediary platform is independent of its social responsibility concern* λ.

Proposition 9 reveals that the financing format decision of the intermediary platform remains unchanged, regardless of its level of social responsibility concern. This is because on the one hand, the intermediary platform under direct financing can always mimic its decision under guarantor financing to achieve at least the same level of overall social payoff. Therefore, it will provide direct financing instead of guarantor financing. On the other hand, the farmer's decision between direct financing and bank financing remains intact by the platform's social responsibility concern because the farmer is assumed to be rational and only cares about his economic profit.

7. Concluding Remarks

In this paper, we examine the performance of an agricultural supply chain from a financial perspective. Specifically, we investigate an agricultural supply chain consisting of a capital-constrained smallholder farmer and an intermediary platform such as JD. The farmer cultivates one type of agricultural product and sells it to consumers *exclusively* through the platform. To ensure the stable functioning of the whole supply chain, the intermediary platform may launch certain financing programs, such as guarantor financing and direct financing, to help the smallholder farmer solve the bottleneck of capital. Therefore, we consider three financing formats–bank, guarantor, and direct financing–and examine how they affect the performance of the whole supply chain. We also derive equilibrium financing schemes from the perspective of the intermediary platform.

In terms of production quantity, we show that the farmer produces the most under guarantor financing or direct financing. Interestingly, under guarantor and direct financing, the farmer's production quantity can be even higher than that in a centralized supply chain–that is, overproduction can be observed. In terms of payoffs, bank financing makes the smallholder farmer worse off. Specifically, the farmer prefers direct financing when the production cost is low but the commission fee is sufficiently high; otherwise, guarantor financing is preferred. For the whole supply chain, any of the three financing formats can be preferred depending on the production cost and the commission fee. The involvement of the intermediary platform to provide the financing service can improve the performance of the entire agricultural supply chain, benefiting the farmer and the whole supply chain. For the intermediary platform, it will not provide any financing service when the farmer's production cost is sufficiently high and the committee fee is low. Instead, it will encourage the farmer to resort to bank financing. Otherwise, the platform will provide direct financing, making the platform weakly better off than guarantor financing. Guarantor financing will be adopted by the platform only when it is also capital-constrained. These results remain robust regardless of whether the intermediary platform is socially concerned about the welfare of the farmer.

When the intermediary platform exhibits social responsibility concern, i.e., caring not only for its own profit, but also for the welfare of the farmer, it charges the farmer a lower interest rate, increasing the farmer's production incentive. As a result, both the farmer and the whole supply chain are better off and the platform also reaps more social payoff. That is, an increase in the level of the platform's social responsibility concern can lead to a *win-win-win* outcome for all parties.

We conclude this paper by discussing some limitations and directions for future research. It would be of empirical value to verify the adoption of the financing formats proposed in this paper using real-world data. The challenges will mainly come from collecting relevant data and teasing out the focal mechanism. It would also be interesting to examine the long-term effect of the financing options. If the farmer goes bankrupt within a season and fails to repay the loan issued by the bank, this indicates that he is a risky candidate, which should affect his creditworthiness. Then, the bank will raise the interest rate and control the risk more strictly when issuing loans the following season. As a result, bank financing and guarantor financing, which require the involvement of a bank, becomes less attractive and more difficult to obtain for the farmer. Consequently, the intermediary platform, which is relatively less risk-sensitive compared with the commercial bank, will need to provide financing services more aggressively for the agricultural supply chain. Such intertemporal interactions should have a significant impact on the decision-making of supply chain participants, and we leave it for future research. It would also be interesting to investigate the possibility for the intermediary platform to borrow from a bank or another financial institution when it faces internal financial constraints to help the farmer alleviate his capital stress. The interest rate may be lower than that provided by the bank to the farmer due to the higher credit rating of the platform. However, in this paper, we consider farmers in developing economies, where the agricultural sector plays a key role and many farmer support programs are implemented. Therefore, the interest rate provided by the bank is already the lowest that allows the bank to break even. That is, in our setting, the interest rate offered by the bank to the platform should be the same as that provided to the farmer. Thus, we expect the equilibrium outcome in this situation to be very similar to our result. As this exploration is beyond the scope of our paper, we leave it for future research.

Acknowledgements

We are grateful to the departmental editor (Prof. J. George Shanthikumar), an anonymous senior editor, and two anonymous referees for very helpful comments and suggestions. The first author Zelong Yi's work was partially supported by the National Natural Science Foundation of China (71801155, 72031004) and the Natural Science Foundation of Guangdong Province (2018A030313938). The corresponding author Yulan Wang acknowledges the financial supports from the National Natural Science Foundation of China (71971184) and the Research Grants Council of Hong Kong (RGC Reference Number: 15504615). The third author Ying-Ju Chen acknowledges the financial support by NSFC/RGC Grant (# N_HKUST615/19).

References

Berhane, G., & Gardebroek, C. (2011). Does microfinance reduce rural poverty? Evidence based on household panel data from northern Ethiopia. American Journal of Agricultural Economics, 93(1), 43-55.

Buzacott, J. A., & Zhang, R. Q. (2004). Inventory management with asset-based financing. Management Science, 50(9), 1274-1292.

Cai, G. G., Chen, X., & Xiao, Z. (2014). The roles of bank and trade credits: Theoretical analysis and empirical evidence. Production and Operations Management, 23(4), 583-598.

Chen, X., & Cai, G. G. (2011). Joint logistics and financial services by a 3PL firm. European Journal of Operational Research, 214(3), 579-587.

Chen, Y., & Gupta, D. (2014). Trade-finance contracts for small-business suppliers and the effect of third-party financing. Working paper.

Chod, J. (2017). Inventory, risk shifting, and trade credit. Management Science, 63(10), 3207-3225.

Cummings, R., Rashid, S., & Gulati, A. (2006). Grain price stabilization experiences in Asia: What have we learned? Food Policy, 31, 302-312.

Dada, M., & Hu, Q. (2008). Financing newsvendor inventory. Operations Research Letters, 36(5), 569-573.

Daniela, R. (2015). Agricultural finance for smallholder farmers: Rethinking traditional microfinance risk and cost management approaches. Columbia University Press.

Deng, S. M., Gu, C. C, Cai, G. G., & Li, Y. H. (2018). Financing multiple heterogeneous suppliers in assembly systems: Buyer finance vs. bank finance. Manufacturing & Service Operations Management, 20(1), 53-69.

Gupta, D., & Wang, L. (2009). A stochastic inventory model with trade credit. Manufacturing & Service Operations Management, 11(1), 4-18.

Hsu, V., Lai, G., & Liang, G. (2019). Innovation of agricultural partnership for dairy farming. Production and Operations Management, 28(12), 3042-3059.

IFC. (2014). Access to finance for smallholder farmers: Learning from the experiences of microfinance institutions in Latin America. International Finance Corporation.

Jing, B., Chen, X., & Cai, G. G. (2012). Equilibrium financing in a distribution channel with capital constraint. Production and Operations Management, 21(6), 1090-1101.

Jing, B., & Seidmann, A. (2014). Finance sourcing in a supply chain. Decision Support Systems, 58, 15-20.

Khandker, S. R. (2005). Microfinance and poverty: Evidence using panel data from Bangladesh. The World Bank Economic Review, 19(2), 263-286.

Kouvelis, P., & Zhao, W. (2011). The newsvendor problem and price-only contract when bankruptcy costs exist. Production and Operations Management, 20(6), 921-936.

Kouvelis, P., & Zhao, W. (2012). Financing the newsvendor: supplier vs. bank, and the structure of optimal trade credit contracts. Operations Research, 60(3), 566-580.

Kouvelis, P., & Zhao, W. (2016). Supply chain contract design under financial constraints and bankruptcy costs. Management Science, 62(8), 2341-2357.

Kouvelis, P., & Zhao, W. (2018). Who should finance the supply chain? Impact of credit ratings on supply chain decisions. Manufacturing & Service Operations Management, 20(1), 19-35.

Lai, G., Debo, L. G., & Sycara, K. (2009). Sharing inventory risk in supply chain: The implication of financial constraint. Omega, 37(4), 811-825.

Lai, G., Xiao, W., & Yang, J. (2012). Supply chain performance under market valuation: An operational approach to restore efficiency. Management Science, 58(10), 1933-1951.

Lariviere, M. A. (2006). A note on probability distributions with increasing generalized failure rates. Operations Research, 54(3), 602-604.

Lariviere, M. A., & Porteus, E. L. (2001). Selling to the newsvendor: An analysis of price-only contracts. Manufacturing & Service Operations Management, 3(4), 293-305.

Lee, H. L., & Tang, C. S. (2017). Socially and environmentally responsible value chain innovations: New operations management research opportunities. Management Science, 64(3), 983-996.

Li, L., Shubik, M., & Sobel, M. J. (2013). Control of dividends, capital subscriptions, and physical inventories. Management Science, 59(5), 1107-1124.

Meyer, R. L. (2014). Innovative microfinance: Potential for serving rural markets sustainably. In *Finance for Food: Towards New Agricultural and Rural Finance*; Springer: Berlin/Heidelberg, Germany.

Rajan, R. G., & Zingales, L. (1996). Financial dependence and growth (No. w5758). National Bureau of Economic Research.

Shi, J. P. (2013). China SME Finance Report 2013.

Sodhi, M. S., & Tang, C. S. (2011). Social enterprises as supply-chain enablers for the poor. Socio-Economic Planning Sciences, 45(4), 146-153.

Sodhi, M. S., & Tang, C. S. (2014). Supply-chain research opportunities with the poor as suppliers or distributors in developing countries. Production and Operations Management, 23(9), 1483-1494.

Tang, C. S., Yang, S. A., & Wu, J. (2018). Sourcing from suppliers with financial constraints and performance risk. Manufacturing & Service Operations Management, 20(1), 70-84.

Thierry, M., Omer, Z., & Mylene, K. (2011). Financing smallholder farmers and rural entrepreneurs through IFAD-supported operations in the Near East and North Africa. New Directions for Smallholder Agriculture, 24-25.

Tunca, T. I., & Zhu, W. M. (2018). Buyer intermediation in supplier finance. Management Science, 64(12), 5631-5650.

Xu, X., & Birge, J. R. (2004). Joint production and financing decisions: Modeling and analysis. Working paper.

Yan, N. N., Sun, B. W., Zhang, H., & Liu, Q. C. (2016). A partial credit guarantee contract in a capital-constrained supply chain: Financing equilibrium and coordinating strategy. International Journal of Production Economics, 173, 122-133.

Yang, S. A., & Birge, J. R. (2013). How inventory is (should be) financed: Trade credit in supply chains with demand uncertainty and costs of financial distress. Working paper.

Yang, S. A., Birge, J. R., & Parker, R. P. (2015). The supply chain effects of bankruptcy. Management Science, 61(10), 2320-2338.

Zhen, X., Shi, D., Li, Y., & Zhang, C. (2020). Manufacturer's financing strategy in a dual-channel supply chain: Third-party platform, bank, and retailer credit financing. Transportation Research Part E: Logistics and Transportation Review, 133, 101820.

Zhou, W., Lin, T. T., & Cai, G. G. (2020). Guarantor financing in a four-party supply chain with leadership influence. Production and Operations Management, 29(9), 2035-2056.

Online Appendix

"Financing an Agricultural Supply Chain with a Capital-Constrained Smallholder Farmer in Developing Economies"

Proof of Lemma 1: Let $\widehat{\xi} = \frac{c(1+r)q_b}{1-t}$. By (1), we have

$$cq_b = \mathsf{E}[\min\{c(1+r)q_b, \min\{\xi, q_b\}(1-t)\}] = \mathsf{E}_{\xi\le\widehat{\xi}}[\xi(1-t)] + \mathsf{E}_{\xi>\widehat{\xi}}[c(1+r)q_b],$$

which means $\mathsf{E}_{\xi>\widehat{\xi}}[c(1+r)q_b] = cq_b - \mathsf{E}_{\xi\le\widehat{\xi}}[\xi(1-t)]$. Then, (2) becomes

$$\begin{aligned}\Pi_f^b(q_b) &= \mathsf{E}[\min\{\xi, q_b\}(1-t) - c(1+r)q_b]^+ = \mathsf{E}_{\widehat{\xi}\le\xi\le q_b}[\xi(1-t)] + \mathsf{E}_{\xi>q_b}[q_b(1-t)] - \mathsf{E}_{\xi\ge\widehat{\xi}}[c(1+r)q_b] \\ &= \mathsf{E}_{\widehat{\xi}\le\xi\le q_b}[\xi(1-t)] + \mathsf{E}_{\xi>q_b}[q_b(1-t)] - cq_b + \mathsf{E}_{\xi\le\widehat{\xi}}[\xi(1-t)] \\ &= \mathsf{E}_{\xi\le q_b}[\xi(1-t)] + \mathsf{E}_{\xi>q_b}[q_b(1-t)] - cq_b = \mathsf{E}[\min\{\xi, q_b\}(1-t)] - cq_b.\end{aligned}$$

Then, the farmer's problem reduces to the traditional newsvendor model without financial constraint. The marginal revenue is $1-t$ and the production cost is c. We immediately obtain the *critical fractile* solution: $\overline{F}(q_b^*) = \frac{c}{1-t}$, which means q_b^* decreases in t. As a result, by noting that in a centralized chain $\overline{F}(q^c) = c$, we have $q_b^* < q^c$.

Proof of Proposition 1: By Lemma 1, we obtain a one-to-one correspondence between q_b^* and t: $\overline{F}(q_b^*) = \frac{c}{1-t}$. Then, according to the Implicit Function Theorem, we have

$$\frac{dq_b^*(t)}{dt} = -\frac{c}{(1-t)^2 f(q_b^*(t))} = -\frac{c}{(\frac{c}{\overline{F}(q_b^*(t))})^2 f(q_b^*(t))} = -\frac{(\overline{F}(q_b^*(t)))^2}{cf(q_b^*(t))} = -\frac{\overline{F}(q_b^*(t))}{cg(q_b^*(t))} < 0.$$

Note that the intermediary platform's profit a function of t is as follows:

$$\Pi_i^{b*}(t) = \left[\int_0^{q_b^*(t)} \overline{F}(\xi)d\xi\right] t.$$

Then, taking the first-order derivative of the profit function with respect to t, we have

$$\frac{d\Pi_i^{b*}(t)}{dt} = \overline{F}(q_b^*(t))t\frac{dq_b^*(t)}{dt} + \int_0^{q_b^*(t)} \overline{F}(\xi)d\xi = -\frac{(\overline{F}(q_b^*(t)))^2 t}{cg(q_b^*(t))} + \int_0^{q_b^*(t)} \overline{F}(\xi)d\xi,$$

which decreases in t, by the assumption of the increasing failure rate (IFR). Hence, the profit function is concave with respect to t and it first increases and then decreases in t.

Moreover, by the proof of Lemma 1, we have

$$\Pi_f^{b*}(t) = \mathsf{E}[\min\{\xi, q_b^*(t)\}(1-t)] - cq_b^*(t),$$

which increases in q_b^* and hence decreases in t. Similarly, for the supply chain, we have

$$\Pi_{sc}^{b*}(t) = \mathsf{E}[\min\{\xi, q_b^*(t)\}] - cq_b^*(t),$$

which increases in q_b^* and hence decreases in t.

Proof of Lemma 2: Let $\widetilde{\xi} = \frac{cq_g}{1-t}$. By (3), we have

$$\Pi_f^g(q_g) = \mathsf{E}[\min\{\xi, q_g\}(1-t) - cq_g]^+ = \int_{\widetilde{\xi}}^{q_g} (\xi(1-t) - cq_g)dF(\xi) + \int_{q_g}^{\infty} (q_g(1-t) - cq_g)dF(\xi),$$

and

$$\frac{d\Pi_f^g(q_g)}{dq_g} = (1-t-c)\overline{F}(q_g) + c(\overline{F}(q_g) - \overline{F}(\widetilde{\xi})) = (1-t)\left[\overline{F}(q_g) - \frac{c}{1-t}\overline{F}\left(\frac{cq_g}{1-t}\right)\right].$$

Hence, for the smallholder farmer, when the FOC is satisfied, there must exist a q_g^* satisfying

$$\overline{F}(q_g^*) = \frac{c}{1-t}\overline{F}\left(\frac{cq_g^*}{1-t}\right); \text{ that is, } \overline{F}(q_g^*) = \widetilde{c}\overline{F}(\widetilde{c}q_g^*). \tag{9}$$

Next, we show the solution of the FOC is unique. Define $y(q_g) = q_g\overline{F}(q_g)$. Then, the FOC is equivalent to $y(q_g) = y(\widetilde{c}q_g)$. Note that

$$\frac{dy(q_g)}{dq_g} = \overline{F}(q_g) - q_g f(q_g) = \overline{F}(q_g)\left(1 - \frac{q_g f(q_g)}{\overline{F}(q_g)}\right) = \overline{F}(q_g)(1 - G(q_g)).$$

By the assumption that $G(q_g)$ increases in q_g, we immediately obtain that $\frac{dy(q_g)}{dq_g}$ crosses zero at most once. If it indeed crosses, $\frac{dy(q_g)}{dq_g}$ is first positive and then negative. As such, $y(q_g)$ first increases and then decreases in q_g. Then, $y(q_g)$ is unimodal with respect to q_g, with the maximum at q^0 satisfying $G(q^0) = 1$. To proceed, we provide a visual illustration in Figure 9, where $y(q_g)$ increases in q_g when $q_g < q^0$ and decreases in q_g when $q_g > q^0$. Thus, given t (or $\widetilde{c}$), q_g^* must be unique, which satisfies $y(q_g^*) = y(\widetilde{c}q_g^*)$ or $\overline{F}(q_g^*) = \widetilde{c}\overline{F}(\widetilde{c}q_g^*)$. Moreover, we have $\widetilde{c}q_g^* \le q^0 \le q_g^*$. When $\widetilde{c} = 1$, q_g^* coincides with $\widetilde{c}q_g^*$ and we have $q_g^* = q^0$. Note that as q_g^* increases, $\widetilde{c}q_g^*$ must decrease, and thus, it follows that $\widetilde{c}(q_g^*) = \frac{\widetilde{c}q_g^*}{q_g^*}$ decreases in q_g^*. As a result, we have q_g^* decreases in $\widetilde{c}$, $\widetilde{c}q_g^*$ increases in $\widetilde{c}$ and $q_g^*\overline{F}(q_g^*)$ increases in $\widetilde{c}$. By the assumption that $G(q_g)$ increases in q_g, we generally have $G(\widetilde{c}q_g^*) \le 1 \le G(q_g^*)$. Specifically, when $\widetilde{c} = 1$, we have $G(\widetilde{c}q_g^*) = G(q_g^*) = 1$.

Proof of Lemma 3: For a uniform distribution over the interval $[0, a]$, we have $f(\xi) = \frac{1}{a}$, $F(\xi) = \frac{\xi}{a}$, $\overline{F}(\xi) = 1 - \frac{\xi}{a}$, $g(\xi) = \frac{1}{a-\xi}$ and $G(\xi) = \frac{\xi}{a-\xi}$. By $\overline{F}(q_d^*) = \widetilde{c}\overline{F}(\widetilde{c}q_d^*)$, we obtain $q_d^* = \frac{a}{1+\widetilde{c}}$. Thus, $G(q_d^*) = \frac{1}{\widetilde{c}}$ and $G(\widetilde{c}q_d^*) = \widetilde{c}$. Then, $z(\widetilde{c}) = -\frac{\widetilde{c}}{1+\widetilde{c}}$ and $z'(\widetilde{c}) = -\frac{1}{(1+\widetilde{c})^2} < 0$. Hence, $z(\widetilde{c})$ decreases in $\widetilde{c}$.

For an exponential distribution with a parameter τ over the interval $[0, +\infty)$, we have $f(\xi) = \tau e^{-\tau\xi}$, $F(\xi) = 1 - e^{-\tau\xi}$, $\overline{F}(\xi) = e^{-\tau\xi}$, $g(\xi) = \tau$ and $G(\xi) = \tau\xi$. By $\overline{F}(q_d^*) = \widetilde{c}\overline{F}(\widetilde{c}q_d^*)$, we obtain $q_d^* = \frac{\ln\widetilde{c}}{\tau(\widetilde{c}-1)}$. Then, $G(q_d^*) = \frac{\ln\widetilde{c}}{\widetilde{c}-1}$ and $G(\widetilde{c}q_d^*) = \frac{\widetilde{c}\ln\widetilde{c}}{\widetilde{c}-1}$. Thus, $z(\widetilde{c}) = \frac{1}{\ln\widetilde{c}} - \frac{\widetilde{c}}{\widetilde{c}-1}$ and $z'(\widetilde{c}) = -\frac{1}{(\ln\widetilde{c})^2\widetilde{c}} + \frac{1}{(\widetilde{c}-1)^2} < 0$. Hence, $z(\widetilde{c})$ decreases in $\widetilde{c}$.

Proof of Proposition 2: (i) By noting $\widetilde{\xi}^*(t) = \frac{cq_g^*(t)}{1-t} = \widetilde{c}q_g^*(t)$, we have

$$\Pi_i^{g*}(t) = -\Pi_f^{g*}(t) + \Pi_{sc}^{g*}(t) = -\mathsf{E}[\min\{\xi, q_g^*(t)\}(1-t) - cq_g^*(t)]^+ + \mathsf{E}[\min\{\xi, q_g^*(t)\}] - cq_g^*(t)$$

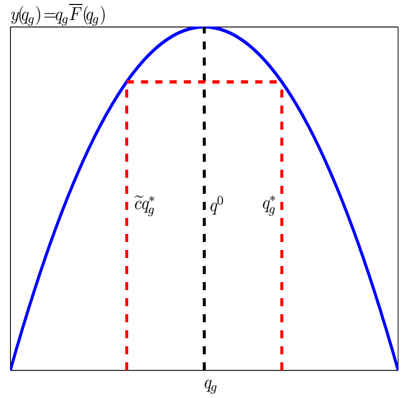


Figure 9 **An illustration of $y(q_g)$ as a function of q_g.**

$$
\begin{aligned}
&= -\left[\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} (\xi(1-t) - cq_g^*(t))dF(\xi) + \int_{q_g^*(t)}^{\infty} (q_g^*(t)(1-t) - cq_g^*(t))dF(\xi)\right] \\
&\quad + \left[\int_0^{q_g^*(t)} \xi dF(\xi) + \int_{q_g^*(t)}^{\infty} q_g^*(t)dF(\xi)\right] - cq_g^*(t) \\
&= -\left[\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} \xi(1-t)dF(\xi) + \int_{q_g^*(t)}^{\infty} q_g^*(t)(1-t)dF(\xi) - \int_{\widetilde{\xi}^*(t)}^{\infty} cq_g^*(t)dF(\xi)\right] \\
&\quad + \left[\int_0^{q_g^*(t)} \xi dF(\xi) + \int_{q_g^*(t)}^{\infty} q_g^*(t)dF(\xi)\right] - cq_g^*(t).
\end{aligned}
$$

By (9) and the Implicit Function Theorem, we have

$$
\frac{dq_g^*(t)}{dt} = \frac{q_g^*(t)(1-G(\widetilde{c}q_g^*(t)))}{G(\widetilde{c}q_g^*(t)) - G(q_g^*(t))}\frac{\widetilde{c}}{c}. \tag{10}
$$

Then, we have

$$
\begin{aligned}
\frac{d\Pi_i^{g*}(t)}{dt} &= -\left[\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} -\xi dF(\xi) - \int_{q_g^*(t)}^{\infty} q_g^*(t)dF(\xi)\right] + (\overline{F}(q_g^*(t)) - c)\frac{dq_g^*(t)}{dt} \\
&= \left[\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} \xi dF(\xi) + q_g^*(t)\overline{F}(q_g^*(t))\right] + (\overline{F}(q_g^*(t)) - c)\frac{q_g^*(t)(1-G(\widetilde{c}q_g^*(t)))}{G(\widetilde{c}q_g^*(t)) - G(q_g^*(t))}\frac{\widetilde{c}}{c} \\
&= (\overline{F}(q_g^*(t)) - c)q_g^*(t)\frac{\widetilde{c}}{c}\left[\frac{\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} \xi dF(\xi) + q_g^*(t)\overline{F}(q_g^*(t))}{(\overline{F}(q_g^*(t)) - c)q_g^*(t)\widetilde{c}}c + \frac{1-G(\widetilde{c}q_g^*(t))}{G(\widetilde{c}q_g^*(t)) - G(q_g^*(t))}\right] \\
&:= (\overline{F}(q_g^*(t)) - c)q_g^*(t)\frac{\widetilde{c}}{c}(y_1(t) + y_2(t)).
\end{aligned}
$$

Based on the sign of $\overline{F}(q_g^*(t)) - c$, we have the following two cases:

(a). If $\overline{F}(q_g^*(t)) - c > 0$, i.e., $q_g^*(t)$ is small and t is large, we have

$$
y_1(t) = \frac{\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} \xi dF(\xi) + q_g^*(t)\overline{F}(q_g^*(t))}{(\overline{F}(q_g^*(t)) - c)q_g^*(t)\widetilde{c}}c = \frac{\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} \xi dF(\xi)}{(\overline{F}(q_g^*(t)) - c)q_g^*(t)}(1-t) + \frac{\overline{F}(q_g^*(t))}{\overline{F}(q_g^*(t)) - c}(1-t).
$$

As t increases, by the proof of Lemma 2 and noting that $\widetilde{c}$ increases in t, we have $q_g^*(t)$ decreases, $\widetilde{\xi}^*(t) = \widetilde{c}q_g^*(t)$ increases and $y(q_g^*(t)) = q_g^*(t)\overline{F}(q_g^*(t))$ increases. Then, $y_1(t)$ decreases in t. Assumption A1 implies that $y_2(t) := \frac{1-G(\widetilde{c}q_g^*(t))}{G(\widetilde{c}q_g^*(t))-G(q_g^*(t))}$ also decreases in t. Then, we have $y_1(t)+y_2(t)$ decreases in t. Hence, $\frac{d\Pi_i^{g*}(t)}{dt}$ crosses zero at most once. If it crosses, it is first positive and then negative.

(b). If $\overline{F}(q_g^*(t)) - c < 0$, i.e., $q_g^*(t)$ is large and t is small, we have $y_1(t) < 0$ and $y_2(t) < 0$. Hence, $\frac{d\Pi_i^g(t)}{dt} > 0$ and $\Pi_i^g(t)$ increases in t.

By combining (a) $\overline{F}(q_g^*(t)) - c > 0$ and (b) $\overline{F}(q_g^*(t)) - c < 0$, we have $\Pi_i^{g*}(t)$ generally first increases and then decreases in t.

(ii) For the farmer, we have

$$\Pi_f^{g*}(t) = \mathsf{E}[\min\{\xi, q_g^*(t)\}(1-t) - cq_g^*(t)]^+ = -\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} \xi(1-t)dF(\xi) - \int_{q_g^*(t)}^{\infty} q_g^*(t)(1-t)dF(\xi) + \int_{\widetilde{\xi}^*(t)}^{\infty} cq_g^*(t)dF(\xi)$$

and

$$\frac{d\Pi_f^{g*}(t)}{dt} = -\int_{\widetilde{\xi}^*(t)}^{q_g^*(t)} \xi dF(\xi) - q_g^*(t)\overline{F}(q_g^*(t)) < 0.$$

Then, we have $\Pi_f^{g*}(t)$ decreases in t.

(iii) Similarly, for the supply chain, we have

$$\Pi_{sc}^{g*}(t) = \mathsf{E}[\min\{\xi, q_g^*(t)\}] - cq_g^*(t) = \left[\int_0^{q_g^*(t)} \xi dF(\xi) + \int_{q_g^*(t)}^{\infty} q_g^*(t)dF(\xi)\right] - cq_g^*(t)$$

and

$$\frac{d\Pi_{sc}^g(t)}{dt} = (\overline{F}(q_g^*(t)) - c)\frac{dq_g^*(t)}{dt} = (\overline{F}(q_g^*(t)) - c)\frac{q_g^*(t)(1-G(\widetilde{c}q_g^*(t)))}{G(\widetilde{c}q_g^*(t)) - G(q_g^*(t))}\frac{\widetilde{c}}{c} = (\overline{F}(q_g^*(t)) - c)q_g^*(t)\frac{\widetilde{c}}{c}y_2(t).$$

Based on the sign of $\overline{F}(q_g^*(t)) - c$, we have the following two cases:

(a). If $\overline{F}(q_g^*(t)) - c > 0$, noting that $y_2(t)$ decreases in t, we have $\frac{d\Pi_{sc}^{g*}(t)}{dt}$ crosses zero at most once. If it crosses, it is first positive and then negative.

(b). If $\overline{F}(q_g^*(t)) - c < 0$, noting that $y_2(t) < 0$, we have $\frac{d\Pi_{sc}^{g*}(t)}{dt} > 0$ and $\Pi_{sc}^{g*}(t)$ increases in t.

By combining (a) $\overline{F}(q_g^*(t)) - c > 0$ and (b) $\overline{F}(q_g^*(t)) - c < 0$, we have $\Pi_{sc}^{g*}(t)$ generally first increases and then decreases in t.

Proof of Lemma 4: Let $\widehat{\xi} = \frac{c(1+r)q_d}{1-t}$. By (5), we have

$$\begin{aligned}\Pi_f^d(q_d) &= \mathsf{E}[\min\{\xi, q_d\}(1-t) - c(1+r)q_d]^+ = \mathsf{E}_{\widehat{\xi}\le\xi\le q_d}[\xi(1-t) - c(1+r)q_d] + \mathsf{E}_{\xi>q_d}[q_d(1-t) - c(1+r)q_d] \\ &= \int_{\widehat{\xi}}^{q_d} (\xi(1-t) - c(1+r)q_d)dF(\xi) + \int_{q_d}^{\infty} (q_d(1-t) - c(1+r)q_d)dF(\xi).\end{aligned}$$

Then, we have

$$\frac{d\Pi_f^d(q_d)}{dq_d} = ((1-t) - c(1+r))\overline{F}(q_d) + c(1+r)(\overline{F}(q_d) - \overline{F}(\widehat{\xi})) = (1-t)\left[\overline{F}(q_d) - \frac{c(1+r)}{1-t}\overline{F}\left(\frac{c(1+r)q_d}{1-t}\right)\right].$$

Hence, the smallholder farmer's optimal production quantity q_d^* must satisfy the FOC as follows:

$$\overline{F}(q_d^*) = \frac{c(1+r)}{1-t}\overline{F}\left(\frac{c(1+r)q_d^*}{1-t}\right);\ \text{that is,}\ \overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*). \tag{11}$$

Next, we show the uniqueness of q_d^*. Define $y(q_d) = q_d\overline{F}(q_d)$. Then, the FOC in (11) is equivalent to $y(q_d) = y(\widehat{c}q_d)$. Note that

$$\frac{dy(q_d)}{dq_d} = \overline{F}(q_d) - q_d f(q_d) = \overline{F}(q_d)\left(1 - \frac{q_d f(q_d)}{\overline{F}(q_d)}\right) = \overline{F}(q_d)(1 - G(q_d)).$$

By the assumption that $G(q_d)$ increases in q_d, we obtain that $\frac{dy(q_d)}{dq_d}$ crosses zero at most once. If it crosses, $\frac{dy(q_d)}{dq_d}$ is first positive and then negative. As such, $y(q_d)$ first increases and then decreases in q_d. Then, $y(q_d)$ is unimodal with respect to q_d, with the maximum q^0 satisfying $G(q^0) = 1$. We proceed by providing a visual illustration in Figure 10, where $y(q_d)$ increases in q_d when $q_d < q^0$ and decreases in q_d when $q_d > q^0$. Then, given t and r (or $\widehat{c}$), q_d^* must be unique, which satisfies $y(q_d^*) = y(\widehat{c}q_d^*)$ or $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$. Moreover, we have $\widehat{c}q_d^* \le q^0 \le q_d^*$. When $\widehat{c} = 1$, q_d^* coincides with $\widehat{c}q_d^*$ and we have $q_d^* = q^0$. It is noted that, as q_d^* increases, $\widehat{c}q_d^*$ must decrease, and it follows that $\widehat{c}(q_d^*) = \frac{\widehat{c}q_d^*}{q_d^*}$ decreases in q_d^*. As a result, we have q_d^* decreases in $\widehat{c}$, $\widehat{c}q_d^*$ increases in $\widehat{c}$ and $q_d^*\overline{F}(q_d^*)$ increases in $\widehat{c}$. By the assumption that $G(q_d)$ increases in q_d, we generally have $G(\widehat{c}q_d^*) \le 1 \le G(q_d^*)$. Specifically, when $\widehat{c} = 1$, we have $G(\widehat{c}q_d^*) = G(q_d^*) = 1$.

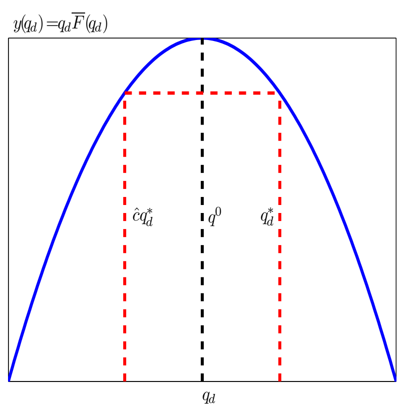


Figure 10 **An illustration of $y(q_d)$ as a function of q_d.**

Proof of Lemma 5: Let $c_r = c(1+r)$. Then, $\widehat{\xi} = \frac{c(1+r)q_d}{1-t} = \frac{c_r q_d}{1-t}$ and $\widehat{c} = \frac{c(1+r)}{1-t} = \frac{c_r}{1-t}$. Given t, by the proof of Lemma 4, the farmer's optimization problem in (5) can be written as

$$\begin{aligned}\Pi_f^d(q_d) &= \int_{\widehat{\xi}}^{q_d} (\xi(1-t) - c(1+r)q_d)dF(\xi) + \int_{q_d}^{\infty} (q_d(1-t) - c(1+r)q_d)dF(\xi) \\ &= \int_{\widehat{\xi}}^{q_d} \xi(1-t)dF(\xi) + \int_{q_d}^{\infty} q_d(1-t)dF(\xi) - \int_{\widehat{\xi}}^{\infty} c_r q_d dF(\xi).\end{aligned}$$

By the Envelope Theorem, we have

$$\frac{d\Pi_f^d(q_d^*)}{dc_r} = -\widehat{\xi}^*(1-t)f(\widehat{\xi}^*)\widehat{\xi}^{*\prime} + c_r q_d^* f(\widehat{\xi}^*)\widehat{\xi}^{*\prime} - \int_{\widehat{\xi}^*}^{\infty} q_d^* dF(\xi) = -q_d^*\overline{F}(\widehat{\xi}^*), \tag{12}$$

where $'$ denotes the first-order derivative and $\widehat{\xi}^* = \widehat{c}q_d^*$. By (11) and the Implicit Function Theorem,

$$\frac{dq_d^*}{d\widehat{c}} = \frac{\overline{F}(\widehat{c}q_d^*) - \widehat{c}q_d^* f(\widehat{c}q_d^*)}{\widehat{c}^2 f(\widehat{c}q_d^*) - f(q_d^*)} = \frac{q_d^*(1-G(\widehat{c}q_d^*))}{\widehat{c}(G(\widehat{c}q_d^*) - G(q_d^*))}. \tag{13}$$

Thus, we can obtain

$$\frac{dq_d^*}{dc_r} = \frac{dq_d^*}{d\widehat{c}}\frac{d\widehat{c}}{dc_r} = \frac{q_d^*(1-G(\widehat{c}q_d^*))}{\widehat{c}(G(\widehat{c}q_d^*) - G(q_d^*))}\frac{1}{1-t} = \frac{q_d^*(1-G(\widehat{c}q_d^*))}{c_r(G(\widehat{c}q_d^*) - G(q_d^*))}. \tag{14}$$

Note that the intermediary platform's optimization problem (6) can be written as

$$\Pi_i^d(\widehat{c}) = -\Pi_f^d(\widehat{c}) + \Pi_{sc}^d(\widehat{c}) = -\mathsf{E}[\min\{\xi, q_d^*\} - \widehat{c}q_d^*]^+(1-t) + \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^*,$$

where q_d^* satisfies $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$. Then, by (11) (12) and (14), we have

$$\begin{aligned}\frac{d\Pi_i^d(\widehat{c})}{d\widehat{c}} &= q_d^*\overline{F}(\widehat{\xi}^*)(1-t) + (\overline{F}(q_d^*) - c)\frac{dq_d^*}{d\widehat{c}} = \frac{q_d^*}{\widehat{c}}\left[\widehat{c}\overline{F}(\widehat{\xi}^*)(1-t) + (\overline{F}(q_d^*) - c)\frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*) - G(q_d^*)}\right] \\ &= \frac{q_d^*}{\widehat{c}}(\overline{F}(q_d^*) - c)\left[\frac{\overline{F}(q_d^*)}{\overline{F}(q_d^*) - c}(1-t) + \frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*) - G(q_d^*)}\right] := \frac{q_d^*}{\widehat{c}}(\overline{F}(q_d^*) - c)y(\widehat{c}).\end{aligned}$$

Based on the sign of $\overline{F}(q_d^*) - c$, we have the following two cases:

(a). If $\overline{F}(q_d^*) - c < 0$, i.e., q_d^* is large and $\widehat{c}$ is small, we have $y(\widehat{c}) < 0$ since $G(\widehat{c}q_d^*) - G(q_d^*) < 0$. Hence, $\frac{d\Pi_i^d(\widehat{c})}{d\widehat{c}} > 0$ and $\Pi_i^d(\widehat{c})$ increases in $\widehat{c}$.

(b). If $\overline{F}(q_d^*) - c > 0$, i.e., q_d^* is small and $\widehat{c}$ is large, by Lemma 4, q_d^* decreases in $\widehat{c}$. Thus, $\frac{\overline{F}(q_d^*)}{\overline{F}(q_d^*) - c}$ decreases in $\widehat{c}$. By Assumption A1, we have $z(\widehat{c}) := \frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*) - G(q_d^*)}$ also decreases in $\widehat{c}$. Then, we have $y(\widehat{c})$ decreases in $\widehat{c}$. As a result, $\frac{d\Pi_i^d(\widehat{c})}{d\widehat{c}}$ crosses zero at most once. If it crosses, it is first positive and then negative. Furthermore, we have $y(\widehat{c})$ decreases in t.

By combining (a) $\overline{F}(q_d^*) - c < 0$ and (b) $\overline{F}(q_d^*) - c > 0$, we have $\Pi_i^d(\widehat{c})$ generally first increases and then decreases in $\widehat{c}$, and $\Pi_i^d(\widehat{c})$ is unimodal in $\widehat{c}$. Specifically, by L'Hopital's rule, we have

$$\lim_{\widehat{c}\to 1} z(\widehat{c}) = \lim_{\widehat{c}\to 1}\left[\frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*) - G(q_d^*)}\right] = \lim_{\widehat{c}\to 1}\left[\frac{-G'(\widehat{c}q_d^*)(q_d^* + \widehat{c}\frac{dq_d^*}{d\widehat{c}})}{G'(\widehat{c}q_d^*)(q_d^* + \widehat{c}\frac{dq_d^*}{d\widehat{c}}) - G'(q_d^*)\frac{dq_d^*}{d\widehat{c}}}\right] = \lim_{\widehat{c}\to 1}\left[\frac{-(q_d^* + \frac{q_d^*(1-G(\widehat{c}q_d^*))}{\widehat{c}(G(\widehat{c}q_d^*) - G(q_d^*))})}{q_d^*}\right].$$

Thus, we have

$$\lim_{\widehat{c}\to 1} z(\widehat{c}) = -1 - \lim_{\widehat{c}\to 1} z(\widehat{c}) \implies \lim_{\widehat{c}\to 1} z(\widehat{c}) = -\frac{1}{2}.$$

Then, we have

$$\lim_{\widehat{c}\to 1} y(\widehat{c}) = \lim_{\widehat{c}\to 1}\left[\frac{\overline{F}(q_d^*)}{\overline{F}(q_d^*) - c}(1-t) + \frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*) - G(q_d^*)}\right] = \frac{\overline{F}(q^0)}{\overline{F}(q^0) - c}(1-t) - \frac{1}{2}$$

and

$$\lim_{\widehat{c}\to 1}\frac{d\Pi_i^d(\widehat{c})}{d\widehat{c}}=\frac{q^0}{1-t}(\overline{F}(q^0)-c)\left(\frac{\overline{F}(q^0)}{\overline{F}(q^0)-c}(1-t)-\frac{1}{2}\right)=\frac{q^0(\overline{F}(q^0)(1-2t)+c)}{2(1-t)}.$$

Thus, given t, when $t<\frac{c}{2\overline{F}(q^0)}+\frac{1}{2}$, $\Pi_i^d(\widehat{c})$ always increases in $\widehat{c}$ and the intermediary platform will charge an interest rate such that $\widehat{c}^*=1$; that is, $r^*=\frac{1-t}{c}-1$. Otherwise, $\Pi_i^d(\widehat{c})$ first increases in $\widehat{c}$ and then decreases in $\widehat{c}$, and the optimal interest rate solves

$$\frac{\overline{F}(q_d^*)}{\overline{F}(q_d^*)-c}(1-t)+\frac{1-G(\widehat{c}^*q_d^*)}{G(\widehat{c}^*q_d^*)-G(q_d^*)}=0.$$

Noting that $y(\widehat{c})$ decreases in t, we have $\widehat{c}^*$ (weakly) decreases in t and hence by Lemma 4, q_d^* (weakly) increases in t.

Proof of Proposition 3: To complete this proof, we sequentially examine the two cases:

(i) If $c\le\overline{F}(q^0)$, when $t<\frac{c}{2\overline{F}(q^0)}+\frac{1}{2}$, by Lemma 5, the intermediary platform charges an interest rate such that $\widehat{c}^*=1$, i.e., $r^*=\frac{1-t}{c}-1$. In this case, $\Pi_f^{d*}(t)=0$ and then

$$\Pi_i^{d*}(t)=-\Pi_f^{d*}(t)+\Pi_{sc}^{d*}(t)=\Pi_{sc}^{d*}(t)=\mathsf{E}[\min\{\xi,q_d^*(t)\}]-cq_d^*(t).$$

Note that $q_d^*(t)$ satisfies $G(q_d^*(t))=1$, i.e., $q_d^*=q^0$, and is independent of t. Hence, $\Pi_i^{d*}(t)$ and $\Pi_{sc}^{d*}(t)$ are both independent of t.

When $t\ge\frac{c}{2\overline{F}(q^0)}+\frac{1}{2}$, the intermediary platform charges an interest rate satisfying

$$\frac{\overline{F}(q_d^*(t))}{\overline{F}(q_d^*(t))-c}(1-t)+\frac{1-G(\widehat{c}^*q_d^*(t))}{G(\widehat{c}^*q_d^*(t))-G(q_d^*(t))}=0.$$

Note that, by (13) in the proof of Lemma 5,

$$\frac{dq_d^*(t)}{dt}=\frac{dq_d^*(t)}{d\widehat{c}^*}\frac{d\widehat{c}^*}{dt}=\frac{q_d^*(t)(1-G(\widehat{c}^*q_d^*(t)))}{\widehat{c}^*(G(\widehat{c}^*q_d^*(t))-G(q_d^*(t)))}\frac{\frac{dc_r^*(t)}{dt}(1-t)+c_r^*(t)}{(1-t)^2}=\frac{q_d^*(t)}{\widehat{c}^*}\frac{\overline{F}(q_d^*(t))}{\overline{F}(q_d^*(t))-c}\frac{\frac{dc_r^*(t)}{dt}(1-t)+c_r^*(t)}{1-t}.$$

Also note that

$$\begin{aligned}\Pi_i^{d*}(t)&=-\Pi_f^{d*}(t)+\Pi_{sc}^{d*}(t)=-\mathsf{E}[\min\{\xi,q_d^*(t)\}(1-t)-c_r^*(t)q_d^*(t)]^++\mathsf{E}[\min\{\xi,q_d^*(t)\}]-cq_d^*(t)\\&=-\left[\int_{\widehat{\xi}^*(t)}^{q_d^*(t)}\xi(1-t)dF(\xi)+\int_{q_d^*(t)}^{\infty}q_d^*(t)(1-t)dF(\xi)-\int_{\widehat{\xi}^*(t)}^{\infty}c_r^*(t)q_d^*(t)dF(\xi)\right]\\&\quad+\left[\int_0^{q_d^*(t)}\xi dF(\xi)+\int_{q_d^*(t)}^{\infty}q_d^*(t)dF(\xi)\right]-cq_d^*(t).\end{aligned}$$

Then, by noting $\widehat{\xi}^*(t)=\widehat{c}^*q_d^*(t)$ and applying (11), we have

$$\begin{aligned}\frac{d\Pi_i^{d*}(t)}{dt}&=-\left[\int_{\widehat{\xi}^*(t)}^{q_d^*(t)}-\xi dF(\xi)-\int_{q_d^*(t)}^{\infty}q_d^*(t)dF(\xi)-\int_{\widehat{\xi}^*(t)}^{\infty}q_d^*(t)\frac{dc_r^*(t)}{dt}dF(\xi)\right]+(\overline{F}(q_d^*(t))-c)\frac{dq_d^*(t)}{dt}\\&=\left[\int_{\widehat{\xi}^*(t)}^{q_d^*(t)}\xi dF(\xi)+\int_{q_d^*(t)}^{\infty}q_d^*(t)dF(\xi)+\int_{\widehat{\xi}^*(t)}^{\infty}q_d^*(t)\frac{dc_r^*(t)}{dt}dF(\xi)\right]-\frac{q_d^*(t)}{\widehat{c}^*}\overline{F}(q_d^*(t))\frac{\frac{dc_r^*(t)}{dt}(1-t)+c_r^*(t)}{1-t}\end{aligned}$$

$$= \left[\int_{\widehat{\xi}^*(t)}^{q_d^*(t)} \xi dF(\xi) + \int_{q_d^*(t)}^{\infty} q_d^*(t) dF(\xi) + \int_{\widehat{\xi}^*(t)}^{\infty} q_d^*(t) \frac{dc_r^*(t)}{dt} dF(\xi) \right] - q_d^*(t)\overline{F}(q_d^*(t)) \frac{\frac{dc_r^*(t)}{dt}}{\widehat{c}^*} - q_d^*(t)\overline{F}(q_d^*(t))$$

$$= \int_{\widehat{\xi}^*(t)}^{q_d^*(t)} \xi dF(\xi) + \left[q_d^*(t)\overline{F}(\widehat{c}^* q_d^*(t)) \frac{dc_r^*(t)}{dt} - q_d^*(t)\overline{F}(q_d^*(t)) \frac{\frac{dc_r^*(t)}{dt}}{\widehat{c}^*} \right] = \int_{\widehat{\xi}^*(t)}^{q_d^*(t)} \xi dF(\xi) \geq 0.$$

Thus, $\frac{d\Pi_i^{d*}(t)}{dt} \geq 0$; that is, $\Pi_i^{d*}(t)$ increases in t.

Then, by combining $t < \frac{c}{2\overline{F}(q^0)} + \frac{1}{2}$ and $t \geq \frac{c}{2\overline{F}(q^0)} + \frac{1}{2}$, we have $\Pi_i^{d*}(t)$ always increases in t.

For the supply chain, we have

$$\Pi_{sc}^{d*}(t) = \mathsf{E}[\min\{\xi, q_d^*(t)\}] - cq_d^*(t),$$

which increases in q_d^* and hence increases in t.

(ii) If $c > \overline{F}(q^0)$, we have $t < \frac{c}{2\overline{F}(q^0)} + \frac{1}{2}$ must be satisfied. Then, by Lemma 5, the intermediary platform will charge an interest rate such that $\widehat{c}^* = 1$, i.e., $r^* = \frac{1-t}{c} - 1$. In this case, the intermediary platform will extract all of the supply chain profit and $\Pi_f^{d*}(t) = 0$. Then,

$$\Pi_i^{d*}(t) = -\Pi_f^{d*}(t) + \Pi_{sc}^{d*}(t) = \Pi_{sc}^{d*}(t) = \mathsf{E}[\min\{\xi, q_d^*(t)\}] - cq_d^*(t).$$

Note that $q_d^*(t)$ satisfies $G(q_d^*(t)) = 1$, i.e., $q_d^* = q^0$, and is independent of t. Hence, $\Pi_i^{d*}(t)$ and $\Pi_{sc}^{d*}(t)$ are both independent of t.

Proof of Proposition 4: We start by characterizing q_d^* for a uniform distribution. By Lemma 5, for a uniform distribution over $[0, 1]$, we have $f(\xi) = 1$, $F(\xi) = \xi$, $\overline{F}(\xi) = 1 - \xi$, $g(\xi) = \frac{1}{1-\xi}$ and $G(\xi) = \frac{\xi}{1-\xi}$. By $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$, we obtain $q_d^* = \frac{1}{1+\widehat{c}}$. Thus, $G(q_d^*) = \frac{1}{\widehat{c}}$ and $G(\widehat{c}q_d^*) = \widehat{c}$. Then, when $t < c + \frac{1}{2}$, we have $\widehat{c}^* = 1$, i.e., $r^* = \frac{1-t}{c} - 1$, and $q_d^* = q^0 = \frac{1}{2}$; otherwise, by solving

$$\frac{\overline{F}(q_d^*)}{\overline{F}(q_d^*) - c}(1-t) + \frac{1 - G(\widehat{c}^* q_d^*)}{G(\widehat{c}^* q_d^*) - G(q_d^*)} = \frac{1 - \frac{1}{1+\widehat{c}^*}}{1 - \frac{1}{1+\widehat{c}^*} - c}(1-t) + \frac{1 - \widehat{c}^*}{\widehat{c}^* - \frac{1}{\widehat{c}^*}} = 0,$$

we have $\widehat{c}^* = \frac{1}{t-c} - 1$, i.e., $r^* = \frac{(1+c-t)(1-t)}{c(t-c)} - 1$, and $q_d^* = t - c$. The comparison is then as follows:

(a). Bank financing versus guarantor financing: By Lemma 1, we have $q_b^* = 1 - \frac{c}{1-t}$; by Lemma 2, we have $q_g^* = \frac{1}{1+\widehat{c}} = \frac{1}{1+\frac{c}{1-t}}$. We can then obtain $q_b^* < q_g^*$.

(b). Guarantor financing versus direct financing: By the proof of Proposition 4, when $t < c + \frac{1}{2}$, $q_d^* = q^0 = \frac{1}{2}$; otherwise, $q_d^* = t - c$. Note that $q_g^* = \frac{1}{1+\frac{c}{1-t}}$ decreases in t while q_d^* weakly increases in t. Also note that $q_g^* > \frac{1}{2}$ for any $t \leq 1 - c$. Thus, if $c < \frac{1}{4}$, there exists a critical threshold $\bar{t} = c + 1 - \sqrt{c} \in (c + \frac{1}{2}, 1 - c)$ such that $q_g^* > q_d^*$ when $t < \bar{t}$ and $q_g^* < q_d^*$ when $t > \bar{t}$; otherwise, we have $q_g^* > q_d^*$ for any $t \leq 1 - c$. For the latter case, letting $\bar{t} = 1 - c$, the result still holds.

(c). Bank financing versus direct financing: If $c \leq \frac{1}{2}$, we have $q_b^* > q_d^*$ at $t = 0$. Note that $q_b^* = 1 - \frac{c}{1-t}$ decreases in t while q_d^* increases in t. Also note that at $t = 0$, $q_b^* = 1 - \frac{c}{1-t} = 1 - c \geq \frac{1}{2}$ if $c \leq \frac{1}{2}$; otherwise, $q_b^* < \frac{1}{2}$. Thus, if $c > \frac{1}{2}$, we have $q_b^* < q_d^*$ for any $t \leq 1 - c$. However, if $c \leq \frac{1}{2}$, there exists a critical threshold $\bar{\bar{t}}$ such that $q_b^* > q_d^*$ when $t < \bar{\bar{t}}$ and $q_b^* < q_d^*$ when $t > \bar{\bar{t}}$. Specifically, $\bar{\bar{t}} = \frac{c}{2} + 1 - \frac{\sqrt{c^2+4c}}{2}$ if $c < \frac{1}{6}$ and $\bar{\bar{t}} = 1 - 2c$ if $c > \frac{1}{6}$. If $c > \frac{1}{2}$, we have $q_b^* < q_d^*$ at $t = 0$ and hence $q_b^* < q_d^*$ for any $t \leq 1 - c$.

Also note that $q^c = 1 - c$. Then, we have $q_b^* < q^c$. Moreover, $q_g^* > q^c$ if and only if $\frac{1}{1+\frac{c}{1-t}} > 1 - c$, i.e., $t < c$. As to $q_d^* - q^c$, when $c > \frac{1}{2}$, $q_d^* - q^c = \frac{1}{2} - (1-c) = -\frac{1}{2} + c > 0$ always holds. When $c < \frac{1}{2}$, we have $q_d^* - q^c < t - c - (1-c) = t - 1 \leq 0$ where the equality holds only when $t = 1$.

Proof of Proposition 5: (i). By (3) and (5), under direct financing, the intermediary platform can always charge the same interest rate as the optimal one under guarantor financing (i.e., $r^* = 0$) to draw the same production quantity and generate the same profitability for itself, the farmer and the whole supply chain. Then, from the intermediary platform's perspective, serving as a guarantor is equivalently a special case of providing direct financing to the farmer. Consequently, direct financing always dominates guarantor financing from the viewpoint of platform.

More specifically, under bank financing, noting that $\overline{F}(q_b^*) = \frac{c}{1-t} = \widetilde{c}$, we have

$$\Pi_i^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}t] = \left(q_b^* - \frac{(q_b^*)^2}{2}\right)t = \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right)t = \left((1 - \frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t.$$

Under direct financing, by the proof of Proposition 4, when $t \geq c + \frac{1}{2}$, $q_d^* = t - c$, and by noting that $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$, we have $\widehat{c} = \frac{1-q_d^*}{q_d^*}$, i.e., $1 - t = \frac{c(1+r^*)q_d^*}{1-q_d^*}$. Since $\widehat{c} \leq 1$ is required, we have $q_d^* \geq \frac{1}{2}$. Thus,

$$\begin{aligned}
\Pi_f^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ = \mathsf{E}[\min\{\xi, q_d^*\} - (1-q_d^*)]^+(1-t) \\
&= \left(q_d^* - \frac{1}{2}\right)(1-t) = \left(t - c - \frac{1}{2}\right)(1-t); \text{ and} \\
\Pi_i^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* - \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ \\
&= (1-c)q_d^* - \frac{(q_d^*)^2}{2} - (q_d^* - \frac{1}{2})(1-t) = \frac{(t-c)^2 + (1-t)}{2};
\end{aligned}$$

otherwise, $q_d^* = q^0 = \frac{1}{2}$, $\Pi_f^{d*} = 0$ and $\Pi_i^{d*} = \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* = (1-c)q_d^* - \frac{(q_d^*)^2}{2} = \frac{3}{8} - \frac{c}{2}$. Then, if $t \geq c + \frac{1}{2}$,

$$\Pi_i^{b*} - \Pi_i^{d*} = \left((1 - \frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t - \frac{(t-c)^2 + (1-t)}{2} = -\frac{t^2 - t + 1}{2(1-t)^2}c^2 + tc - \frac{(1-t)^2}{2} < 0,$$

where the inequality holds because the maximal value $-\frac{(1-t)^3}{2t^2-2t+2} < 0$; otherwise,

$$\Pi_i^{b*} - \Pi_i^{d*} = \left((1 - \frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t - \frac{3}{8} + \frac{c}{2} = -\frac{tc^2}{2(1-t)^2} + \frac{c}{2} + \frac{t}{2} - \frac{3}{8},$$

which is greater than zero if and only if $t < \frac{5-\sqrt{5}}{10}$ and $\frac{1-t-\sqrt{5t^2-5t+1}}{2t}(1-t) < c < 1 - t$. Considering the requirement of $\frac{1-t-\sqrt{5t^2-5t+1}}{2t}(1-t) < 1 - t$, we have $t < \frac{1}{4}$. Then, $t < c + \frac{1}{2}$ can be satisfied. Thus, we have $\Pi_i^{b*} > \Pi_i^{d*}$ if and only if $t < \frac{1}{4}$ and $c > \frac{1-t-\sqrt{5t^2-5t+1}}{2t}(1-t)$.

(ii). For the farmer's preference, we first compare guarantor financing versus bank financing. By the proof of Lemma 1 and noting that $\overline{F}(q_b^*) = \frac{c}{1-t} = \widetilde{c}$, we have

$$\Pi_f^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}(1-t)] - cq_b^* = \mathsf{E}[\min\{\xi, q_b^*\}]\frac{c}{\overline{F}(q_b^*)} - cq_b^* = \frac{(q_b^*)^2 c}{2(1-q_b^*)} = \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}}.$$

By noting that $\overline{F}(q_g^*) = \widetilde{c}\overline{F}(\widetilde{c}q_g^*)$, we have $\widetilde{c} = \frac{1-q_g^*}{q_g^*}$, i.e., $1-t = \frac{cq_g^*}{1-q_g^*}$. Then, we have

$$\begin{aligned}\Pi_f^{g*} &= \mathsf{E}[\min\{\xi, q_g^*\}(1-t) - cq_g^*]^+ = \mathsf{E}[\min\{\xi, q_g^*\} - (1-q_g^*)]^+(1-t) \\ &= \mathsf{E}[\min\{\xi, q_g^*\} - (1-q_g^*)]^+ \frac{cq_g^*}{1-q_g^*} = \frac{cq_g^*(q_g^* - \frac{1}{2})}{1-q_g^*} = \frac{(1-\widetilde{c})c}{2(1+\widetilde{c})\widetilde{c}}.\end{aligned}$$

It is straightforward to obtain

$$\Pi_f^{b*} - \Pi_f^{g*} = \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}} - \frac{(1-\widetilde{c})c}{2(1+\widetilde{c})\widetilde{c}} = \frac{(1-\widetilde{c})c}{2\widetilde{c}}(1-\widetilde{c} - \frac{1}{1+\widetilde{c}}) < 0.$$

Then, we have $\Pi_f^{b*} < \Pi_f^{g*}$.

Then, to obtain the farmer's preference, we just need to compare guarantor financing and direct financing. Under direct financing, when $t < c + \frac{1}{2}$, $q_d^* = q^0 = \frac{1}{2}$ and $\Pi_f^{d*} = 0$; otherwise,

$$\Pi_f^{d*} = \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ = \mathsf{E}[\min\{\xi, q_d^*\} - (1-q_d^*)]^+(1-t) = \left(t - c - \frac{1}{2}\right)(1-t).$$

Then, $\Pi_f^{d*} > \Pi_f^{g*}$ if and only if $t > \bar{t} = c + 1 - \sqrt{c}$. That is, when $\max\{c + \frac{1}{2}, c + 1 - \sqrt{c}\} < t < 1 - c$, direct financing is preferred, which holds only if $c < \frac{1}{4}$. Note that, if $c < \frac{1}{4}$, we have $c + 1 - \sqrt{c} > c + \frac{1}{2}$.

(iii). Note that, for each financing format $j = b, g, d$, the corresponding supply chain profit is

$$\Pi_{sc}^{j*} = \mathsf{E}[\min\{\xi, q_j^*\}] - cq_j^* = (1-c)q_j^* - \frac{(q_j^*)^2}{2},$$

which is a quadratic function of q_j^* and achieves the maximal value at $q^c = 1 - c$. Thus, to compare Π_{sc}^{j*} to derive the supply chain's preference, we just need the ordering of q_j^* and q^c, which has been discussed in the proof of Proposition 4. Then, we can further obtain the following results:

(iii-1). $t > c$ and $c < \frac{1}{2}$: By the proof of Proposition 4, we have $q_d^* < q^c$ and $q_b^* < q_g^* < q^c$. The preference can be either guarantor financing or direct financing. In this case, by Lemma 2, $q_g^* = \frac{1}{1+\widetilde{c}} = \frac{1}{1+\frac{c}{1-t}}$; by the proof of Proposition 4, when $t < c + \frac{1}{2}$, $q_d^* = q^0 = \frac{1}{2}$ and $q_d^* = t - c$ otherwise. Note that $q_d^* > q_g^*$ if and only if $t > \bar{t} = c + 1 - \sqrt{c}$. Then, the supply chain preference is direct financing if and only if $t > \bar{t}$. We note that, $c + 1 - \sqrt{c} < 1 - c$ can be satisfied only if $c < \frac{1}{4}$.

(iii-2). $t < c$ and $c < \frac{1}{2}$: In this case, by Lemma 1, $q_b^* = 1 - \frac{c}{1-t}$; by Lemma 2, $q_g^* = \frac{1}{1+\widetilde{c}} = \frac{1}{1+\frac{c}{1-t}}$; by the proof of Proposition 4, $q_d^* = q^0 = \frac{1}{2}$. By the proof of Proposition 4, we have $q_b^* < q_d^* < q^c < q_g^*$ if $t > 1 - 2c$ and $q_d^* < q_b^* < q^c < q_g^*$ if $t < 1 - 2c$. Then, we have two cases: (a). If $t > 1 - 2c$, the preference can be either guarantor financing or direct financing. Note that $q^c - q_d^* < q_g^* - q^c$, i.e., $2(1-c) < \frac{1}{2} + \frac{1}{1+\frac{c}{1-t}}$, if and only if $t < \frac{4c^2+c-1}{4c-1}$. Then, the supply chain preference is direct financing if and only if $t < \frac{4c^2+c-1}{4c-1}$ and $t > 1-2c$. We note that, in this case, $1-2c < \frac{4c^2+c-1}{4c-1}$ can be satisfied only if $\frac{5}{12} < c < \frac{1}{2}$. (b). If $t < 1-2c$, the preference can be either bank financing or guarantor financing. Note that $q^c - q_b^* > q_g^* - q^c$, i.e., $2(1-c) > 1 - \frac{c}{1-t} + \frac{1}{1+\frac{c}{1-t}}$, if and only if $t > \frac{c+1-\sqrt{c^2+1}}{2}$. Then, the supply chain preference is guarantor financing if and only if $t > \frac{c+1-\sqrt{c^2+1}}{2}$ and $t < 1-2c$. We note that, in this case, $\frac{c+1-\sqrt{c^2+1}}{2} < 1-2c$ can be satisfied only if $c < \frac{5}{12}$.

(iii-3). $t < c$ and $c > \frac{1}{2}$: By the proof of Proposition 4, we have $q_b^* < q^c < q_d^* < q_g^*$. The preference can be either bank financing or direct financing. In this case, by Lemma 1, $q_b^* = 1 - \frac{c}{1-t}$; by the proof of Proposition 4, $q_d^* = q^0 = \frac{1}{2}$. Note that $q_d^* - q^c < q^c - q_b^*$, i.e., $\frac{1}{2} + 1 - \frac{c}{1-t} < 2(1-c)$, if and only if $t > \frac{2c-1}{4c-1}$. Then, the supply chain preference is direct financing if and only if $t > \frac{2c-1}{4c-1}$. We note that, in this case, $\frac{2c-1}{4c-1} < 1-c$ can be satisfied only if $c < \frac{3}{4}$.

Summarizing (iii-1) (iii-2) (iii-3) and considering the boundary condition $t \leq 1-c$, we have the following results: the supply chain as a whole prefers bank financing when either $c < \frac{1}{2}$ and $t < \min\left\{\frac{c+1-\sqrt{c^2+1}}{2}, 1-2c\right\}$ or $c > \frac{1}{2}$ and $t < \min\left\{\frac{2c-1}{4c-1}, 1-c\right\}$ and prefers guarantor financing when $\widetilde{t}(c) < t < \widehat{t}(c)$; otherwise, it prefers direct financing. Specifically,

$$\widetilde{t}(c) = \begin{cases} \frac{c+1-\sqrt{c^2+1}}{2}, & \text{when } c < \frac{5}{12}, \\ \frac{4c^2+c-1}{4c-1}, & \text{when } \frac{5}{12} < c < \frac{1}{2}, \end{cases} \quad \text{and} \quad \widehat{t}(c) = \begin{cases} c+1-\sqrt{c}, & \text{when } c < \frac{1}{4}, \\ 1-c, & \text{when} \frac{1}{4} < c < \frac{1}{2}. \end{cases} \tag{15}$$

Proof of Proposition 6: (i). By the proof of Proposition 5, when $t < \bar{t} = c+1-\sqrt{c}$, the farmer prefers guarantor financing, and we have

$$\Pi_f^{g*} = \mathsf{E}[\min\{\xi, q_g^*\}(1-t) - cq_g^*]^+ = \frac{(1-\widetilde{c})c}{2(1+\widetilde{c})\widetilde{c}},$$

which always decreases in $\widetilde{c}$ and hence decreases in t.

When $t > \bar{t} = c+1-\sqrt{c}$, the farmer prefers direct financing. Note that $t \leq 1-c$ is required for the farmer to participate in the production. This implies that the case that direct financing is preferred by the farmer happens only when $\bar{t} = c+1-\sqrt{c} < 1-c$, i.e., $c < \frac{1}{4}$. Moreover, under direct financing, by the proofs of Proposition 4 and Proposition 5, when $t < c + \frac{1}{2}$, $q_d^* = q^0 = \frac{1}{2}$ and $\Pi_f^{d*} = 0$; otherwise, $q_d^* = t - c$, $\Pi_f^{d*} = \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ = \left(t - c - \frac{1}{2}\right)(1-t)$. Thus,

$$\frac{d\Pi_f^{d*}}{dt} = c + \frac{3}{2} - 2t \text{ and } \frac{d^2\Pi_f^{d*}}{dt^2} = -2 < 0,$$

implying Π_f^{d*} achieves the maximal value at $t = \frac{c}{2} + \frac{3}{4}$. Then, if $c + 1 - \sqrt{c} < \frac{c}{2} + \frac{3}{4}$, i.e., $\frac{3}{2} - \sqrt{2} < c < \frac{1}{4}$, the farmer's profit increases in t for $c + 1 - \sqrt{c} < t < \frac{c}{2} + \frac{3}{4}$ and decreases in t for $t > \frac{c}{2} + \frac{3}{4}$. Otherwise, it decreases in t.

(ii). Regarding the platform's profit, we have the following two cases:

(a). $c < \frac{1}{4}$: When $t < \bar{t}$, the farmer prefers guarantor financing and we have

$$\Pi_i^{g*} = \Pi_{sc}^{g*} - \Pi_f^{g*} = \mathsf{E}[\min\{\xi, q_g^*\}] - cq_g^* - \mathsf{E}[\min\{\xi, q_g^*\}(1-t) - cq_g^*]^+ = (1-c)q_g^* - \frac{(q_g^*)^2}{2} - \frac{cq_g^*(q_g^* - \frac{1}{2})}{1 - q_g^*},$$

where $q_g^* = \frac{1}{1+\tilde{c}} = \frac{1}{1+\frac{c}{1-t}}$. It is straightforward to obtain $\frac{dq_g^*}{dt} < 0$. Then, we have

$$\frac{d\Pi_i^{g*}}{dt} = \left(1 - c - q_g^* - c\frac{(2q_g^* - \frac{1}{2})(1-q_g^*) + q_g^*(q_g^* - \frac{1}{2})}{(1-q_g^*)^2}\right)\frac{dq_g^*}{dt},$$

and, at $t = \bar{t} = c + 1 - \sqrt{c}$,

$$\frac{d\Pi_i^{g*}}{dt}\Big|_{t=\bar{t}} = \left(\sqrt{c} - \frac{1}{2}\right)\frac{dq_g^*}{dt}\Big|_{t=\bar{t}} > 0.$$

By the proof of Proposition 2, Π_i^{g*} is unimodal in t, namely, first increasing and then decreasing in t. Thus, Π_i^{g*} increases in t when $t < \bar{t}$.

When $t > \bar{t}$, the farmer prefers direct financing. By the proof of Proposition 3, we have Π_i^{d*} (weakly) increases in t; by the proof of Proposition 5, guarantor financing is (weakly) dominated by direct financing; at $t = \bar{t}$, direct financing generates the same profit level for the platform as guarantor financing. Combining above analysis, the platform's profit always increases in t.

(b). $c > \frac{1}{4}$: In this case, the farmer prefers guarantor financing. Then, we have

$$\Pi_i^{g*} = (1-c)q_g^* - \frac{(q_g^*)^2}{2} - \frac{cq_g^*(q_g^* - \frac{1}{2})}{1-q_g^*},$$

and, at $t = 1-c$,

$$\frac{d\Pi_i^{g*}}{dt}\Big|_{t=1-c} = \left(1 - c - q_g^* - c\frac{(2q_g^* - \frac{1}{2})(1-q_g^*) + q_g^*(q_g^* - \frac{1}{2})}{(1-q_g^*)^2}\right)\frac{dq_g^*}{dt}\Big|_{t=1-c} = \left(\frac{1}{2} - 2c\right)\frac{dq_g^*}{dt}\Big|_{t=1-c} > 0.$$

Again, by noting that Π_i^{g*} is unimodal in t, we have Π_i^{g*} always increases in t.

(iii). Regarding the supply chain profit, we have the following two cases:

(a). $c < \frac{1}{4}$: When $t < \bar{t}$, the farmer prefers guarantor financing and we have

$$\Pi_{sc}^{g*} = \mathsf{E}[\min\{\xi, q_g^*\}] - cq_g^* = (1-c)q_g^* - \frac{(q_g^*)^2}{2},$$

where $q_g^* = \frac{1}{1+\tilde{c}} = \frac{1}{1+\frac{c}{1-t}}$. It is straightforward to obtain $\frac{dq_g^*}{dt} < 0$. Then, we have

$$\frac{d\Pi_{sc}^{g*}}{dt} = (1 - c - q_g^*)\frac{dq_g^*}{dt} = \left(1 - c - \frac{1}{1+\tilde{c}}\right)\frac{dq_g^*}{dt},$$

which is first positive and then negative. It implies that Π_{sc}^{g*} is unimodal with respect to t with the maximal value attained at $1 - c - \frac{1}{1+\tilde{c}} = 0$, i.e., at $t = c < \bar{t}$. Then, Π_{sc}^{g*} increases in t when $t < c$ and decreases in t when $c < t < \bar{t}$.

When $t > \bar{t}$, the farmer prefers direct financing and we have

$$\Pi_{sc}^{d*} = \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* = (1-c)q_d^* - \frac{(q_d^*)^2}{2}.$$

By $c < \frac{1}{4}$ and $t > \bar{t}$, we have $t > c + \frac{1}{2}$. By the proof of Proposition 4, we obtain $q_d^* = t - c$ and

$$\frac{d\Pi_{sc}^{d*}}{dt} = (1 - c - q_d^*)\frac{dq_d^*}{dt} = 1 - t > 0.$$

Then, Π_{sc}^{d*} increases in t when $t > \bar{t}$.

(b). $c > \frac{1}{4}$: In this case, the farmer always prefers guarantor financing. Meanwhile, Π_{sc}^{g*} is unimodal in t with the maximal value attained at $t = c$. Then, Π_{sc}^{g*} increases in t when $t < \min\{c, 1-c\}$ and decreases in t when $c < t < 1-c$.

To sum up, if $c < \frac{1}{4}$, the total profit of the supply chain increases in t when either $t < c$ or $t > c + 1 - \sqrt{c}$; if $c > \frac{1}{4}$, it increases in t when $t < \min\{c, 1-c\}$; otherwise, it decreases in t.

Proof of Proposition 7: (i) When the farmer is non-creditworthy, bank financing is unavailable, and he chooses the financing format provided by the intermediary platform. For the platform, it selects either guarantor financing or direct financing. By Proposition 5, guarantor financing is always dominated by direct financing. Then, it always provides direct financing.

(ii) When the farmer is creditworthy, bank financing is available to the farmer. For the intermediary platform, it selects either guarantor financing or direct financing, competing against bank financing, or chooses no financing and lets the farmer resort to bank financing. Specifically, if guarantor financing is provided, by Proposition 5, guarantor financing always dominates bank financing from the farmer's perspective. Then, the farmer always chooses guarantor financing.

However, if direct financing is provided, the farmer will choose between bank financing and direct financing. By the proof of Lemma 1 and noting that $\overline{F}(q_b^*) = \frac{c}{1-t} = \widetilde{c}$, we have

$$\Pi_f^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}(1-t)] - cq_b^* = \mathsf{E}[\min\{\xi, q_b^*\}]\frac{c}{\overline{F}(q_b^*)} - cq_b^* = \frac{(q_b^*)^2 c}{2(1-q_b^*)} = \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}}$$

and

$$\Pi_i^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}t] = \left(q_b^* - \frac{(q_b^*)^2}{2}\right)t = \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right)t = \left((1 - \frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t.$$

Under direct financing, by the proof of Proposition 4, $q_d^* = \frac{1}{1+\widehat{c}}$, and by noting that $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$, we have $\widehat{c} = \frac{1-q_d^*}{q_d^*}$, i.e., $1 - t = \frac{c(1+r)q_d^*}{1-q_d^*}$. Since $\widehat{c} \le 1$ is required, we have $q_d^* \ge \frac{1}{2}$. Thus,

$$\begin{aligned}
\Pi_f^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r)q_d^*]^+ = \mathsf{E}[\min\{\xi, q_d^*\} - (1-q_d^*)]^+(1-t) \\
&= \left(q_d^* - \frac{1}{2}\right)(1-t) = \frac{1-\widehat{c}}{2(1+\widehat{c})}(1-t); \text{ and} \\
\Pi_i^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* - \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r)q_d^*]^+ \\
&= (1-c)q_d^* - \frac{(q_d^*)^2}{2} - (q_d^* - \frac{1}{2})(1-t) = \frac{t - 2c + 2(1-c)\widehat{c} - (1-t)\widehat{c}^2}{2(1+\widehat{c})^2}.
\end{aligned}$$

Then, for the farmer, he chooses direct financing if and only if $\frac{1-\widehat{c}}{2(1+\widehat{c})}(1-t) \ge \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}}$, i.e., $\widehat{c} \le \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$.

From the intermediary platform's perspective, note that, guarantor financing is a special case of direct financing with $r = 0$ or $\widehat{c} = \widetilde{c}$. That is, under direct financing, the platform can always mimic guarantor financing by setting $r = 0$ or $\widehat{c} = \widetilde{c}$. Also note that the intermediary platform's profit function first increases and then decreases with $\widehat{c}$. Then, to compare guarantor financing and direct financing, we need to compare $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$ and $\widetilde{c}$. More specifically, $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2} - \widetilde{c} = \widetilde{c}^2 \frac{1-\widetilde{c}}{1+(1-\widetilde{c})^2} > 0$. Then, for the intermediary platform, direct financing always dominates guarantor financing.

To proceed, we need to compare direct financing against no financing. The latter choice means the farmer needs to rely on bank financing to raise funds. Based on the parameter values of c and t, we have two cases:

(ii-1) When $t < c + \frac{1}{2}$, we have the intermediary platform's profit increases with $\widehat{c}$. Then, the constraint to induce the farmer to select direct financing, i.e., $\widehat{c} \le \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$, will essentially affect its optimal decision and profitability. By comparing bank financing and direct financing, we have, at $\widehat{c} = \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$,

$$\Pi_i^{d*} - \Pi_i^{b*} = \frac{t - 2c + 2(1-c)\widehat{c} + (1-t)\widehat{c}^2}{2(1+\widehat{c})^2} - \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right) t = -\frac{c^3}{8(1-t)^4}\left(4t^2 - 4t + c\right).$$

As a result, $\Pi_i^{d*} - \Pi_i^{b*} < 0$ when $t < \frac{1}{4}$ and $c > 4t(1-t)$; otherwise, $\Pi_i^{d*} - \Pi_i^{b*} \ge 0$.

(ii-2) When $t \ge c + \frac{1}{2}$, we have the intermediary platform's profit increases with $\widehat{c}$ if $\widehat{c} < \frac{1}{t-c} - 1$ and decreases with $\widehat{c}$ if $\widehat{c} \ge \frac{1}{t-c} - 1$. We further note that, if $c \ge (t - \sqrt{t^2 + 2t - 2})(1-t)$, $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2} - (\frac{1}{t-c} - 1) = -\frac{c(c+2t-2)}{2t^2 + 2(c-2)t + (c^2 - 2c + 2)} - \frac{1}{t-c} + 1 \ge 0$, i.e., $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2} \ge \frac{1}{t-c} - 1$. As such, the constraint to induce the farmer to select direct financing does not affect the intermediary platform's choice of interest rate. By Proposition 5, in this case, the intermediary platform always benefits from providing direct financing, compared with bank financing. Otherwise, if $c < (t - \sqrt{t^2 + 2t - 2})(1-t)$, the optimal interest rate will be at $\widehat{c} = \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$. By comparing bank financing and direct financing, we have

$$\Pi_i^{d*} - \Pi_i^{b*} = \frac{t - 2c + 2(1-c)\widehat{c} + (1-t)\widehat{c}^2}{2(1+\widehat{c})^2} - \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right) t = -\frac{c^3}{8(1-t)^4}\left(4t^2 - 4t + c\right).$$

Then, $\Pi_i^{d*} - \Pi_i^{b*} < 0$ when $t < \frac{1}{4}$ and $c > 4t(1-t)$, which is impossible because $t \ge c + \frac{1}{2}$. As such, the intermediary platform also benefits from providing direct financing in this case.

Then, to summarize, the intermediary platform provides no financing and induces the farmer to take bank financing when the unit commission fee $t < \frac{1}{4}$ and the production cost $c > 4t(1-t)$; otherwise, it provides direct financing.

Proof of Lemma 7: Note that, given r (or equivalently $\widehat{c}$), social responsibility concern does not affect the farmer's decision. That is, q_d^* satisfies $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$. Then, the intermediary platform's optimization problem (8) can be written as

$$\Omega_i^d(\widehat{c}) = -(1-\lambda)\Pi_f^d(\widehat{c}) + \Pi_{sc}^d(\widehat{c}) = -(1-\lambda)\mathsf{E}[\min\{\xi, q_d^*\} - \widehat{c}q_d^*]^+(1-t) + \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^*.$$

By (13), we have

$$\frac{d\Omega_i^d(\widehat{c})}{d\widehat{c}} = (1-\lambda)q_d^*\overline{F}(\widehat{\xi}^*)(1-t) + (\overline{F}(q_d^*) - c)\frac{dq_d^*}{d\widehat{c}} = \frac{q_d^*}{\widehat{c}}\left[(1-\lambda)\widehat{c}\overline{F}(\widehat{\xi}^*)(1-t) + (\overline{F}(q_d^*) - c)\frac{1 - G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*) - G(q_d^*)}\right]$$

$$= \frac{q_d^*}{\widehat{c}}(\overline{F}(q_d^*) - c)\left[\frac{(1-\lambda)\overline{F}(q_d^*)}{\overline{F}(q_d^*) - c}(1-t) + \frac{1 - G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*) - G(q_d^*)}\right] := \frac{q_d^*}{\widehat{c}}(\overline{F}(q_d^*) - c)y(\widehat{c}).$$

Based on the sign of $\overline{F}(q_d^*) - c$, we have the following two cases:

(a). If $\overline{F}(q_d^*) - c < 0$, i.e., q_d^* is large and $\widehat{c}$ is small, we have $y(\widehat{c}) < 0$ since $G(\widehat{c}q_d^*) - G(q_d^*) < 0$. Hence, $\frac{d\Omega_i^d(\widehat{c})}{d\widehat{c}} > 0$ and $\Pi_i^d(\widehat{c})$ increases in $\widehat{c}$.

(b). If $\overline{F}(q_d^*) - c > 0$, i.e., q_d^* is small and $\widehat{c}$ is large, by Lemma 4, q_d^* decreases in $\widehat{c}$. Thus, $\frac{(1-\lambda)\overline{F}(q_d^*)}{\overline{F}(q_d^*)-c}$ decreases in $\widehat{c}$. Note that $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$. Then, Assumption A1 implies that $z(\widehat{c}) := \frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*)-G(q_d^*)}$ decreases in $\widehat{c}$. Then, we have $y(\widehat{c})$ decreases in $\widehat{c}$. As a result, $\frac{d\Omega_i^d(\widehat{c})}{d\widehat{c}}$ crosses zero at most once. If it crosses, it is first positive and then negative.

By combining (a) $\overline{F}(q_d^*) - c < 0$ and (b) $\overline{F}(q_d^*) - c > 0$, we have $\Omega_i^d(\widehat{c})$ generally first increases and then decreases in $\widehat{c}$, and $\Omega_i^d(\widehat{c})$ is unimodal in $\widehat{c}$. Specifically, by L'Hopital's rule,

$$\lim_{\widehat{c}\to 1} z(\widehat{c}) = \lim_{\widehat{c}\to 1}\left[\frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*)-G(q_d^*)}\right] = \lim_{\widehat{c}\to 1}\left[\frac{-G'(\widehat{c}q_d^*)(q_d^* + \widehat{c}\frac{dq_d^*}{d\widehat{c}})}{G'(\widehat{c}q_d^*)(q_d^* + \widehat{c}\frac{dq_d^*}{d\widehat{c}}) - G'(q_d^*)\frac{dq_d^*}{d\widehat{c}}}\right] = \lim_{\widehat{c}\to 1}\left[\frac{-(q_d^* + \frac{q_d^*(1-G(\widehat{c}q_d^*))}{\widehat{c}(G(\widehat{c}q_d^*)-G(q_d^*))})}{q_d^*}\right].$$

Thus, we have

$$\lim_{\widehat{c}\to 1} z(\widehat{c}) = -1 - \lim_{\widehat{c}\to 1} z(\widehat{c}) \implies \lim_{\widehat{c}\to 1} z(\widehat{c}) = -\frac{1}{2}.$$

Then, we have

$$\lim_{\widehat{c}\to 1} y(\widehat{c}) = \lim_{\widehat{c}\to 1}\left[\frac{(1-\lambda)\overline{F}(q_d^*)}{\overline{F}(q_d^*)-c}(1-t) + \frac{1-G(\widehat{c}q_d^*)}{G(\widehat{c}q_d^*)-G(q_d^*)}\right] = \frac{(1-\lambda)\overline{F}(q^0)}{\overline{F}(q^0)-c}(1-t) - \frac{1}{2}$$

and

$$\lim_{\widehat{c}\to 1}\frac{d\Omega_i^d(\widehat{c})}{d\widehat{c}} = \frac{q^0}{1-t}(\overline{F}(q^0)-c)\left(\frac{(1-\lambda)\overline{F}(q^0)}{\overline{F}(q^0)-c}(1-t) - \frac{1}{2}\right) = \frac{q^0(\overline{F}(q^0)(2(1-\lambda)(1-t)-1)+c)}{2(1-t)}.$$

Thus, given t, when $t < \frac{c}{2(1-\lambda)\overline{F}(q^0)} + \frac{1-2\lambda}{2(1-\lambda)}$, $\Omega_i^d(\widehat{c})$ always increases in $\widehat{c}$ and the intermediary platform will charge an interest rate such that $\widehat{c}^* = 1$; that is, $r^* = \frac{1-t}{c} - 1$. Otherwise, $\Omega_i^d(\widehat{c})$ first increases in $\widehat{c}$ and then decreases in $\widehat{c}$, and the optimal interest rate solves

$$\frac{(1-\lambda)\overline{F}(q_d^*)}{\overline{F}(q_d^*)-c}(1-t) + \frac{1-G(\widehat{c}^*q_d^*)}{G(\widehat{c}^*q_d^*)-G(q_d^*)} = 0.$$

Proof of Corollary 1: For a uniform distribution over the interval $[0,1]$, by $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$, we obtain $q_d^* = \frac{1}{1+\widehat{c}}$. Thus, $G(q_d^*) = \frac{1}{\widehat{c}}$ and $G(\widehat{c}q_d^*) = \widehat{c}$. Then, by Lemma 7, when $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $\lambda < \frac{2c-2t+1}{2(1-t)}$), we have $\widehat{c}^* = 1$, i.e., $r^* = \frac{1-t}{c} - 1$, and $q_d^* = q^0 = \frac{1}{2}$; otherwise, by solving

$$\frac{(1-\lambda)\overline{F}(q_d^*)}{\overline{F}(q_d^*)-c}(1-t) + \frac{1-G(\widehat{c}^*q_d^*)}{G(\widehat{c}^*q_d^*)-G(q_d^*)} = (1-\lambda)\frac{1-\frac{1}{1+\widehat{c}^*}}{1-\frac{1}{1+\widehat{c}^*}-c}(1-t) + \frac{1-\widehat{c}^*}{\widehat{c}^* - \frac{1}{\widehat{c}^*}} = 0,$$

we have $\widehat{c}^* = \frac{1}{t+\lambda(1-t)-c} - 1$, i.e., $r^* = \frac{((1-\lambda)(1-t)+c)(1-t)}{c(t+\lambda(1-t)-c)} - 1$, and $q_d^* = t + \lambda(1-t) - c$. We can quickly obtain q_d^* (weakly) increases in λ but r^* (weakly) decreases in λ.

To proceed, if $t \geq \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $\lambda \geq \frac{2c-2t+1}{2(1-t)}$),

$$\begin{aligned}\Pi_f^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ = \mathsf{E}[\min\{\xi, q_d^*\} - (1-q_d^*)]^+(1-t) \\ &= \left(q_d^* - \frac{1}{2}\right)(1-t) = \left(t + \lambda(1-t) - c - \frac{1}{2}\right)(1-t);\end{aligned}$$

otherwise, if $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $\lambda < \frac{2c-2t+1}{2(1-t)}$), $q_d^* = q^0 = \frac{1}{2}$, $\Pi_f^{d*} = 0$. Then, we have Π_f^{d*} (weakly) increases in λ. Moreover, if $t \geq \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $\lambda \geq \frac{2c-2t+1}{2(1-t)}$),

$$\begin{aligned}\Omega_i^{d*} &= \lambda\left(t + \lambda(1-t) - c - \frac{1}{2}\right)(1-t) + \frac{(t-c)^2 - \lambda^2(1-t)^2 + (1-t)}{2} \\ &= \frac{(1-t)^2}{2}\lambda^2 - (1-t)\left(c - t + \frac{1}{2}\right)\lambda + \frac{(c-t)^2 + (1-t)}{2},\end{aligned}$$

which increases in λ; otherwise, if $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $\lambda < \frac{2c-2t+1}{2(1-t)}$), $\Omega_i^{d*} = \frac{3}{8} - \frac{c}{2}$, which is independent of λ. As such, Ω_i^{d*} (weakly) increases in λ.

For the whole supply chain, we have

$$\Pi_{sc}^{d*} = \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* = (1-c)q_d^* - \frac{(q_d^*)^2}{2}.$$

Then, if $t \geq \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $\lambda \geq \frac{2c-2t+1}{2(1-t)}$),

$$\Pi_{sc}^{d*} = (1-c)(t + \lambda(1-t) - c) - \frac{(t + \lambda(1-t) - c)^2}{2} = -\frac{(1-t)^2}{2}\lambda^2 + (1-t)^2\lambda + \frac{(c-t)(c+t-2)}{2},$$

which increases in λ; otherwise, if $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $\lambda < \frac{2c-2t+1}{2(1-t)}$), $\Pi_{sc}^{d*} = \frac{3}{8} - \frac{c}{2}$, which is independent of λ. As such, Π_{sc}^{d*} also (weakly) increases in λ.

Proof of Lemma 8: Here we compare the production quantities under the three financing formats:

(a). Bank financing versus guarantor financing: By Lemma 1, we have $q_b^* = 1 - \frac{c}{1-t}$; by Lemma 2, we have $q_g^* = \frac{1}{1+\tilde{c}} = \frac{1}{1+\frac{c}{1-t}}$. We can then obtain $q_b^* < q_g^*$.

(b). Guarantor financing versus direct financing: By Corollary 1, when $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$, $q_d^* = q^0 = \frac{1}{2}$; otherwise, $q_d^* = t + \lambda(1-t) - c$. Note that $q_g^* = \frac{1}{1+\frac{c}{1-t}}$ decreases in t while q_d^* weakly increases in t. Also note that $q_g^* > \frac{1}{2}$ for any $t \leq 1-c$. Thus, if $c < \frac{1}{2(2-\lambda)}$, there exists a critical threshold $\bar{t}' = \frac{2(1+c) - \lambda(2+c) - \sqrt{c(c\lambda^2 - 4\lambda + 4)}}{2(1-\lambda)} \in (\frac{1+2c-2\lambda}{2(1-\lambda)}, 1-c)$ such that $q_g^* > q_d^*$ when $t < \bar{t}'$ and $q_g^* < q_d^*$ when $t > \bar{t}'$; otherwise, $q_g^* > q_d^*$ for any $t \leq 1-c$. For the latter case, letting $\bar{t}' = 1-c$, the result still holds.

(c). Bank financing versus direct financing: If $c \leq \frac{1}{2}$, we have $q_b^* > q_d^*$ at $t = 0$. Note that $q_b^* = 1 - \frac{c}{1-t}$ decreases in t while q_d^* increases in t. Also note that at $t = 0$, $q_b^* = 1 - \frac{c}{1-t} = 1 - c \geq \frac{1}{2}$ if $c \leq \frac{1}{2}$; otherwise, $q_b^* < \frac{1}{2}$. Thus, if $c > \frac{1}{2}$, we have $q_b^* < q_d^*$ for any $t \leq 1-c$. However, if $c \leq \frac{1}{2}$, there exists a critical threshold $\bar{\bar{t}}'$ such that $q_b^* > q_d^*$ when $t < \bar{\bar{t}}'$ and $q_b^* < q_d^*$ when $t > \bar{\bar{t}}'$. Specifically, $\bar{\bar{t}}' = \frac{2+c-2\lambda - \sqrt{c(4+c-4\lambda)}}{2(1-\lambda)}$ if $c < \frac{1}{2(3-2\lambda)}$ and $\bar{\bar{t}}' = 1-2c$ if $c > \frac{1}{2(3-2\lambda)}$. If $c > \frac{1}{2}$, we have $q_b^* < q_d^*$ at $t = 0$ and hence $q_b^* < q_d^*$ for any $t \leq 1-c$.

Also note that $q^c = 1-c$. Then, we have $q_b^* < q^c$. Moreover, $q_g^* > q^c$ if and only if $\frac{1}{1+\frac{c}{1-t}} > 1-c$, i.e., $t < c$. As to $q_d^* - q^c$, when $c > \frac{1}{2}$, $q_d^* - q^c = \frac{1}{2} - (1-c) = -\frac{1}{2} + c > 0$ always holds. When $c < \frac{1}{2}$, we have $q_d^* - q^c < t + \lambda(1-t) - c - (1-c) = (1-\lambda)(t-1) \le 0$ where the equality holds when $t = 1$.

Proof of Proposition 8: (i). With social responsibility concern, from the platform's perspective, serving as a guarantor is a special case of providing direct financing to the farmer. Consequently, direct financing always dominates guarantor financing from the viewpoint of platform.

More specifically, under bank financing, noting that $\overline{F}(q_b^*) = \frac{c}{1-t} = \widetilde{c}$, we have

$$\Pi_i^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}t] = \left(q_b^* - \frac{(q_b^*)^2}{2}\right)t = \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right)t = \left((1-\frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t; \text{ and}$$

$$\Pi_f^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}(1-t)] - cq_b^* = \mathsf{E}[\min\{\xi, q_b^*\}]\frac{c}{\overline{F}(q_b^*)} - cq_b^* = \frac{(q_b^*)^2 c}{2(1-q_b^*)} = \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}} = \frac{(1-t-c)^2}{2(1-t)}.$$

Under direct financing, by Corollary 1, if $t \ge \frac{1+2c-2\lambda}{2(1-\lambda)}$, $q_d^* = t + \lambda(1-t) - c$, and by noting that $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$, we have $\widehat{c} = \frac{1-q_d^*}{q_d^*}$, i.e., $1-t = \frac{c(1+r^*)q_d^*}{1-q_d^*}$. Note that $\widehat{c} \le 1$ is required for the participation of the farmer, which implies that $q_d^* > \frac{1}{2}$. Thus, if $t \ge \frac{1+2c-2\lambda}{2(1-\lambda)}$,

$$\begin{aligned}
\Pi_f^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ = \mathsf{E}[\min\{\xi, q_d^*\} - (1-q_d^*)]^+(1-t) \\
&= \left(q_d^* - \frac{1}{2}\right)(1-t) = \left(t + \lambda(1-t) - c - \frac{1}{2}\right)(1-t); \text{ and} \\
\Pi_i^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* - \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ \\
&= (1-c)q_d^* - \frac{(q_d^*)^2}{2} - (q_d^* - \frac{1}{2})(1-t) = \frac{(t-c)^2 - \lambda^2(1-t)^2 + (1-t)}{2};
\end{aligned}$$

otherwise, if $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$, $q_d^* = q^0 = \frac{1}{2}$, $\Pi_f^{d*} = 0$ and $\Pi_i^{d*} = \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* = (1-c)q_d^* - \frac{(q_d^*)^2}{2} = \frac{3}{8} - \frac{c}{2}$. Then, if $t \ge \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $c \le t(1-\lambda) + \lambda - \frac{1}{2}$),

$$\begin{aligned}
\Omega_i^{b*} - \Omega_i^{d*} &= \lambda\frac{(1-t-c)^2}{2(1-t)} + \left((1-\frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t \\
&\quad -\lambda\left(t + \lambda(1-t) - c - \frac{1}{2}\right)(1-t) - \frac{(t-c)^2 - \lambda^2(1-t)^2 + (1-t)}{2} \\
&= -\frac{t\lambda - \lambda - t + t^2 + 1}{2\left(1-t\right)^2}c^2 + \left(t - \lambda + \lambda\left(1-t\right)\right)c - \frac{1}{2}\left(1-\lambda\right)^2\left(1-t\right)^2 < 0;
\end{aligned}$$

otherwise, if $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$ (i.e., $c > t(1-\lambda) + \lambda - \frac{1}{2}$),

$$\begin{aligned}
\Omega_i^{b*} - \Omega_i^{d*} &= \lambda\frac{(1-t-c)^2}{2(1-t)} + \left((1-\frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t - \frac{3}{8} + \frac{c}{2} \\
&= -\frac{t - \lambda + t\lambda}{2(1-t)^2}c^2 + (\frac{1}{2} - \lambda)c + \frac{1}{8}(4t + 4\lambda - 4t\lambda - 3),
\end{aligned}$$

which is greater than zero if and only if $\frac{(1-t)(1-2\lambda) - \sqrt{5t^2 - 5t + 5t\lambda - 4t^2\lambda + 1 - \lambda}}{2(t-\lambda+t\lambda)}(1-t) < c < 1-t$. Considering the requirement of $\frac{(1-t)(1-2\lambda) - \sqrt{5t^2 - 5t + 5t\lambda - 4t^2\lambda + 1 - \lambda}}{2(t-\lambda+t\lambda)}(1-t) < 1-t$, we have $t < \frac{1}{4}$. Then,

$t < \frac{1+2c-2\lambda}{2(1-\lambda)}$ can always be satisfied regardless the value of λ. Thus, we have $\Omega_i^{b*} > \Omega_i^{d*}$ if and only if $t < \frac{1}{4}$ and $c > \frac{(1-t)(1-2\lambda)-\sqrt{5t^2-5t+5t\lambda-4t^2\lambda+1-\lambda}}{2(t-\lambda+t\lambda)}(1-t)$.

(ii). For the farmer's preference, we first compare guarantor financing versus bank financing. By the proof of Lemma 1 and noting that $\overline{F}(q_b^*) = \frac{c}{1-t} = \widetilde{c}$, we have

$$\Pi_f^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}(1-t)] - cq_b^* = \mathsf{E}[\min\{\xi, q_b^*\}]\frac{c}{\overline{F}(q_b^*)} - cq_b^* = \frac{(q_b^*)^2 c}{2(1-q_b^*)} = \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}}.$$

By noting that $\overline{F}(q_g^*) = \widetilde{c}\overline{F}(\widetilde{c}q_g^*)$, we have $\widetilde{c} = \frac{1-q_g^*}{q_g^*}$, i.e., $1-t = \frac{cq_g^*}{1-q_g^*}$. Then, we have

$$\begin{aligned}\Pi_f^{g*} &= \mathsf{E}[\min\{\xi, q_g^*\}(1-t) - cq_g^*]^+ = \mathsf{E}[\min\{\xi, q_g^*\} - (1-q_g^*)]^+(1-t) \\ &= \mathsf{E}[\min\{\xi, q_g^*\} - (1-q_g^*)]^+\frac{cq_g^*}{1-q_g^*} = \frac{cq_g^*(q_g^* - \frac{1}{2})}{1-q_g^*} = \frac{(1-\widetilde{c})c}{2(1+\widetilde{c})\widetilde{c}}.\end{aligned}$$

It is straightforward to obtain

$$\Pi_f^{b*} - \Pi_f^{g*} = \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}} - \frac{(1-\widetilde{c})c}{2(1+\widetilde{c})\widetilde{c}} = \frac{(1-\widetilde{c})c}{2\widetilde{c}}(1-\widetilde{c}-\frac{1}{1+\widetilde{c}}) = -\frac{(1-\widetilde{c})\widetilde{c}^2 c}{2(1+\widetilde{c})} < 0.$$

Then, we have $\Pi_f^{b*} < \Pi_f^{g*}$.

Then, to obtain the farmer's preference, we just need to compare guarantor financing and direct financing. Under direct financing, when $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$, $q_d^* = q^0 = \frac{1}{2}$ and $\Pi_f^{d*} = 0$; otherwise,

$$\Pi_f^{d*} = \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r^*)q_d^*]^+ = \mathsf{E}[\min\{\xi, q_d^*\} - (1-q_d^*)]^+(1-t) = \left(t + \lambda(1-t) - c - \frac{1}{2}\right)(1-t).$$

Then, we have $\Pi_f^{d*} > \Pi_f^{g*}$ if and only if $t > \bar{t} = \frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)}$. That is, when $\max\{\frac{1+2c-2\lambda}{2(1-\lambda)}, \frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)}\} < t < 1-c$, direct financing is preferred by the farmer, which holds only if $c < \frac{1}{2(2-\lambda)}$. Note that, if $c < \frac{1}{2(2-\lambda)}$, we have $\frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)} > \frac{1+2c-2\lambda}{2(1-\lambda)}$.

(iii). Similar to the proof of Proposition 5, for each financing format $j = b, g, d$, we have

$$\Pi_{sc}^{j*} = \mathsf{E}[\min\{\xi, q_j^*\}] - cq_j^* = (1-c)q_j^* - \frac{(q_j^*)^2}{2},$$

which is a quadratic function of q_j^* and achieve the maximal value at $q^c = 1-c$. Thus, to compare Π_{sc}^{j*} to derive the supply chain's preference, we need the ordering of q_j^* and q^c, which has been discussed in the proof of Lemma 8. Then, we can further obtain the following results:

(iii-1). $t > c$ and $c < \frac{1}{2}$: By the proof of Lemma 8, we have $q_d^* < q^c$ and $q_b^* < q_g^* < q^c$. The preference can be either guarantor financing or direct financing. By Lemma 2, $q_g^* = \frac{1}{1+\widetilde{c}} = \frac{1}{1+\frac{c}{1-t}}$; by the proof of Corollary 1, when $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$, $q_d^* = q^0 = \frac{1}{2}$ and $q_d^* = t + \lambda(1-t) - c$ otherwise. Note that $q_d^* > q_g^*$ if and only if $t > \bar{t}' = \frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)}$. Then, the supply chain preference is direct financing if and only if $t > \bar{t}'$. Note that $\frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)} < 1-c$ can be satisfied only if $c < \frac{1}{2(2-\lambda)}$.

(iii-2). $t < c$ and $c < \frac{1}{2}$: In this case, by Lemma 1, $q_b^* = 1 - \frac{c}{1-t}$; by Lemma 2, $q_g^* = \frac{1}{1+\widetilde{c}} = \frac{1}{1+\frac{c}{1-t}}$; by the proof of Corollary 1, when $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$, $q_d^* = q^0 = \frac{1}{2}$ and $q_d^* = t + \lambda(1-t) - c$ otherwise. By the proof of Lemma 8, we have $q_b^* < q_d^* < q^c < q_g^*$ if $t > t^q$ and $q_d^* < q_b^* < q^c < q_g^*$ if $t < t^q$. Specifically, $t^q = \min\left\{\frac{c-2\lambda+2-\sqrt{c(c-4\lambda+4)}}{2(1-\lambda)}, 1-2c\right\}$; that is, $t^q = \frac{c-2\lambda+2-\sqrt{c(c-4\lambda+4)}}{2(1-\lambda)}$ if $c < \frac{1}{2(3-2\lambda)}$ and $t^q = 1-2c$ otherwise. Then, we have two cases:

(a). If $t < t^q$, the preference can be either bank financing or guarantor financing. Note that $q^c - q_b^* > q_g^* - q^c$, i.e., $2(1-c) > 1 - \frac{c}{1-t} + \frac{1}{1+\frac{c}{1-t}}$, if and only if $t > \frac{c+1-\sqrt{c^2+1}}{2}$. Then, the supply chain preference is guarantor financing if and only if $\frac{c+1-\sqrt{c^2+1}}{2} < t < t^q$. Specifically, for $\lambda > \frac{9}{10}$, $\frac{c+1-\sqrt{c^2+1}}{2} < t^q$ holds only if $c < \frac{-(\lambda-1)(\lambda-3)+(\lambda+1)\sqrt{(\lambda-1)(\lambda-9)}}{4\lambda}$; otherwise, it holds only if $c < \frac{5}{12}$.

(b). If $t > t^q$, the preference can be either guarantor financing or direct financing. Note that, for $c < \frac{1}{2(3-2\lambda)}$, $q^c - q_d^* > q_g^* - q^c$, i.e., $2(1-c) > t + \lambda(1-t) - c + \frac{1}{1+\frac{c}{1-t}}$, if and only if $t > \frac{-c\lambda-2\lambda+2+\sqrt{c(4c+4\lambda-4c\lambda+c\lambda^2-4)}}{2(1-\lambda)}$. Then, the supply chain preference is guarantor financing if and only if $t > \frac{-c\lambda-2\lambda+2+\sqrt{c(4c+4\lambda-4c\lambda+c\lambda^2-4)}}{2(1-\lambda)}$. Similarly, for $c > \frac{1}{2(3-2\lambda)}$, $q^c - q_d^* > q_g^* - q^c$, i.e., $2(1-c) > \frac{1}{2} + \frac{1}{1+\frac{c}{1-t}}$, if and only if $t > \frac{4c^2+c-1}{4c-1}$. Then, the supply chain preference is guarantor financing if and only if $t > \frac{4c^2+c-1}{4c-1}$. To summarize, for $\lambda > \frac{9}{10}$, guarantor financing is preferred by the platform if and only if $t > \frac{-c\lambda-2\lambda+2+\sqrt{c(4c+4\lambda-4c\lambda+c\lambda^2-4)}}{2(1-\lambda)}$ when $\frac{-(\lambda-1)(\lambda-3)+(\lambda+1)\sqrt{(\lambda-1)(\lambda-9)}}{4\lambda} < c < \frac{3\lambda+\sqrt{9\lambda^2-8\lambda}}{8\lambda}$ or $t > \frac{4c^2+c-1}{4c-1}$ when $\frac{3\lambda+\sqrt{9\lambda^2-8\lambda}}{8\lambda} < c < \frac{1}{2}$; for $\lambda \le \frac{9}{10}$, guarantor financing is preferred by the platform if and only if $t > \frac{4c^2+c-1}{4c-1}$ when $\frac{5}{12} < c < \frac{1}{2}$.

(iii-3). $t < c$ and $c > \frac{1}{2}$: By the proof of Lemma 8, we have $q_b^* < q^c < q_d^* < q_g^*$. The preference can be either bank financing or direct financing. By Lemma 1, we have $q_b^* = 1 - \frac{c}{1-t}$; by the proof of Corollary 1 and noting that $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$ in this case, we have $q_d^* = q^0 = \frac{1}{2}$. Note that $q_d^* - q^c < q^c - q_b^*$, i.e., $\frac{1}{2} + 1 - \frac{c}{1-t} < 2(1-c)$, if and only if $t > \frac{2c-1}{4c-1}$. Then, the supply chain preference is direct financing if and only if $t > \frac{2c-1}{4c-1}$. We note that $\frac{2c-1}{4c-1} < 1-c$ can be satisfied only if $c < \frac{3}{4}$.

By summarizing (iii-1) (iii-2) (iii-3) and considering the boundary condition $t \le 1-c$, we have the following results: the supply chain as a whole prefers bank financing when either $c < \frac{1}{2}$ and $t < \min\left\{\frac{c+1-\sqrt{c^2+1}}{2}, \frac{c-2\lambda+2-\sqrt{c(c-4\lambda+4)}}{2(1-\lambda)}, 1-2c\right\}$ or $c > \frac{1}{2}$ and $t < \min\left\{\frac{2c-1}{4c-1}, 1-c\right\}$ and prefers guarantor financing when $\widetilde{t}'(c) < t < \widehat{t}'(c)$; otherwise, it prefers direct financing. Specifically,

$$\widetilde{t}'(c) = \begin{cases} \frac{c+1-\sqrt{c^2+1}}{2}, & \text{when } \lambda \le \frac{9}{10} \text{ and } c < \frac{5}{12}, \\ \frac{4c^2+c-1}{4c-1}, & \text{when } \lambda \le \frac{9}{10} \text{ and } c \ge \frac{5}{12}, \\ \frac{c+1-\sqrt{c^2+1}}{2}, & \text{when } \lambda > \frac{9}{10} \text{ and } c < \frac{-(\lambda-1)(\lambda-3)+(\lambda+1)\sqrt{(\lambda-1)(\lambda-9)}}{4\lambda}, \\ \frac{-c\lambda-2\lambda+2+\sqrt{c(4c+4\lambda-4c\lambda+c\lambda^2-4)}}{2(1-\lambda)}, & \text{when } \lambda > \frac{9}{10} \text{ and } \frac{-(\lambda-1)(\lambda-3)+(\lambda+1)\sqrt{(\lambda-1)(\lambda-9)}}{4\lambda} \le c \le \frac{3\lambda+\sqrt{9\lambda^2-8\lambda}}{8\lambda}, \\ \frac{4c^2+c-1}{4c-1}, & \text{othwerwise.} \end{cases} \tag{16}$$

and

$$\widehat{t}'(c) = \begin{cases} \frac{2(1+c)-\lambda(2+c)-\sqrt{c(c\lambda^2-4\lambda+4)}}{2(1-\lambda)}, & \text{when } c < \frac{1}{2(2-\lambda)}, \\ 1-c, & \text{othwerwise.} \end{cases} \tag{17}$$

Proof of Proposition 9: (i) When the farmer is non-creditworthy, bank financing is unavailable, and the farmer chooses the financing format provided by the intermediary platform. For the platform, it selects either guarantor financing or direct financing. By Proposition 8, guarantor financing is always dominated by direct financing. Then, it always provides direct financing.

(ii) When the farmer is creditworthy, bank financing is available to the farmer. For the intermediary platform, it selects either guarantor financing or direct financing, competing against bank financing, or chooses no financing and lets the farmer resort to bank financing. Specifically, if guarantor financing is provided, by Proposition 8, it always dominates bank financing from the farmer's perspective. Then, the farmer always chooses guarantor financing.

However, if direct financing is provided, the farmer will choose between bank financing and direct financing. By the proof of Lemma 1 and noting that $\overline{F}(q_b^*) = \frac{c}{1-t} = \widetilde{c}$, we have

$$\Pi_f^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}(1-t)] - cq_b^* = \mathsf{E}[\min\{\xi, q_b^*\}]\frac{c}{\overline{F}(q_b^*)} - cq_b^* = \frac{(q_b^*)^2 c}{2(1-q_b^*)} = \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}}$$

and

$$\Pi_i^{b*} = \mathsf{E}[\min\{\xi, q_b^*\}t] = \left(q_b^* - \frac{(q_b^*)^2}{2}\right)t = \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right)t = \left((1-\frac{c}{1-t}) - \frac{(1-\frac{c}{1-t})^2}{2}\right)t.$$

Under direct financing, by noting that $\overline{F}(q_d^*) = \widehat{c}\overline{F}(\widehat{c}q_d^*)$ and $q_d^* = \frac{1}{1+\widehat{c}}$, we have $\widehat{c} = \frac{1-q_d^*}{q_d^*}$, i.e., $1-t = \frac{c(1+r)q_d^*}{1-q_d^*}$. Since $\widehat{c} \le 1$ is required, we have $q_d^* \ge \frac{1}{2}$. Thus,

$$\begin{aligned}
\Pi_f^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r)q_d^*]^+ = \mathsf{E}[\min\{\xi, q_d^*\} - (1-q_d^*)]^+(1-t) \\
&= \left(q_d^* - \frac{1}{2}\right)(1-t) = \frac{1-\widehat{c}}{2(1+\widehat{c})}(1-t); \text{ and} \\
\Pi_i^{d*} &= \mathsf{E}[\min\{\xi, q_d^*\}] - cq_d^* - \mathsf{E}[\min\{\xi, q_d^*\}(1-t) - c(1+r)q_d^*]^+ \\
&= (1-c)q_d^* - \frac{(q_d^*)^2}{2} - (q_d^* - \frac{1}{2})(1-t) = \frac{t - 2c + 2(1-c)\widehat{c} - (1-t)\widehat{c}^2}{2(1+\widehat{c})^2}.
\end{aligned}$$

Then, for the farmer, he chooses direct financing if and only if $\frac{1-\widehat{c}}{2(1+\widehat{c})}(1-t) \ge \frac{(1-\widetilde{c})^2 c}{2\widetilde{c}}$, i.e., $\widehat{c} \le \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$.

From the intermediary platform's perspective, note that, guarantor financing is a special case of direct financing with $r = 0$ or $\widehat{c} = \widetilde{c}$. That is, under direct financing, the platform can always mimic guarantor financing by setting $r = 0$ or $\widehat{c} = \widetilde{c}$. Also note that the intermediary platform's overall "social payoff" first increases and then decreases with $\widehat{c}$. Then, to compare guarantor financing and direct financing, we need to compare $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$ and $\widetilde{c}$. More specifically, $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2} - \widetilde{c} = \widetilde{c}^2 \frac{1-\widetilde{c}}{1+(1-\widetilde{c})^2} > 0$. Then, for the intermediary platform, direct financing always dominates guarantor financing.

To proceed, we need to compare direct financing against no financing. The latter choice means the farmer needs to rely on bank financing to raise funds. Based on the parameter values of c and t, we have two cases:

(ii-1) When $t < \frac{1+2c-2\lambda}{2(1-\lambda)}$, we have the intermediary platform's overall "social payoff" increases with $\widehat{c}$. Then, the constraint to induce the farmer to select direct financing, i.e., $\widehat{c} \le \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$, will essentially affect its optimal decision and profitability. By comparing bank financing and direct financing, we have, at $\widehat{c} = \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$,

$$\begin{aligned}\Omega_i^{d*} - \Omega_i^{b*} &= \frac{t-2c+2(1-c)\widehat{c}+(1-t)\widehat{c}^2}{2(1+\widehat{c})^2} + \lambda\frac{1-\widehat{c}}{2(1+\widehat{c})}(1-t) - \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right)t - \lambda\frac{(1-\widetilde{c})^2 c}{2\widetilde{c}} \\ &= -\frac{c^3}{8(1-t)^4}\left(4t^2-4t+c\right).\end{aligned}$$

As a result, $\Omega_i^{d*} - \Omega_i^{b*} < 0$ when $t < \frac{1}{4}$ and $c > 4t(1-t)$; otherwise, $\Omega_i^{d*} - \Omega_i^{b*} \geq 0$.

(ii-2) When $t \geq \frac{1+2c-2\lambda}{2(1-\lambda)}$, we have the intermediary platform's profit increases with $\widehat{c}$ if $\widehat{c} < \frac{1}{t+\lambda(1-t)-c} - 1$ and decreases with $\widehat{c}$ if $\widehat{c} \geq \frac{1}{t+\lambda(1-t)-c} - 1$. We further note that, if $c \geq (t - \sqrt{t^2+2(1-\lambda)t-2(1-\lambda)})(1-t)$, $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2} - (\frac{1}{t+\lambda(1-t)-c} - 1) = -\frac{c(c+2t-2)}{2t^2+2(c-2)t+(c^2-2c+2)} - \frac{1}{t+\lambda(1-t)-c} + 1 \geq 0$, i.e., $\frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2} \geq \frac{1}{t+\lambda(1-t)-c} - 1$. As such, the constraint to induce the farmer to select direct financing does not affect the intermediary platform's choice of interest rate. By Proposition 8, in this case, the intermediary platform always benefits from providing direct financing, compared with bank financing. Otherwise, if $c < (t - \sqrt{t^2+2(1-\lambda)t-2(1-\lambda)})(1-t)$, the optimal interest rate will be at $\widehat{c} = \frac{1-(1-\widetilde{c})^2}{1+(1-\widetilde{c})^2}$. By comparing bank financing and direct financing, we have

$$\begin{aligned}\Omega_i^{d*} - \Omega_i^{b*} &= \frac{t-2c+2(1-c)\widehat{c}+(1-t)\widehat{c}^2}{2(1+\widehat{c})^2} + \lambda\frac{1-\widehat{c}}{2(1+\widehat{c})}(1-t) - \left((1-\widetilde{c}) - \frac{(1-\widetilde{c})^2}{2}\right)t - \lambda\frac{(1-\widetilde{c})^2 c}{2\widetilde{c}} \\ &= -\frac{c^3}{8(1-t)^4}\left(4t^2-4t+c\right).\end{aligned}$$

Then, $\Omega_i^{d*} - \Omega_i^{b*} < 0$ when $t < \frac{1}{4}$ and $c > 4t(1-t)$, which is impossible because $t \geq \frac{1+2c-2\lambda}{2(1-\lambda)}$. As such, the intermediary platform also benefits from providing direct financing in this case.

Then, to summarize, the intermediary platform provides no financing and induces the farmer to take bank financing when the unit commission fee $t < \frac{1}{4}$ and the production cost $c > 4t(1-t)$; otherwise, it provides direct financing.