Impact of Social Interactions on Duopoly Competition with Quality Considerations

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We study the impacts of social interactions on competing firms' quality differentiation, pricing decisions, and profit performance. Two forms of social interactions are identified and analyzed: 1) market expansion effect (MEE) – the total market expands as a result of both firms' sales; and 2) value enhancement effect (VEE) – a consumer gains additional utility of purchasing from one firm based on this firm's previous and/or current sales volume. We consider a two-stage duopoly competition framework, in which both firms select quality levels in the first stage simultaneously and engage in a two-period price competition in the second stage. In the main model, we assume that each firm sets a single price and commits to it across two selling periods. We find that both forms of social interactions tend to lower prices and intensify price competition for given quality levels. However, MEE weakens the product quality differentiation and is benign to both high-quality and low-quality firms. It also benefits consumers and improves social welfare. By contrast, VEE enlarges the quality differentiation and only benefits high-quality firm, but is particularly malignant to the low-quality firm. It further reduces the consumers' monetary surplus. Such impact is consistent regardless of whether the VEE interactions involve previous or current consumers. We further discuss several model extensions including dynamic pricing, combined social effects, and various cost structures, and verify that the aforementioned impacts of MEE and VEE are qualitatively robust to those extensions. Our results provide important managerial insights for firms in competitive markets and suggest that they need to not only be aware of the consumers' social interactions, but also, more importantly, distinguish the predominant form of the interactions so as to apply proper marketing strategies.

Key words: duopoly competition; social interactions; product differentiation; pricing

1. Introduction

The proliferation of social interactions among consumers nowadays has been unprecedented in both breadth and depth. Almost every product can be discussed and recommended by people via various kinds of social activities, especially through internet. Consumers' awareness and valuation concerning a product are thus constantly influenced by those social interactions. Due to their significant impacts on consumer behavior, social interactions should be taken into account by firms when making important managerial decisions such as product design and pricing, especially by those whose products heavily rely on the social activities.

While social interactions in general affect the market diffusion process in certain aspects, the exact underlying mechanism depends on the particular form of the interactions. In this paper, we focus on two major forms of social interactions that are commonly observed in practice and exhibit different ways of influencing the consumer behavior and market dynamics. First, the consumer-to-consumer interactions may be simply informative, bringing certain things to people's attention and expand the product awareness. For instance, newly released products are often introduced to potential consumers by the existing consumers through social contagions such as word-of-mouth communications (Libai et al. 2009). It is noteworthy that these interpersonal communications are not necessarily brand specific. As a result, the whole potential market of a certain kind of products could be enlarged with more informed consumers. We refer to this form of social interactions as the *market expansion effect*.

Second, interacting with other consumers may improve a consumer's utility towards a certain product. Various behavioral and psychological factors could contribute to the positive effect of such interactions. The well-documented network effect (Economides 1996), which enhances consumers' utility through the cascade of positive externalities when they consume social goods, serves as a good example. For another example, consumers of a specific product may derive additional utility from the purchase if many others have already purchased/used the same product. After all, the crowd's behavior imposes a considerable impact on consumers' preferences (Laja 2019). We remark that, depending on the context, consumers may gain the extra utility by interacting with those who are purchasing the product currently and/or have purchased previously[1]. This form of social interactions is referred to as the *value enhancement effect*.

An illustrating example is the so-called MOBAs (multiplayer online battle arena games) in the video game industry. The MOBAs belong to the genre of real-time strategy games, where two teams (multiple players each) battle against each other with every player controlling one character. The hottest MOBAs include Heroes Evolved, DOTA 2 and League of Legends, etc. Abundant social interactions are present in the community of game players. On the one hand, by the spread of the words on the gaming forums (NeoGAF, etc.), more people become aware of MOBAs and join in as potential players everyday. Since most MOBAs have similar settings, potential players can

[1] This phenomenon is prevalent in practice. When consumers shop online, they observe past sales, which is usually provided by the e-commerce platforms such as Taobao and eBay. When consumers visit a physical store/restaurant, the present crowd size is a direct indication of current sales. In both cases, the consumers' utility is enhanced by the popularity (despite possible time lag) of the product.

communicate with players of one MOBA but eventually choose to play another. Due to this word-of-mouth spillover (Peres and Van den Bulte 2014), the potential market of all MOBA publishers could be expanded by the informative social interactions, which exemplifies the market expansion effect. On the other hand, players of a MOBA can derive enhanced utility from interacting with peer players of the same game. Such value enhancement effect is generally achieved in two ways: Players find it more fun to play with more people in the real time; they can also gain a sense of belonging to a larger community, which includes the previous players, when exchanging game experiences with peers on the forum. No matter how the players derive the additional utility, the enhanced value originates mainly from playing the same MOBA game.

It is worth noting that which form of social interactions is prevailing depends on the specific characteristics of the industry. In a mature industry where market potential remains stable, the market expansion effect may be weak and the value enhancement effect is more prominent. By contrast, for an emerging industry where product awareness is limited, market growth could be significant due to the market expansion effect in the early stage. For example, when firms materialize an innovative idea into business (e.g., sharing platforms, cloud services, etc.), they first devote efforts to the awareness expansion among consumers and then take advantage of the grown market. Given the profound impacts of social interactions on market dynamic and consumer behavior, competing firms should exploit them in effective ways when making management and marketing decisions, such as product quality design and pricing. Motivated by the above discussions, our research aims to address the following questions: How should competing firms select their quality levels and prices in the market that is influenced by social interactions? How do different forms of social interactions affect the firms' quality differentiation, profits, and consumers' surplus?

To address the above questions, we build a two-stage duopoly competition model with the consideration of social interactions. In Stage 1, the two firms simultaneously choose product quality levels with an exogenous upper bound determined by the current technology capacity. In Stage 2, two firms make price decisions and sell products through two periods. We consider that each firm sets and commits to a single price throughout both periods in our main model, and examine the inter-temporal pricing scheme as a model extension in which firms can dynamically set prices in each period. Moreover, we consider two forms of social interactions: (1) *Market expansion effect* (*MEE*), which enlarges the total market size for both firms through informative interactions that expand product awareness; and (2) *value enhancement effect* (*VEE*), which exclusively improves the consumer utility towards one particular product via dedicated social interactions with consumers of the same product. We further categorize VEE into two types: VEE via social interactions with consumers in *current* period (VEE-C) and with consumers in *previous* period (VEE-P).

We investigate the above forms of social interactions separately to distill their individual impacts. In each scenario, we solve for the unique pure-strategy sub-game perfect equilibrium, in which one firm selects a high quality level (high-quality firm) and the other selects a lower quality level (low-quality firm). In a monopolistic setting, both MEE and VEE seem to favor the firm via different mechanisms. Yet, in a competitive environment, questions regarding whether they can truly benefit firms and how they affect firms' product quality differentiation and profits remain unclear. As such, the main purpose of our research is to provide a systematic investigation on this matter. Interestingly, our study reveals that although both MEE and VEE intensify the price competition, they have completely distinct impacts on the firms' quality differentiation and profits, as well as the consumer surplus and the overall social welfare. We elaborate these findings as follows.

Under the market expansion effect, the equilibrium quality differentiation level is smaller than the benchmark case without social interactions, and decreases in the strength of MEE. In this case, the low-quality firm would increase quality and better utilize the informative social interactions to generate higher sales, even though doing so may lead to smaller differentiation and more intense competition. Moreover, due to the demand spillover induced by MEE, both firms' profits are improved. Hence, MEE is benign to the duopoly despite the intensified competition. In addition, we find that MEE also benefits consumers and can improve the social welfare.

By contrast, under the value enhancement effect, quality differentiation is enlarged and increases in the strength of VEE, because the competition becomes so fierce and the low-quality firm prefers to further differentiate by choosing lower quality and focus on lower-end market, rather than increasing quality to boost the impact of VEE. Moreover, VEE reduces the low-quality firm's profit but improves the high-quality firm's profit, and the profit gap between the two firms is increasing as VEE becomes stronger, positing the Matthew effect of accumulated advantage. After all, a consumer's utility towards one product can be enhanced only by the interactions with consumers of the same product, and the high-quality firm can gain more advantages from VEE while low-quality firm loses competitive edge. Therefore, although these social interactions always benefits a firm in a monopoly setting, it is not necessarily true in a competitive environment. Lastly, we find that VEE reduces consumers' monetary surplus and may hurt the the monetary part of social welfare. The above results remain the same regardless of the type of VEE (VEE-C or VEE-P).

Finally, we also provide further discussions on several model extensions, including models with dynamic pricing scheme, combined effects of both MEE and VEE, and cost considerations, respectively. We not only show that the main results are qualitatively robust in different model extensions, but also gain additional insights regarding the incorporated model ingredient.

To conclude, we summarize our main contributions and insights. Although social interactions have been widely studied in the marketing literature, how different forms of interactions affect competing firms' quality differentiation and equilibrium profits remain unclear. Hence, we contribute by systematically examining and contrasting two major forms of social interactions, leading to a multi-faceted analysis of the impetus for the dynamic changes in consumers' purchasing behaviors and the market potential. Our results suggest that, when firms consider product quality and pricing strategies in a competitive environment, they should not only be aware of the consumers' social interactions, but also, more importantly, distinguish the predominant form of the interactions so as to apply proper marketing strategies.

The rest of the paper is organized as follows. We position our work in the related literature in Section 2. Section 3 describes the model setup and Section 4 discusses the results. Additional discussions on model extensions are provided in Section 5. We conclude by summarizing the managerial insights in Section 6. All proofs and additional results are relegated to the appendices.

2. Literature Review

This paper is mainly related to two streams of literature: Product differentiation and social interactions. First of all, product differentiation, especially in the contexts of quality design and pricing in competitive environments, has been studied extensively in the marketing and economics literature. In a duopoly setting, the seminal paper, Shaked and Sutton (1982), shows that product differentiation is adopted to soften price competition. This result can be generalized to include different cost structures (Lehmann-Grube 1997) or special functional properties of costs (Chambers et al. 2006). Moreover, many studies have been conducted on this topic in various settings with interesting features incorporated, such as multi-attribute product positioning (Vandenbosch and Weinberg 1995), market entry decisions (Donnenfeld and Weber 1992), imperfect information for consumers (Jing 2007), and product-line design with consumer anticipated regret (Zou et al. 2020). In the similar vein, our paper examines the strategic implications of product differentiation when firms compete in the market with different forms of social interactions. As such, we contribute by addressing the product differentiation issue in competitive settings from a new perspective that has been practically often observed, yet never thoroughly studied before.

Secondly, our paper also contributes to the broad stream of literature on social interactions (Godes et al. 2005, Hartmann et al. 2008), especially their impacts on new product diffusion and adoption (Iyengar et al. 2011, Kuksov and Liao 2019, Katona et al. 2011). Regarding the nature of social interactions, Van den Bulte and Lilien (2001) summarize four different underlying mechanisms of social influence proposed by the existing literature. Two of those mechanisms, i.e., information transfer and performance network effect, are considered to be most relevant to our motivating settings of MEE and VEE, respectively. Moreover, Hartmann et al. (2008) discuss social spillover and social multiplier engendered from social interactions, which underscore the aforementioned two mechanisms regarding how consumers' actions are influenced by the peers. These papers share a common approach: The impact of social interactions on consumer behavior is studied through multiple mechanisms and the predominant mechanism is context-dependent.

Similarly, we focus on two major forms of social interactions in this paper, which are exemplified by the word-of-mouth effect (e.g., Godes and Mayzlin 2004, Godes 2017) and the network effect (e.g., Katz and Shapiro 1985, 1986, Economides 1996). Relevant to our context, the impact of a specific form of social interactions on a firm's competitive strategy has been studied in the literature. For example, Xie and Sirbu (1995) investigate the impact of demand externality on competing firms' prices and profits under dynamic pricing. Chen and Xie (2007) study the strategic implication of cross market network effect and consumer loyalty on competing firms' profitability. Doganoglu (2003) studies dynamic price competition with consumption externality under horizontal differentiation. Zhang and Sarvary (2015) examine the horizontal differentiation between firms with user generated content, which displays a strong network effect. Kuksov and Liao (2019) study a firm's optimal choice of product variety in the presence of word-of-mouth through opinion leaders' product evaluations. We differ from the above papers by systematically examining and, more importantly, contrasting the impacts of different forms of social interactions on competing firms' vertical product differentiation and profit.

Moreover, there are studies that look at multiple forms of social interactions, but tackle different business problems with distinct settings. For example, when exploring financial implications of network externalities, Goldenberg et al. (2010) note the distinction and separate the network effect from the word-of-mouth effect in their study. Kamada and Ory (2018) study the use of free contract and referral rewards to encourage word-of-mouth about the existence of a network product. Particularly, Godes (2017) examines two types of word-of-mouth communication and their interactions with a monopoly firm's quality decision. Our work is in line with Godes (2017) in the rationale of categorizing the two forms of social interactions, but is also different in several important aspects. First, we focus on competing firms' quality differentiation decision which is not a concern in the monopoly setting of Godes (2017). Second, we view social interactions as a dynamic factor and formulate a multi-period model, whereas Godes (2017) does not. Third, the value enhancement effect in our paper not only can arise from the persuasive word-of-mouth defined in Godes (2017), but may also capture other ways of social interactions, e.g., the network externality, that affect consumers' utility through interacting with either current or previous consumers.

To sum up, our paper contributes to the aforementioned streams of literature by uncovering the profound impacts of different forms of social interactions on competing firms' product design, price decisions, and profit, as well as the welfare implications. Our results suggest that conventional wisdom on the strategic adoption of product quality differentiation should be applied with caution. Instead, firms facing competition should identify the prevailing form of social interactions, understand its impact, and adjust their product design and pricing decisions accordingly.

3. Model Preliminaries

In this section, we outline the model setup by describing the major players and game sequence in Section 3.1, and present the benchmark model as well as its results in Section 3.2. The notations used in this paper are summarized on the first page of the Online Appendix.

3.1. Model Setup

Firms. We consider two competing firms selling vertically differentiated products in a market through two periods and each firm offers a single product. The firms need to decide the quality levels of their own products. We denote the firm with higher quality level q_H as firm H and lower quality level q_L as firm L, respectively. Assume that $0 \leq q_L < q_H \leq \bar{q}$, where $\bar{q}$ represents the upper bound of the quality restricted by the overall technology level of the industry. Without loss of generality, we normalize $\bar{q}$ to 1.

After selecting the quality levels for their products, the two firms engage in a two-period price competition. Let p_{in} denote firm i's price for period n, where $n \in \{1, 2\}$ and $i \in \{H, L\}$. We consider two different pricing schemes, both of which are commonly seen in practice. (1) *Committed pricing*: Each firm sets a single price at the beginning of the first period, and commits to it throughout the two periods (i.e., $p_{i1} = p_{i2}$ for $i \in \{H, L\}$). (2) *Dynamic pricing*: Firms are allowed to set inter-temporal prices in the beginning of each period. We focus on committed pricing as our main model in Section 4, and then study dynamic pricing as a model extension in Section 5.1 to investigate how the endowed pricing flexibility affects the main results. In addition, we assume that both firms have zero cost in our main model for analytical brevity. We will relax this assumption as a model extension in Section 5.3 and numerically verify that all the main results hold qualitatively when both marginal and fixed costs are considered. As such, firm i's total profit in two periods is given by $\pi_i = p_{i1}d_{i1} + p_{i2}d_{i2}$, where d_{in} denote firm i's demand in period n, for $n \in \{1, 2\}$ and $i \in \{H, L\}$.

Consumers. In period n, each consumer in the market will purchase at most one unit of the product from the two firms. We focus on non-durable goods in our setting where consumers with purchase needs will buy immediately without strategic waiting. More discussions on the issue of strategic purchase timing are provided in Section 6. The utility of purchasing from firm i is denoted by $u_{in}(\theta, q_i, p_{in})$, where θ represents the consumer's willingness-to-pay for quality and is assumed to be uniformly distributed over $[0,1]$, q_i is the product quality of firm i, and p_{in} is the price in period n. Consumers' utility of outside option is normalized to zero. A consumer will purchase from firm i in period n if and only if $u_{in} > \max\{u_{jn}, 0\}$, where $n \in \{1,2\}$, $i, j \in \{H, L\}$ and $i \neq j$.

In our model, the market diffusion process and the consumer utility structure are endogenously affected by social interactions across the two periods. We consider two forms of social interactions among consumers that have different mechanisms in influencing the potential market size and consumer utility (cf. Van den Bulte and Lilien 2001): *Market expansion effect* (MEE) and *value enhancement effect* (VEE). More specifically, the *market expansion effect* refers to the phenomenon that the total potential market size could be enlarged due to the word-of-mouth effect and the cross-brand communications among consumers (Libai et al. 2009). That is, the total market size of the second period increases in the total sales from the first period. On the other hand, the *value enhancement effect* refers to the phenomenon that consumers who purchase the product can gain additional utility through social interactions with other consumers of the same product. In addition, we further categorize VEE into two types according to the source of the additional utility: (1) Value enhancement effect from *current* consumers (VEE-C) — additional utility is gained from the social interactions with consumers in the current period who are purchasing the same product; and (2) value enhancement effect from *previous* consumers (VEE-P) — additional utility is gained from the social interactions with consumers from the previous period who have purchased the same product. We remark that the network externality is one common interpretation of VEE, but our model admits broader interpretations that involve certain product-exclusive social interactions and takes the time-lag effect of the interactions into account. Hence, VEE could be viewed as a more general framework to capture the social interactions related impact on consumers' utility.

Next, we capture MEE and VEE in the model setup. First, MEE affects the total market size by expanding the product awareness. The total market size in period n is given by:

$$D_n = \tilde{D}_n + r(d_{H,n-1} + d_{L,n-1}), n \in \{1,2\}.$$

Here, $\tilde{D}_n$ denotes base market size in period n without MEE, $r > 0$ represents the strength of MEE, and d_{in} is the demand of firm i in period n for $i \in \{H, L\}$. Note that $d_{H0} = d_{L0} = 0$. Hence, in period

1, the total market only consists of the base $\tilde{D}_1$, i.e., $D_1 = \tilde{D}_1$. In period 2, the total market consists of the base $\tilde{D}_2$ and an additional group of consumers induced by the informative social interactions. Specifically, new consumers are made aware of the products by the peers who made purchase in period 1 and then become part of the potential market in period 2. It is noteworthy that both firms contribute to this awareness expansion process with their respective existing consumers. Moreover, for analytical brevity, we assume that $\tilde{D}_n = 1$ for $n \in \{1, 2\}$. Our results are qualitatively robust when we consider $\tilde{D}_n$ differs in each period (i.e., $\tilde{D}_1 \neq \tilde{D}_2$).

Second, VEE improves the consumers' utility of purchasing from a firm. The utility of purchasing product i in period n is given by

$$u_{in} = \theta q_i - p_{in} + t_c d_{in} + t_p d_{i,n-1}, \text{ for } i \in \{H, L\} \text{ and } n \in \{1, 2\}.$$

Here, the parameters $t_c \geq 0$ and $t_p \geq 0$ represent the strength of VEE from current consumers (VEE-C) and previous consumers (VEE-P), respectively. While MEE creates mutual benefits for both firms, VEE features an exclusive utility boost induced via social interactions. Indeed, the consumers' utility towards a product is positively affected only by the sales of that product (in the current period and/or the previous period).[2]

To recap, the two forms of social interactions influence the market dynamics from two interrelated dimensions: The market expansion effect enlarges the total pie for both firms, whereas the value enhancement effect dictates the division of the pie. We remark that the parameters r, t_c, and t_p are assumed to be positive in our model setup. They can be relaxed to be negative to capture the negative impacts of social interactions due to congestion. See Section 6 for more discussions.

Sequence of Events. There are two stages in our model, with detailed game sequences given below.

Stage 1 – Quality Decision. The two firms simultaneously decide their respective product quality levels q_H and q_L within the feasible range $[0, \bar{q}]$. We remark that our results in the main model are found to be invariant whether the firms make quality decisions simultaneously or sequentially.[3]

Stage 2 – Price Competition. Given the selected quality levels, the two firms engage in a two-period price competition. If committed pricing scheme is adopted, the two firms simultaneously set their respective prices across the two periods at the beginning of period 1, i.e., $p_{H1} = p_{H2} = p_H$ and

[2] A consumer who promotes a product to others might base on the overall utility derived from purchasing the product, rather than the product quality alone (see, e.g., Kuksov and Xie 2010).

[3] For the case of simultaneous quality decision, the firm choosing high quality is called firm H and the one choosing low quality is called firm L, and the equilibrium is unique up to a role swap. For the case of sequential quality decision, in the equilibrium, the first mover will choose to be firm H and decide q_H and the follower will be firm L and decide q_L. These two cases yield the same equilibrium outcome in the main model.

$p_{L1} = p_{L2} = p_L$. If dynamic pricing scheme is used, the two firms simultaneously set their own prices at the beginning of period n, i.e., p_{Hn} and p_{Ln}, for $n \in \{1,2\}$. After observing firms' quality and prices, consumers make purchase decisions to maximize their utilities in each period.

3.2. Benchmark Case

To reveal the impacts of social interactions on the duopoly competition, we first analyze the benchmark case, where no social interaction exists (i.e., $r = t_c = t_p = 0$). In this case, the market dynamics are removed as the two periods are independent and identical, and the two different pricing schemes lead to the same equilibrium outcome. Moreover, the benchmark case is identical to the model studied in Tirole (1988). We use superscript "b" to denote this benchmark case and solve it via backward induction. Given quality q_H and q_L, the two firms' prices of each period are:

$$p_H^b(q_H, q_L) = \frac{2q_H(q_H - q_L)}{4q_H - q_L} \quad \text{and} \quad p_L^b(q_H, q_L) = \frac{q_L(q_H - q_L)}{4q_H - q_L}.$$

Moreover, the two firms' demands and profits in period n, for $n \in \{1,2\}$, are given by:

$$(d_{Hn}^b(q_H, q_L), d_{Ln}^b(q_H, q_L)) = (\frac{2q_H}{4q_H - q_L}, \frac{q_H}{4q_H - q_L}) \quad \text{and}$$

$$(\pi_{Hn}^b(q_H, q_L), \pi_{Ln}^b(q_H, q_L)) = (\frac{4q_H^2(q_H - q_L)}{(4q_H - q_L)^2}, \frac{q_H q_L(q_H - q_L)}{(4q_H - q_L)^2}).$$

Lastly, we solve for the firms' quality decisions in Stage 1 and use "$\hat{}$" over a symbol to denote the final equilibrium outcome. In the final equilibrium, the firms' quality levels, prices, and total profits over the two selling periods are respectively given by:

$$(\hat{q}_H^b, \hat{q}_L^b) = (1, \frac{4}{7}), \quad (\hat{p}_H^b, \hat{p}_L^b) = (\frac{1}{4}, \frac{1}{14}), \quad \text{and} \quad (\hat{\pi}_H^b, \hat{\pi}_L^b) = (\frac{7}{24}, \frac{1}{24}).$$

4. Equilibrium Analysis

In this section, we analyze the main model under the committed pricing scheme, i.e., $p_{H1} = p_{H2} = p_H$ and $p_{L1} = p_{L2} = p_L$. To single out the individual impact of various forms of social interactions, we separately investigate the market expansion effect in Section 4.1 and the value enhancement effect in Section 4.2. In the latter case, we further scrutinize VEE from current consumers and VEE from previous consumers, respectively. For each scenario, we solve for the pure-strategy Sub-game Perfect Equilibrium (SPE) of the two-stage game via backward induction and examine the respective impacts of different forms of social interactions. Finally, in Section 4.3, we compare and contrast the results under scenarios MEE and VEE to provide managerial insights on product design and pricing decisions in the presence of social interactions in a competitive environment.

4.1. Market Expansion Effect

Suppose only the market expansion effect is present, i.e., $r > 0$ and $t_c = t_p = 0$. In this case, the potential market size is expanded in period 2 as more consumers are drawn to the two products, but the consumers' utilities retain the same structure. Here, social interactions are informative and contribute to awareness expansion, and thus display a spillover effect on the total market size. In what follows, we will first solve the firms' price decisions in Stage 2 and then the firms' quality decisions in Stage 1 to characterize the final equilibrium. The final equilibrium outcomes are written as functions of r, with superscript "M" referring to "*market* expansion effect". Moreover, we assume $0 < r \leq 1$ so that the market increment never exceeds the previous total sales. This assumption ensures that the market will not be fully covered with both firms having positive demands in each period (i.e., $d_{in} > 0$ and $d_{Hn} + d_{Ln} < 1$ for $i = H, L$ and $n = 1, 2$), which is not only commonly observed in practice, but also shown by Proposition 1 as the unique equilibrium outcome in our setting. We refer to conditions $t_c = t_p = 0$ and $0 < r \leq 1$ as the MEE case.

Pricing Decisions. We first analyze the price competition in the second stage for given quality levels q_H and q_L.

Firm H's and firm L's demands in period n are $d^M_{Hn} = D_n(1 - \bar{\theta}^M_H)$ and $d^M_{Ln} = D_n(\bar{\theta}^M_H - \bar{\theta}^M_L)$, respectively, for $n \in \{1, 2\}$, where $\bar{\theta}^M_H = (p_H - p_L)/(q_H - q_L)$ represents the consumer who is indifferent between purchasing high-quality and low-quality product, and $\bar{\theta}^M_L = p_L/q_L$ represents the consumer who is indifferent between purchasing low-quality product and nothing. Given q_H and q_L, firm i selects its price by maximizing its total profit, i.e., solving $\max_{p_i} p_i(d^M_{i1} + d^M_{i2})$.

Lemma 1 below establishes the existence and uniqueness of the sub-game equilibrium prices, denoted by $p^M_H(q_H, q_L, r)$ and $p^M_L(q_H, q_L, r)$, respectively. Moreover, let $d^M_i(q_H, q_L, r)$ and $\pi^M_i(q_H, q_L, r)$ be firm i's ($i \in \{H, L\}$) corresponding total demand and profit of the two periods. Then, Lemma 1 further discusses how these equilibrium outcomes are affected by the strength of MEE, r.

Lemma 1. *In the MEE case, given the firms' quality levels $0 \leq q_L < q_H \leq 1$, the firms' sub-game equilibrium prices are characterized by a unique pair $(p^M_H(q_H, q_L, r), p^M_L(q_H, q_L, r))$ with detailed expressions shown in Appendix A.1. Moreover, the following statements hold.*

(*a*) *$p^M_H(q_H, q_L, r)$ and $p^M_L(q_H, q_L, r)$ decrease in r.*

(*b*) *$d^M_H(q_H, q_L, r)$ and $d^M_L(q_H, q_L, r)$ increase in r.*

(*c*) *$\pi^M_H(q_H, q_L, r)$ and $\pi^M_L(q_H, q_L, r)$ increase in r.*

Lemma 1 reveals the critical role that MEE plays in the duopoly price competition for fixed quality levels. First, Lemma 1(a) shows that both firms decrease their prices when MEE gets stronger.

As r increases, each firm can benefit more from a larger market coverage in period 1. Indeed, since the total demand of the two firms in period 1 is given by $(1 - p_L/q_L)$, firm L tends to cut its price p_L to expand market coverage and amplify the effect of market expansion. Such a pricing strategy, however, is deemed as aggressive and thus intensifies price competition to some extent. As a response, firm H also cuts its price to maintain its own first-period demand. Second, Lemma 1(b) shows that each firm's total demand increases as MEE becomes stronger. On the one hand, each firm decreases price to expand the market coverage as r increases; on the other hand, stronger MEE leads to larger market size in the second period. Hence, both firms are able to capture more total demand as r increases. Third, by Lemma 1(c), although a stronger MEE intensifies the price competition and decreases both firms' profit margin, it eventually benefits both firms. In fact, as r increases, both firms suffer from profit loss in the first period due to the intensified competition, but will enjoy a larger profit increase in the second period with the expanded market size; and, as a result, firms' total profits increase in r. This indicates that MEE's positive impact, which is mainly manifested in the second period, outweighs the firms' sacrifice in the first period profits.

Quality Decisions. Given the firms' price decisions in Stage 2, we fold back to solve the firms' quality decisions in Stage 1. In this stage, firm i selects quality q_i to maximize its profit $\pi_i^M(q_H, q_L, r) = p_i^M(q_H, q_L, r) d_i^M(q_H, q_L, r)$, for $i \in \{H, L\}$. Proposition 1 shows the existence and uniqueness of the firms' equilibrium quality decisions ($\hat{q}_H^M(r)$ and $\hat{q}_L^M(r)$). By substituting the equilibrium quality levels into the corresponding sub-game equilibrium, we can derive the firms' final equilibrium prices ($\hat{p}_i^M(r)$), demands ($\hat{d}_i^M(r)$), and profits ($\hat{\pi}_i^M(r)$), for $i \in \{H, L\}$. Then, Proposition 1 further investigates how those final equilibrium outcomes are influenced by the strength of MEE, r.

Proposition 1. *In the MEE case, there exists a unique equilibrium, in which firm H's quality is $\hat{q}_H^M = 1$ and firm L's quality $0 < \hat{q}_L^M(r) < 1$ is given in Appendix A.1. In the equilibrium, the market is partially covered and each firm has positive demand in each period. Moreover, the following statements hold.*

(*a*) *$\hat{q}_L^M(r)$ increases in r, and $\hat{q}_L^M(r) > \hat{q}_L^M(0) = \hat{q}_L^b$.*

(*b*) *$\hat{p}_H^M(r)$ and $\hat{p}_L^M(r)$ decrease in r; $\hat{p}_H^M(r) < \hat{p}_H^b$ and $\hat{p}_L^M(r) < \hat{p}_L^b$.*

(*c*) *$\hat{d}_H^M(r)$ and $\hat{d}_L^M(r)$ increase in r.*

(*d*) *$\hat{\pi}_H^M(r)$ and $\hat{\pi}_L^M(r)$ increase in r. Moreover, $\hat{\pi}_H^M(r) - \hat{\pi}_H^b > \hat{\pi}_L^M(r) - \hat{\pi}_L^b > 0$ whereas $(\hat{\pi}_L^M(r) - \hat{\pi}_L^b)/\hat{\pi}_L^b > (\hat{\pi}_H^M(r) - \hat{\pi}_H^b)/\hat{\pi}_H^b > 0$.*

For any given q_L, firm H's profit always increases in q_H since a higher q_H not only improves consumers' utility but also enlarges quality differentiation between the firms and thereby reduces competition. Thus, firm H's optimal quality reaches the upper bound of the quality level (i.e.,

$\hat{q}_H^M = 1$) in the equilibrium, and the degree of quality differentiation between the two firms in this case is simply $1 - \hat{q}_L^M(r)$. One may intuit that firm L would reduce the quality and further differentiate itself in the presence of MEE since the price competition is intensified (see Lemma 1). However, Proposition 1(a) indicates the opposite: Firm L's quality actually increases as MEE becomes stronger and is always higher than that in the benchmark case without MEE (i.e., $\hat{q}_L^M(r) > \hat{q}_L^M(0) = \hat{q}_L^b$). Such a result can be explained in the following way. Although higher q_L implies lower quality differentiation and intensifies quality competition, it also attracts more low-end consumers and increases the total market coverage, which in turn expands the market size through MEE. As the benefit of expanded market size outweighs the drawback of intensified quality competition, firm L increases its quality as MEE becomes stronger. It is noteworthy that the quality change brought by MEE is continuous in r. Hence, $\hat{q}_L^M(r) > \hat{q}_L^M(r=0) = \hat{q}_L^b$ holds for every $r \in (0,1]$.

Proposition 1(b) further shows that both firms' equilibrium prices decrease in the strength of MEE and are lower than the benchmark prices. The reason for this result is twofold. First, as shown in Lemma 1, for given quality levels, higher r induces firm L to reduce price to expand the market coverage, which intensifies price competition and forces firm H to follow suit. Second, as mentioned above, the equilibrium quality differentiation level decreases in r, resulting in fiercer competition and lower prices from both firms. As such, the fact that firms' prices decrease in r is driven by both the intensified price competition and the reduced quality differentiation.

As MEE becomes stronger, firm L increases quality and both firms decrease prices to expand the market coverage in order to fully utilize MEE, which leads to a higher total demand for each firm (i.e., Proposition 1(c)). Hence, although the competition is intensified, both firms can benefit from stronger MEE due to the enlarged total demand (i.e., Proposition 1(d)). It is noteworthy that, when comparing the profits to the benchmark case, firm H's *absolute* increase is higher than firm L's whereas firm L enjoys a higher *relative* (percentage) increase.

Welfare Implications. Finally, we conclude this section by studying how MEE affects consumers' total surplus and the social welfare. Let $\hat{D}_2^M(r) = 1 + r(\hat{d}_{H1}^M(r) + \hat{d}_{L1}^M(r))$ be the equilibrium total market size of period 2. We define the equilibrium *consumer surplus* of period 1 and 2 with MEE as $CS_1^M(r) = \int_{\bar{\theta}_H^M}^1 (\theta - \hat{p}_H^M(r))d\theta + \int_{\bar{\theta}_L^M}^{\bar{\theta}_H^M} (\theta \hat{q}_L^M(r) - \hat{p}_L^M(r))d\theta$ and $CS_2^M(r) = \hat{D}_2^M(r) CS_1^M(r)$, respectively, where θ_H^M and θ_L^M are previously defined in the pricing decision stage. Then, $CS^M(r) = CS_1^M(r) + CS_2^M(r)$ is the total consumer surplus in two periods and $SW^M(r) = CS^M(r) + \hat{\pi}_H^M(r) + \hat{\pi}_L^M(r)$ is the social welfare in the MEE case. The effect of MEE is characterized in Corollary 1 below.[4]

[4] Note that we focus on total consumer surplus of the MEE case. Indeed, even examining consumer average surplus by dividing the market size (i.e., $CS_n^M(r)/\hat{D}_n^M(r), n \in \{1, 2\}$), we can still show all the results in Corollary 1 hold.

COROLLARY 1. *The following statements hold in the MEE case.*

(a) $CS_1^M(r)$, $CS_2^M(r)$, *and* $CS^M(r)$ *all increase in* r*; and* $CS^M(r) > CS^b$*;*

(b) $SW^M(r)$ *increases in* r*; and* $SW^M(r) > SW^b$*.*

As shown in Corollary 1, consumers always benefit from stronger MEE. In fact, the increased quality of firm L, the reduced prices, and the enlarged market size at period 2 altogether contribute to the positive impact of MEE. As such, it is interesting to notice that stronger MEE would benefit both firms and their consumers, a desirable *win-win* outcome that improves the social welfare.

4.2. Value Enhancement Effect

In this section, we study the value enhancement effect, which is categorized into two types. VEE-C refers to the interactions with peers who purchase the same product in the current period, and VEE-P captures the interactions with those who purchased the same product in the previous period. These two types of VEE share similar nature but differ in the source of social interactions. They may or may not co-exist for different products in practice. To distill the individual impact of each one, we analyze VEE-C and VEE-P separately. As shown in the sequel, the central results for the VEE-C case and the VEE-P case are qualitatively similar. Hence, we devote Section 4.2.1 to a detailed investigation on VEE-C, which parallels with Section 4.1. Then we use Section 4.2.2 to briefly report the relevant findings for VEE-P and, more importantly, to discuss the similarities and distinctions between the systematic impacts of the two types of VEE.

4.2.1. Value Enhancement Effect from Current C onsumers. Consider the scenario VEE-C, in which $r = t_p = 0$ and $t_c > 0$. The total market size is 1 in each period, but consumers' utility towards a firm's product positively depends on that firm's demand in the current period. We use superscript "VC" to represent the case of *value* enhancement effect from *current* consumers. Since the first period has no impact on the second period under VEE-C, the two periods are independent, and each firm's demand and profit in the two periods are identical (i.e., $d_{i1}^{VC} = d_{i2}^{VC}$ and $\pi_{i1}^{VC} = \pi_{i2}^{VC}$ for $i \in \{H, L\}$). Similar to the analysis in the MEE case, we first solve the price competition in Stage 2, and then characterize firms' quality decisions in Stage 1 to obtain the final equilibrium outcomes, which are written as functions of the strength of VEE-C, t_c. Furthermore, we simplify the analysis by assuming that t_c is a small fraction, i.e. $t_c \leq \bar{t}_c$, where $\bar{t}_c$ is given in Appendix A.2. This assumption guarantees that the market is not fully covered and each firm has positive demand in the final equilibrium, which is commonly observed and consistent with many practical situations. In the following, we refer to conditions $r = t_p = 0$ and $0 < t_c \leq \bar{t}_c$ as the VEE-C case.

Pricing Decisions. We analyze the duopoly price competition in the second stage for given quality levels. Note that, unlike the MEE case, the market may be fully covered and one of the firms may have zero demand in the sub-game equilibrium for some quality levels, because consumers' utility is changed in the VEE-C case. However, as shown later in Proposition 2, in the final equilibrium, the market is partially covered and each firm has positive demand. This indicates that any quality levels that lead to a fully covered market or zero demand of either firm cannot be a final equilibrium. Hence, for expositional brevity, we exclude the off-equilibrium cases in the following sub-game discussion by assuming partial market coverage and positive demand for each firm.

Given firms' quality levels q_H and q_L, it is straightforward to see that firms' demands in period n are $d_{Hn}^{VC} = 1 - \bar{\theta}_H^{VC}$ and $d_{Ln}^{VC} = \bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}$, for $n \in \{1,2\}$, where $\bar{\theta}_H^{VC} = [q_L(p_H - p_L) - (q_L + p_H - t_c)t_c]/[q_L(q_H - q_L) - (q_L + q_H - t_c)t_c]$ represents the consumer who is indifferent between purchasing high-quality and low-quality product, and $\bar{\theta}_L^{VC} = [p_L(q_H - q_L) - (p_L + p_H - t_c)t_c]/[q_L(q_H - q_L) - (q_L + q_H - t_c)t_c]$ captures the consumer who is indifferent between purchasing low-quality product and nothing. Hence, firm i's problem is to maximize its total profit, i.e., $\max_{p_i} p_i(d_{i1}^{VC} + d_{i2}^{VC})$, $i \in \{H, L\}$. In Lemma 2 below, we characterize the firms' sub-game equilibrium prices $p_H^{VC}(q_H, q_L, t_c)$ and $p_L^{VC}(q_H, q_L, t_c)$ in Stage 2, and then discuss the impact of t_c on firm i's price, total demand ($d_i^{VC}(q_H, q_L, t_c)$), and total profit ($\pi_i^{VC}(q_H, q_L, t_c)$), for $i \in \{H, L\}$.

Lemma 2. *In the VEE-C case, given the quality levels q_H and q_L, the firms' sub-game equilibrium prices are characterized by a unique pair $(p_H^{VC}(q_H, q_L, t_c), p_H^{VC}(q_H, q_L, t_c))$, with the detailed expressions shown in Appendix A.2. Moreover, the following statements hold.*

(*a*) *$p_L^{VC}(q_H, q_L, t_c)$ decreases in t_c, whereas $p_H^{VC}(q_H, q_L, t_c)$ may increase or decrease in t_c;*
(*b*) *$d_H^{VC}(q_H, q_L, t_c)$ increases in t_c, whereas $d_L^{VC}(q_H, q_L, t_c)$ may increase or decrease in t_c;*
(*c*) *Both $\pi_H^{VC}(q_H, q_L, t_c)$ and $\pi_L^{VC}(q_H, q_L, t_c)$ may increase or decrease in t_c.*

There are several useful takeaways from Lemma 2 that can help us understand the important role VEE-C plays in the price competition with fixed quality levels. With VEE-C, consumers' utility improves as the product sales increase. That is, product i gains additional value via social interactions, $t_c d_{in}$, in period n, for $i \in \{H, L\}$ and $n \in \{1,2\}$. However, since firm H has quality advantage and can captures more demand than firm L, the high-quality product can gain higher social value than the low-quality one. Consequently, despite the additional social value received by the low-quality product, firm L ends up falling to a more disadvantageous position in the price competition, as VEE-C makes firm H even more competitive. In this sense, VEE-C actually favors the high-quality product and lets firm H gain more competitive edge. Therefore, by Lemma 2(a), regardless of the given quality levels, firm L has to reduce its price to compete for consumers as VEE-C gets stronger. Firm H, by contrast, may either raise or cut its price depending on the quality levels. Moreover, we can further show that firm H's price decreases in t_c when $q_H = 1$, implying that a stronger VEE-C can result in a more competitive environment.

The same rationale explains the properties of firms' demands in Lemma 2(b). As firm H can obtain more added social value via VEE-C, it has more advantage to gain higher demand. However, although the low-quality product's additional social value increases as VEE-C becomes stronger, firm L indeed becomes less competitive. Hence, its demand may be higher or lower as t_c increases, depending on the quality levels. Finally, Lemma 2(c) shows that both firms' profits may increase or decrease in t_c for given quality levels. Even though VEE-C renders an exclusive positive effect by improving consumers' utilities for both products, the two firms may or may not benefit from a stronger VEE-C, as it may intensify the price competition and reduce both firms' profit margins.

Quality Decisions. Given the firms' sub-game equilibrium price decisions, we proceed to solve for firms' quality decisions in Stage 1. In this stage, firm i chooses quality q_i to maximize its profit $\pi_i^{VC}(q_H, q_L, t_c) = p_i^{VC}(q_H, q_L, t_c) d_i^{VC}(q_H, q_L, t_c)$, for $i \in \{H, L\}$. Proposition 2 first establishes the existence and uniqueness of the final equilibrium, and then examines how the strength of VEE-C (t_c) influences the final equilibrium outcomes, including firm i's equilibrium quality ($\hat{q}_i^{VC}(t_c)$), price ($\hat{p}_i^{VC}(t_c)$), demand ($\hat{d}_i^{VC}(t_c)$), and profit ($\hat{\pi}_i^{VC}(t_c)$), for $i \in \{H, L\}$.

Proposition 2. *In the VEE-C case, there exists a unique equilibrium, in which firm H's quality is $\hat{q}_H^{VC} = 1$ and firm L's quality $0 < \hat{q}_L^{VC}(t_c) < 1$ is given in Appendix A.2. In the equilibrium, the market is partially covered and each firm has positive demand in each period. Moreover, the following statements hold.*

(a) *$\hat{q}_L^{VC}(t_c)$ decreases in t_c, and $\hat{q}_L^{VC}(t_c) < \hat{q}_L^{VC}(0) = \hat{q}_L^b$.*
(b) *$\hat{p}_H^{VC}(t_c)$ increases in t_c and $\hat{p}_L^{VC}(t_c)$ decreases in t_c. In addition, $\hat{p}_H^{VC}(t_c) > \hat{p}_H^b$ and $\hat{p}_L^{VC}(t_c) < \hat{p}_L^b$.*
(c) *$\hat{d}_H^{VC}(t_c)$ increases in t_c, and $\hat{d}_L^{VC}(t_c)$ may decrease or increase in t_c.*
(d) *$\hat{\pi}_H^{VC}(t_c)$ increases in t_c and $\hat{\pi}_L^{VC}(t_c)$ decreases in t_c; moreover, $\hat{\pi}_H^{VC}(t_c) - \hat{\pi}_H^b > 0 > \hat{\pi}_L^{VC}(t_c) - \hat{\pi}_L^b$.*

As firm H's profit increases in q_H for any given q_L, it is optimal to set q_H as the upper bound (i.e., $\hat{q}_H^{VC} = 1$). Hence, $1 - \hat{q}_L^{VC}(t_c)$ represents the quality differentiation between the two firms. Different from the MEE case, Proposition 2(a) shows that firm L's equilibrium quality level decreases as VEE-C becomes stronger and is lower than the benchmark case. As mentioned, the presence of VEE-C provides uneven additional social values to the products and tends to intensify the price war, leaving firm L in an undesirable situation as it loses competitiveness to firm H. To combat, firm L has to reduce the product quality to further differentiate from firm H. In other words, with VEE-C, firm L simply cannot effectively exploit consumers' enhanced utility; instead, it has to reduce its quality to target on the lower-end consumers. Similar to the MEE case, firm L's equilibrium quality without VEE-C coincides with the benchmark, i.e., $\hat{q}^{VC}(t_c = 0) = \hat{q}^b_L$. Thus, $\hat{q}^{VC}_L(t_c) < \hat{q}^b_L$ holds.

Compared to the sub-game results shown in Lemma 2, the impact of VEE-C on the final equilibrium outcomes exhibit some differences. By Lemma 2(a), for given quality levels, firm H's price may increase or decrease in t_c. In particular, p^{VC}_H decreases in t_c for $q_H = 1$ as mentioned previously. However, Proposition 2(b) shows that firm H's final equilibrium price always increases in t_c. This seemingly contradictory finding can be understood as follows. On the one hand, for given $q_H = 1$ and $q_L < 1$, stronger VEE-C decreases firm H's price; on the other hand, stronger VEE-C decreases firm L's quality, which incentivizes firm H to increase its price. The latter impact turns out to outweigh the former, and firm H's equilibrium price $\hat{p}^{VC}_H(t_c)$ increases in t_c as a result.

Proposition 2(c) characterizes interesting properties of firms' equilibrium demands. Due to the strengthened advantageous position, firm H's equilibrium demand $\hat{d}^{VC}_H(t_c)$ is larger than that in the benchmark model, and always increases as VEE-C becomes stronger; but firm L's equilibrium demand $\hat{d}^{VC}_L(t_c)$ could be lower than the benchmark case. It is noteworthy that, although the market is still partially covered, the total market coverage from the two firms under VEE-C is larger than that in benchmark case, i.e. $\hat{d}^{VC}_H(t_c) + \hat{d}^{VC}_L(t_c) > \hat{d}^b_H + \hat{d}^b_L$. Hence, firm H's demand increase always dominates firm L's demand loss, if there is any.

Finally, in contrast to the previous result that MEE induces mutual benefits, Proposition 2(d) shows that firms cannot simultaneously benefit from a stronger VEE-C. In fact, as VEE-C becomes stronger, firm H always enjoys an increased profit that is higher than the benchmark case, whereas firm L's profit decreases and is lower than the benchmark case. Such Matthew effect of accumulated advantage indicates that VEE-C is competitive in nature and is particularly detrimental to the low-quality firm. In other words, being a high-quality firm is a persistent advantage when consumers actively engage in value-enhancing social interactions with their peers in the current period.

Welfare Implications. Finally, we study how VEE-C affects both the consumers and the social welfare. We first define consumers' total surplus in period n, including the additional utility caused by VEE-C, as $CS^{VC}_n(t_c) = \int_{\bar{\theta}^{VC}_H}^{1} (\theta - \hat{p}^{VC}_H(t_c) + t_c \hat{d}^{VC}_{Hn}(t_c)) d\theta + \int_{\bar{\theta}^{VC}_L}^{\bar{\theta}^{VC}_H} (\theta \hat{q}^{VC}_L(t_c) - \hat{p}^{VC}_L(t_c) + t_c \hat{d}^{VC}_{Ln}(t_c)) d\theta$, where $\bar{\theta}^{VC}_H$ and $\bar{\theta}^{VC}_L$ are given previously in the pricing decision stage, for $n = 1, 2$. Then, we exclude the additional social utility and define consumers' monetary surplus in each period as $CM^{VC}_1(t_c) = CM^{VC}_2(t_c) = \int_{\bar{\theta}^{VC}_H}^{1} (\theta - \hat{p}^{VC}_H(t_c)) d\theta + \int_{\bar{\theta}^{VC}_L}^{\bar{\theta}^{VC}_H} (\theta \hat{q}^{VC}_L(t_c) - \hat{p}^{VC}_L(t_c)) d\theta$. Let $CM^{VC}(t_c) = CM^{VC}_1(t_c) + CM^{VC}_2(t_c)$ and $CS^{VC}(t_c) = CS^{VC}_1(t_c) + CS^{VC}_2(t_c)$ be the consumer monetary surplus and total surplus of two periods, respectively.

We further define $SW^{VC}(t_c) = \hat{\pi}_H^{VC}(t_c) + \hat{\pi}_L^{VC}(t_c) + CS^{VC}(t_c)$ as the social welfare under VEE-C. Note that we can divide the social welfare into two parts:

$$SW^{VC}(t_c) = \underbrace{\hat{\pi}_H^{VC}(t_c) + \hat{\pi}_L^{VC}(t_c) + CM^{VC}(t_c)}_{\text{Monetary Term}} + \underbrace{(CS^{VC}(t_c) - CM^{VC}(t_c))}_{\text{Social Utility Term}}.$$

Let $SM^{VC}(t_c) = \hat{\pi}_H^{VC}(t_c) + \hat{\pi}_L^{VC}(t_c) + CM^{VC}(t_c)$ represent the monetary term of social welfare that excludes consumers' additional social utility. Corollary 2 summarizes the social impacts of VEE-C.

Corollary 2. *The following statements hold in the VEE-C case.*

(*a*) *$CM^{VC}(t_c)$ decreases in t_c, and $CM^{VC}(t_c) < CM^{VC}(0) = CM^b$;*

(*b*) *$CS^{VC}(t_c)$ first decreases and then increases in t_c;*

(*c*) *$SM^{VC}(t_c)$ decreases in t_c, and $SM^{VC}(t_c) < SM^b$;*

(*d*) *$SW^{VC}(t_c)$ increases in t_c, and $SW^{VC}(t_c) > SW^b$.*

Under VEE-C, consumers' total surplus (i.e., $CS^{VC}(t_c)$) includes the monetary surplus and the additional social utility. We find that although consumers' additional social utility increases as VEE-C becomes stronger, their monetary surplus always decreases and their total surplus could be lower compared to the benchmark without VEE-C. The lowered quality from firm L and increased price from firm H are the main drivers of this result, outweighing the fact that firm L's price becomes lower. Similarly, social welfare contains the monetary part and consumers' social utility part under VEE-C. As VEE-C becomes stronger, the reduction in both firm L's profit and consumer monetary surplus reduces the monetary term of social welfare, despite the increased profit of firm H, as shown in Corollary 2(c). However, the overall social welfare would increase in t_c since consumers' additional social utility becomes higher as t_c increases (see Corollary 2(d)). The social impacts of VEE-C are in sharp contrast to the MEE case, which induces a win-win outcome for both firms and consumers (see Corollary 1). As such, the distinct nature of social interactions indirectly affects consumers via its impact on competing firms' product design and pricing decisions.

4.2.2. Value Enhancement Effect from Previous Consumers. Now, we turn to study the scenario when only VEE-P is present, i.e., $r = t_c = 0$ and $t_p > 0$. Similar to VEE-C, VEE-P can also improve consumers' utility exclusively for the chosen product; however, in the VEE-P case, there is a time lag between demand realization and the value-enhancing interactions, as the firm's first-period demand affects consumers' utility in the second period. We take the same analytical approach to solve the game. Particularly, we assume that t_p is a small fraction, i.e. $t_p \leq \bar{t}_p$, where $\bar{t}_p$ is given in Appendix A.3. Hereafter, the VEE-P case means $r = t_c = 0$ and $0 < t_p \leq \bar{t}_p$, and we write the equilibrium outcomes with superscript "*VP*" referring to "*value* enhancement effect from *previous* consumers". We have mentioned previously that the main results in scenario VEE-P are qualitatively similar as in VEE-C. Therefore, in what follows, we will briefly outline the study on the VEE-P case without unnecessary reiteration. After presenting each main result, we will discuss the similarities and, more importantly, highlight the distinctions between the two types of VEE.

Pricing Decisions. In Stage 2, firms choose prices for given q_H and q_L to maximize their profits. Again, we exclude the off-equilibrium cases in the following discussion for expositional brevity by assuming partial market coverage and positive demand for each firm. The sub-game equilibrium outcome is examined by the following lemma, which is in parallel with Lemma 2 in Section 4.2.1.

Lemma 3. *In the VEE-P case, given the quality levels q_H and q_L, the firms' sub-game equilibrium prices are characterized by a unique pair $(p_H^{VP}(q_H, q_L, t_p), p_H^{VP}(q_H, q_L, t_p))$, with the detailed expressions shown in Appendix A.3. Moreover, the following statements hold.*

(*a*) *Both $p_H^{VP}(q_H, q_L, t_p)$ and $p_L^{VP}(q_H, q_L, t_p)$ decrease in t_p;*

(*b*) *$d_H^{VP}(q_H, q_L, t_p)$ increases in t_p, whereas $d_L^{VP}(q_H, q_L, t_p)$ may increase or decrease in t_p;*

(*c*) *Both $\pi_H^{VP}(q_H, q_L, t_p)$ and $\pi_L^{VP}(q_H, q_L, t_p)$ may increase or decrease in t_p.*

Consistent with the VEE-C case, the presence of VEE-P improves consumers' utilities by providing additional social value to the product, and the high-quality product gains higher social value and enjoys a more advantageous position whereas firm L loses its competitive edge. Thus, the impact of VEE-P is almost identical to that of VEE-C in this regard. However, comparing Lemma 3 and Lemma 2, one distinction stands out of the overall similarities. Specifically, Lemma 3(a) states that $p_H^{VP}(q_H, q_L, t_p)$ decreases in t_p in the VEE-P case, whereas Lemma 2(a) says that $p_H^{VC}(q_H, q_L, t_c)$ may increase or decrease in t_c in the VEE-C case. In other words, for given quality levels, different types of VEE have different impacts on firm H's price. Since the value enhancement is exclusive to the chosen product, both types of VEE are favorable to firm H. As a result, firm H may actually increase the price as VEE-C gets stronger. However, firm H is shown to always cut price as VEE-P becomes stronger. This result reveals an important distinction between VEE-C and VEE-P: The time lag for the value enhancement to take effect in the VEE-P case weakens firm H's advantage. Indeed, if additional social value is from interacting with the previous consumers, then firm H can only enjoy the benefit in period 2, and it has to drop price for given quality levels in order to capture sufficient first-period sales to effectively exploit value enhancement in period 2.

Quality Decisions. Given the firms' price decisions in Stage 2, we now solve the first stage of the game. The existence and uniqueness of the equilibrium quality decisions are established by

Proposition 3, which is a counterpart of Proposition 2 in Section 4.2.1. Additionally, the impact of the strength of VEE-P, i.e., t_p, on the final equilibrium outcomes is also studied by Proposition 3.

Proposition 3. *In the VEE-P case, there exists a unique equilibrium, in which firm H's quality is $\hat{q}_H^{VP} = 1$ and firm L's quality $0 < \hat{q}_L^{VC}(t_p) < 1$ is given in Appendix A.3. In the equilibrium, the market is partially covered and each firm has positive demand in each period. Moreover, the following statements hold.*

(a) $\hat{q}_L^{VP}(t_p)$ *decreases in* t_p*, and* $\hat{q}_L^{VP}(t_p) < \hat{q}_L^{VP}(0) = \hat{q}_L^b$.

(b) $\hat{p}_H^{VP}(t_p)$ *increases in* t_p *and* $\hat{p}_L^{VP}(t_p)$ *decreases in* t_p*. In addition,* $\hat{p}_H^{VP}(t_p) > \hat{p}_H^b$ *and* $\hat{p}_L^{VP}(t_p) < \hat{p}_L^b$.

(c) $\hat{d}_H^{VP}(t_p)$ *increases in* t_p*, and* $\hat{d}_L^{VP}(t_p)$ *may decrease or increase in* t_p.

(d) $\hat{\pi}_H^{VP}(t_p)$ *increases in* t_p *and* $\hat{\pi}_L^{VP}(t_p)$ *decreases in* t_p*; and* $\hat{\pi}_H^{VP}(t_p) - \hat{\pi}_H^b > 0 > \hat{\pi}_L^{VP}(t_p) - \hat{\pi}_L^b$.

Comparing the parallel results listed in Propositions 2 and 3, it is immediate that both VEE-C and VEE-P exhibit qualitatively identical impacts on the firms' equilibrium decisions and outcomes. This is because both effects unilaterally improve consumers' utility towards a chosen product, with the only difference that the additional utility gain is originated from interacting with current consumers in VEE-C and previous consumers in VEE-P. Hence, value-enhancing social interactions, regardless of its sources, always reduces firm L's quality and enlarges the degree of quality differentiation. Besides, we again have $\hat{q}_L^{VP}(t_p) < \hat{q}^{VP}(t_p = 0) = \hat{q}_L^b$. Furthermore, both types of VEE tend to increase firm H's profit and reduce firm L's profit in the final equilibrium. Hence, the Matthew effect of accumulated advantage emerges, benefiting firm H at the expense of hurting firm L.

The above discussions indicate that the impacts of the value enhancement effect on competing firms' quality and price decisions and profits are consistent and robust, regardless of whether there is a time lag between the demand realization and social interactions. Given the consistent impacts, we conjecture that the individual impacts of VEE-C and VEE-P remain unchanged when both effects are present. Indeed, Section 5.2 provides a comprehensive numerical study to check and confirm our prediction with more detailed discussions.

Welfare Implications. Finally, we check the properties of consumer monetary surplus (i.e, $CM^{VP}(t_p)$), consumer total surplus (i.e, $CS^{VP}(t_p)$), social welfare (i.e., $SW^{VP}(t_p)$), and the monetary term of social welfare (i.e., $SM^{VP}(t_p)$) in the VEE-P case, which are defined in a similar way as their counterparts under VEE-C. The effects of t_p are captured by Corollary 3.

Corollary 3. *The following statements hold in the VEE-P case.*

(a) $CM^{VP}(t_p)$ *decreases in* t_p*; and* $CM^{VP}(t_p) < CM^{VP}(0) = CM^b$*;*

(b) $CS^{VP}(t_p)$ *first decreases and then increases in* t_p*;*

(c) $SM^{VP}(t_p)$ *may decrease or increase in* t_p*;*

(d) $SW^{VP}(t_p)$ increases in t_p, and $SW^{VP}(t_p) > SW^b$.

Comparing Corollary 3 with Corollary 2, there are three interesting observations. First, both VEE-C and VEE-P hurt consumers' monetary surplus and could reduce their overall surplus, because firm L lowers the quality and firm H raises the price (even though firm L cuts its price). Second, both VEE-C and VEE-P lead to higher social welfare due to the increased consumers' additional social utility and firm H's profit. Finally, stronger VEE-C always reduces the monetary term of social welfare, but stronger VEE-P may either increase or decrease it. This difference is due to the one-period lag for the value-enhancing interactions to take effect in the VEE-P case. Therefore, while both types of VEE hurt consumers' monetary surplus, the monetary term of social welfare is possible to increase only under VEE-P.

4.3. Comparisons between MEE and VEE

In this section, we compare and contrast the main results obtained in Sections 4.1 and 4.2 to shed light on the similarities and differences of the two fundamental forms of social interactions. Although both MEE and VEE introduce positive impacts to the market, we emphasize the importance of identifying the prevailing form of social interactions. Our results can provide managerial insights into how competing firms should leverage the social interactions to guide their product design and pricing strategies. Among others, we highlight the following major findings.

First, a stronger MEE strengthens the level of product quality differentiation (i.e., larger $q_H - q_L$), while a stronger VEE weakens it (i.e., smaller $q_H - q_L$). Note that firm H always selects quality at the upper bound under both cases and only firm L's quality is influenced by MEE or VEE. Both effects are more powerful with higher demands: MEE can generate a larger market size if total demand of previous period is higher, whereas VEE can lead to higher consumer utility if demand of current/previous period is larger. Given that a higher product quality can generate larger demand, one may intuit that firm L will select a higher quality level to exploit social interactions. However, based on our findings, this intuition is valid only in the MEE case. Indeed, firm L's equilibrium quality in the MEE case is higher than the benchmark value $4/7$ and increases in the strength of MEE (i.e., r). This shows that selecting a higher quality level to utilize MEE is more beneficial to firm L than further differentiating itself to soften the competition. However, the opposite is true under VEE: Firm L's quality is always lower than $4/7$ and decreases in the strength of VEE (i.e., either t_c or t_p). Different from MEE, VEE favors firm H at the expense of hurting firm L, since it provides higher social value to high-quality product than low-quality one. To alleviate such a disadvantage, firm L chooses to reduce its quality to further differentiate from firm H and soften the competition, rather than being aggressive to increase quality and exploit VEE. As such, the effectiveness of VEE to firm L is compromised since it simply does not have enough competitive edge to leverage VEE and has to lower the quality to survive the competition.

Second, both MEE and VEE tend to intensify the price competition. Given pre-determined quality levels, firms are likely to reduce prices in the second stage regardless of the present effect, which intensifies the ensuing price competition. Moreover, recall that both MEE and VEE are more effective with higher demand, which can be induced via lower prices. One may think that both MEE and VEE should incentivize firms to reduce prices to effectively exploit the consumers' social interactions. Although our results support this intuition, we find that the underlying reasons are essentially different: MEE incentivizes firm L to take an active role and aggressively reduce price to amplify the ensuing market expansion, and firm H in turn decreases the price to retain its own market share; whereas VEE provides more advantages to firm H and thus forces firm L to passively reduce its price in order to keep market share under the intensified competition.

Third, both MEE and VEE benefit firm H; whereas MEE benefits, but VEE hurts, firm L. Although both effects introduce positive factors and intensify price competition, they lead to different implications on firms' profit performance. The competition in MEE is benign to both firms due to the spillover effect in market growth, and the enlarged market size benefits both firms. As such, despite the intensified competition, both firms' profits will increase as MEE becomes stronger. On the other hand, the intensified competition under VEE induces the Matthew effect, where firm H is better off but firm L is worse off; and, furthermore, their profit gap increases in the strength of VEE. Our findings reveal that, different from the MEE's win-win outcome, VEE makes high-quality product more appealing but is malignant to low-quality product's survival in the market.

Finally, MEE benefits consumers and improves social welfare, whereas VEE could hurt consumers' monetary surplus. Consumers benefit from stronger MEE due to firm L's improved quality, the reduced prices, and the inflated market size in period 2. Therefore, consumer surplus increases as MEE becomes stronger. As MEE leads to a win-win-win outcome, the social welfare is also improved. This indicates that the intensified competition in MEE is benign not only to firms but also to consumers. Quite the opposite, consumers' monetary surplus drops and their total surplus may also decline under VEE since firm L's quality decreases and firm H's price increases. This shows that the intensified competition caused by VEE is malignant not only to firm L but also to consumers.

5. Additional Discussions

In this section, we provide additional discussions on the key model assumptions and related issues. Specifically, we study how dynamic pricing scheme may affect our main results in Section 5.1, investigate the combined effects of MEE and VEE in Section 5.2, and extend our base model to include costs in Section 5.3. In the subsequent analysis and discussions, we will demonstrate our main results remain qualitatively valid and also provide additional managerial insights.

5.1. Model with Dynamic Pricing Scheme

Our main model in Section 4 assumes that each firm sets a single price and commits to it across two selling periods. In reality, however, firms in different industries may adopt different pricing schemes, among which dynamic pricing is also commonly observed. For example, new sharing economy platforms or new social goods producers may offer discounted prices at early stage and then return back to the normal prices later. In this section, we examine dynamic pricing in which both firms simultaneously decide their selling prices at the beginning of each period. Adopting the same approach, we study MEE and VEE separately by solving the competition model with dynamic pricing scheme. We focus on the following two aspects: (1) We replicate the investigations on the impact of the social interactions (i.e., r, t_c, and t_p) on firms' equilibrium decisions; and (2) we compare the results under different pricing schemes to examine how pricing flexibility affects the equilibrium outcomes in the presence of social interactions. In this way, we could understand both the qualitative and the quantitative changes of our results under dynamic pricing. We briefly report our main findings in the following.

First, our main results in Section 4 continue to hold qualitatively when considering the model with dynamic pricing. As such, we claim that the impacts of social interactions on competing firms' quality and price decisions and profits are robust with respect to firms' pricing schemes. For simplicity, we do not repeat the results here; instead, we show the detailed results in Appendix B.

Second, more interesting results arise when we quantitatively compare the respective equilibrium outcomes under the two pricing schemes to distill the impact of pricing flexibility and its interplay with social interactions. We elaborate our findings below.

5.1.1. Market Expansion Effect. We keep our convention in notation and use the superscript "DM" to represent "*dynamic* pricing with *market* expansion effect". Hence, $\hat{q}_i^{DM}(r)$, $\hat{p}_{in}^{DM}(r)$, $\hat{d}_i^{DM}(r)$, and $\hat{\pi}_i^{DM}(r)$ denote firm i's equilibrium quality, price of period n, total demand, and total profit, respectively. First, we emphasize that the impacts of MEE on the final equilibrium under dynamic pricing remain qualitatively unchanged as those from committed pricing. Then, by comparing the equilibrium outcomes under the two pricing schemes, we reveal how pricing flexibility affects firms' decisions and profits under MEE in Proposition 4.

Proposition 4. *The following statements hold in the MEE case for $i \in \{H, L\}$.*

(a) *$\hat{q}_H^M = \hat{q}_H^{DM} = 1$ and $\hat{q}_L^M(r) > \hat{q}_L^{DM}(r) > \hat{q}_L^b$, i.e., firms are more differentiated under dynamic pricing.*

(b) *Each firm's committed price is between its first-period and second-period dynamic prices, i.e., $\hat{p}_{i1}^{DM}(r) < \hat{p}_i^M(r) < \hat{p}_{i2}^{DM}(r)$.*

(c) *Each firm's total demand is larger under dynamic pricing, i.e., $\hat{d}_i^{DM}(r) > \hat{d}_i^M(r) > \hat{d}_i^b$.*

(d) *Each firm's profit is higher under dynamic pricing, i.e., $\hat{\pi}_i^{DM}(r) > \hat{\pi}_i^M(r) > \hat{\pi}_i^b$.*

The comparison between the two pricing schemes in the MEE case, as characterized in Proposition 4, has several interesting implications. Parts (a) and (b) together reveal how firms' quality and price decisions are affected by dynamic pricing. Specifically, firm H chooses the same quality level under both pricing schemes, whereas firm L chooses lower quality under dynamic pricing than committed pricing; and each firm's committed price is higher than its first-period dynamic price, but is lower than its second-period one. This shows that, with pricing flexibility, firm L exploits MEE with a different approach. Instead of raising quality level to attract more consumers, it now lowers the first-period price to induce large sales, which is more effective. In period 2, the prices are set higher under dynamic pricing due to the end-of-horizon effect, which allows firms to maximize their profits. Moreover, since firm L does not have to increase quality too much under dynamic pricing (compared to committed pricing), the quality differentiation is larger and the competition is softer. Indeed, as shown in Proposition 4(c) and (d), both firms' profits and total demands are higher than those under committed pricing. Therefore, pricing flexibility amplifies the overall impact of MEE and benefits both firms. However, a close look at the percentage increase in profit shows that $(\hat{\pi}_i^{DM}(r) - \hat{\pi}_i^M(r))/\hat{\pi}_i^M(r)$ is below 0.3% for $0 < r \leq 1$ and $i \in \{H, L\}$. Hence, the difference between dynamic pricing and committed pricing is only modest.

5.1.2. Value Enhancement Effect. We first remark that all the equilibrium outcomes and results for VEE-C remains the same regardless of the adopted pricing scheme, because social interactions under VEE-C occur only within the current period, making each period independent and identical. Thus, only VEE-P is relevant in this section. Similarly as before, we assume that the parameter $t_p < \bar{t}_p$ so that the market is partially covered and each firm has positive demand in the equilibrium. Moreover, all the notations are defined in the same manner, except that here we use the superscript "*DVP*" to represent "*dynamic* pricing with *value* enhancement effect from *previous* consumers". Again, we remark that the impacts of VEE-P on the final equilibrium under dynamic and committed pricing are qualitatively the same. Then, we compare the equilibrium outcomes of the VEE-P case under the two pricing schemes in Proposition 5.

Proposition 5. *The following statements hold in the VEE-P case.*

(a) $\hat{q}_H^{VP} = \hat{q}_H^{DVP} = 1$*, and there exists* $\tilde{t}$ *such that* $\hat{q}_L^{VP}(t_p) < \hat{q}_L^{DVP}(t_p) < \hat{q}_L^b$ *if* $t_p < \tilde{t}$ *and* $\hat{q}_L^{DVP}(t_p) < \hat{q}_L^{VP}(t_p) < \hat{q}_L^b$ *otherwise.*

(b) Each firm's committed price is between its first-period and second-period dynamic prices, i.e., $\hat{p}_{i1}^{DVP}(t_p) < \hat{p}_i^{VP}(t_p) < \hat{p}_{i2}^{DVP}(t_p)$*, for* $i = H, L$*.*

(c) Firm H's total demand is lower under dynamic pricing, i.e., $\hat{d}_H^{DVP}(t_p) < \hat{d}_H^{VP}(t_p)$*, whereas firm L's is higher, i.e.,* $\hat{d}_L^{DVP}(t_p) > \hat{d}_L^{VP}(t_p)$*.*

(d) Firm L's profit is higher under dynamic pricing, i.e., $\hat{\pi}_L^{VP}(t_p) < \hat{\pi}_L^{DVP}(t_p) < \hat{\pi}_L^b$*; for firm H, there exists a threshold* $\hat{t}$ *such that* $\hat{\pi}_H^{VP}(t_p) > \hat{\pi}_H^{DVP}(t_p) > \hat{\pi}_H^b$ *if* $t_p < \hat{t}$ *and* $\hat{\pi}_H^{DVP}(t_p) > \hat{\pi}_H^{VP}(t_p) > \hat{\pi}_H^b$ *otherwise.*

Proposition 5(b) confirms the rationale mentioned in the MEE case. That is, endowed with the pricing flexibility, firms cut the first-period prices to generate high sales and enhance consumers' utility via VEE-P, and set higher prices in period 2 (the end period) to avoid unnecessary competition and revenue loss. In addition, Proposition 5(a) shows that when t_p is sufficiently small, firm L does not have to set a very low quality level as it does under committed pricing, since dynamic pricing can already help firm L avoid fierce competition against firm H. By contrast, when t_p is relatively large, firm L has to set an even lower quality to further differentiate from firm H.

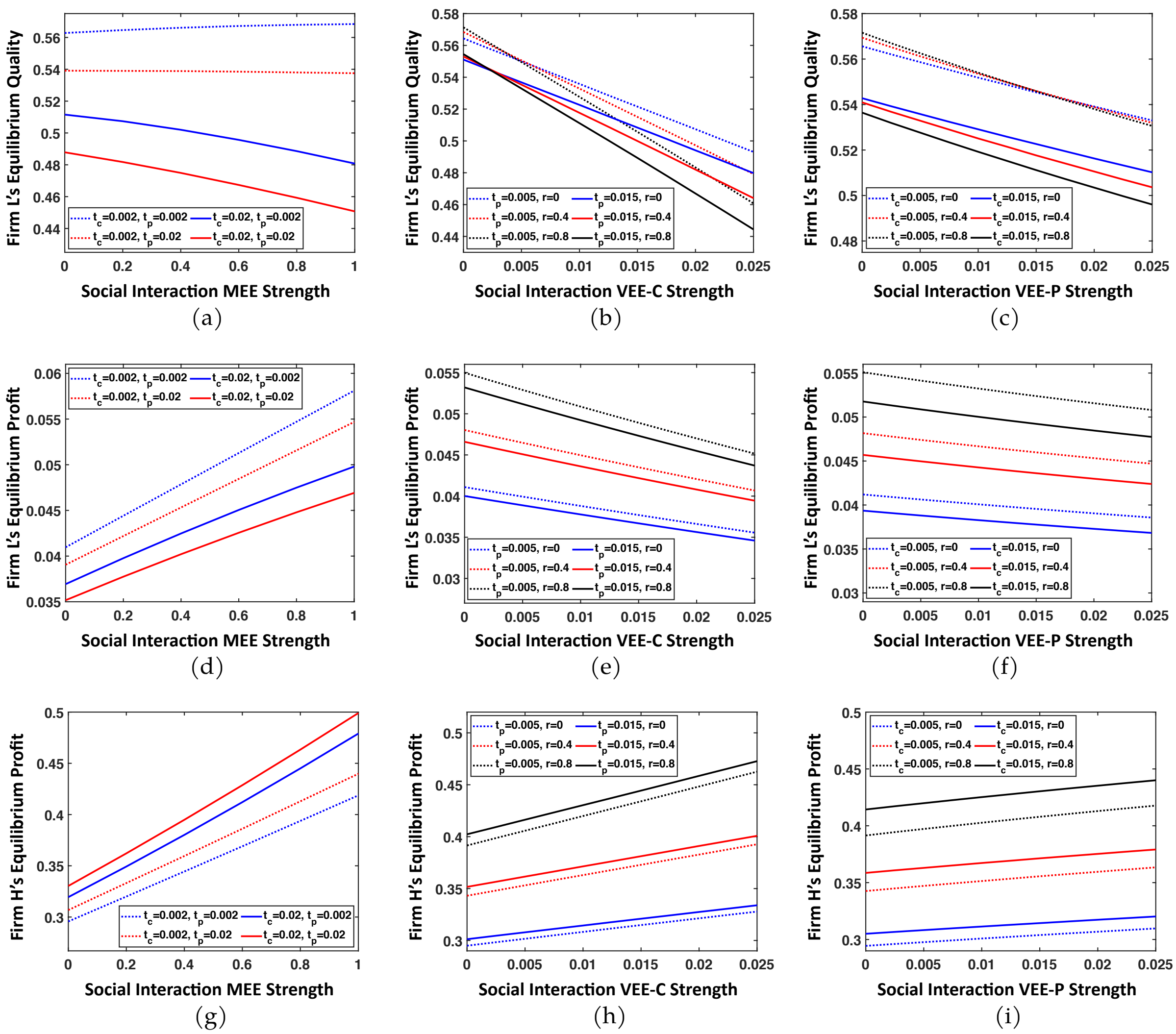
Recall that, under committed pricing, VEE-P benefits firm H at the expense of hurting firm L, positing the Matthew effect. Although this phenomenon persists under dynamic pricing, it is weakened to some extent. Specifically, by Proposition 5(c) and (d), firm L captures more demand whereas firm H loses demand under dynamic pricing; moreover, firm L's profit is improved but firm H's profit may decline. To wit, the pricing flexibility helps firm L combat the negative impact of VEE-P and improves its competitive edge. Thus, comparing to committed pricing under VEE-P, dynamic pricing can either lead to a Pareto improvement for both firms, or create a more balanced duopoly competition by alleviating the Matthew effect. Finally, we note that the percentage change in profit between the two pricing schemes is not significant in magnitude. Particularly, for firm L, $0 < (\hat{\pi}_L^{DVP}(t_p) - \hat{\pi}_L^{VP}(t_p))/\hat{\pi}_L^{VP}(t_p) < 3.5\%$ and, for firm H, $|\hat{\pi}_H^{DVP}(t_p) - \hat{\pi}_H^{VP}(t_p)|/\hat{\pi}_H^{VP}(t_p) < 1.5\%$.

5.2. Model with Combined Effects of MEE and VEE

In this subsection, we investigate the situation where both MEE and VEE (including VEE-C and VEE-P) exist. The model can be solved in the same manner as before. However, the presence of both MEE and VEE greatly increases the complexity of the problem, making the analytical approach intractable. Therefore, we resort to numerical analysis. In the sequel, we focus on $r \in [0, 1]$, $t_c \in [0, 0.025]$, and $t_p \in [0, 0.025]$ (recall that t_c and t_p are small fractions). Our analysis particularly focuses on the degree of product differentiation and the firms' equilibrium profits. Moreover, we will only discuss the analysis for committed pricing; in fact, similar numerical experiments have been conducted for dynamic pricing, and the observations are qualitatively identical.

We conduct extensive studies with numerous instances. For each instance, we obtain firms' equilibrium quality levels and profits, and then compare them with their counterparts where only one form of social interactions exists. Figure 1 below illustrates our main results on this matter. Note that Figures 1(a)-(c) focus on the combined effects on firm L's quality level, which reflects product differentiation between the two firms (firm H's equilibrium quality is always 1 because there is no cost), and Figures 1(d)-(i) show the interplay of multiple effects on firms' profits.

Figure 1 **Combined Effect of Social Interactions under Committed Pricing.**



By and large, our previous findings in Section 4 are robust. For example, in Figure 1(a), we observe that, when VEE-C is relatively weak (i.e., $t_c = 0.002$), the product differentiation is reduced as MEE becomes stronger. Hence, our result from Proposition 1(a) is qualitatively retained. Similarly, Figures 1(b) and (c) support the robustness of our results in Propositions 2(a) and 3(a) with respect to the possible existence of MEE. Moreover, observations drawn from the firms' profit curves in Figures 1(d)-(i) are also consistent with our previous results. Therefore, the combined effect of MEE and VEE may be decomposed into the individual effects and the exact implication hinges on their respective magnitudes.

Apart from confirming the robustness of our main results, the numerical analysis also highlights two additional observations that complement our previous results: (1) The original impact of MEE on quality differentiation could be distorted in the presence of relatively strong VEE. As illustrated by the two solid curves associated with $t_c = 0.02$ in Figure 1(a), firm L tends to decrease quality as MEE becomes stronger. Rather than a means to facilitate the market expansion, the product quality in this case becomes an instrument for firm L to alleviate fierce competition induced by strong VEE ($t_c = 0.02$). (2) VEE-C has a stronger impact on firms' quality differentiation and profits than VEE-P. This is revealed by the comparison between the dotted and the solid curves in Figure 1. Such an observation echoes with our previous discussion on the time-lag effect that contributes to a weaker impact of value enhancement from interacting with previous consumers.

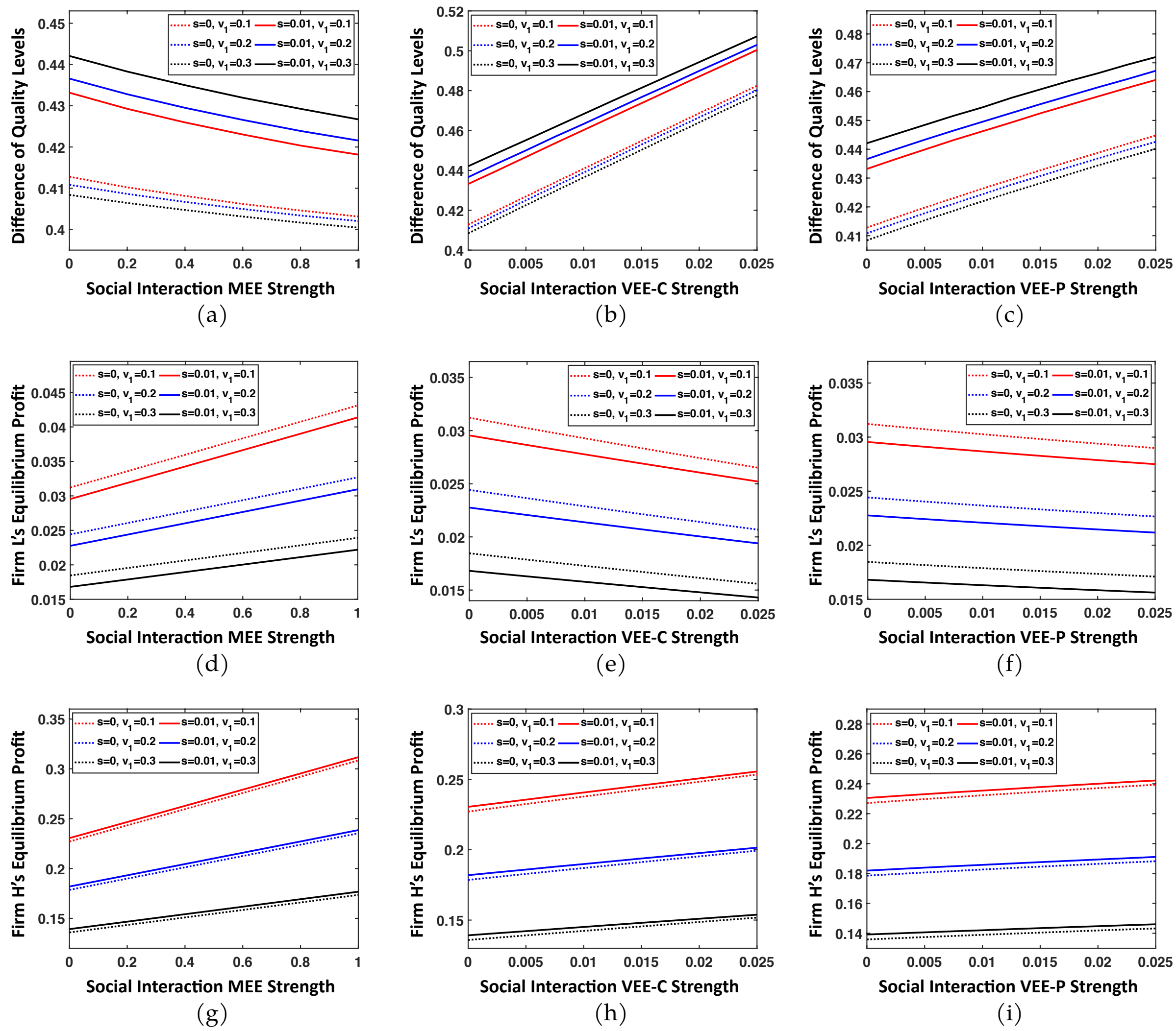
5.3. Model with Cost Considerations

In this subsection, we extend our main model to include cost considerations for the firms. Typically, when firms invest in the product quality and run the production process, they incur both fixed setup cost and per-unit marginal cost. To capture these costs, we assume that, for a firm with product quality q and demand d, the total cost incurred is given by $C(q,d) = S(q)\delta(d) + V(q,d)$, where $\delta(d)$ equals to 1 if $d > 0$ and 0 otherwise. In accordance with the classic assumption that the quality dependent set up cost $S(q)$ is convex increasing in q, we assume $S(q) = sq^2/2$ for some $s \geq 0$. In addition, $V(q,d)$ represents the total production cost. We focus on the linear functional form and assume $V(q,d) = (v_0 + v_1 q)d$ for some $v_0, v_1 \geq 0$.[5] Firms are assumed to have the same cost structure, and the same parameters apply to both firms.

Given the above specified cost function, we conduct extensive numerical experiments to test whether the respective impacts of MEE, VEE-C and VEE-P on firms' equilibrium product differentiation and profits are robust when costs are considered. To purely focus on the cost considerations, we follow the main model of committed pricing and do not involve combined effects of social interactions. Since the results are quite consistent across different combinations of parameters, we only present the results for a few representative instances ($v_0 = 0.01$, $v_1 \in \{0.1, 0.2, 0.3\}$ and $s \in \{0, 0.01\}$) in Figure 2, and highlight some major observations below.

[5] We have repeated our study with other alternatives, e.g., quadratic functional form, and drawn the same conclusion.

Figure 2 Impact of Social Interactions under Committed Pricing with Cost Considerations. (Fix $v_0 = 0.01$).



In Figures 2(a)-(c), we observe that the monotonicity of the quality differentiation $\hat{q}_H^k - \hat{q}_L^k$ ($k \in \{M, VC, VP\}$) is consistent with our previous results as described in Propositions 1(a), 2(a) and 3(a) (note that firm H's equilibrium quality is always 1 without cost in the main model, but may be less than 1 with cost considerations). Specifically, the quality differentiation decreases as MEE becomes stronger but as VEE-C/VEE-P becomes weaker. This shows the robustness of our main result even when we consider the fixed setup cost and marginal production cost.

Second, the individual impact of each form of social interaction on firms' profits with cost considerations is the same as before, which is presented in Figures 2(d)-(i). Specifically, stronger MEE improves both firms' profits, whereas stronger VEE-C or VEE-P only benefits firm H but hurts firm L. Therefore, our results are quite robust with respect to the general cost structure. In addition, Figures 2(d)-(i) further reveal the impacts of costs on firms' profits: (1) Both firms are negatively affected by the marginal cost factor, as their profits are lower for larger v_1; and (2) the setup cost factor (s) has different impacts on different firms, as larger s may benefit firm H but hurt firm L.

In sum, we extend our model to include fixed setup cost and marginal production cost, both of which depend on the quality level. We numerically confirm that our main results are qualitatively robust, which indicates that it is a legitimate simplification to assume zero cost in our main model. Moreover, our findings also shed light on the impacts of cost parameters on firms' profits.

6. Concluding Remarks

Consumers' social interactions are commonly seen in many marketplaces. Our paper aims to understand how different forms of social interactions affect duopoly firms' product differentiation, pricing decisions, and profits in a competitive environment. Specifically, we focus on two forms of commonly observed social interactions: The market expansion effect that causes the total market size to increase for competing firms through social interactions that expand products awareness, and the value enhancement effect that exclusively increases the consumers' utility towards one firm when interacting with consumers in either current or previous period.

Using a two-stage multi-period duopoly competition model, we uncover an interesting interplay between product quality differentiation and consumers' social interactions. Indeed, quality differentiation not only bears the purpose of alleviating the price competition, but also has impacts on the effective exploitation of the consumer-to-consumer contagions. Our results reveal that it is important for firms to distinguish the aforementioned two forms of social interactions, because although the price competition is intensified in both cases, the strategic implications on the product differentiation and profitability are quite different. Specifically, the quality differentiation level between the duopoly decreases in the strength of MEE, but increases in that of VEE. Moreover, MEE is benign to firms and makes consumers better off, fostering a healthy competitive environment (win-win situation); however, VEE (both VEE-C and VEE-P) benefits the high-quality firm at the expense of hurting both the low-quality firm and consumers, leading to the Matthew effect of accumulated advantage. Hence, firms' quality and price decisions as well as their profitability eventually depend on which form of the social interactions is more prominent.

As the main managerial insights, we connect consumers' social interactions with competing firms' product design and pricing decisions, and highlight the importance of distinguishing different forms of social interactions. Our results can be applied to many industries where social interactions are prevalent and product quality is a critical decision. Consider our motivating example of the MOBAs marketplace (or video game industry in general). Here, informative social interactions include players' introducing games to the community on gaming forums. Hence, if this form of interactions is strong, publishers may consider selecting a relatively aggressive product quality level to exploit MEE, such as improving the level of gaming experience, character design, in-game purchase options and so forth. However, from the high-quality publisher's standpoint, it may employ various marketing strategies to actively boost the value enhancement effect, thereby leveraging the advantageous market position endowed by VEE.

In addition, competing firms should also pay close attention to the possible evolution of the prominent form of social interactions and adjust their marketing strategies accordingly. For instance, in the infant stage of bike-sharing industry, the market expansion effect is prevalent, because the future market growth could be potentially significant and major bike-sharing companies (such as Mobike and Ofo) aim to introduce this innovative service to potential consumers to rapidly enlarge the overall bike-sharing market. When the industry becomes mature, the market size remains stable as almost all potential consumers are aware of the products. Then, social interactions would go through a paradigm shift such that exchanging user experiences with specific product prevails (Ouyang 2017). In this case, the value enhancement effect could become dominant, and efforts should be primarily devoted to product differentiation and branding. As Mobike CEO, Mr. Wang Xiaofeng, commented, "*without a strong differentiation from similar products, the users will eventually abandon you*" (Hu 2017). As such, firms may focus on enlarging product differentiation against their opponents to leverage and benefit from the value-enhancing social interactions.

To conclude, we discuss a couple of interesting model extensions as future research directions. In contrast to the current non-durable goods setting where consumers purchase immediately without strategic waiting, we could assume the focal product is durable and consider the consumers' strategic waiting behavior. As suggested by Coase (1972), consumers may wait for future price reduction under dynamic pricing; moreover, under VEE-P, they may delay their purchase to join into a potentially larger user base later so that their utility could be further strengthened. To study the aforementioned strategic behaviors, we could incorporate consumers' heterogeneous patience levels so that impatient ones will find it too costly to wait. As such, in each period, there will be a group of impatient consumers who choose to purchase, and social interactions will continue to take effect in this case. Hence, we conjecture that our main findings regarding the impacts of social interactions will hold qualitatively. Moreover, the interaction between consumers' strategic waiting and social effects may generate new interesting insights, and we leave the thorough analysis for this case as future research.

Second, while we focus on positive social interactions in the main paper, it is also practically possible that the strength of the social interactions may be negative, especially for the case of VEE. For example, when all orders or service requests are processed by a common server (e.g., the internet bandwidth for all players in a game or the logistic provider that serves multiple online retailers), too many current users may result in server congestion, which negatively affects consumers' utility. In this case, we find that, when the parameters r, t_c, or t_p are negative but close to zero, our results are exactly reversed. That is, for MEE, $\hat{q}_L^M$ is lower than the benchmark when r is negative, and for VEE, $\hat{q}_L^{VC}$ and $\hat{q}_L^{VP}$ are higher than the benchmarks when t_c and t_p are negative, respectively. When the social interactions have negative impacts, each firm may prefer a lower demand in the first period, and the competition is softened, which is opposite to the findings under positive social interactions. While we can partially extend our findings to the aforementioned situation, we leave the comprehensive analysis for the negative social interactions as future research.

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Appendices to "Impact of Social Interactions on Duopoly Competition with Quality Considerations"

The appendices are divided into two parts. We present the proofs for the main model under the committed pricing scheme (Section 4) in Part A. In Part B, we present the additional findings and proofs for the results under the dynamic pricing scheme (Section 5.1). Table 1 below summarizes the notations used in the paper.

Table 1 Summary of Notations

Symbol	Description
"M"	the scenario of committed pricing with MEE
"VC"	the scenario of VEE-C under both committed pricing and dynamic pricing strategies
"VP"	the scenario of committed pricing with VEE-P
"DM"	the scenario of dynamic pricing with MEE
"DVP"	the scenario of dynamic pricing with VEE-P
"b"	benchmark case without MEE or VEE
r	the strength of MEE
t_c	the strength of VEE-C
t_p	the strength of VEE-P
p_i^s	committed price of firm i under scenario $s \in \{M, VP, VC\}$
p_{in}^s	price of firm i in period $n = 1, 2$, under scenario $s \in \{DM, DVP\}$
d_{in}^s	demand of firm i in period $n = 1, 2$, under scenario $s \in \{M, VP, VC, DM, DVP\}$
d_i^s	total demand of firm i in the two periods under scenario $s \in \{M, VP, VC, DM, DVP\}$
π_{in}^s	profit of firm i in period $n = 1, 2$, under scenario $s \in \{M, VP, VC, DM, DVP\}$
π_i^s	total profit of firm i in two periods under scenario $s \in \{M, VP, VC, DM, DVP\}$
q_i^s	quality level of firm i under scenario $s \in \{M, VP, VC, DM, DVP\}$
"$\hat{\ }$"	"$\hat{\ }$" over above symbols to denote the corresponding final equilibrium

A. The Committed Pricing Scheme

A.1. Market Expansion Effect (MEE)

Proof of Lemma 1: Given $t_c = t_p = 0$ and $0 < r \le 1$ and $0 \le q_L < q_H \le 1$, firm i decides its price, p_i, to maximize its total profit:

$$\max_{p_i \ge 0} p_i(d_{i1}^M + d_{i2}^M), \tag{A1}$$

where $i = H, L$. We will show in Proposition 1 that the market is partially covered and each firm has positive demand in the equilibrium. Consumers will purchase from firm H if $u_{Hn} > \max\{0, u_{Ln}\}$

and from firm L if $u_{Ln} > \max\{0, u_{Hn}\}$ in period $n = 1, 2$. Thus, the demand of each firm in each period can be solved as

$$d_{H1}^M = 1 - \frac{p_H - p_L}{q_H - q_L}, d_{L1}^M = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L},$$
$$d_{H2}^M = (1 + rd_{H1}^M + rd_{L1}^M)\left(1 - \frac{p_H - p_L}{q_H - q_L}\right), \text{ and } d_{L2}^M = (1 + rd_{H1}^M + rd_{L1}^M)\left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right). \quad \text{(A2)}$$

Given the above demands, one can easily verify that firm i's total profit $p_i(d_{i1}^M + d_{i2}^M)$ is concave in p_i, i.e., $\frac{\partial^2(p_i(d_{i1}^M + d_{i2}^M))}{\partial p_i^2} < 0$, for $i = H, L$. Thus, solving the two firms' first-order-conditions together, i.e., $\frac{\partial(p_H(d_{H1}^M + d_{H2}^M))}{\partial p_H} = 0$ and $\frac{\partial(p_L(d_{L1}^M + d_{L2}^M))}{\partial p_L} = 0$, leads to the the sub-game equilibrium prices:

$$p_H^M(q_H, q_L, r) = \frac{q_L\left(8q_H - 10rq_H - q_L(2-r) - X_1\right) + 12rq_H^2}{8(3q_H - q_L)r} \quad \text{and}$$
$$p_L^M(q_H, q_L, r) = \frac{q_L\left(6rq_H + 8q_H - q_L(3r+2) - X_1\right)}{4(3q_H - q_L)r}, \quad \text{(A3)}$$

where $X_1 = \sqrt{q_L^2(r-2)^2 - 4q_Hq_L\left(r^2 + 2r + 8\right) + 4q_H^2(3r^2 + 12r + 16)}$. Plugging Equations (A3) into Equations (A2), the demand of each firm in each period can be written as:

$$d_{H1}^M(q_H, q_L, r) = \frac{q_L\left(8q_H - 10rq_H - q_L(2-r) - X_1\right) + 12rq_H^2}{8(3q_H - q_L)(q_H - q_L)r},$$
$$d_{L1}^M(q_H, q_L, r) = \frac{q_L\left(4q_H(3-r) - q_L(2-r) - X_1\right) + 2q_H\left(X_1 - 8q_H\right)}{8(3q_H - q_L)(q_H - q_L)r},$$
$$d_{H2}^M(q_H, q_L, r) = \frac{\left(X_1 - q_L(r+2) + q_H(6r+4)\right)\left(q_L\left(q_L(r-2) - X_1 - 10rq_H + 8q_H\right) + 12rq_H^2\right)}{32(3q_H - q_L)^2(q_H - q_L)r}, \text{ and}$$
$$d_{L2}^M(q_H, q_L, r) = \frac{\left(X_1 - q_L(r+2) + q_H(6r+4)\right)\left(q_L\left(q_L(r-2) - 4rq_H + 12q_H - X_1\right) + 2q_H\left(X_1 - 8q_H\right)\right)}{32(3q_H - q_L)^2(q_H - q_L)r}. \quad \text{(A4)}$$

Moreover, the total demand of firm i is given by $d_i^M(q_H, q_L, r) = d_{i1}^M(q_H, q_L, r) + d_{i2}^M(q_H, q_L, r)$, and firm i's total profit is $\pi_i^M(q_H, q_L, r) = p_i^M(q_H, q_L, r)d_i^M(q_H, q_L, r)$, for $i = H, L$.

Note that the sub-game equilibrium prices, demands, and profits are all differentiable. For $r \in (0, 1]$ and $0 \le q_L < q_H \le 1$, taking derivative with respect to r and after simplification, one can verify that the followings hold

$$\frac{\partial p_H^M(q_H, q_L, r)}{\partial r} < 0, \quad \frac{\partial p_L^M(q_H, q_L, r)}{\partial r} < 0,$$
$$\frac{\partial d_H^M(q_H, q_L, r)}{\partial r} > 0, \quad \frac{\partial d_L^M(q_H, q_L, r)}{\partial r} > 0,$$
$$\frac{\partial \pi_H^M(q_H, q_L, r)}{\partial r} > 0, \text{ and } \quad \frac{\partial \pi_L^M(q_H, q_L, r)}{\partial r} > 0.$$

This completes the proof of Lemma 1. □

Proof of Proposition 1: We complete the proof in three steps: **Step 1.** Solve the potential equilibrium given that the market is partially covered and each firm has positive demand; **Step 2.** Show the existence of the equilibrium derived in Step 1; **Step 3.** Show the uniqueness of the equilibrium.

Step 1. We first solve the potential equilibrium given that the market is partially covered and each firm has positive demand. We have solved the firms' pricing decisions for given quality levels in the proof of Lemma 1. Given the firms' pricing decisions in Stage 2, we proceed to solve for firms' quality decisions in the first stage:

$$\max_{q_H \in (q_L,1]} \pi_H^M(q_H,q_L,r) = p_H^M(q_H,q_L,r)d_H^M(q_H,q_L,r), \text{ and}$$
$$\max_{q_L \in [0,q_H)} \pi_L^M(q_H,q_L,r) = p_L^M(q_H,q_L,r)d_L^M(q_H,q_L,r),$$

where $p_i^M(q_H,q_L,r)$ and $d_i^M(q_H,q_L,r)$ are given by Equations (A3) and (A4), respectively.

First, one can easily verify that $\pi_H^M(q_H,q_L,r)$ increases in q_H for any $q_L \in [0,1)$ and $r \in (0,1]$, i.e., $\frac{d\pi_H^M(q_H,q_L,r)}{dq_H} > 0$. Thus, firm H's optimal quality is $\hat{q}_H^M = 1$ for any $q_L \in [0,1)$ and $r \in (0,1]$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L. Directly checking firm L's profit function, we find that it is strictly concave in q_L, i.e., $\frac{\partial^2 \pi_L^M(1,q_L,r)}{\partial q_L^2} < 0$, for any $r \in (0,1]$ and $q_L \in [0,1)$. Moreover, one can verify that $\frac{\partial \pi_L^M(1,q_L,r)}{\partial q_L}|_{q_L=0.1} > 0$ and $\frac{\partial \pi_L^M(1,q_L,r)}{\partial q_L}|_{q_L=0.9} < 0$. Therefore, there exists a unique $\hat{q}_L^M(r) \in (0.1,0.9)$ that can be solved from $\frac{\partial \pi_L^M(1,q_L,r)}{\partial q_L} = 0$ and maximizes $\pi_L^M(1,q_L,r)$.

Hence, given that the market is partially covered and each firm has positive demand, the potential equilibrium outcome is: $q_H = \hat{q}_H^M = 1$, $q_L = \hat{q}_L^M(r)$, $p_H = \hat{p}_H^M(r) = p_H^M(1,\hat{q}_L^M(r),r)$, and $p_L = \hat{p}_L^M(r) = p_L^M(1,\hat{q}_L^M(r),r)$. Let $\sigma^M = (\hat{q}_H^M, \hat{q}_L^M(r), \hat{p}_H^M(r), \hat{p}_L^M(r))$ denote this potential equilibrium, and let $\hat{d}_i^M(r) = d_i^M(1,\hat{q}_L^M(r),r)$ and $\hat{\pi}_i^M(r) = \hat{p}_i^M(r)\hat{d}_i^M(r)$ denote firm i's total demand and profit at the potential equilibrium σ^M.

Step 2. To show the existence of the equilibrium σ^M, we will show that σ^M is a Sub-game Perfect Equilibrium (SPE). We just need to verify that σ^M satisfies the no-deviation requirements of SPE: (1) Given firms' quality $0 \le q_L < q_H \le 1$ and firm i's price $p_i^M(q_H,q_L,r)$, firm j's price decision will not deviate from $p_j^M(q_H,q_L,r)$ to other prices; and (2) given both firms' prices $(p_H^M(q_H,q_L,r), p_L^M(q_H,q_L,r))$ in Stage 2 and firm i's quality $\hat{q}_i^M(r)$, firm j's quality will not deviate from $\hat{q}_j^M(r)$ to other qualities, i.e., $\hat{\pi}_j^M(r) \ge \max_{q_j \neq \hat{q}_j^M(r)} \pi_j^M(q_j, \hat{q}_i^M(r), r)$, for $i,j \in \{H,L\}$ and $i \neq j$.

For requirement (1), we just need to show that

$$\pi_j^M(q_H,q_L,r) \ge \max_{p_j \neq p_j^M(q_H,q_L,r)} p_j d_j^M(p_j|q_H,q_L,p_i^M(q_H,q_L,r)), \text{for } i,j \in \{H,L\} \text{ and } i \neq j, \tag{A5}$$

where the general demand functions are given by:

$$d_{H1}^M = (1-\max\{\frac{p_H-p_L}{q_H-q_L}, \frac{p_H}{q_H}\})^+, \quad d_{L1}^M = (\min\{1, \frac{p_H-p_L}{q_H-q_L}\} - \frac{p_L}{q_L})^+,$$
$$d_{H2}^M = (1+rd_{H1}^M + rd_{L1}^M)d_{H1}^M, \quad \text{and} \quad d_{L2}^M = (1+rd_{H1}^M + rd_{L1}^M)d_{L1}^M.$$

Firm i's total demand of two periods is $d_i^M = d_{i1}^M + d_{i2}^M$. Note that the above demand functions include all situations that market is partially covered or fully covered and both firms have positive demands or one of them has zero demand. By straightforward but tedious algebraic analysis, we can verify that given $r \in (0,1]$ and firm i's price $p_i^M(q_H,q_L,r)$, firm j's optimal price is $p_j^M(q_H,q_L,r)$ and thus will not deviate, i.e., the inequality (A5) holds. Thus, requirement (1) is satisfied.

For requirement (2), we have already shown in Step 1 that firm H's optimal quality is $q_H = 1$ for any $q_L \in [0,1)$ and that firm L's optimal quality is $q_L = \hat{q}_L^M(r)$ given $q_H = 1$. Thus, requirement (2) holds. Hence, we have shown that σ^M is an SPE.

Step 3. We show the uniqueness of the equilibrium σ^M. Note that in Step 1, we have derived a unique equilibrium σ^M by backward induction under the condition that the market is partially covered and each firm has positive demand. Thus, we just need to show that there does not exist any equilibrium if the above condition does not hold, i.e., the market is fully covered or one of the firms has zero demand. Note that in the case of MEE, $d_{H2} + d_{L2} = 1 + r(d_{H1} + d_{L1})$ if and only if $d_{H1} + d_{L1} = 1$, i.e., the market is fully covered in period 2 if and only if the market is fully covered in period 1; and $d_{i2} = 0$ if and only $d_{i1} = 0$, i.e., firm i has zero demand in period 2 if and only if it has zero demand in period 1. Thus, to show the uniqueness, we need to verify that: (1) Any strategy that leads to $d_{H1} + d_{L1} = 1$ is not an equilibrium; and (2) any strategy that leads to $d_{H1} = 0$ or $d_{L1} = 0$ is not an equilibrium.

If $d_{H1} + d_{L1} = 1$, it implies that the consumer with $\theta = 0$ makes the purchase. That is, either $p_L = 0$ or $p_H = 0$. It is obvious that any strategy with $p_L = 0$ or $p_H = 0$ cannot be an equilibrium since one of the firm's profit will be zero and always has incentive to deviate to a small enough price to earn a positive profit. Thus, (1) holds. Similarly, if $d_{i1} = 0$, then firm i's profit is zero. It is obvious that any strategy with $d_{i1} = 0$ cannot be an equilibrium outcome since firm i has incentive to deviate to a lower price to get a positive demand and earn a positive profit. Thus (2) holds.

Hence, we conclude that in the equilibrium, the market is partially covered and each firm has positive demand (i.e., $d_{H1} > 0$, $d_{L1} > 0$, $d_{H2} > 0$, $d_{L2} > 0$, and $d_{H1} + d_{L1} < 1$). Combined with Steps 1 and 2, it shows that σ^M is the unique equilibrium. In what follows, we prove Parts (a)-(d). Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}_L^M(r)$ is the unique solution of $\frac{\partial \pi_L^M(1,q_L,r)}{\partial q_L} = 0$ for $q_L \in [0,1)$, by the Implicit Function Theorem, we have $\frac{\partial \hat{q}_L^M(r)}{\partial r} = -\frac{\partial^2 \pi_L^M(1,q_L,r)}{\partial q_L \partial r} / \frac{\partial^2 \pi_L^M(1,q_L,r)}{\partial q_L^2}|_{q=\hat{q}_L^M(r)}$. We have already shown that $\frac{\partial^2 \pi_L^M(1,q_L,r)}{\partial q_L^2} < 0$ for any $q_L \in [0,1)$ and $r \in (0,1]$. One can also verify that $\frac{\partial^2 \pi_L^M(1,q_L,r)}{\partial q_L \partial r} > 0$ for any $q_L \in [0,0.6)$ and $r \in (0,1]$. Moreover, we find that $\hat{q}_L^M(r) < 0.6$ for any $r \in (0,1]$ since $\frac{\partial \pi_L^M(1,q_L,r)}{\partial q_L} < 0$ for any $q_L \in [0.6,1)$ and $r \in (0,1]$. Therefore, $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$ for $r \in (0,1]$.

Part (b): Note that $\hat{p}_i^M(r) = p_i^M(1, \hat{q}_L^M(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_i^M(r)}{\partial r} = (\frac{\partial p_i^M(1,q_L,r)}{\partial r} + \frac{\partial p_i^M(1,q_L,r)}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r})|_{q_L=\hat{q}_L^M(r)}$. In the proof of Lemma 1, we have shown that $\frac{\partial p_i^M(1,q_L,r)}{\partial r} < 0$. In Part (a) above, we have shown that $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$. Moreover, one can also verify that $\frac{\partial p_H^M(1,q_L,r)}{\partial q_L} < 0$ for any $q_L \in [0,1)$ and $r \in (0,1]$, and that $\frac{\partial p_L^M(1,q_L,r)}{\partial q_L} < 0$ for any $q_L \in (0.55,1)$ and $r \in (0,1]$. Since $\hat{q}_L^M(r)$ increases in r, we have $\hat{q}_L^M(r) \geq \hat{q}_L^M(0) = \hat{q}^b = 4/7 > 0.55$. Thus $\frac{\partial p_L^M(1,q_L,r)}{\partial q_L}|_{q_L=\hat{q}_L^M(r)} < 0$ holds for any $r \in$

$(0,1]$. Combining all the above statements, we deduce that $\frac{\partial \hat{p}_i^M(r)}{\partial r} < 0$ for any $r \in (0,1]$, as desired. Therefore, $\hat{p}_i^M(r)$ decreases in r and $\hat{p}_i^M(r) < \hat{p}_i^M(0) = \hat{p}_i^b$ for $r \in (0,1]$ and $i = H, L$.

Part (c): Note that $\hat{d}_i^M(r) = d_i^M(1, \hat{q}_L^M(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^M(r)}{\partial r} = (\frac{\partial d_i^M(1,q_L,r)}{\partial r} + \frac{\partial d_i^M(1,q_L,r)}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r})|_{q_L=\hat{q}_L^M(r)}$. In the proof of Lemma 1, we have shown that $\frac{\partial d_i^M(1,q_L,r)}{\partial r} > 0$. In Part (a) above, we have shown that $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$. Moreover, one can verify that $\frac{\partial d_i^M(1,q_L,r)}{\partial q_L} > 0$. Thus, $\frac{\partial \hat{d}_i^M(r)}{\partial r} > 0$ for $i = H, L$.

Part (d): Note that $\hat{\pi}_i^M(r) = \pi_i^M(1, \hat{q}_L^M(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{\pi}_i^M(r)}{\partial r} = (\frac{\partial \pi_i^M(1,q_L,r)}{\partial r} + \frac{\partial \pi_i^M(1,q_L,r)}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r})|_{q_L=\hat{q}_L^M(r)}$. One can verify that $\frac{\partial \pi_H^M(1,q_L,r)}{\partial r} + \frac{\partial \pi_H^M(1,q_L,r)}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$ for any $q_L \in [0,1)$ and $r \in (0,1]$. Thus, $\frac{\partial \hat{\pi}_H^M(r)}{\partial r} > 0$. In the proof of Lemma 1, we have shown that $\frac{\partial \pi_L^M(1,q_L,r)}{\partial r} > 0$. In Part (a) above, we have shown that $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$. Moreover, $\frac{\partial \pi_L^M(1,q_L,r)}{\partial q_L}|_{q_L=\hat{q}_L^M(r)} = 0$. Thus, $\frac{\partial \hat{\pi}_L^M(r)}{\partial r} > 0$.

Next, we show that $\hat{\pi}_H^M(r) - \hat{\pi}_L^M(r)$ increases in r. By the chain rule: $\frac{\partial(\hat{\pi}_H^M(r) - \hat{\pi}_L^M(r))}{\partial r} = \left(\frac{\partial(\pi_H^M(1,q_L,r) - \pi_L^M(1,q_L,r))}{\partial r} + \frac{\partial(\pi_H^M(1,q_L,r) - \pi_L^M(1,q_L,r))}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r}\right)|_{q_L=\hat{q}_L^M(r)}$. First, the definition of $\hat{q}_L^M(r)$ indicates that $\frac{\pi_L^M(1,q_L,r)}{\partial q_L}|_{q_L=\hat{q}_L^M(r)} = 0$; second, one can verify that $\frac{\partial(\pi_H^M(1,q_L,r) - \pi_L^M(1,q_L,r))}{\partial r} + \frac{\partial \pi_H^M(1,q_L,r)}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$ for any $q_L \in [0,1)$ and $r \in (0,1]$. Thus, $\frac{\partial(\hat{\pi}_H^M(r) - \hat{\pi}_L^M(r))}{\partial r} > 0$, which implies $\hat{\pi}_H^M(r) - \hat{\pi}_H^b > \hat{\pi}_L^M(r) - \hat{\pi}_L^b > 0$, for $r \in (0,1]$.

Finally, we show that $\frac{\hat{\pi}_L^M(r)}{\hat{\pi}_L^b} - \frac{\hat{\pi}_H^M(r)}{\hat{\pi}_H^b}$ increases in r. By the chain rule,

$$\frac{\partial\left(\frac{\hat{\pi}_L^M(r)}{\hat{\pi}_L^b} - \frac{\hat{\pi}_H^M(r)}{\hat{\pi}_H^b}\right)}{\partial r} = \left(\frac{\partial(\frac{\pi_L^M(1,q_L,r)}{\hat{\pi}_L^b} - \frac{\pi_H^M(1,q_L,r)}{\hat{\pi}_H^b})}{\partial r} + \frac{\partial(\frac{\pi_L^M(1,q_L,r)}{\hat{\pi}_L^b} - \frac{\pi_H^M(1,q_L,r)}{\hat{\pi}_H^b})}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r}\right)|_{q_L=\hat{q}_L^M(r)}.$$

First, $\frac{\pi_L^M(1,q_L,r)}{\partial q_L}|_{q_L=\hat{q}_L^M(r)} = 0$; second, one can verify that $\frac{\partial\left(\frac{\pi_L^M(1,q_L,r)}{\hat{\pi}_L^b} - \frac{\pi_H^M(1,q_L,r)}{\hat{\pi}_H^b}\right)}{\partial r} + \frac{\partial\left(-\frac{\pi_H^M(1,q_L,r)}{\hat{\pi}_H^b}\right)}{\partial q_L}\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$ for any $q_L \in (0.5, 0.6)$ and $r \in (0,1]$; third, $0.5 < \hat{q}_L^M(0) \leq \hat{q}_L^M(r) \leq \hat{q}_L^M(1) < 0.6$ for any $r \in (0,1]$. Thus, we have $\frac{\partial\left(\frac{\hat{\pi}_L^M(r)}{\hat{\pi}_L^b} - \frac{\hat{\pi}_H^M(r)}{\hat{\pi}_H^b}\right)}{\partial r} > 0$, which implies that $(\hat{\pi}_L^M(r) - \hat{\pi}_L^b)/\hat{\pi}_L^b > (\hat{\pi}_H^M(r) - \hat{\pi}_H^b)/\hat{\pi}_H^b > 0$ for all $r \in (0,1]$. This completes the proof. $\square$

Proof of Corollary 1: **Part (a):** Let $CS_{in}^M(r)$ denote total consumer surplus of purchasing from firm i in period n in the equilibrium. Thus, $CS_{H1}^M(r)$ and $CS_{L1}^M(r)$ are given by:

$$\begin{aligned}
CS_{H1}^M(r) &= \int_{\bar{\theta}_H^M}^1 (\theta - \hat{p}_H^M(r))d\theta = (1 - 2\hat{p}_H^M(r) + \bar{\theta}_H^M)(1 - \bar{\theta}_H^M)/2 \\
&= (1 - 2\hat{p}_H^M(r) + \frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)})(1 - \frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)})/2 \quad \text{and} \\
CS_{L1}^M(r) &= \int_{\bar{\theta}_L^M}^{\bar{\theta}_H^M} (\theta \hat{q}_L^M(r) - \hat{p}_L^M(r))d\theta = (\hat{q}_L^M(r)(\bar{\theta}_H^M + \bar{\theta}_L^M) - 2\hat{p}_L^M(r))(\bar{\theta}_H^M - \bar{\theta}_L^M)/2 \\
&= (\hat{q}_L^M(r)(\frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)} + \frac{\hat{p}_L^M(r)}{\hat{q}_L^M(r)}) - 2\hat{p}_L^M(r))(\frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)} - \frac{\hat{p}_L^M(r)}{\hat{q}_L^M(r)})/2.
\end{aligned}$$

The total consumer surplus in each period is then given by $CS_1^M(r) = CS_{H1}^M(r) + CS_{L1}^M(r)$ and $CS_2^M(r) = (1 + r\hat{d}_{H1}^M(r) + r\hat{d}_{L1}^M(r))CS_1^M(r)$, and total consumer surplus of two periods is $CS^M(r) =$

$CS_1^M(r) + CS_2^M(r)$. We can show that $\frac{dCS_{H1}^M(r)}{dr} > 0$ and $\frac{dCS_{L1}^M(r)}{dr} > 0$. Thus $\frac{dCS_1^M(r)}{dr} > 0$. Moreover one can easily verify that $\frac{d(1+r\hat{d}_{H1}^M(r)+r\hat{d}_{L1}^M(r))}{dr} > 0$. Thus, $\frac{dCS_2^M(r)}{dr} > 0$. Hence, $\frac{dCS^M(r)}{dr} > 0$. Therefore, $CS_1^M(r)$, $CS_2^M(r)$, and $CS^M(r)$ all increase in r. Let CS^b denote the total consumer surplus of two periods in the benchmark case, i.e., $CS^b = CS^M(0)$. Thus, $CS^M(r) > CS^M(0) = CS^b$ for $r \in (0,1]$.

Part (b): Total social welfare is defined as $SW^M(r) = CS^M(r) + \pi_H^M(r) + \pi_L^M(r)$. We have already shown that $CS^M(r)$ increases in r in (a), and that $\pi_H^M(r)$ and $\pi_L^M(r)$ increase in r in Proposition 1. Hence, $SW^M(r)$ increases in r. $\square$

A.2. Value Enhancement Effect from Current Consumers (VEE-C)

Proof of Lemma 2: Given $r = t_p = 0$ and $t_c > 0$ and $0 \le q_L < q_H \le 1$, firm i decides its price p_i to maximize its total profit:

$$\max_{p_i \ge 0} p_i(d_{i1}^{VC} + d_{i2}^{VC}),$$

where $i = H, L$. In the VEE-C case, each period is identical; thus, each firm's demand is the same in each period (i.e., $d_{H1}^{VC} = d_{H2}^{VC}$ and $d_{L1}^{VC} = d_{L2}^{VC}$). If firm i has zero demand in one period, then it has zero demand in both periods and zero total profit. Thus, it is obvious that in the equilibrium, each firm must has positive demand in each period, i.e., $d_{in} > 0$ for $i = H, L$ and $n = 1, 2$. However, due to the value enhancement effect in each period, it is not obvious that in the equilibrium whether the market is fully covered or not. We will focus on the case where the market is partially covered in the following proof; and we will show in the proof of Proposition 2 that there is no equilibrium in the case where the market is fully covered and thus the market being partially covered is the unique equilibrium outcome when t_c is small enough.

Given that the market is partially covered, let $\bar{\theta}_H^{VC} \in (0,1)$ denote the consumer who is indifferent in purchasing from firm H and firm L and $\bar{\theta}_L^{VC} \in (0, \bar{\theta}_H^{VC})$ denote the consumer who is indifferent in purchasing from firm L and not purchasing. Then, the two firms' demands in each period are given by $d_{H1}^{VC} = d_{H2}^{VC} = 1 - \bar{\theta}_H^{VC}$ and $d_{L1}^{VC} = d_{L2}^{VC} = \bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}$, respectively; and a consumer's utility of purchasing from firm H is $u_{Hn} = \theta q_H - p_H + t_c(1 - \bar{\theta}_H^{VC})$ and the utility of purchasing from firm L is $u_{Ln} = \theta q_L - p_L + t_c(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC})$ in period $n = 1, 2$. Thus, $\bar{\theta}_H^{VC}$ and $\bar{\theta}_L^{VC}$ can be solved from:

$$\bar{\theta}_H^{VC} q_H - p_H + t_c(1 - \bar{\theta}_H^{VC}) = \bar{\theta}_H^{VC} q_L - p_L + t_c(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}) \quad \text{and}$$
$$\bar{\theta}_L^{VC} q_L - p_L + t_c(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}) = 0.$$

Hence, we have:

$$\bar{\theta}_H^{VC} = \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} \text{ and } \bar{\theta}_L^{VC} = \frac{t_c(p_H - t_c) + p_L(q_L - q_H + t_c)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}. \tag{A6}$$

Each firm's demand in each period is then given as below:

$$d_{H1}^{VC} = d_{H2}^{VC} = 1 - \bar{\theta}_H^{VC} = 1 - \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} \quad \text{and}$$
$$d_{L1}^{VC} = d_{L2}^{VC} = \bar{\theta}_H^{VC} - \bar{\theta}_L^{VC} = \frac{p_L(q_H - t_c) + q_L(t_c - p_H)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}. \tag{A7}$$

Then, firm i's profit is given by $2p_i d_{i1}^{VC}$. Checking the second-order derivative, we find that firm i's total profit is concave in p_i, i.e., $\frac{\partial^2 2p_i d_{i1}^{VC}}{\partial p_i^2} < 0$, for $i = H, L$. Solving the two firms' first-order conditions together, i.e., $\frac{\partial(2p_H d_{H1}^{VC})}{\partial p_H} = 0$ and $\frac{\partial(2p_L d_{L1}^{VC})}{\partial p_L} = 0$, we get the following prices:

$$p_H^{VC}(q_H, q_L, t_c) = \frac{2q_H^2 (q_L - t_c) - 2q_H (t_c q_L - t_c^2 + q_L^2) + t_c q_L^2}{4q_H (q_L - t_c) - 4t_c q_L + 4t_c^2 - q_L^2} \quad \text{and}$$
$$p_L^{VC}(q_H, q_L, t_c) = \frac{q_L (q_H (t_c - q_L) + 2t_c q_L - 2t_c^2 + q_L^2)}{-4q_H (q_L - t_c) + 4t_c q_L - 4t_c^2 + q_L^2}. \tag{A8}$$

$p_H = p_H^{VC}(q_H, q_L, t_c)$ and $p_L = p_L^{VC}(q_H, q_L, t_c)$ satisfy the conditions of this case $0 < \bar{\theta}_L^{VC} < \bar{\theta}_H^{VC} < 1$ if $0 < t_c < T^{VC}(q_H, q_L)$, where $T^{VC}(q_H, q_L) = (2q_H + q_L - \sqrt{4q_H^2 - 4q_H q_L + 9q_L^2})/4$. Plugging Equations (A8) into Equations (A7), the demand of each firm in each period can be written as:

$$d_{H1}^{VC}(q_H, q_L, t_c) = d_{H2}^{VC}(q_H, q_L, t_c) = \frac{(q_L - t_c)(2q_H^2 (q_L - t_c) - 2q_H (t_c q_L - t_c^2 + q_L^2) + t_c q_L^2)}{(-4q_H (q_L - t_c) + 4t_c q_L - 4t_c^2 + q_L^2)(q_H (t_c - q_L) + t_c q_L - t_c^2 + q_L^2)} \quad \text{and}$$
$$d_{L1}^{VC}(q_H, q_L, t_c) = d_{L2}^{VC}(q_H, q_L, t_c) = \frac{q_L (q_H^2 (q_L - t_c) - q_H (3t_c q_L - 3t_c^2 + q_L^2) + t_c (2t_c q_L - 2t_c^2 + q_L^2))}{(-4q_H (q_L - t_c) + 4t_c q_L - 4t_c^2 + q_L^2)(q_H (t_c - q_L) + t_c q_L - t_c^2 + q_L^2)}. \tag{A9}$$

Firm i's total demand of two periods is given by $d_i^{VC}(q_H, q_L, t_c) = d_{i1}^{VC}(q_H, q_L, t_c) + d_{i2}^{VC}(q_H, q_L, t_c)$, and its total profit is $\pi_i^{VC}(q_H, q_L, t_c) = p_i^{VC}(q_H, q_L, t_c) d_i^{VC}(q_H, q_L, t_c)$, for $i \in \{H, L\}$.

Given that the market is partially covered and each firm has positive demand (i.e., $0 < \bar{\theta}_L^{VC} < \bar{\theta}_H^{VC} < 1$), taking derivative with respect to t_c (note that the sub-game equilibrium outcomes are all differentiable), one can verify that, after simplification:

$$\frac{\partial p_L^{VC}(q_H, q_L, t_c)}{\partial t_c} < 0 \quad \text{and} \quad \frac{\partial d_H^{VC}(q_H, q_L, t_c)}{\partial t_c} > 0.$$

$\frac{\partial p_H^{VC}(q_H, q_L, t_c)}{\partial t_c}$, $\frac{\partial d_L^{VC}(q_H, q_L, t_c)}{\partial t_c}$, $\frac{\partial \pi_H^{VC}(q_H, q_L, t_c)}{\partial t_c}$, and $\frac{\partial \pi_L^{VC}(q_H, q_L, t_c)}{\partial t_c}$ could be > 0, $= 0$ or < 0. Thus, $p_L^{VC}(q_H, q_L, t_c)$ decreases in t_c, $d_H^{VC}(q_H, q_L, t_c)$ increases in t_c; and $p_H^{VC}(q_H, q_L, t_c)$, $d_L^{VC}(q_H, q_L, t_c)$, $\pi_H^{VC}(q_H, q_L, t_c)$, and $\pi_L^{VC}(q_H, q_L, t_c)$ may increase or decrease in t_c. This completes the proofs of Parts (a)-(c). $\square$

Proof of Proposition 2: Similar to the proof of Proposition 1, we complete the proof in three steps: **Step 1.** Solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period; **Step 2.** Show the existence of the equilibrium derived in Step 1 when t_c is small enough; **Step 3.** Show the uniqueness of the equilibrium when t_c is small enough. In the end, we will provide the upper bound for t_c that guarantees t_c is small enough and the equilibrium uniquely exists.

Step 1. We first solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period. We have solved the firms' pricing decisions for given quality levels in the proof of Lemma 2. Given the firms' pricing decisions in Stage 2, we proceed to solve firms' quality decisions in the first stage:

$$\max_{q_H \in (q_L,1]} \pi_H^{VC}(q_H,q_L,t_c) = p_H^{VC}(q_H,q_L,t_c) d_H^{VC}(q_H,q_L,t_c) \quad \text{and} \quad \max_{q_L \in [0,q_H)} \pi_L^{VC}(q_H,q_L,t_c) = p_L^{VC}(q_H,q_L,t_c) d_L^{VC}(q_H,q_L,t_c),$$

where $p_i^{VC}(q_H,q_L,t_c)$ and $d_i^{VC}(q_H,q_L,t_c)$ are given by Equations (A8) and (A9), respectively.

First, one can easily verify that given the market is partially covered and each firm has positive demand (i.e., $0 < \bar{\theta}_L^{VC} < \bar{\theta}_H^{VC} < 1$), $\pi_H^{VC}(q_H,q_L,t_c)$ increases in q_H for any $q_L \in [0,1)$ and $q_H > t_c$ i.e., $\frac{d\pi_H^{VC}(q_H,q_L,t_c)}{dq_H} > 0$. Thus for $q_H > t_c$, the optimal quality is $q_H = 1$. Moreover, it is easy to verify that $\pi_H^{VC}(q_H,q_L,t_c) < \pi_H^{VC}(1,q_L,t_c)$ for any $q_L < q_H \le t_c$ and $q_L \in [0,1)$. Thus, firm H's global optimal quality is $\hat{q}_H^{VC} = 1$ for any $q_L \in [0,1)$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L. Directly checking firm L's profit function, we find that $\pi_L^{VC}(1,q_L,t_c)$ first increases and then decreases in q_L if $0 < t_c < \frac{23-3\sqrt{41}}{40} \approx 0.0948$. Therefore, there exists a unique $\hat{q}_L^{VC}(t_c)$ that can be solved from $\frac{\partial \pi_L^{VC}(1,q_L,t_c)}{\partial q_L} = 0$ when t_c is small enough.

Hence, when t_c is small enough, given that the market is partially covered and each firm has positive demand in each period, the potential equilibrium outcome is: $q_H = \hat{q}_H^{VC} = 1$, $q_L = \hat{q}_L^{VC}(t_c)$, $p_H = \hat{p}_H^{VC}(t_c) = p_H^{VC}(1,\hat{q}_L^{VC}(t_c),t_c)$, and $p_L = \hat{p}_L^{VC}(t_c) = p_L^{VC}(1,\hat{q}_L^{VC}(t_c),t_c)$. Let $\sigma^{VC} = (\hat{q}_H^{VC}, \hat{q}_L^{VC}(t_c), \hat{p}_H^{VC}(t_c), \hat{p}_L^{VC}(t_c))$ denote this potential equilibrium, and let $\hat{d}_i^{VC}(t_c) = d_i^{VC}(1,\hat{q}_L^{VC}(t_c),t_c)$ and $\hat{\pi}_i^{VC}(t_c) = \hat{p}_i^{VC}(t_c)\hat{d}_i^{VC}(t_c)$ be firm i's total demand and profit at the potential equilibrium σ^{VC}.

Step 2. To show the existence of the equilibrium σ^{VC}, we will show that σ^{VC} is a Sub-game Perfect Equilibrium (SPE) when t_c is small enough. We just need to verify that σ^{VC} satisfies the no-deviation requirements of SPE: (1) Given firms' quality $0 \le q_L < q_H \le 1$ and firm i's price $p_i^{VC}(q_H,q_L,t_c)$, firm j's price decision will not deviate from $p_j^{VC}(q_H,q_L,t_c)$ to any other price. (2) Given both firms' pricing strategies $(p_H^{VC}(q_H,q_L,t_c), p_L^{VC}(q_H,q_L,t_c))$ in Stage 2 and firm i's quality $\hat{q}_i^{VC}(r)$, firm j's quality decision will not deviate from $\hat{q}_j^{VC}(t_c)$ to any other quality, i.e., $\hat{\pi}_j^{VC}(t_c) \ge \max_{q_j \neq \hat{q}_j^{VC}(t_c)} \pi_j^{VC}(q_j, \hat{q}_i^{VC}(t_c), t_c)$, for $i,j \in \{H,L\}$ and $i \neq j$.

To show (1), we just need to show

$$\pi_j^{VC}(q_H,q_L,t_c) \ge \max_{p_j \neq p_j^{VC}(q_H,q_L,t_c)} p_j d_j^{VC}(p_j | q_H, q_L, p_i^{VC}(q_H,q_L,t_c)), \text{for } i,j \in \{H,L\} \text{ and } i \neq j, \tag{A10}$$

where the general demand functions in period $n = 1,2$ are given by:

$$(d_{Hn}^{VC}(q_H,q_L,p_H,p_L), d_{Ln}^{VC}(q_H,q_L,p_H,p_L)) = \begin{cases} (1 - \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}, \frac{p_L(q_H - t_c) + q_L(t_c - p_H)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}), & \text{if } (p_H,p_L) \in C_1 \\ (1 - \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c}, \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c}), & \text{if } (p_H,p_L) \in C_2 \\ (1 - \frac{p_H - t_c}{q_H - t_c}, 0), & \text{if } (p_H,p_L) \in C_3 \\ (1,0), & \text{if } (p_H,p_L) \in C_4 \\ (0, 1 - \frac{p_L - t_c}{q_L - t_c}), & \text{if } (p_H,p_L) \in C_5 \\ (0,1), & \text{if } (p_H,p_L) \in C_6 \end{cases}$$

Note that the above demand functions include all the situations that market is partially or fully covered and each firm has positive or zero demand. In the above equation, C_i $(i = 1, 2, ..., 6)$ is a set of conditions that defines a region for the price pair: $C_1 = \{0 < \frac{t_c(p_H - t_c) + p_L(q_L - q_H + t_c)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} < \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} < 1\}$, in which the market is partially covered and each firm has positive demand; $C_2 = \{0 < \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c} < 1, t_c \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c} \geq p_L\}$, in which the market is fully covered and each firm has positive demand; $C_3 = \{0 < \frac{p_H - t_c}{q_H - t_c} < 1, \frac{p_H - t_c}{q_H - t_c} q_L - p_L < 0\}$, in which the market is partially covered and firm L has no demand; $C_4 = \{t_c \geq p_H\}$, in which the market is fully covered and firm L has no demand; $C_5 = \{0 < \frac{p_L - t_c}{q_L - t_c} < 1, q_H - p_H \leq q_L - p_L + t_c(1 - \frac{p_L - t_c}{q_L - t_c})\}$, in which the market is partially covered and firm H has no demand; and $C_6 = \{t_c \geq p_L, q_H - p_H \leq q_L - p_L + t_c\}$, in which the market is fully covered and firm H has no demand. Firm i's total demand of two periods is given by $d_i^{VC} = d_{i1}^{VC} + d_{i2}^{VC}$, for $i \in \{H, L\}$. Through straightforward yet tedious algebraic analysis, we can verify that firm L will not deviate from $p_L^{VC}(q_H, q_L, t_c)$ when t_c is small enough (i.e., $0 < t_c < T^{VC}(q_H, q_L)$ and $t_c < \frac{23 - 3\sqrt{41}}{40} \approx 0.0948$); and firm H will not deviate from $p_H^{VC}(q_H, q_L, t_c)$ when $t_c < T_1^{VC}(q_H, q_L)$, where $T_1^{VC}(q_H, q_L)$ is the smallest root of the polynomial $-16t_c^6(5q_H + 2q_L) + 4t_c^5(42q_H q_L + 32q_H^2 - 5q_L^2) - 8t_c^4(35q_H^2 q_L - 3q_H q_L^2 + 10q_H^3 - 6q_L^3) + t_c^3(184q_H^3 q_L + 61q_H^2 q_L^2 - 162q_H q_L^3 + 16q_H^4 + 9q_L^4) + t_c^2 q_L(-97q_H^3 q_L + 153q_H^2 q_L^2 + q_H q_L^3 - 40q_H^4 - 17q_L^4) + t_c q_L^2(q_H - q_L)^2(33q_H q_L + 32q_H^2 - 5q_L^2) + 16t_c^7 + 8q_H q_L^6 - 24q_H^2 q_L^5 + 24q_H^3 q_L^4 - 8q_H^4 q_L^3 = 0$. Thus, inequality (A10) holds and requirement (1) is satisfied when t_c is small enough.

For requirement (2), we have shown in Step 1 that when $t_c < 0.0948$, firm H's optimal quality is $q_H = 1$ for any $q_L \in [0, 1)$ and firm L's optimal quality is $q_L = \hat{q}_L^{VC}(t_c)$ given $q_H = 1$. Thus, both firms will not deviate and requirement (2) holds. Hence, we have shown that σ^{VC} is an SPE when t_c is small enough.

Step 3. We will show the uniqueness of the equilibrium σ^{VC} when t_c is small enough. Note that in Step 1 we have derived the unique equilibrium σ^{VC} by backward induction under the condition that the market is partially covered and each firm has positive demand in each period. Thus, we just need to show that there does not exist any equilibrium if the above condition does not hold, i.e., the market is fully covered or one of the firms has zero demand. Since the two periods are identical, we just need to verify that: (1) Any strategy that leads to $d_{H1} = 0$ or $d_{L1} = 0$ is not an equilibrium. (2) Any strategy that leads to $d_{H1} + d_{L1} = 1$ is not an equilibrium.

If $d_{i1} = 0$, then $d_{i2} = 0$ and thus firm i's profit is zero. It is obvious that any strategy with $d_{i1} = 0$ cannot be an equilibrium outcome since firm i has incentive to deviate to a lower price to get a positive demand and earn a positive profit. Thus (1) holds.

If $d_{H1}+d_{L1}=1$, then the market is fully covered. We will show that there does not exist any SPE in this case by contradiction. Assume there exists an SPE in this case, then we can solve it by backward induction. Let $\bar{\theta}^{VC}_{H,F}\in(0,1)$ denote the consumer who is indifferent in purchasing from firm H and firm L. Since the market is fully covered by the two firms, then each firm's demand in each period is given by $d^{VC}_{H1}=d^{VC}_{H2}=1-\bar{\theta}^{VC}_{H,F}$ and $d^{VC}_{L1}=d^{VC}_{L2}=\bar{\theta}^{VC}_{H,F}$. Thus, $\bar{\theta}^{VC}_{H,F}$ can be solved from $\bar{\theta}^{VC}_{H,F}q_H-p_H+t_c(1-\bar{\theta}^{VC}_{H,F})=\bar{\theta}^{VC}_{H,F}q_L-p_L+t_c\bar{\theta}^{VC}_{H,F}$. Thus, we get $\bar{\theta}^{VC}_{H,F}=\frac{p_H-p_L-t_c}{q_H-q_L-2t_c}$. Note that in this case, consumer with $\theta=0$ will purchase from firm L and has non-negative utility, i.e., $u_{L1}|_{\theta=0}=-p_L+t_c\bar{\theta}^{VC}_{H,F}\geq 0$. Each firm's demand in each period is then given as below:

$$d^{VC}_{H1}=d^{VC}_{H2}=1-\bar{\theta}^{VC}_{H,F}=1-\frac{p_H-p_L-t_c}{q_H-q_L-2t_c}\text{ and }d^{VC}_{L1}=d^{VC}_{L2}=\bar{\theta}^{VC}_{H,F}=\frac{p_H-p_L-t_c}{q_H-q_L-2t_c}.$$

In Stage 2, given quality levels $0\leq q_L<q_H\leq 1$, firm i decides price p_i to maximize its total profit $2p_id^{VC}_{i1}$. Checking the derivative, we find that firm i's total profit is concave in $p_i$if $q_H-q_L-2t_c>0$ and increases in p_i if $q_H-q_L-2t_c<0$, i.e., $\frac{\partial^2(2p_id^{VC}_{i1})}{\partial p_i^2}<0$ if $q_H-q_L-2t_c>0$ and $\frac{\partial(2p_id^{VC}_{i1})}{\partial p_i}>0$ if $q_H-q_L-2t_c<0$, for $i=H,L$. When $q_H-q_L-2t_c>0$, solving the two firms' first-order conditions together (i.e., $\frac{\partial(2p_Hd^{VC}_{H1})}{\partial p_H}=0$ and $\frac{\partial(2p_Ld^{VC}_{L1})}{\partial p_L}=0$), we can get the following prices:

$$p^{VC}_{H,F}(q_H,q_L,t_c)=\frac{2q_H-2q_L-3t_c}{3}\quad\text{and}\quad p^{VC}_{L,F}(q_H,q_L,t_c)=\frac{q_H-q_L-3t_c}{3}.$$

However, the above prices violate the conditions of $0<\bar{\theta}^{VC}_{H}=\frac{p_H-p_L-t_c}{q_H-q_L-2t_c}<1$ and $u_{L1}|_{\theta=0}=-p_L+t_c\bar{\theta}^{VC}_{H}\geq 0$. When $q_H-q_L-2t_c<0$, firm i's profit increases in p_i and thus its optimal price is on the boundary in this case. We find that for any p_H, there does not exist optimal p_L for firm L that satisfies $0<\bar{\theta}^{VC}_{H}=\frac{p_H-p_L-t_c}{q_H-q_L-2t_c}<1$ and $u_{L1}|_{\theta=0}=-p_L+t_c\bar{\theta}^{VC}_{H}\geq 0$. Thus, no pricing equilibrium exists in the case of fully covered market. That is, any strategy that leads to $d_{H1}+d_{L1}=1$ cannot be an equilibrium. Therefore, (2) holds.

Hence, we conclude that in the equilibrium, the market is partially covered and each firm has positive demand in each period (i.e., $d_{Hi}>0$, $d_{Li}>0$, and $d_{Hi}+d_{Li}<1$, for $i=1,2$).

Combining Steps 1-3, we show that σ^{VC} is the unique SPE when t_c is small enough. Moreover, in the main body of the paper and the appendix , we define $\bar{t}_c=0.07$ and assume $0<t_c\leq\bar{t}_c$ to make sure t_c is small enough which guarantees the existence and uniqueness of the equilibrium σ^{VC}.

Finally, we proceed to prove parts (a)-(d) below. Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}^{VC}_L(t_c)$ is the unique solution to $\frac{\partial\pi^{VC}_L(1,q_L,t_c)}{\partial q_L}=0$, by the Implicit Function Theorem, we have: $\frac{\partial\hat{q}^{VC}_L(t_c)}{\partial t_c}=-\frac{\partial^2\pi^{VC}_L(1,q_L,t_c)}{\partial q_L\partial t_c}/\frac{\partial^2\pi^{VC}_L(1,q_L,t_c)}{\partial q_L^2}|_{q_L=\hat{q}^{VC}_L(t_c)}$. One can verify that $\frac{\partial^2\pi^{VC}_L(1,q_L,t_c)}{\partial q_L^2}<0$ and $\frac{\partial^2\pi^{VC}_L(1,q_L,t_c)}{\partial q_L\partial t_c}<0$ for any $q_L\in[0.26,0.6]$ and $0<t_c\leq\bar{t}_c$. Moreover, we find that $0.26<\hat{q}^{VC}_L(t_c)<0.6$

holds for $0 < t_c \le \bar{t}_c$, since $\frac{\partial \pi_L^{VC}(1,q_L,t_c)}{\partial q_L} < 0$ for any $q_L \in [0.6, 1)$ and $\frac{\partial \pi_L^{VC}(1,q_L,t_c)}{\partial q_L} > 0$ for any $q_L \in [0, 0.26]$. Therefore, $\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} < 0$.

Part (b): Note that $\hat{p}_i^{VC}(t_c) = p_i^{VC}(1, \hat{q}_L^{VC}(t_c), t_c)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_i^{VC}(t_c)}{\partial t_c} = (\frac{\partial p_i^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial p_i^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c})|_{q_L=\hat{q}_L^{VC}(t_c)}$. One can verify that $\frac{\partial p_H^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial p_H^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial p_L^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial p_L^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} < 0$ for any $q_L \in [0.26, 0.6]$ and $0 < t_c \le \bar{t}_c$. As already shown in Part (a) above, $\hat{q}_L^{VC}(t_c) \in (0.26, 0.6)$. Therefore, $\frac{\partial \hat{p}_H^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial \hat{p}_L^{VC}(t_c)}{\partial t_c} < 0$. Hence, $\hat{p}_H^{VC}(t_c) > \hat{p}_H^{VC}(0) = \hat{p}_H^b$ and $\hat{p}_L^{VC}(t_c) < \hat{p}_L^{VC}(0) = \hat{p}_L^b$ for $0 < t_c \le \bar{t}_c$.

Part (c): Note that $\hat{d}_i^{VC}(t_c) = d_i^{VP}(1, \hat{q}_L^{VC}(t_c), t_c)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{VC}(t_c)}{\partial t_c} = (\frac{\partial d_i^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial d_i^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c})|_{q_L=\hat{q}_L^{VC}(t_c)}$. One can verify that $\frac{\partial d_H^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial d_H^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} > 0$ for any for any $q_L \in [0.26, 0.6]$ and $0 < t_c \le \bar{t}_c$. Since $\hat{q}_L^{VC}(t_c) \in (0.26, 0.6)$, we have $\frac{\partial \hat{d}_H^{VC}(t_c)}{\partial t_c} > 0$, i.e, $\hat{d}_H^{VC}(t_c)$ increases in t_c. Moreover, one can verify that $\frac{\partial d_L^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial d_L^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c}$ is negative when t_c is close to 0 and is positive when t_c is close to $\bar{t}_c$. Thus, $\hat{d}_L^{VC}(t_c)$ may decrease or increase in t_c.

Part (d): Note that $\hat{\pi}_i^{VC}(t_c) = \pi_i^{VC}(1, \hat{q}_L^{VC}(t_c), t_c)$. By the chain rule, $\frac{\partial \hat{\pi}_i^{VC}(t_c)}{\partial t_c} = (\frac{\partial \pi_i^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial \pi_i^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c})|_{q_L=\hat{q}_L^{VC}(t_c)}$. One can verify that $\frac{\partial \pi_H^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial \pi_H^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial \pi_L^{VC}(1,q_L,t_c)}{\partial t_c} + \frac{\partial \pi_L^{VC}(1,q_L,t_c)}{\partial q_L}\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} < 0$ for $q_L \in (0.26, 0.6)$ and $0 < t_c \le \bar{t}_c$. Since $\hat{q}_L^{VC}(t_c) \in (0.26, 0.6)$, we have $\frac{\partial \hat{\pi}_H^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial \hat{\pi}_L^{VC}(t_c)}{\partial t_c} < 0$, i.e., $\hat{\pi}_H^{VC}(t_c)$ increases in t_c and $\hat{\pi}_L^{VC}(t_c)$ decreases in t_c. Therefore, $\hat{\pi}_H^{VC}(t_c) - \hat{\pi}_H^b > 0 > \hat{\pi}_L^{VC}(t_c) - \hat{\pi}_L^b$. $\square$

Proof of Corollary 2: **Part (a):** Let $CM_{in}^{VC}(t_c)$ denote consumer monetary surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$CM_{H1}^{VC}(t_c) = CM_{H2}^{VC}(t_c) = \int_{\bar{\theta}_H^{VC}}^{1} (\theta - \hat{p}_H^{VC}(t_c))d\theta = (1 - 2\hat{p}_H^{VC}(t_c) + \bar{\theta}_H^{VC})(1 - \bar{\theta}_H^{VC})/2 \quad \text{and}$$
$$CM_{L1}^{VC}(t_c) = CM_{L2}^{VC}(t_c) = \int_{\bar{\theta}_L^{VC}}^{\bar{\theta}_H^{VC}} (\theta\hat{q}_L^{VC}(t_c) - \hat{p}_L^{VC}(t_c))d\theta = (\hat{q}_L^{VC}(t_c)(\bar{\theta}_H^{VC} + \bar{\theta}_L^{VC}) - 2\hat{p}_L^{VC}(t_c))(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC})/2,$$

where $\bar{\theta}_H^{VC} = \frac{\hat{p}_L^{VC}(t_c)\hat{q}_L^{VC}(t_c) + (\hat{p}_H^{VC}(t_c) - t_c)(t_c - \hat{q}_L^{VC}(t_c))}{\hat{q}_L^{VC}(t_c)^2 + t_c\hat{q}_L^{VC}(t_c) + (t_c - \hat{q}_L^{VC}(t_c)) - t_c^2}$ and $\bar{\theta}_L^{VC} = \frac{t_c(\hat{p}_H^{VC}(t_c) - t_c) + \hat{p}_L^{VC}(t_c)(\hat{q}_L^{VC}(t_c) - 1 + t_c)}{\hat{q}_L^{VC}(t_c)^2 + t_c\hat{q}_L^{VC}(t_c) + (t_c - \hat{q}_L^{VC}(t_c)) - t_c^2}$. $\bar{\theta}_H^{VC}$ is the consumer who is indifferent in purchasing from firm H and firm L; $\bar{\theta}_L^{VC}$ is the consumer who is indifferent in purchasing from firm L and not purchasing. Consumer monetary surplus in period n is then given by $CM_n^{VC}(t_c) = CM_{Hn}^{VC}(t_c) + CM_{Ln}^{VC}(t_c)$, and total consumer monetary surplus of two periods is given by $CM^{VC}(t_c) = CM_1^{VC}(t_c) + CM_2^{VC}(t_c)$.

We can show that $\frac{dCM_{Hn}^{VC}(t_c)}{dt_c} < 0$ and $\frac{dCM_{Ln}^{VC}(t_c)}{dt_c} < 0$ for $0 < t_c \le \bar{t}_c$ and $n = 1, 2$. Thus $\frac{dCM_1^{VC}(t_c)}{dt_c} < 0$, $\frac{dCM_2^{VC}(t_c)}{dt_c} < 0$, and $\frac{dCM^{VC}(t_c)}{dt_c} < 0$. Therefore, $CM_1^{VC}(t_c)$, $CM_2^{VC}(t_c)$, and $CM^{VC}(t_c)$ all decrease in t_c. Let CM^b denote the total consumer monetary surplus of two periods in the benchmark case, i.e., $CM^b = CM^{VC}(0)$. Thus, $CM^{VC}(t_c) < CM^{VC}(0) = CM^b$ for $0 < t_c \le \bar{t}_c$.

Part (b): Let $CS_{in}^{VC}(t_c)$ denote consumer total surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$\begin{aligned}
CS_{Hn}^{VC}(t_c) &\int_{\bar{\theta}_H^{VC}}^{1}(\theta - \hat{p}_H^{VC}(t_c) + t_c\hat{d}_{Hn}^{VC}(t_c))d\theta \\
&= \frac{1}{2}(1 - 2\hat{p}_H^{VC}(t_c) + 2t_c\hat{d}_{Hn}^{VC}(t_c) + \bar{\theta}_H^{VC})(1 - \bar{\theta}_H^{VC}) \quad \text{and} \\
CS_{Ln}^{VC}(t_c) &= \int_{\bar{\theta}_L^{VC}}^{\bar{\theta}_H^{VC}}(\theta\hat{q}_L^{VC}(t_c) - \hat{p}_L^{VC}(t_c) + t_c\hat{d}_{Ln}^{VC}(t_c))d\theta \\
&= \frac{1}{2}(\hat{q}_L^{VC}(t_c)(\bar{\theta}_H^{VC} + \bar{\theta}_L^{VC}) - 2\hat{p}_L^{VC}(t_c) + 2t_c\hat{d}_{Ln}^{VC}(t_c))(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}), \textit{ for } n = 1,2.
\end{aligned}$$

Consumer total surplus in period n is given by $CS_n^{VC}(t_c) = CS_{Hn}^{VC}(t_c) + CS_{Ln}^{VC}(t_c)$, and consumer total surplus of two periods is given by $CS^{VC}(t_c) = CS_1^{VC}(t_c) + CS_2^{VC}(t_c)$.

Given $0 < t_c \leq \bar{t}_c$, we can show that $\frac{dCS_{Hn}^{VC}(t_c)}{dt_c} < 0$ if $t_c < 0.0128246$ and $\frac{dCS_{Hn}^{VC}(t_c)}{dt_c} \geq 0$ otherwise; and $\frac{dCS_{Ln}^{VC}(t_c)}{dt_c} < 0$, for $n = 1,2$. Moreover, we can also show that $\frac{dCS^{VC}(t_c)}{dt_c} < 0$ if $t_c < 0.0603381$ and $\frac{dCS^{VC}(t_c)}{dt_c} \geq 0$ otherwise. Therefore, $CS^{VC}(t_c)$ first decreases and then increases in t_c.

Part (c): The monetary term of social welfare is defined as $SM^{VC}(t_c) = \pi_H^{VC}(t_c) + \pi_L^{VC}(t_c) + CM^{VC}(t_c)$. We have already shown that $CM^{VC}(t_c)$ decreases in t_c in part (a), and that $\pi_H^{VC}(t_c)$ increases in t_c and $\pi_L^{VC}(t_c)$ decreases in t_c in Proposition 2(d). We can verify that $\frac{dSM^{VC}(t_c)}{dt_c} < 0$, indicating $SM^{VC}(t_c)$ decreases in t_c. Thus, $SM^{VC}(t_c) < SM^{VC}(0) = SM^b$.

Part (d): The social welfare is defined as $SW^{VC}(t_c) = \pi_H^{VC}(t_c) + \pi_L^{VC}(t_c) + CS^{VC}(t_c)$. Given $0 < t_c \leq \bar{t}_c$, we can verify that $\frac{dSW^{VC}(t_c)}{dt_c} > 0$, indicating $SW^{VC}(t_c)$ increases in t_c, Thus, $SW^{VC}(t_c) > SW^{VC}(0) = SW^b$. $\square$

A.3. Value Enhancement Effect from Previous Consumers (VEE-P)

Proof of Lemma 3: Given $r = t_c = 0$ and $t_p > 0$ and $0 \leq q_L < q_H \leq 1$, firm i decides its price p_i to maximize its total profit:

$$\max_{p_i \geq 0} p_i d_{i1}^{VP} + p_i d_{i2}^{VP}, \text{ where } i = H, L.$$

It is obvious that in the equilibrium $p_H > 0$ and $p_L > 0$, which indicates that the market is partially covered by the two firms in period 1 since a consumer with $\theta = 0$ always receives negative utility of purchasing from either firm. Moreover, if firm i has zero demand in period 1, then there is no value enhancement effect for firm i in period 2 and its demand in period 2 is also zero, which leads to zero total profit. Thus, in the equilibrium, each firm must has positive demand in period 1, i.e., $0 < \frac{p_L}{q_L} < \frac{p_H - p_L}{q_H - q_L} < 1$. Thus, the demand of each firm in period 1 can be solved as below:

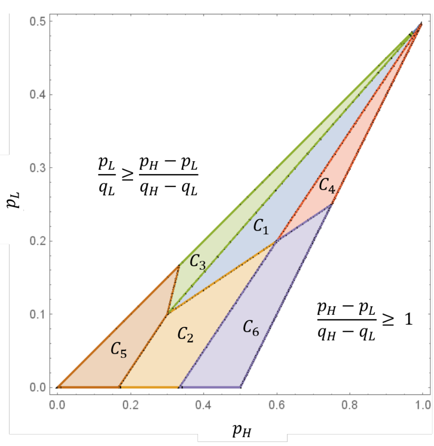
$$d_{H1}^{VP} = 1 - \frac{p_H - p_L}{q_H - q_L} \quad \text{and} \quad d_{L1}^{VP} = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}. \tag{A11}$$

However, due to the value enhancement effect, in period 2, it is not obvious that in the equilibrium whether the market is fully covered or not and whether each firm has zero demand or not. The general demand of each firm in period 2 can be solved as below:

$$(d_{H2}^{VP}, d_{L2}^{VP}) = \begin{cases} (1 - \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}, \frac{(p_H - t d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} - \frac{p_L - t_p d_{L1}^{VP}}{q_L}), & \text{if } (p_H, p_L) \in C_1; \\ (1 - \frac{(p_H - t d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}, \frac{(p_H - t d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}), & \text{if } (p_H, p_L) \in C_2; \\ (1 - (p_H - t_p d_{H1}^{VP}), 0), & \text{if } (p_H, p_L) \in C_3; \\ (0, 1 - \frac{p_L - t d_{L1}^{VP}}{q_L}), & \text{if } (p_H, p_L) \in C_4; \\ (1, 0), & \text{if } (p_H, p_L) \in C_5; \\ (0, 1), & \text{if } (p_H, p_L) \in C_6. \end{cases} \tag{A12}$$

In the above, C_i $(i = 1, 2, \cdots, 6)$ is a set of conditions that defines a region for the price pair: $C_1 = \{0 < \frac{(p_L - t_p d_{L1}^{VP})}{q_L} < \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} < 1\}$; $C_2 = \{\frac{(p_L - t_p d_{L1}^{VP})}{q_L} \leq 0 < \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} < 1\}$; $C_3 = \{0 \leq p_H - t_p d_{H1}^{VP} \leq \frac{(p_L - t_p d_{L1}^{VP})}{q_L}\}$; $C_4 = \{0 \leq p_L - t_p d_{L1}^{VP} \leq \min\{q_L, q_L - q_H + (p_H - t_p d_{H1}^{VP})\}\}$; $C_5 = \{p_H - t d_{H1}^{VP} \leq \min\{0, p_L - t_p d_{L1}^{VP}\}\}$; and $C_6 = \{p_L - t_p d_{L1}^{VP} \leq \min\{0, q_L - q_H + (p_H - t_p d_{H1}^{VP})\}\}$. See Figure A1 for an illustration. We will say "under Condition i" if $(p_H, p_L) \in C_i$. Under Condition 1, the market is not fully covered and each firm has positive demand in period 2; under Condition 2, the market is fully covered in period 2 and each firm has positive demand in period 2; under other conditions, one of the firms has zero demand in period 2.

Figure A1 Six Cases for Demand in Period 2 under VEE-P



Note: in the figure, $q_H = 1$ and $q_L = 0.5$, and C_i presents Condition i for $i = 1, 2, ..., 6$.

Next, we solve the firms' pricing decisions for given quality in the equilibrium in two steps. First, we find all the possible sub-game equilibrium candidates in each region of C_i. Second, we show when and which candidate could be an equilibrium outcome. We will show that when t_p is small enough, the unique equilibrium only exists in the region C_1 where the market is partially covered and each firm has positive demand in each period.

Under Condition 1: $0 < \frac{(p_L - t_p d_{L1}^{VP})}{q_L} < \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} < 1$. Each firm's profit can be written as:

$$\pi_H^{VP}(q_H, q_L, t_p, p_H, p_L) = p_H\left(1 - \frac{p_H - p_L}{q_H - q_L}\right) + p_H\left(1 - \frac{(p_H - t_p(1 - \frac{p_H - p_L}{q_H - q_L})) - (p_L - t_p(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}))}{q_H - q_L}\right), \text{ and}$$

$$\pi_L^{VP}(q_H, q_L, t_p, p_L, p_H) =$$
$$p_L\left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right) + p_L\left(\frac{(p_H - t_p(1 - \frac{p_H - p_L}{q_H - q_L})) - (p_L - t_p(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}))}{q_H - q_L} - \frac{p_L - t_p(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L})}{q_L}\right).$$

One can verify that firm i's total profit is concave in p_i, i.e., $\frac{\partial^2 \pi_i^{VP}}{\partial p_i^2} < 0$, for $i = H, L$. Suppose that there exists an equilibrium outcome $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$ under Condition 1, which satisfies $\frac{\partial \pi_H^{VP}}{\partial p_H}|_{p_H = p_H^{VP}(q_H, q_L, t_p), p_L = p_L^{VP}(q_H, q_L, t_p)} = 0$ and $\frac{\partial \pi_L^{VP}}{\partial p_L}|_{p_H = p_H^{VP}(q_H, q_L, t_p), p_L = p_L^{VP}(q_H, q_L, t_p)} = 0$. Thus, $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$ can be solved as below:

$$p_H^{VP}(q_H, q_L, t_p) = \frac{(q_H - q_L)\left(4q_H^3(2q_L + t_p) + 2q_H^2\left(t_p^2 - 8q_L^2\right) + q_H q_L\left(-2q_L t_p + 8q_L^2 - t_p^2\right) + q_L^2 t_p(t_p - 2q_L)\right)}{8q_H^3(2q_L + t_p) + q_H^2(4q_L t_p - 36q_L^2 + 7t_p^2) + 2q_H q_L(-4q_L t_p + 12q_L^2 - t_p^2) + q_L^2(-4q_L t_p - 4q_L^2 + 7t_p^2)},$$
$$p_L^{VP}(q_H, q_L, t_p) = \frac{q_L(q_H - q_L)\left(2q_H^2(2q_L + t_p) + q_H\left(-2q_L t_p - 8q_L^2 + t_p^2\right) - 3q_L t_p^2 + 4q_L^3\right)}{8q_H^3(2q_L + t_p) + q_H^2(4q_L t_p - 36q_L^2 + 7t_p^2) + 2q_H q_L(-4q_L t_p + 12q_L^2 - t_p^2) + q_L^2(-4q_L t_p - 4q_L^2 + 7t_p^2)}. \tag{A13}$$

One can verify that $p_H = p_H^{VP}(q_H, q_L, t_p)$ and $p_L = p_L^{VP}(q_H, q_L, t_p)$ satisfy Condition 1 iff $0 \leq t_p < T^{VP}(q_H, q_L)$, where $T^{VP}(q_H, q_L)$ is the third (or largest) root of the polynomial $(q_L + q_H)^2 t_p^3 + (-5q_L^3 + 2q_H q_L^2 + q_H^2 q_L + 2q_H^3) t_p^2 + (2q_H q_L^3 - 4q_H^2 q_L^2 + 2q_H^3 q_L) t_p - 4q_H^3 q_L^2 + 4q_L^5 - 12q_H q_L^4 + 12q_H^2 q_L^3 = 0$. Thus, $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$ is a possible equilibrium candidate when $0 \leq t_p < T^{VP}(q_H, q_L)$.

Under Conditions 2-6: Similar as the analysis under Condition 1, we can solve the equilibrium candidates under each condition and verify whether this equilibrium satisfies the condition requirements. Through tedious but straightforward algebraic analysis, we can show that when $0 \leq t_p < T^{VP}(q_H, q_L)$, there does not exist any equilibrium under Conditions 2-6. Detailed analysis is available from authors upon request.

Therefore, for any given $0 \leq q_L < q_H \leq 1$ and $0 \leq t_p < T^{VP}(q_H, q_L)$, the firms' p rices are $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$, under which the market is partially covered and each firm has positive demand in each period. In the proof of Proposition 3, we will further show that when t_p is small enough, the market is partially covered and each firm has positive demand in each period in the SPE. Plugging Equations (A13) into Equation (A12), the demand of each firm in each period can be written as:

$$
\begin{aligned}
d_{H1}^{VP}(q_H,q_L,t_p) &= \frac{4q_H^3\left(2q_L+t_p\right)+q_H^2\left(6q_Lt_p-16q_L^2+5t_p^2\right)+8q_Hq_L^2\left(q_L-t_p\right)+q_L^2t_p\left(3t_p-2q_L\right)}{8q_H^3\left(2q_L+t_p\right)+q_H^2\left(4q_Lt_p-36q_L^2+7t_p^2\right)+2q_Hq_L\left(-4q_Lt_p+12q_L^2-t_p^2\right)+q_L^2\left(-4q_Lt_p-4q_L^2+7t_p^2\right)},\\
d_{L1}^{VP}(q_H,q_L,t_p) &= \frac{2q_H^3\left(2q_L+t_p\right)+q_H^2\left(2q_Lt_p-8q_L^2+t_p^2\right)+2q_Hq_L\left(-q_Lt_p+2q_L^2+t_p^2\right)+q_L^2t_p\left(t_p-2q_L\right)}{8q_H^3\left(2q_L+t_p\right)+q_H^2\left(4q_Lt_p-36q_L^2+7t_p^2\right)+2q_Hq_L\left(-4q_Lt_p+12q_L^2-t_p^2\right)+q_L^2\left(-4q_Lt_p-4q_L^2+7t_p^2\right)},\\
d_{H2}^{VP}(q_H,q_L,t_p) &= \frac{X_3-q_Hq_L\left(-10q_L^2t_p+3q_Lt_p^2+8q_L^3+2t_p^3\right)+q_L^2t_p\left(-3q_Lt_p+2q_L^2+2t_p^2\right)}{\left(q_H-q_L\right)X_2},\\
d_{L2}^{VP}(q_H,q_L,t_p) &= \frac{X_4+q_Hq_L^2\left(-8q_L^2t_p+5q_Lt_p^2-4q_L^3+t_p^3\right)+q_L^3t_p\left(q_Lt_p+2q_L^2-3t_p^2\right)}{q_L\left(q_H-q_L\right)X_2},
\end{aligned}
\tag{A14}
$$

where $X_2=\left(8q_H^3\left(2q_L+t_p\right)-q_H^2\left(36q_L^2-7t_p^2-4q_Lt_p\right)+2q_Hq_L\left(12q_L^2-t_p^2-4q_Lt_p\right)-q_L^2\left(4q_Lt_p+4q_L^2-7t_p^2\right)\right)$, $X_3=4q_H^4\left(2q_L+t_p\right)+q_H^3\left(6q_Lt_p-24q_L^2+7t_p^2\right)+q_H^2\left(-22q_L^2t_p-q_Lt_p^2+24q_L^3+4t_p^3\right)$,
and $X_4=2q_H^4\left(3q_Lt_p+2q_L^2+t_p^2\right)+q_H^3\left(-16q_L^2t_p-q_Lt_p^2-12q_L^3+t_p^3\right)+q_H^2q_L\left(16q_L^2t_p-7q_Lt_p^2+12q_L^3-3t_p^3\right)$.
Firm i's total demand of two periods is given by $d_i^{VP}(q_H,q_L,t_p)=d_{i1}^{VP}(q_H,q_L,t_p)+d_{i2}^{VP}(q_H,q_L,t_p)$, and its total profit is $\pi_i^{VP}(q_H,q_L,t_p)=p_i^{VP}(q_H,q_L,t_p)d_i^{VP}(q_H,q_L,t_p)$, for $i\in\{H,L\}$.
For $0\le q_L<q_H\le 1$ and $0<t_p<T^{VP}(q_H,q_L)$, taking derivative with respect to t_p (note that the sub-game equilibrium outcomes are all differentiable), one can verify that, after simplification:

$$
\frac{\partial p_H^{VP}(q_H,q_L,t_p)}{\partial t_p}<0,\quad \frac{\partial p_L^{VP}(q_H,q_L,t_p)}{\partial t_p}<0,\quad \text{and } \frac{\partial d_H^{VP}(q_H,q_L,t_p)}{\partial t_p}>0.
$$

However, $\frac{\partial d_L^{VP}(q_H,q_L,t_p)}{\partial t_p}$, $\frac{\partial \pi_H^{VP}(q_H,q_L,t_p)}{\partial t_p}$, and $\frac{\partial \pi_L^{VP}(q_H,q_L,t_p)}{\partial t_p}$ could be >0, $=0$ or <0. Thus, $p_H^{VP}(q_H,q_L,t_p)$ and $p_L^{VP}(q_H,q_L,t_p)$ decrease in t_p; $d_H^{VP}(q_H,q_L,t_p)$ increases in t_p; and $d_L^{VP}(q_H,q_L,t_p)$, $\pi_H^{VP}(q_H,q_L,t_p)$, and $\pi_L^{VP}(q_H,q_L,t_p)$ may increase or decrease in t_p. This completes the proofs of Parts (a)-(c). □

Proof of Proposition 3: Similar to the proof of Proposition 1, we complete the proof in three steps: **Step 1.** Solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period; **Step 2.** Show the existence of the equilibrium derived in Step 1 when t_p is small enough; **Step 3.** Show the uniqueness of the equilibrium when t_p is small enough. In the end, we will provide the upper bound for t_p guarantees t_p is small enough and the equilibrium uniquely exists.

Step 1. We first solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period. We have solved the firms' pricing decisions for given quality levels in the proof of Lemma 3. Given the firms' pricing decisions in Stage 2, we proceed to solve firms' quality decisions in the first stage:

$$
\begin{aligned}
&\max_{q_H\in(q_L,1]} \pi_H^{VP}(q_H,q_L,t_p)=p_H^{VP}(q_H,q_L,t_p)d_H^{VP}(q_H,q_L,t_p), \text{ and}\\
&\max_{q_L\in[0,q_H)} \pi_L^{VP}(q_H,q_L,t_p)=p_L^{VP}(q_H,q_L,t_p)d_L^{VP}(q_H,q_L,t_p),
\end{aligned}
$$

where $p_i^{VP}(q_H, q_L, t_p)$ and $d_i^{VP}(q_H, q_L, t_p)$ are given by Equations (A13) and (A14), respectively.

First, one can easily verify that $\pi_H^{VP}(q_H, q_L, t_p)$ increases in q_H for any $q_L \in [0,1)$ and $t_p > 0$, i.e., $\frac{d\pi_H^{VP}(q_H,q_L,t_p)}{dq_H} > 0$. Thus, firm H's optimal quality is $\hat{q}_H^{VP} = 1$ for any $q_L \in [0,1)$ and $t_p > 0$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L. Directly checking firm L's profit function, we find that $\pi_L^{VP}(1, q_L, t_p)$ first increases and then decreases in q_L for $0 < t_p < T^{VP}(1, q_L)$. Therefore, there exists a unique $\hat{q}_L^{VP}(t_p)$ that can be solved from $\frac{\partial \pi_L^{VP}(1,q_L,t_p)}{\partial q_L} = 0$.

Therefore, given that the market is partially covered and each firm has positive demand in each period, the potential equilibrium outcome is: $q_H = \hat{q}_H^{VP} = 1$, $q_L = \hat{q}_L^{VP}(t_p)$, $p_H = \hat{p}_H^{VP}(t_p) = p_H^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$, and $p_L = \hat{p}_L^{VP}(t_p) = p_L^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. Let $\sigma^{VP} = (\hat{q}_H^{VP}, \hat{q}_L^{VP}(t_p), \hat{p}_H^{VP}(t_p), \hat{p}_L^{VP}(t_p))$ denote this potential equilibrium, and let $\hat{d}_i^{VP}(t_p) = d_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$ and $\hat{\pi}_i^{VP}(t_p) = \hat{p}_i^{VP}(t_p)\hat{d}_i^{VP}(t_p)$ denote firm i's total demand and profit at the potential equilibrium σ^{VP}.

Step 2. To show the existence of the equilibrium σ^{VP}, we will show that σ^{VP} is a Sub-game Perfect Equilibrium (SPE) when t_p is small enough. We just need to verify that σ^{VP} satisfies the no-deviation requirements of SPE: (1) Given firms' quality $0 \le q_L < q_H \le 1$ and firm i's price $p_i^{VP}(q_H, q_L, t_p)$, firm j's price decision will not deviate from $p_j^{VP}(q_H, q_L, t_p)$ to any other price. (2) Given both firms' pricing strategies ($p_H^{VP}(q_H, q_L, t_p)$ and $p_L^{VP}(q_H, q_L, t_p)$) in Stage 2 and firm i's quality $\hat{q}_i^{VP}(t_p)$, firm j's quality decision will not deviate from $\hat{q}_j^{VP}(t_p)$ to any other quality, i.e., $\hat{\pi}_j^{VP}(t_p) \ge \max_{q_j \neq \hat{q}_j^{VP}(t_p)} \pi_j^{VP}(q_j, \hat{q}_i^{VP}(t_p), t_p)$, for $i, j \in \{H, L\}$ and $i \neq j$.

To show (1), we just need to show that

$$\pi_j^{VP}(q_H, q_L, t_p) \ge \max_{p_j \neq p_j^{VP}(q_H,q_L,t_p)} p_j d_j^{VP}(p_j | q_H, q_L, p_i^{VP}(q_H, q_L, t_p)), \text{ for } i, j \in \{H, L\} \text{ and } i \neq j, \tag{A15}$$

where the general demand functions are given by:

$$\begin{aligned}
d_{H1}^{VP} &= (1 - \max\{\frac{p_H - p_L}{q_H - q_L}, \frac{p_H}{q_H}\})^+, \quad d_{L1}^{VP} = (\min\{1, \frac{p_H - p_L}{q_H - q_L}\} - \frac{p_L}{q_L})^+, \\
d_{H2}^{VP} &= (1 - \max\{\frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}, \frac{p_H - t_p d_{H1}^{VP}}{q_H}\})^+, \text{ and} \\
d_{L2}^{VP} &= (\min\{1, \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}\} - \frac{p_L - t_p d_{L1}^{VP}}{q_L})^+.
\end{aligned}$$

Firm i's total demand of two periods is given by $d_i^{VP} = d_{i1}^{VP} + d_{i2}^{VP}$, for $i = H, L$. Note that the above demand functions include all the situations that market is partially covered or fully covered and both firms have positive demand or zero demand in some period. Through straightforward yet tedious algebraic analysis, we can verify that given $0 < t_p < T^{VP}(q_H, q_L)$ and firm i's price $p_i^{VP}(q_H, q_L, t_p)$, firm j's optimal price is $p_j^{VP}(q_H, q_L, t_p)$ and thus will not deviate. That is, the inequality (A15) holds. Thus, requirement (1) holds.

For requirement (2), we have already shown in Step 1 that firm H's optimal quality is $q_H = 1$ for any $q_L \in [0,1)$ and that firm L's optimal quality is $q_L = \hat{q}_L^{VP}(t_p)$ given $q_H = 1$. Thus, requirement (2) holds. Hence, we have shown that σ^{VP} is an SPE.

Step 3. We will show the uniqueness of the equilibrium σ^{VP} when t_p is small enough. Note that in Step 1 we have derived the unique equilibrium σ^{VP} by backward induction under the condition that the market is partially covered and each firm has positive demand in each period. Thus, we just need to show that there does not exist any equilibrium if the above condition does not hold, i.e., the market is fully covered or one of the firms has zero demand in period 1 or 2. That is, to show the uniqueness, we need to verify that: (1) Any strategy that leads to $d_{H1} + d_{L1} = 1$ is not an equilibrium; (2) any strategy that leads to $d_{H1} = 0$ or $d_{L1} = 0$ is not an equilibrium; (3) any strategy that leads to $d_{H2} + d_{L2} = 1$ is not an equilibrium; and (4) any strategy that leads to $d_{H2} = 0$ or $d_{L2} = 0$ is not an equilibrium.

If $d_{H1} + d_{L1} = 1$, then that means the consumer with $\theta = 0$ makes the purchase in period 1. That is, either $p_L = 0$ or $p_H = 0$. It is obvious that any strategy with $p_L = 0$ or $p_H = 0$ cannot be an equilibrium since one of the firm's profit will be zero and always has incentive to deviate to a small enough price to earn a positive profit. Thus, (1) holds.

If $d_{i1} = 0$, then $d_{i2} = 0$ since there is no value enhancement effect in period 2 and $u_{i2} = u_{i1}$, for $i = H, L$. Thus, firm i's profit is zero. It is obvious that any strategy with $d_{i1} = 0$ cannot be an equilibrium outcome since firm i has incentive to deviate to a lower price to get a positive demand in period 1 and earn a positive profit. Thus (2) holds.

If $d_{H2} + d_{L2} = 1$ or $d_{H2} = 0$ or $d_{L2} = 0$, then the equilibrium is in the region $C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6$. In the proof of Lemma 3, we have shown that there does not exist any price equilibrium of Stage 2 in region C_i for given quality $0 \leq q_L < q_H \leq 1$ when t_p is small enough. Thus, (3) and (4) hold.

Hence, we conclude that in the equilibrium, the market is partially covered and each firm has positive demand in each period (i.e., $d_{Hi} > 0$, $d_{Li} > 0$, and $d_{Hi} + d_{Li} < 1$, for $i = 1, 2$).

Combining Steps 1-3, we show that σ^{VP} is the unique equilibrium when t_p is small enough. In the main body of the paper and the appendix, we define that $\bar{t}_p = 0.1$ and assume $0 < t_p \leq \bar{t}_p$ to make sure t_p is small enough to guarantee the existence and uniqueness of the equilibrium σ^{VP}.

Finally, we proceed to prove Parts (a)-(d) below. Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}_L^{VP}(t_p)$ is the unique solution of $\frac{\partial \pi_L^{VP}(1, q_L, t_p)}{\partial q_L} = 0$ for $0 < t < T^{VP}(1, q_L)$, by the Implicit Function Theorem, we have: $\frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} = -\frac{\partial^2 \pi_L^{VP}(1, q_L, t_p)}{\partial q_L \partial t_p} / \frac{\partial^2 \pi_L^{VP}(1, q_L, t_p)}{\partial q_L^2}|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify that $\frac{\partial^2 \pi_L^{VP}(1,q_L,t_p)}{\partial q_L^2} < 0$ and $\frac{\partial^2 \pi_L^{VP}(1,q_L,t_p)}{\partial q_L \partial t_p} < 0$ for any $q_L \in [0, 0.65]$ and $0 < t_p \le \bar{t}_p$. Moreover, we find that $\hat{q}_L^{VP}(t_p) < 0.65$ for $0 < t_p \le \bar{t}_p$, since $\frac{\partial \pi_L^{VP}(1,q_L,t_p)}{\partial q_L} < 0$ for any $q_L \in [0.65, 1)$. Therefore, $\frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} < 0$.

Part (b): Note that $\hat{p}_i^{VP}(t_p) = p_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$, for $i = H, L$. By the Chain Rule, $\frac{\partial \hat{p}_i^{VP}(t_p)}{\partial t_p} = (\frac{\partial p_i^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial p_i^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p})|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify that $\frac{\partial p_H^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial p_H^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial p_L^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial p_L^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} < 0$ for any $q_L \in [0, 0.65]$ and $0 < t_p \le \bar{t}_p$. As already shown in Part (a) above, $\hat{q}_L^{VP}(t_p) < 0.65$. Therefore, $\frac{\partial \hat{p}_H^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial \hat{p}_L^{VP}(t_p)}{\partial t_p} < 0$. Hence, $\hat{p}_H^{VP}(t_p) > \hat{p}_H^{VP}(0) = \hat{p}_H^b$ and $\hat{p}_L^{VP}(t_p) < \hat{p}_L^{VP}(0) = \hat{p}_L^b$ for $0 < t_p \le \bar{t}_p$.

Part (c): Note that $\hat{d}_i^{VP}(t_p) = d_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{VP}(t_p)}{\partial t_p} = (\frac{\partial d_i^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial d_i^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p})|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify that $\frac{\partial d_H^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial d_H^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} > 0$ for any for any $q_L \in [0, 0.65]$ and $0 < t_p \le \bar{t}_p$. Since $\hat{q}_L^{VP}(t_p) < 0.65$, we have $\frac{\partial \hat{d}_H^{VP}(t_p)}{\partial t_p} > 0$, i.e, $\hat{d}_H^{VP}(t_p)$ increases in t_p. Moreover, one can verify that $\frac{\partial d_L^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial d_L^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p}$ is negative when t_p is close to 0 and is positive when t_p is close to $\bar{t}_p$. Thus, $\hat{d}_L^{VP}(t_p)$ may decrease or increase in t_p.

Part (d): Note that $\hat{\pi}_i^{VP}(t_p) = \pi_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. By the chain rule, $\frac{\partial \hat{\pi}_i^{VP}(t_p)}{\partial t_p} = (\frac{\partial \pi_i^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial \pi_i^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p})|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify that $\frac{\partial \pi_H^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial \pi_H^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial \pi_L^{VP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial \pi_L^{VP}(1,q_L,t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} < 0$ for $q_L \in [0, 0.65]$ and $0 < t_p \le \bar{t}_p$. Since $\hat{q}_L^{VP}(t_p) < 0.65$, we have $\frac{\partial \hat{\pi}_H^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial \hat{\pi}_L^{VP}(t_p)}{\partial t_p} < 0$, i.e., $\hat{\pi}_H^{VP}(t_p)$ increases in t_p and $\hat{\pi}_L^{VP}(t_p)$ decreases in t_p. Therefore, $\hat{\pi}_H^{VP}(t_p) - \hat{\pi}_H^b > 0 > \hat{\pi}_L^{VP}(t_p) - \hat{\pi}_L^b$. $\square$

Proof of Corollary 3: Part (a): Let $CM_{in}^{VP}(t_p)$ denote consumer monetary surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$
\begin{aligned}
CM_{H1}^{VP}(t_p) &= \int_{\bar{\theta}_{H1}^{VP}}^{1} (\theta - \hat{p}_H^{VP}(t_p)) d\theta \\
&= (1 - 2\hat{p}_H^{VP}(t_p) + \bar{\theta}_{H1}^{VP})(1 - \bar{\theta}_{H1}^{VP})/2 \\
&= (1 - 2\hat{p}_H^{VP}(t_p) + \frac{\hat{p}_H^{VP}(t_p) - \hat{p}_L^{VP}(t_p)}{1 - \hat{q}_L^{VP}(t_p)})(1 - \frac{\hat{p}_H^{VP}(t_p) - \hat{p}_L^{VP}(t_p)}{1 - \hat{q}_L^{VP}(t_p)})/2, \\
CM_{L1}^{VP}(t_p) &= \int_{\bar{\theta}_{L1}^{VP}}^{\bar{\theta}_{H1}^{VP}} (\theta \hat{q}_L^{VP}(t_p) - \hat{p}_L^{VP}(t_p)) d\theta \\
&= (\hat{q}_L^{VP}(t_p)(\bar{\theta}_{H1}^{VP} + \bar{\theta}_{L1}^{VP}) - 2\hat{p}_L^{VP}(t_p))(\bar{\theta}_{H1}^{VP} - \bar{\theta}_{L1}^{VP})/2 \\
&= (\hat{q}_L^{VP}(t_p)(\frac{\hat{p}_H^{M}(t_p) - \hat{p}_L^{M}(t_p)}{1 - \hat{q}_L^{VP}(t_p)} + \frac{\hat{p}_L^{VP}(t_p)}{\hat{q}_L^{VP}(t_p)}) - 2\hat{p}_L^{VP}(t_p))(\frac{\hat{p}_H^{VP}(t_p) - \hat{p}_L^{VP}(r)}{1 - \hat{q}_L^{VP}(t_p)} - \frac{\hat{p}_L^{VP}(t_p)}{\hat{q}_L^{VP}(t_p)})/2, \\
CM_{H2}^{VP}(t_p) &= \int_{\bar{\theta}_{H2}^{VP}}^{1} (\theta - \hat{p}_H^{VP}(t_p)) d\theta \\
&= (1 - 2\hat{p}_H^{VP}(t_p) + \bar{\theta}_{H2}^{VP})(1 - \bar{\theta}_{H2}^{VP})/2, \quad \text{and} \\
CM_{L2}^{VP}(t_p) &= \int_{\bar{\theta}_{L2}^{VP}}^{\bar{\theta}_{H2}^{VP}} (\theta \hat{q}_L^{VP}(t_p) - \hat{p}_L^{VP}(t_p)) d\theta \\
&= (\hat{q}_L^{VP}(t_p)(\bar{\theta}_{H2}^{VP} + \bar{\theta}_{L2}^{VP}) - 2\hat{p}_L^{VP}(t_p))(\bar{\theta}_{H2}^{VP} - \bar{\theta}_{L2}^{VP})/2,
\end{aligned}
$$

where $\bar{\theta}_{H2}^{VP} = \frac{(\hat{p}_H^{VP}(t_p) - t_p \hat{d}_{H1}^{VP}(t_p)) - (\hat{p}_L^{VP}(t_p) - t_p \hat{d}_{L1}^{VP}(t_p))}{1 - \hat{q}_L^{VP}(t_p)}$ and $\bar{\theta}_{L2}^{VP} = \frac{\hat{p}_L^{VP}(t_p) - t_p \hat{d}_{L1}^{VP}(t_p)}{\hat{q}_L^{VP}(t_p)}$. The consumer monetary surplus in period n is given by $CM_n^{VP}(t_p) = CM_{Hn}^{VP}(t_p) + CM_{Ln}^{VP}(t_p)$ for $n = 1, 2$, and total consumer monetary surplus of two periods is given by $CM^{VP}(t_p) = CM_1^{VP}(t_p) + CM_2^{VP}(t_p)$.

We can show that $\frac{dCM_{H1}^{VP}(t_p)}{dt_p} < 0$, $\frac{dCM_{L1}^{VP}(t_p)}{dt_p} < 0$, $\frac{dCM_{H2}^{VP}(t_p)}{dt_p} < 0$ and $\frac{dCM_{L2}^{VP}(t_p)}{dt_p} < 0$ for $0 < t_p \leq \bar{t}_p$. Thus $\frac{dCM_1^{VP}(t_p)}{dt_p} < 0$, $\frac{dCM_2^{VP}(t_p)}{dt_p} < 0$, and $\frac{dCM^{VP}(t_p)}{dt_p} < 0$. Therefore, $CM_1^{VP}(t_p)$, $CM_2^{VP}(t_p)$, and $CM^{VP}(t_p)$ all decrease in t_p. Let CM^b denote the total consumer monetary surplus of two periods in the benchmark case, i.e., $CM^b = CM^{VP}(0)$. Thus, $CM^{VP}(t_p) < CM^{VP}(0) = CM^b$ for $0 < t_p \leq \bar{t}_p$.

Part (b): Let $CS_{in}^{VP}(t_p)$ denote consumer total surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$
\begin{aligned}
CS_{H1}^{VP}(t_p) &= CM_{H1}^{VP}(t_p), \quad CS_{L1}^{VP}(t_p) = CM_{L1}^{VP}(t_p), \\
CS_{H2}^{VP}(t_p) &= \int_{\bar{\theta}_{H2}^{VP}}^{1} (\theta - \hat{p}_H^{VP}(t_p) + t_p \hat{d}_{H1}^{VP}(t_p)) d\theta \\
&= \frac{1}{2}(1 - 2\hat{p}_H^{VP}(t_p) + 2t_p \hat{d}_{H1}^{VP}(t_p) + \bar{\theta}_{H2}^{VP})(1 - \bar{\theta}_{H2}^{VP}) \quad \text{and} \\
CS_{L2}^{VP}(t_p) &= \int_{\bar{\theta}_{L2}^{VC}}^{\bar{\theta}_{H2}^{VP}} (\theta \hat{q}_L^{VP}(t_p) - \hat{p}_L^{VP}(t_p) + t_p \hat{d}_{L1}^{VP}(t_p)) d\theta \\
&= \frac{1}{2}(\hat{q}_L^{VP}(t_p)(\bar{\theta}_{H2}^{VP} + \bar{\theta}_{L2}^{VP}) - 2\hat{p}_L^{VP}(t_p) + 2t_p \hat{d}_{L1}^{VP}(t_p))(\bar{\theta}_{H2}^{VP} - \bar{\theta}_{L2}^{VP}), \textit{ for } n = 1, 2.
\end{aligned}
$$

Consumer total surplus in period n is given by $CS_n^{VP}(t_p) = CS_{Hn}^{VP}(t_p) + CS_{Ln}^{VP}(t_p)$, and consumer total surplus of two periods is given by $CS^{VP}(t_p) = CS_1^{VP}(t_p) + CS_2^{VP}(t_p)$.

Given $0 < t_p \leq \bar{t}_p$, we can show that $\frac{dCS^{VP}(t_p)}{dt_p} < 0$ if $t_p < 0.0382251$ and $\frac{dCS^{VP}(t_p)}{dt_p} \geq 0$ otherwise. Thus, $CS^{VP}(t_p)$ first decreases and then increases in t_p.

Part (c): The monetary term of social welfare is defined as $SM^{VP}(t_p) = \pi_H^{VP}(t_p) + \pi_L^{VP}(t_p) + CM^{VP}(t_p)$. We have shown that $CM^{VP}(t_p)$ decreases in t_p in (a), and that $\pi_H^{VP}(t_p)$ increases in t_p and $\pi_L^{VP}(t_p)$ decreases in t_p in Proposition 3(d). We can verify that $\frac{dSM^{VP}(t_p)}{dt_p}$ could be ≥ 0 or ≤ 0. Thus $SM^{VP}(t_p)$ may increase or decrease in t_p.

Part (d): The social welfare is defined as $SW^{VP}(t_p) = \pi_H^{VP}(t_p) + \pi_L^{VP}(t_p) + CS^{VP}(t_p)$. Given $0 < t_p \leq \bar{t}_p$, we can verify that $\frac{dSW^{VP}(t_p)}{dt_p} > 0$, indicating $SW^{VP}(t_p)$ increases in t_p, Thus, $SW^{VP}(t_p) > SW^{VP}(0) = SW^b$. $\square$

B. The Dynamic Pricing Scheme

In this section, we analyze the model with dynamic pricing scheme, and show the equilibrium results parallel to those in our main model. Since each period is independent and identical in the case of VEE-C, the model and results are exactly the same under different pricing schemes. Hence, in the sequel, we only focus on MEE in Section B.1 and VEE-P in Section B.2. Note that the following results are proved along with Propositions 4 and 5.

B.1. Market Expansion Effect (MEE)

We show the results and proofs of the MEE case under dynamic pricing in this section. Similar to the analysis under committed pricing scheme, we solve the equilibrium of MEE by backward induction. First, for given firms' quality levels $0 \leq q_L < q_H \leq 1$, we solve the two firms' pricing decisions and summarize the results in Lemma B1, which is directly comparable to Lemma 1.

LEMMA B1. *Suppose that firms adopt dynamic pricing strategy. In the MEE case (i.e., $t_c = t_p = 0$ and $0 < r \leq 1$), for any given $0 \leq q_L < q_H \leq 1$, the firms' sub-game equilibrium prices in period n are uniquely given by $(p_{Hn}^{DM}(q_H, q_L, r), p_{Ln}^{DM}(q_H, q_L, r))$, for $n = 1, 2$.*

Next, we proceed to solve the quality decisions in Stage 1, and summarize the final equilibrium and the corresponding result in Proposition B1, a counterpart of Proposition 1.

PROPOSITION B1. *Suppose that firms adopt dynamic pricing strategy. In the MEE case (i.e., $t_c = t_p = 0$ and $0 < r \leq 1$), there exists a unique equilibrium, in which firm H's quality is $\hat{q}_H^{DM} = 1$ and firm L's quality $0 < \hat{q}_L^{DM}(r) < 1$. In the equilibrium, the market is not fully covered and each firm has positive demand. Moreover, the following statements hold.*

(a) $\hat{q}_L^{DM}(r)$ increases in r.

(b) $\hat{p}_{in}^{DM}(r)$ decreases in r and $\hat{p}_{i1}^{DM}(r) < \hat{p}_{i2}^{DM}(r) < \hat{p}_H^b$, for $i = H, L$ and $n = 1, 2$.

(c) $\hat{d}_H^{DM}(r)$ and $\hat{d}_L^{DM}(r)$ increase in r.

(d) $\hat{\pi}_H^{DM}(r)$ and $\hat{\pi}_L^{DM}(r)$ increase in r. Moreover, $\hat{\pi}_H^{DM}(r) - \hat{\pi}_H^b > \hat{\pi}_L^{DM}(r) - \hat{\pi}_L^b > 0$ while $0 < (\hat{\pi}_H^{DM}(r) - \hat{\pi}_H^b)/\hat{\pi}_H^b < (\hat{\pi}_L^{DM}(r) - \hat{\pi}_L^b)/\hat{\pi}_L^b$.

Proof of Lemma B1 and Proposition B1: We remark that the proof of Lemma B1 and Proposition B1 is very similar to that of Lemma 1 and Proposition 1, and therefore we suppress some steps for succinct exhibition. We solve the equilibrium of the MEE case under dynamic pricing by backward induction in three steps: **Step 1.** Given the quality levels and demands in period 1, we solve the price decisions in period 2 of Stage 2. **Step 2.** Given the quality levels, we solve the price decisions in period 1 of Stage 2. **Step 3.** We solve the quality decisions in Stage 1.

Step 1: Price Decisions in Period 2. When $t_c = t_p = 0$ and $r \in (0, 1]$, given that $0 \leq q_L < q_H \leq 1$ and each firm's demand in period 1, d_{H1}^{DM} and d_{L1}^{DM}, firm i decides p_{i2} to maximize its profit of period 2:

$$\max_{p_{i2} \geq 0} \pi_{i2}^{DM} = p_{i2} d_{i2}^{DM}, \quad \text{for } i = H, L.$$

In the equilibrium, $p_{H2} > 0$ and $p_{L2} > 0$ hold, and thus the market will not be fully covered by the two firms in period 2. Moreover, in the equilibrium, each firm's demand is positive in period 2, i.e., $0 < \frac{p_{L2}}{q_L} < \frac{p_{H2} - p_{L2}}{q_H - q_L} < 1$. Therefore, two firms' demands in period 2 are given by

$$d_{H2}^{DM} = (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \left(1 - \frac{p_{H2} - p_{L2}}{q_H - q_L}\right) \quad \text{and} \quad d_{L2}^{DM} = (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \left(\frac{p_{H2} - p_{L2}}{q_H - q_L} - \frac{p_{L2}}{q_L}\right).$$

One can verify that π_{i2}^{DM} is concave in p_{i2}, i.e., $\frac{\partial^2 \pi_{i2}^{DM}}{\partial p_{i2}^2} < 0$. Solving the two firms' first-order-conditions together leads to the prices in period 2 as below:

$$p_{H2}^{DM}(q_H, q_L) = \frac{2q_H(q_H - q_L)}{4q_H - q_L} \quad \text{and} \quad p_{L2}^{DM}(q_H, q_L) = \frac{q_L(q_H - q_L)}{4q_H - q_L}. \tag{B1}$$

The corresponding demand and profit of each firm in period 2 are given as below:

$$\begin{aligned}
d_{H2}^{DM}(q_H, q_L, r, d_{H1}^{DM}, d_{L1}^{DM}) &= (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \frac{2q_H}{4q_H - q_L}, \\
d_{L2}^{DM}(q_H, q_L, r, d_{L1}^{DM}, d_{H1}^{DM}) &= (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \frac{q_H}{4q_H - q_L}, \\
\pi_{H2}^{DM}(q_H, q_L, r, d_{H1}^{DM}, d_{L1}^{DM}) &= (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \frac{4q_H^2(q_H - q_L)}{(4q_H - q_L)^2}, \text{ and} \\
\pi_{L2}^{DM}(q_H, q_L, r, d_{L1}^{DM}, d_{H1}^{DM}) &= (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \frac{q_H q_L(q_H - q_L)}{(4q_H - q_L)^2}.
\end{aligned} \tag{B2}$$

Step 2: Price Decisions in Period 1. Given the firms' prices in period 2, firm i decides price in period 1, i.e., p_{i1}, to maximize its total profit of two periods:

$$\max_{p_{i1} \geq 0} \pi_i^{DM} = p_{i1} d_{i1}^{DM} + \pi_{i2}^{DM}(q_H, q_L, r, d_{i1}^{DM}, d_{j1}^{DM}), \tag{B3}$$

where $i, j = H, L$ and $i \neq j$, and the demands in period 1 are given by

$$(d_{H1}^{DM}, d_{L1}^{DM}) = \begin{cases}
(1 - \frac{p_{H1} - p_{L1}}{q_H - q_L}, \frac{p_{H1} - p_{L1}}{q_H - q_L} - \frac{p_{L1}}{q_L}), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_1; \\
(1 - \frac{p_{H1} - p_{L1}}{q_H - q_L}, \frac{p_{H1} - p_{L1}}{q_H - q_L}), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_2; \\
(1 - \frac{p_{H1}}{q_H}, 0), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_3; \\
(0, 1 - \frac{p_{L1}}{q_L}), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_4; \\
(1, 0), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_5; \\
(0, 1), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_6.
\end{cases}$$

In the above, $\bar{C}_i$ $(i = 1, 2, ..., 6)$ is a set of conditions that defines a region for the price pair in period 1. More specifically, $\bar{C}_1 = \{0 < \frac{p_{L1}}{q_L} < \frac{p_{H1} - p_{L1}}{q_H - q_L} < 1\}$; $\bar{C}_2 = \{\frac{p_{L1}}{q_L} \leq 0 < \frac{p_{H1} - p_{L1}}{q_H - q_L} < 1\}$; $\bar{C}_3 = \{0 \leq \frac{p_{H1}}{q_H} \leq \frac{p_{L1}}{q_L}\}$; $\bar{C}_4 = \{0 \leq p_{L1} \leq q - 1 + p_{H1}\}$; $\bar{C}_5 = \{p_{H1} \leq \min\{0, p_{L1}\}\}$; and $\bar{C}_6 = \{p_{L1} \leq \min\{0, q_L - q_H + p_{H1}\}\}$. We refer to $(p_{H1}, p_{L1}) \in \bar{C}_i$ simply as "under Condition i". Under Condition 1, the market is not fully covered and each firm has positive demand in period 1; under Condition 2, the market is fully covered and each firm has positive demand in period 1; under other conditions, one of the firms has zero demand in period 1.

We can apply similar approach as in the proof of Lemma 1 to show that the market is partially covered and each firm's demand is positive in period 1, i.e., $0 < \frac{p_{L1}}{q} < \frac{p_{H1} - p_{L1}}{1 - q} < 1$. Therefore, we just focus on Condition 1. Under Condition 1, one can easily verify that firm i's total profit is concave in p_{i1}; and thus the firms' first-period prices can be solved from the first-order conditions and are given by:

$$\begin{aligned}
p_{H1}^{DM}(q_H, q_L, r) &= \frac{q_H\,(q_H - q_L)\,(32q_H^2 + (r+2)q_L^2 - (r+16)q_H q_L)}{(4q_H - q_L)^3} \text{ and} \\
p_{L1}^{DM}(q_H, q_L, r) &= \frac{q_L\,(q_H - q_L)\,(2(r-4)q_H q_L - 2(r-8)q_H^2 + q_L^2)}{(4q_H - q_L)^3}.
\end{aligned} \tag{B4}$$

Therefore, the corresponding demand of each firm in period 1 is

$$\begin{aligned}
d_{H1}^{DM}(q_H, q_L, r) &= \frac{q_H\,(32q_H^2 + (r+2)q_L^2 - (r+16)q_H q_L)}{(4q_H - q_L)^3} \quad \text{and} \\
d_{L1}^{DM}(q_H, q_L, r) &= \frac{q_H\,(2(r+8)q_H^2 + (r+1)q_L^2 - (3r+8)q_H q_L)}{(4q_H - q_L)^3}.
\end{aligned} \tag{B5}$$

Plugging Equations (B5) into Equations (B2) yields the second-period demands as functions of (q_H, q_L, r):

$$
\begin{aligned}
d_{H2}^{DM}(q_H, q_L, r) &= (1 + \frac{rq_H\left(-4(r+6)q_Hq_L + 2(r+24)q_H^2 + (2r+3)q_L^2\right)}{(4q_H - q_L)^3})\frac{2q_H}{4q_H - q_L} \text{ and} \\
d_{L2}^{DM}(q_H, q_L, r) &= (1 + \frac{rq_H\left(-4(r+6)q_Hq_L + 2(r+24)q_H^2 + (2r+3)q_L^2\right)}{(4q_H - q_L)^3})\frac{q_H}{4q_H - q_L}.
\end{aligned} \tag{B6}
$$

Moreover, firm i's total demand of two periods $d_i^{DM}(q_H, q_L, r) = d_{i1}^{DM}(q_H, q_L, r) + d_{i2}^{DM}(q_H, q_L, r)$ and its total profit $\pi_i^{DM}(q_H, q_L, r) = p_{i1}^{DM}(q_H, q_L, r)d_{i1}^{DM}(q_H, q_L, r) + p_{i2}^{DM}(q_H, q_L, r)d_{i2}^{DM}(q_H, q_L, r)$.

Step 3. Quality Decisions. Given the firms' dynamic pricing decisions in Stage 2, we proceed to solve firms' quality decisions in Stage 1:

$$
\max_{q_H \in (q_L, 1]} \pi_H^{DM}(q_H, q_L, r) \quad \text{and} \quad \max_{q_L \in [0, q_H)} \pi_L^{DM}(q_H, q_L, r).
$$

First, one can easily verify that $\pi_H^{DM}(q_H, q_L, r)$ increases in q_H for any $q_L \in [0, 1)$ and $r \in (0, 1]$, i.e., $\frac{d\pi_H^{DM}(q_H, q_L, r)}{dq_H} > 0$. Thus, firm H's optimal quality is $\hat{q}_H^{DM} = 1$ for any $q_L \in [0, 1)$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L. Directly checking firm L's profit function, we find that $\pi_L^{DM}(1, q_L, r)$ is concave in q_L, i.e., $\frac{d^2\pi_L^{DM}(1, q_L, r)}{dq_L^2} < 0$, for any $q_L \in [0, 1)$ and $r \in (0, 1]$. Therefore, firm L's optimal quality $\hat{q}_L^{DM}(r)$ is solved from $\frac{d\pi_L^{DM}(1, q_L, r)}{dq_L} = 0$.

Thus, the equilibrium outcome is: $q_H = \hat{q}_H^{DM} = 1$, $q_L = \hat{q}_L^{DM}(r)$, $p_{i1} = \hat{p}_{i1}^{DM}(r) = p_{i1}^{DM}(1, \hat{q}_L^{DM}(r), r)$, and $p_{i2} = \hat{p}_{i2}^{DM}(r) = p_{i2}^{DM}(1, \hat{q}_L^{DM}(r))$, for $i = H, L$. Let $\hat{d}_{in}^{DM}(r) = d_{in}^{DM}(1, \hat{q}_L^{DM}(r), r)$ and $\hat{\pi}_{in}^{DM}(r) = \hat{p}_{in}^{DM}(r)\hat{d}_{in}^{DM}(r)$ denote firm i's demand and profit in period n for $i = H, L$ and $n = 1, 2$. Firm i's total demand and total profit are given by $\hat{d}_i^{DM}(r) = \hat{d}_{i1}^{DM}(r) + \hat{d}_{i2}^{DM}(r)$ and $\hat{\pi}_i^{DM}(r) = \hat{\pi}_{i1}^{DM}(r) + \hat{\pi}_{i2}^{DM}(r)$. Note that we can apply similar approach as in the proof of Proposition 1 to show the existence and uniqueness of the above equilibrium in which the market is partially covered and each firm's demand is positive in each period. The detailed proof is omitted and is available from authors upon request. The proof of Proposition B1(a)-(d) are provided in the proof of Proposition 4. $\square$

Proof of Proposition 4: Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}_L^{DM}(r)$ is the unique solution of $\frac{\partial\pi_L^{DM}(1, q_L, r)}{\partial q_L} = 0$, by the Implicit Function Theorem, we have: $\frac{\partial\hat{q}_L^{DM}(r)}{\partial r} = -\frac{\partial^2\pi_L^{DM}(1, q_L, r)}{\partial q_L\partial r} / \frac{\partial^2\pi_L^{DM}(1, q_L, r)}{\partial q_L^2}|_{q=\hat{q}_L^{DM}(r)}$. We have already shown that $\frac{\partial^2\pi_L^{DM}(1, q_L, r)}{\partial q_L^2} < 0$. Besides, we can verify that $\frac{\partial^2\pi_L^{DM}(1, q_L, r)}{\partial q_L\partial r} > 0$ for any $q_L \in (0, 0.6)$. Moreover, since $\frac{\partial\pi_L^{DM}(1, q_L, r)}{\partial q_L} < 0$ for any $q_L \in [0.6, 1)$, we deduce that $\hat{q}_L^{DM}(r) < 0.6$. Therefore, $\frac{\partial\hat{q}_L^{DM}(r)}{\partial r} > 0$, i.e. $\hat{q}_L^{DM}(r)$ increases in r and $\hat{q}_L^{DM}(r) > \hat{q}^b$ for $r \in (0, 1]$.

Next, we show that $\hat{q}_L^M(r) > \hat{q}_L^{DM}(r)$ for any $r \in (0, 1]$. One can verify that $\frac{\partial\pi_L^M(1, q_L, r)}{\partial q_L} > \frac{\partial\pi_L^{DM}(1, q_L, r)}{\partial q_L}$ for any $q_L \in [0.4, 1]$; moreover, $\frac{\partial\pi_L^M(1, q_L, r)}{\partial q_L} > 0$ and $\frac{\partial\pi_L^{DM}(1, q_L, r)}{\partial_L} > 0$ for any $q_L \in (0, 0.4]$. Since $\hat{q}_L^M(r)$ is the unique solution to $\frac{\partial\pi_L^M(1, q_L, r)}{\partial q_L} = 0$ and $\hat{q}_L^{DM}(r)$ to $\frac{\partial\pi_L^{DM}(1, q_L, r)}{\partial q_L} = 0$ (concavity of the profit functions), we have $\hat{q}_L^M(r) > \hat{q}_L^{DM}(r) > \hat{q}_L^b$.

Part (b): Note that $\hat{p}_{i1}^{DM}(r) = p_{i1}^{DM}(1, \hat{q}_L^{DM}(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_{i1}^{DM}(r)}{\partial r} = (\frac{\partial p_{i1}^{DM}(1,q_L,r)}{\partial r} + \frac{\partial p_{i1}^{DM}(1,q_L,r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r})|_{q=\hat{q}_L^{DM}(r)}$. One can verify that $\frac{\partial p_{i1}^{DM}(1,q_L,r)}{\partial r} < 0$. We already have $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$. Moreover, one can verify that $\frac{\partial p_{H1}^{DM}(1,q_L,r)}{\partial q_L} < 0$ for any $q_L \in (0, 1]$, and that $\frac{\partial p_{L1}^{DM}(1,q_L,r)}{\partial q_L} < 0$ for any $q_L \in (0.56, 1)$. Since $\hat{q}_L^{DM}(r)$ increases in r, we have $\hat{q}_L^{DM}(r) \geq \hat{q}^{DM}(0) = \hat{q}^b = 4/7 > 0.56$. Thus, $\frac{\partial p_{L1}^{DM}(1,q_L,r)}{\partial q_L}|_{q_L=\hat{q}_L^{DM}(r)} < 0$. Based on all these facts, we show that $\frac{\partial \hat{p}_{i1}^{DM}(r)}{\partial r} < 0$. Therefore, $\hat{p}_{i1}^{DM}(r)$ decreases in r and $\hat{p}_{i1}^{DM}(r) < \hat{p}_{i1}^{DM}(0) = \hat{p}_i^b$ for $r \in (0, 1]$ and $i = H, L$.

Similarly, note that $\hat{p}_{i2}^{DM}(r) = p_{i2}^{DM}(1, \hat{q}_L^{DM}(r))$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_{i2}^{DM}(r)}{\partial r} = (\frac{\partial p_{i2}^{DM}(1,q_L)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r})|_{q_L=\hat{q}_L^{DM}(r)}$. We already have $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$. Moreover, we verify that $\frac{\partial p_{H2}^{DM}(1,q_L)}{\partial q_L} < 0$ for any $q_L \in (0, 1]$, and that $\frac{\partial p_{L2}^{DM}(1,q_L)}{\partial q_L} < 0$ for any $q_L \in (0.56, 1)$. Thus, $\frac{\partial \hat{p}_{i2}^{DM}(r)}{\partial r} < 0$. Therefore, $\hat{p}_{i2}^{DM}(r)$ decreases in r and $\hat{p}_{i2}^{DM}(r) < \hat{p}_{i2}^{DM}(0) = \hat{p}_i^b$ for $r \in (0, 1]$.

Next, we will show that $\hat{p}_{i1}^{DM}(r) < \hat{p}_i^M(r) < \hat{p}_{i2}^{DM}(r)$ for any $r \in (0, 1]$. Due to concavity, $\hat{q}_L^{DM}(r)$ is the unique solution of $\frac{\partial \pi_L^{DM}(1,q_L,r)}{\partial q_L} = 0$ for $q_L \in [0, 1)$; that is $\hat{q}_L^{DM}(r)$ is the first (or smallest) root of the polynomial $q_L^5(11r + 14) + q_L^4 (8r^2 - 116r - 232) + q_L^3 (28r^2 + 492r + 1472) + q_L^2 (-96r^2 - 1232r - 4352) + q_L (76r^2 + 1856r + 5632) - 16r^2 - 768r - 2048 = 0$. Similarly, $\hat{q}_L^M(r)$ can be solved as the first (or smallest) root of the polynomial $2q_L^5 (r^3 - 9r^2 + 20r - 28) + q_L^4 (-9r^3 + 72r^2 - 76r + 480) + 2q_L^3 (12r^3 - 87r^2 - 232r - 884) + q_L^2 (-11r^3 + 396r^2 + 1596r + 2992) - 4q_L (15r^3 + 168r^2 + 520r + 640) + 36r^3 + 288r^2 + 768r + 768 = 0$. Substituting the equilibrium quality to $p_{i1}^{DM}(1, \hat{q}_L^{DM}(r), r)$ (Equation (B4)), $p_{i2}^{DM}(1, \hat{q}_L^{DM}(r))$ (Equation (B1)), and $p_i^M(1, \hat{q}_L^M(r), r)$ (Equation (A3)), and after some straightforward simplifications, we can verify that $\hat{p}_{i1}^{DM}(r) < \hat{p}_i^M(r) < \hat{p}_{i2}^{DM}(r) < \hat{p}_i^b$, for $i = H, L$.

Part (c): Note that $\hat{d}_i^{DM}(r) = d_i^{DM}(1, \hat{q}_L^{DM}(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{DM}(r)}{\partial r} = (\frac{\partial d_i^{DM}(1,q_L,r)}{\partial r} + \frac{\partial d_i^{DM}(1,q_L,r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r})|_{q_L=\hat{q}_L^{DM}(r)}$. We already have $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$, and can verify that $\frac{\partial d_i^{DM}(1,q_L,r)}{\partial r} > 0$ and $\frac{\partial d_i^{DM}(1,q_L,r)}{\partial q_L} > 0$. Thus, $\frac{\partial \hat{d}_i^{DM}(r)}{\partial r} > 0$, i.e., $\hat{d}_i^{DM}(r)$ increases in r for $i = H, L$.

Similar to Part (c) above, substituting $\hat{q}_L^{DM}(r)$ into $d_i^{DM}(1, \hat{q}_L^{DM}(r), r)$ and $\hat{q}_L^M(r)$ into $d_i^M(1, \hat{q}_L^M(r), r)$, we can verify that $\hat{d}_i^{DM}(r) > \hat{d}_i^M(r) > \hat{d}_i^b$ for $r \in (0, 1]$ and $i = H, L$.

Part (d): Note that $\hat{\pi}_i^{DM}(r) = \pi_i^{DM}(1, \hat{q}_L^{DM}(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{\pi}_i^{DM}(r)}{\partial r} = (\frac{\partial \pi_i^{DM}(1,q_L,r)}{\partial r} + \frac{\partial \pi_i^{DM}(1,q_L,r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r})|_{q_L=\hat{q}_L^{DM}(r)}$. One can verify that $\frac{\partial \pi_H^{DM}(1,q_L,r)}{\partial r} + \frac{\partial \pi_H^{DM}(1,q_L,r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$ for any $q_L \in (0, 1]$. Thus, $\frac{\partial \hat{\pi}_H^{DM}(r)}{\partial r} > 0$. One can also verify that $\frac{\partial \pi_L^{DM}(1,q_L,r)}{\partial r} > 0$. Since $\frac{\partial \pi_L^{DM}(1,q_L,r)}{\partial q_L}|_{q_L=\hat{q}_L^{DM}(r)} = 0$, we conclude that $\frac{\partial \hat{\pi}_L^{DM}(r)}{\partial r} > 0$. Therefore, both $\hat{\pi}_H^{DM}(r)$ and $\hat{\pi}_L^{DM}(r)$ increase in r.

Substituting $\hat{q}_L^{DM}(r)$ into $\pi_i^{DM}(1, \hat{q}_L^{DM}(r), r)$ and $\hat{q}_L^M(r)$ into $\pi_i^M(1, \hat{q}_L^M(r), r)$, we can verify that $\hat{\pi}_i^{DM}(r) > \hat{\pi}_i^M(r) > \hat{\pi}_i^b$ for any $r \in (0, 1]$ and $i = H, L$.

This completes the proof of Proposition 4 and Proposition B1 parts(a)-(d). $\square$

B.2. Value Enhancement Effect from Previous Consumers (VEE-P)

We show the results and proofs of the VEE-P case under dynamic pricing in this section. Similar to the analysis under committed pricing scheme, we solve the equilibrium of VEE-P by backward induction. First, for given firms' quality levels, we solve the pricing decisions and summarize the results in Lemma B2, which is comparable to Lemma 3. Then we solve the quality decisions in Stage 1 and show results of final equilibrium in Proposition B2, which is comparable to Proposition 3.

LEMMA B2. *Suppose that firms adopt dynamic pricing strategy. In the VEE-P case, for any given* $0 \le q_L < q_H \le 1$*, the firms' equilibrium prices in period* n *are uniquely given by* $(p^{DVP}_{Hn}(q_H, q_L, t_p), p^{DVP}_{Ln}(q_H, q_L, t_p))$*, for* $n = 1, 2$.

PROPOSITION B2. *Suppose that firms adopt dynamic pricing strategy. In the VEE-P case, there exists a unique equilibrium, in which firm H's quality is* $\hat{q}^{DVP}_H = 1$ *and firm L's quality* $0 < \hat{q}^{DVC}_L(t_p) < 1$*. In the equilibrium, the market is not fully covered and each firm has positive demand. Moreover, the following statements hold.*

(*a*) $\hat{q}^{DVP}_L(t_p)$ *decreases in* t_p.

(*b*) $\hat{p}^{DVP}_{H1}(t_p)$ *first decreases in* t_p *and then increases in* t_p; $\hat{p}^{DVP}_{L1}(t_p)$ *decreases in* t_p; $\hat{p}^{DVP}_{H1}(t_p) < \hat{p}^b_H$ *and* $\hat{p}^{DVP}_{L1}(t_p) < \hat{p}^b_L$; $\hat{p}^{DVP}_{H2}(t_p)$ *and* $\hat{p}^{DVP}_{L2}(t_p)$ *increase in* t_p; $\hat{p}^{DVP}_{H2}(t_p) > \hat{p}^b_H$ *and* $\hat{p}^{DVP}_{L2}(t_p) > \hat{p}^b_L$.

(*c*) $\hat{d}^{DVP}_H(t_p)$ *increases in* t_p; $\hat{d}^{DVP}_L(t_p)$ *first decreases and then increases in* t_p.

(*d*) $\hat{\pi}^{DVP}_H(t_p)$ *increases in* t_p *and* $\hat{\pi}^{DVP}_L(t_p)$ *decreases in* t_p; $\hat{\pi}^{DVP}_H(t_p) - \hat{\pi}^b_H > 0 > \hat{\pi}^{DVP}_L(t_p) - \hat{\pi}^b_L$.

Proof of Lemma B2 and Proposition B2: We remark that the proof of Lemma B2 and Proposition B2 is very similar to that of Lemma 3 and Proposition 3, and therefore we suppress some steps for succinct exhibition. We solve the equilibrium of the VEE-P case under dynamic pricing by backward induction in three steps: **Step 1.** Given the quality levels and the demands in period 1, we solve the price decisions in period 2 of Stage 2. **Step 2.** Given the quality levels, we solve the price decisions in period 1 of Stage 2. **Step 3.** We solve the firms' quality decisions in Stage 1.

Step 1: Price Decisions in Period 2. When $r = t_c = 0$ and $t_p > 0$ and is small enough, given that $0 \le q_L < q_H \le 1$ and each firm's demand in period 1, d^{DVP}_{H1} and d^{DVP}_{L1}, firm i decides price in period 2 to maximize its second-period profit: $\max_{p_{i2} \ge 0} \pi^{DVP}_{i2} = p_{i2} d^{DVP}_{i2}$, for $i = H, L$.

Similar to the analysis under committed pricing, we only focus on the case that the market is partially covered, which is true when t_p is sufficiently small. Thus, the demands of period 2 are:

$$d^{DVP}_{H2} = 1 - \frac{(p_{H2} - t_p d^{DVP}_{H1}) - (p_{L2} - t_p d^{DVP}_{L1})}{q_H - q_L} \quad \text{and} \quad d^{DVP}_{L2} = \frac{(p_{H2} - t d^{DVP}_{H1}) - (p_{L2} - t_p d^{DVP}_{L1})}{q_H - q_L} - \frac{p_{L2} - t_p d^{DVP}_{L1}}{q_L}.$$

One can easily verify that π_{i2}^{DVP} is concave in p_{i2}, i.e., $\frac{\partial^2 \pi_{i2}^{DVP}}{\partial p_{i2}^2} < 0$. Solving the two firms' first-order-conditions together leads to the prices in period 2 as below:

$$\begin{aligned} p_{H2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{2q_H^2 + (2t_p q_H - t_p q_L)d_{H1}^{DVP} - t_p q_H d_{L1}^{DVP} - 2q_H q_L}{4q_H - q_L} \text{ and} \\ p_{L2}^{DVP}(q_H, q_L, t_p, d_{L1}^{DVP}, d_{H1}^{DVP})) &= \frac{q_H q_L + (2t_p q_H - t_p q_L)d_{L1}^{DVP} - t_p q_L d_{H1}^{DVP} - q_L^2}{4q_H - q_L}. \end{aligned} \tag{B7}$$

The corresponding demand and profit of each firm in period 2 are as below:

$$\begin{aligned} d_{H2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{2q_H^2 + (2t_p q_H - t_p q_L)d_{H1}^{DVP} - t_p q_H d_{L1}^{DVP} - 2q_H q_L}{4q_H^2 - 5q_H q_L + q_L^2}, \\ d_{L2}^{DVP}(q_H, q_L, t_p, d_{L1}^{DVP}, d_{H1}^{DVP}) &= \frac{(2t_p q_H^2 - t_p q_H q_L)d_{L1}^{DVP} - t_p q_H q_L d_{H1}^{DVP} + q_H^2 q_L - q_H q_L^2}{q_L(4q_H^2 - 5q_H q_L + q_L^2)}, \\ \pi_{H2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{((t_p q_L - 2t_p q_H)d_{H1}^{DVP} + t_p q_H d_{L1}^{DVP} + 2q_H q_L - 2q_H^2)^2}{(4q_H - q_L)^2(q_H - q_L)}, \text{ and} \\ \pi_{L2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{q_H((2t_p q_H - t_p q_L)d_{L1}^{DVP} - t_p q_L d_{H1}^{DVP} + q_H q_L - q_L^2)^2}{(4q_H - q_L)^2(q_H - q_L)q_L}. \end{aligned} \tag{B8}$$

Step 2: Price Decisions in Period 1. Given the firms' prices in period 2, firm i decides price in period 1, i.e., p_{i1}, to maximize its total profit of two periods:

$$\max_{p_{i1} \geq 0} \pi_i^{DVP} = p_{i1} d_{i1}^{DVP} + \pi_{i2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}), \text{ for } i, j = H, L \text{ and } i \neq j.$$

Again, following the same logic as in the proof of Proposition 3, we can show that the market is partially covered and each firm's demand is positive in each period in the equilibrium for sufficiently small t_p. Thus, each firm's demand in period 1 can be written as: $d_{H1}^{DVP} = 1 - \frac{p_{H1} - p_{L1}}{q_H - q_L}$ and $d_{L1}^{DVP} = \frac{p_{H1} - p_{L1}}{q_H - q_L} - \frac{p_L}{q_L}$. For sufficiently small t_p (i.e., $0 < t_p < T^{DVP}(q_H, q_L) = \frac{4q_L q_H^2 - 5q_H q_L^2 + q_L^3}{2q_H^2 - q_H q_L + q_L^2}$), π_i^{DVP} is concave in p_{i1}, and thus firms' prices in period 1 can be solved from the first-order-conditions, i.e., $\frac{d\pi_H^{DVP}}{dp_{H1}} = 0$ and $\frac{d\pi_L^{DVP}}{dp_{L1}} = 0$. Hence, the prices in period 1 are given as below:

$$p_{H1}^{DVP}(q_H, q_L, t_p) = \frac{2q_H(X_5 + X_6)}{X_7} \quad \text{and} \quad p_{L1}^{DVP}(q_H, q_L, t_p) = \frac{q_L X_5}{X_7}, \tag{B9}$$

where $X_5 = -q_H q_L^3 \left(-24q_L^2 t_p^2 + 4q_L t_p^3 + 15q_L^4 + 4t_p^4\right) + q_H^2 q_L^2 \left(8q_L^3 t_p - 98q_L^2 t_p^2 + 12q_L t_p^3 + 87q_L^4 + 16t_p^4\right) - q_H^3 q_L \left(40q_L^3 t_p - 168q_L^2 t_p^2 + 4q_L t_p^3 + 245q_L^4 + 20t_p^4\right) + 32q_H^6 \left(-q_L t_p + 2q_L^2 - t_p^2\right) + 8q_H^5 \left(5q_L^2 t_p + 11q_L t_p^2 - 30q_L^3 + 3t_p^3\right) + 4q_H^4 \left(6q_L^3 t_p - 37q_L^2 t_p^2 + q_L t_p^3 + 87q_L^4 + 6t_p^4\right) - 2q_L^6 t_p^2 + q_L^8$, $X_6 = t_p \left(-q_H q_L + 2q_H^2 + q_L^2\right) (q_H q_L \left(-3q_L^2 t_p + 10q_L t_p^2 - 10q_L^3 + 2t_p^3\right) + 8q_H^4 \left(2q_L + t_p\right) - 2q_H^3 \left(11q_L t_p + 20q_L^2 + 3t_p^2\right) + q_H^2 \left(17q_L^2 t_p - 10q_L t_p^2 + 33q_L^3 - 6t_p^3\right) - 2q_L^3 t_p^2 + q_L^5)$, and $X_7 = 64q_H^6 \left(4q_L^2 - t_p^2\right) - 16q_H^5 \left(48q_L^3 - 7q_L t_p^2\right) + 24q_H^4 \left(-10q_L^2 t_p^2 + 36q_L^4 + t_p^4\right) + q_H^3 \left(238q_L^3 t_p^2 - 20q_L t_p^4 - 464q_L^5\right) + q_H^2 \left(-118q_L^4 t_p^2 + 16q_L^2 t_p^4 + 129q_L^6\right) - 2q_H \left(-13q_L^5 t_p^2 + 2q_L^3 t_p^4 + 9q_L^7\right) - 2q_L^6 t_p^2 + q_L^8$.

Given the firms' prices in period 1, the corresponding demands in period 1 are as below:

$$\begin{aligned} d_{H1}^{DVP}(q_H, q_L, t_p) &= 1 - \frac{p_{H1}^{DVP}(q_H, q_L, t_p) - p_{L1}^{DVP}(q_H, q_L, t_p)}{q_H - q_L}, \text{and} \\ d_{L1}^{DVP}(q_H, q_L, t_p) &= \frac{p_{H1}^{DVP}(q_H, q_L, t_p) - p_{L1}^{DVP}(q_H, q_L, t_p)}{q_H - q_L} - \frac{p_{L1}^{DVP}(q_H, q_L, t_p)}{q_L}. \end{aligned} \tag{B10}$$

The corresponding price, demand, and profit of each firm in period 2 are obtained by substituting Equations (B10) into Equations (B7) and (B8):

$$
\begin{aligned}
p_{H2}^{DVP}(q_H,q_L,t_p) &= p_{H2}^{DVP}(q_H,q_L,t_p,d_{H1}^{DVP}(q_H,q_L,t_p),d_{L1}^{DVP}(q_H,q_L,t_p)), && \text{(B11a)}\\
p_{L2}^{DVP}(q_H,q_L,t_p) &= p_{L2}^{DVP}(q_H,q_L,t_p,d_{L1}^{DVP}(q_H,q_L,t_p),d_{L1}^{DVP}(q_H,q_L,t_p)), && \text{(B11b)}\\
d_{H2}^{DVP}(q_H,q_L,t_p) &= d_{H2}^{DVP}(q_H,q_L,t_p,d_{H1}^{DVP}(q_H,q_L,t_p),d_{L1}^{DVP}(q_H,q_L,t_p)), && \text{(B11c)}\\
d_{L2}^{DVP}(q_H,q_L,t_p) &= d_{L2}^{DVP}(q_H,q_L,t_p,d_{L1}^{DVP}(q_H,q_L,t_p),d_{L1}^{DVP}(q_H,q_L,t_p)), && \text{(B11d)}\\
\pi_{H2}^{DVP}(q_H,q_L,t_p) &= \pi_{H2}^{DVP}(q_H,q_L,t_p,d_{H1}^{DVP}(q_H,q_L,t_p),d_{L1}^{DVP}(q_H,q_L,t_p)), \text{ and} && \text{(B11e)}\\
\pi_{L2}^{DVP}(q_H,q_L,t_p) &= \pi_{L2}^{DVP}(q_H,q_L,t_p,d_{L1}^{DVP}(q_H,q_L,t_p),d_{L1}^{DVP}(q_H,q_L,t_p)), && \text{(B11f)}
\end{aligned}
$$

Moreover, firm i's total demand is $d_i^{DVP}(q_H,q_L,t_p) = d_{i1}^{DVP}(q_H,q_L,t_p) + d_{i2}^{DVP}(q_H,q_L,t_p)$ and its total profit is $\pi_i^{DVP}(q_H,q_L,t_p) = p_{i1}^{DVP}(q_H,q_L,t_p)d_{i1}^{DVP}(q_H,q_L,t_p) + p_{i2}^{DVP}(q_H,q_L,t_p)d_{i2}^{DVP}(q_H,q_L,t_p)$.

Step 3: Quality Decisions. Given the firms' pricing decision in Stage 2, we proceed to solve firms' quality decisions in Stage 1:

$$\max_{q_H\in(q_L,1]} \pi_H^{DVP}(q_H,q_L,t_p) \quad \text{and} \quad \max_{q_L\in[0,q_H)} \pi_L^{DVP}(q_H,q_L,t_p).$$

First, one can easily verify that $\pi_H^{DVP}(q_H,q_L,t_p)$ increases in q_H given that the market is partially covered and each firm has positive demand for sufficiently small t_p. Thus, firm H's optimal quality is $\hat{q}_H^{DVP} = 1$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L. Taking derivative with respect to q_L, one can verify that $\pi_L^{DVP}(1,q_L,t_p)$ is concave in q_L given that the market is partially covered and each firm has positive demand for sufficiently small t_p. Therefore, firm L's optimal quality $\hat{q}_L^{DVP}(t_p)$ is solved from $\frac{d\pi_L^{DVP}(1,q_L,t_p)}{dq_L} = 0$.

Thus, the equilibrium outcome is: $q_H = \hat{q}_H^{DVP} = 1$, $q_L = \hat{q}_L^{DVP}(t_p)$, and $p_{in} = \hat{p}_{in}^{DVP}(t_p) = p_{ni}^{DM}(1,\hat{q}_L^{DVP}(t_p),t_p)$, for $i = H,L$ and $n = 1,2$. Let $\hat{d}_{in}^{DVP}(t_p) = d_{in}^{DVP}(1,\hat{q}_L^{DVP}(t_p),t_p)$ and $\hat{\pi}_{in}^{DVP}(t_p) = \hat{p}_{in}^{DVP}(t_p)\hat{d}_{in}^{DVP}(t_p)$ denote firm i's demand and profit in period n. Firm i's total demand and total profit are given by $\hat{d}_i^{DVP}(t_p) = \hat{d}_{i1}^{DVP}(t_p) + \hat{d}_{i2}^{DVP}(t_p)$ and $\hat{\pi}_i^{DVP}(t_p) = \hat{\pi}_{i1}^{DVP}(t_p) + \hat{\pi}_{i2}^{DVP}(t_p)$.

Note that we can apply similar approach as in the proof of Proposition 3 to show the existence and uniqueness of the above equilibrium in which the market is partially covered and each firm's demand is positive in each period. The detailed proof is omitted and available from authors upon request. The proof of Proposition B2 parts(a)-(d) are provided in proof of Proposition 5. $\square$

Proof of Proposition 5: Note that all the final equilibrium outcomes are differentiable.

Part (a): $\hat{q}_L^{DVP}(t_p)$ is the unique solution of $\frac{\partial\pi_L^{DVP}(1,q_L,t_p)}{\partial q_L} = 0$. By the Implicit Function Theorem, we have $\frac{\partial\hat{q}_L^{DVP}(t_p)}{\partial t_p} = -\frac{\partial^2\pi_L^{DVP}(1,q_L,t_p)}{\partial q_L\partial t_p} / \frac{\partial^2\pi_L^{DVP}(1,q_L,t_p)}{\partial q_L^2}|_{q_L=\hat{q}_L^{DVP}(t_p)}$. We have already shown $\frac{\partial^2\pi_L^{DVP}(1,q_L,t_p)}{\partial q_L^2} < 0$, and can verify that $\frac{\partial^2\pi_L^{DVP}(1,q_L,t_p)}{\partial q_L\partial t_p} < 0$ for any $q_L \in (0.2,0.77)$ and $t_p \in (0,\bar{t}_p]$. Moreover, we find that $\hat{q}_L^{DVP}(t_p) \in (0.2,0.77)$ for any $t_p \in (0,\bar{t}_p]$. Therefore, $\frac{\partial\hat{q}_L^{DVP}(t_p)}{\partial t_p} < 0$, i.e. $\hat{q}_L^{DVP}(t_p)$ decreases in t_p and $\hat{q}_L^{DVP}(t_p) < \hat{q}_L^b$.
Next, we compare $\hat{q}_L^{DVP}(t_p)$ with $\hat{q}_L^{VP}(t_p)$. Directly solving $\frac{\partial\pi_L^{DVP}(1,q_L,t_p)}{\partial q_L} = 0$ and $\frac{\partial\pi_L^{VP}(1,q_L,t_p)}{\partial q_L} = 0$, we can check that $\hat{q}_L^b > \hat{q}_L^{DVP}(t_p) > \hat{q}_L^{VP}(t_p)$ when $t_p < \tilde{t}$ and $\hat{q}_L^{DVP}(t_p) < \hat{q}_L^{VP}(t_p) < \hat{q}_L^b$ when $\tilde{t} < t_p < \bar{t}_p$, where $\tilde{t} \approx 0.0829548$.

Part (b): Note that $\hat{p}_{in}^{DVP}(t_p) = p_{in}^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$, for $i = H, L$ and $n = 1, 2$. By the chain rule, $\frac{\partial \hat{p}_{in}^{DVP}(t_p)}{\partial t_p} = (\frac{\partial p_{in}^{DVP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial p_{in}^{DVP}(1,q_L,t_p)}{\partial q_L}\frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p})|_{q_L=\hat{q}_L^{DVP}(t_p)}$. Similar to the previous analysis, one can verify that: 1) $\hat{p}_{H1}^{DVP}(t_p)$ first decreases and then increases in t_p; 2) $\hat{p}_{L1}^{DVP}(t_p)$ decreases in t_p; and 3) $\hat{p}_{H2}^{DVP}(t_p)$ and $\hat{p}_{L2}^{DVP}(t_p)$ increase in t_p. Comparing with the benchmark prices, we can further show that $\hat{p}_{i1}^{DVP}(t_p) < \hat{p}_i^b$ and $\hat{p}_{i2}^{DVP}(t_p) > \hat{p}_i^b$, for $i = H, L$.

Next, substitute $\hat{q}_L^{DVP}(t_p)$ into $p_{i1}^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ (Equations (B9)) and $p_{i2}^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ (Equations (B11a) and (B11b)); and substitute $\hat{q}_L^{VP}(t_p)$ into $p_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$ (from Equations (A13)). After some tedious but straightforward simplifications, one can verify that $\hat{p}_{H1}^{DVP}(t_p) < \hat{p}_H^b < \hat{p}_H^{VP}(t_p) < \hat{p}_{H2}^{DVP}(t_p)$ and $\hat{p}_{L1}^{DVP}(t_p) < \hat{p}_L^{VP}(t_p) < \hat{p}_L^b < \hat{p}_{L2}^{DVP}(t_p)$.

Part (c): Note that $\hat{d}_i^{DVP}(t_p) = d_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{DVP}(t_p)}{\partial t_p} = (\frac{\partial d_i^{DVP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial d_i^{DVP}(1,q_L,t_p)}{\partial q_L}\frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p})|_{q_L=\hat{q}_L^{DVP}(t_p)}$. Taking similar approach, we can check that: 1) $\hat{d}_H^{DVP}(t_p)$ increases in t_p; and 2) $\hat{d}_L^{DVP}(t_p)$ first decreases and then increases in t_p.

Next, substitute $\hat{q}_L^{DVP}(t_p)$ into $d_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ and $\hat{q}_L^{VP}(t_p)$ into $d_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. After some straightforward yet tedious simplifications, one can verify that $\hat{d}_H^{DVP}(t_p) < \hat{d}_H^{VP}(t_p)$ and $\hat{d}_L^{DVP}(t_p) > \hat{d}_L^{VP}(t_p)$.

Part (d): Note that $\hat{\pi}_i^{DVP}(t_p) = \pi_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{\pi}_i^{DVP}(t_p)}{\partial t_p} = (\frac{\partial \pi_i^{DVP}(1,q_L,t_p)}{\partial t_p} + \frac{\partial \pi_i^{DVP}(1,q_L,t_p)}{\partial q_L}\frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p})|_{q_L=\hat{q}_L^{DVP}(t_p)}$. Again, via similar approach, we can verify that $\hat{\pi}_H^{DVP}(t_p)$ increases in t_p and $\hat{\pi}_L^{DVP}(t_p)$ decreases in t_p.

Substitute $\hat{q}_L^{DVP}(t_p)$ into $\pi_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ and $\hat{q}_L^{VP}(t_p)$ into $\pi_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. After tedious but straightforward simplifications, one can verify that: 1) $\hat{\pi}_L^b > \hat{\pi}_L^{DVP}(t_p) > \hat{\pi}_L^{VP}(t_p)$ for any $t_p \in (0, \bar{t}_p]$; and 2) $\hat{\pi}_H^{VP}(t_p) > \hat{\pi}_H^{DVP}(t_p) > \hat{\pi}_H^b$ when $t_p < \hat{t}$ and $\hat{\pi}_H^{DVP}(t_p) > \hat{\pi}_H^{VP}(t_p) > \hat{\pi}_H^b$ when $\hat{t} < t_p \leq \bar{t}_p$, where $\hat{t} \approx 0.058082$.

This completes the proof of Proposition 5 and Proposition B2 parts(a)-(d). $\square$