https://doi.org/10.1016/j.trc.2020.102761

Submitted to *Transportation Research Part C*
manuscript

Evacuating metro passengers via the urban bus system under uncertain disruption recovery time and heterogeneous risk-taking behaviour

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The bus-bridging service has always faced problems in severe emergencies or catastrophes that require the large-scale evacuation of passengers. This paper provides an alternative evacuation scheme which uses the urban bus network in the case of common metro service disruptions; this is modeled by minimizing the total cost of the affected metro passengers, through jointly selecting the bus lines and frequencies. The uncertain recovery time of the service disruption and the heterogeneous risk-taking behavior of the affected metro passengers are incorporated in the scheme. Therefore, we build a linkage between the evacuation service design and the risk-taking behavior of passengers. A heuristic algorithm is proposed to calculate the optimal evacuation scheme. A numerical experiment using a real-world network is conducted to illustrate the validity of the model and algorithm.

Key words: Metro disruption, heterogeneous risk-taking behaviour, bus-bridging service, passenger evacuation

1. Introduction

1.1. Research motivations

Metro services play an increasingly important role in many mega-cities across the world (Kang et al. 2015, Sun et al. 2016, Weng et al. 2014). However, with the excessive utilization and large-scale expansion of the metro network, service disruptions have become frequent. For example, over 100 metro service disruptions occurred in Shanghai in 2015 (http://service.shmetro.com/lcywgz/index.htm). In Beijing, the number of disruptive incidents in the first quarter of 2015 was up to 146% over that of the previous year (more than one metro service disruption happened every two days) (Sun et al. 2016). In practice, the transportation agencies are mainly concerned about severe emergencies or catastrophes, and design an urgent evacuation scheme. The transportation agencies in many mega-cities have designed the contingency plan according to the estimated delay time or emergency level (GOSC 2015). In Singapore, the metro operators adopt different response strategies according to the disruption delay. When the delay is estimated to exceed 30 minutes, the bus-bridging service should be activated with 12-minute headway in peak hours or 15-minute headway in an off-peak hour. The bus-bridging service would cover 20% to 100% of the affected metro stations, according to the estimated time delay(MOT 2011).

However, to our knowledge, agencies seldom conscientiously implement a contingency plan to cope with common metro disruptions, which are less severe and interrupt the regular service for several minutes (e.g., 5 minutes) to an hour. For example, an agency would not be willing to activate the bus-bridging service for disruption caused by a door failure, even though the door failure would result in a delay of over half an hour. It is indeed very costly to invest in the fleet and provide a bus-bridging service for some common disruptions. Therefore, we aim to investigate whether the urban bus system can be adopted as one of the alternative evacuation schemes.

It is practically challenging to evacuate the affected metro passengers using the urban bus system during disruption. Unlike in common disruptions, in severe emergencies, all the affected passengers are assumed to leave the disrupted metro lines. The abnormal passenger flows under common metro disruptions would depend on individuals' behavior, which is affected by the disruption type, availability and utility of the alternative routes, and individual perception of the delay time. Hence, it is necessary to develop a model to analyze the abnormal behavior of the affected passengers. The metro service disruptions are attributed to various factors, such as the failure of signals, vehicles, the power system, rail track and doors, and falling objects or passengers. Notably, the uncertain recovery time of a metro service disruption highly depends on related types of these factors. For instance, the mean and variance of the recovery time resulting from a power or signal failure are significantly higher than for the other types. Figure 1 shows the causing factors and the corresponding percentages of the Beijing metro service disruptions in 2015. According to Figure 1 (a), about half of the disruptions are caused by signal failure. The recovery time distributions caused by different factors are quite different from each other. It can be seen from Figure 1 (b) that the agency spends much more time to fix the problems caused by signal and power failures than the other factors. Furthermore, the passengers would have different perceptions of the recovery time under different types of disruptions. They generally expect that the disruptions caused by door failure would be resolved sooner than those caused by signal or power failure. The perceptions of passengers are also affected by their journey times (Sun et al. 2016).

Another challenge when adopting an urban bus system is to select the bus lines from the urban bus network and determine their frequencies. Unlike the traditional bus-bridging service, the network configuration and the available capacity of the existing urban bus system are the main constraints that determine the evacuation scheme. It is not always efficient to evacuate the affected passengers by utilizing the available capacity of the urban bus system as much as possible, as this is also dependent on the number of the affected metro passengers transferring to the urban bus system. The design of the evacuation scheme using urban buses is a systematical decision problem of jointly selecting the bus lines and corresponding frequencies, while considering the behavior of the affected metro passengers and the constraints of the current urban bus and metro network.

1.2. Literature review

The literature on metro incident management can be classified into two categories, i.e., pre-disruption analyses and post-disruption emergency rescue planning. The former aims to enhance the metro service resilience by identifying the sources of the disruptions, estimating the system delay, analyzing the passengers' response behavior, and designing a robust metro service network. The latter category, on the other hand, aims to develop a real-time re-scheduling plan and a bus bridging scheme to evacuate the affected passengers in the disrupted metro systems. The findings of the former studies often provide useful insights for designing an efficient evacuation strategy for the post-disruption scenarios.

There is abundant literature focusing on the pre-disruption analyses. For example, Weng et al. (2014, 2015) developed several models to predict the incident delay caused by the aforementioned occurrence factors. Goverde (2010) proposed the delay propagation algorithm to investigate the dominant effect of the secondary delay caused by disruption to a large-scale metro network. The robust network design problem attempted to minimize the effect of the service disruption by using a well-designed metro network topology (Laporte et al. 1999, De Los Santos et al. 2012). In recent years, many empirical studies have been conducted to model metro passengers' behavior in the face of service disruption. Among them, using Spanish railway data, Dellolio et al. (2013) found that the metro passengers' behavior is mainly affected by the emergency type, journey duration, duration of the disruption, passenger type, and information availability. Kattan et al. (2019) pointed out that the severe metro service disruptions would generate both short-term and long-term change in the travelers' behavior. Sun et al. (2016) classified the choices adopted by the metro passengers facing a service disruption into three alternatives: leaving the metro system, detouring in the metro system, and doing nothing but waiting for the system's recovery.

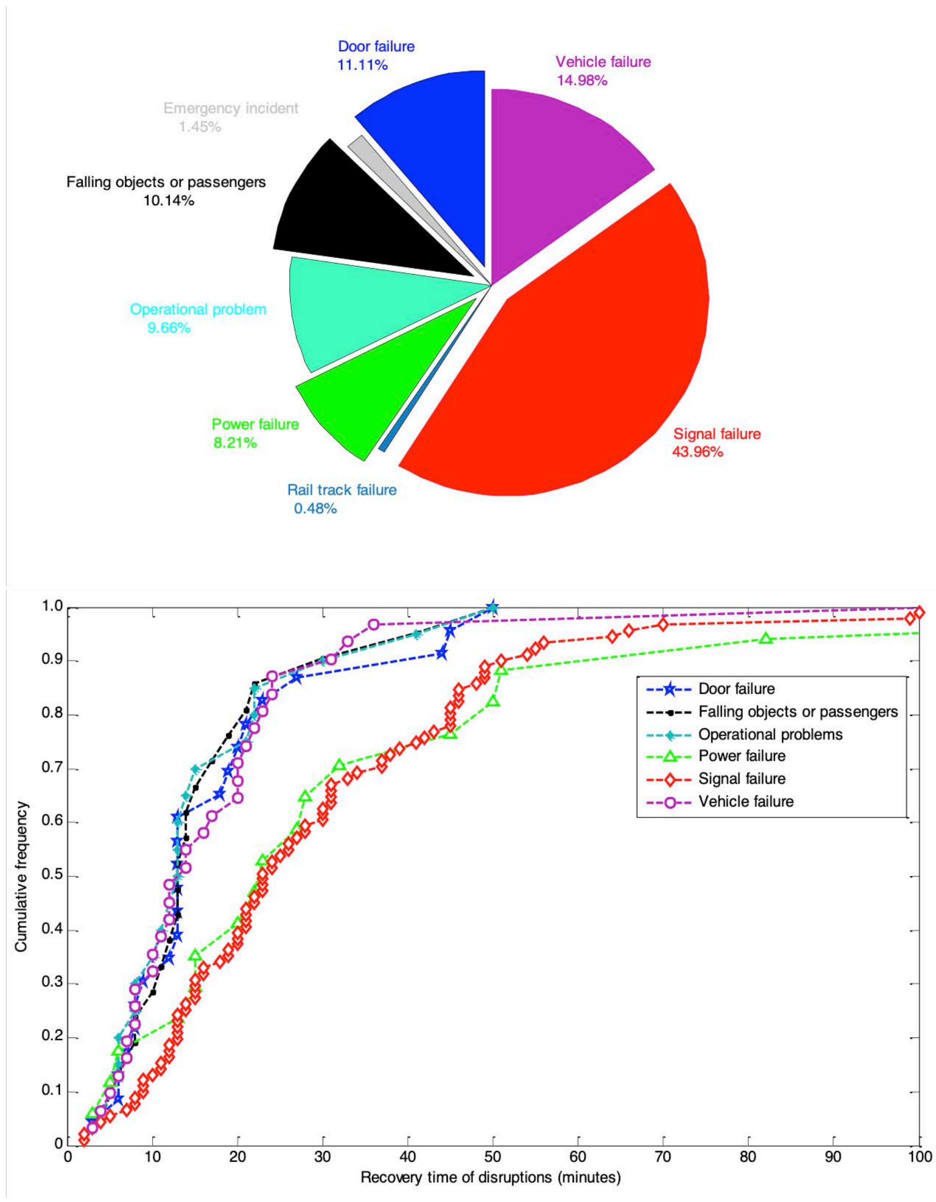


Figure 1 **The causes and recovery time distribution of disruptions to Beijing Metro in 2015 (data collected from Beijing Metro official Weibo)**

As for the plans, Pender et al. (2012) interviewed 48 international passenger transit rail organizations and summarized several commonly adopted response strategies to disruptions, including bus-bridging services, transferring passengers to other rail lines, and tracking crossovers. Jespersen et al. (2009) discussed three sub-problems related to disruption response management for railway systems: timetable adjustment, rolling-stock rescheduling, and crew rescheduling (Almodovar and Garcia 2013, Spliet et al. 2014, Malucelli and Tresoldi 2019). The most prevalent strategy in practice is called "bus-bridging services", which aim to bridge the affected metro links by using additional buses (Kepaptsoglou and Karlaftis 2009, Kepaptsoglou et al. 2010, Wang et al. 2014, Deng et al. 2018). Jin et al. (2014, 2016) proposed a viable method to enhance the metro network's resilience to disruptions, with the integration of public bus services. Zhang and Lo (2018) investigated the initiation time of the bus-bridging service, incorporated into the uncertain disruption recovery time. The uncertainty of the recovery time was described by a probability distribution. Gu et al. (2018) designed a flexible bus-bridging strategy, which does not restrict a bus to serve one bus route with a given frequency. This strategy can efficiently deal with the unplanned metro disruptions and uncertain arrivals of the affected passengers. Furthermore, Liand et al. (2019) designed a robust bus-bridging service to incorporate the uncertainty of the bus travel time.

The issue of the metro-bus interaction or coordination has attracted much attention in recent years. Liand et al. (2019) noted the oversaturation problem in some megacities, and proposed that the bus transit network design problems related to the uncertainty of the metro network. They aimed to mitigate the imbalance present in the modal use caused by the development of megacities in recent years. Dou et al. (2019) proposed the parallel shuttle-bus service design for planned metro shutdown, including aspects of the route design, terminal selection, berth allocation, and bus deployment, to minimize the inconvenience caused to passengers. Guo et al. (2018) proposed a mixed-integer programming model to design a smooth synchronization of the first-trains and the bus service. Then, Kang et al. (2019) addressed the method of bridging the passengers in the last trains with a bus service, and proposed a mixed-integer linear programming model to solve the train-bus coordination problem. The metro-bus coordination design is more important than the synchronization of the pure bus system, in terms of the recent development of megacities (Chu et al. 2019). Zhang and Lo (2020) addressed the issue of designing a contract with a bus company for the provision of a substitute bus service under metro disruption, including its availability and service start time guarantee, and its pricing schemes, under the uncertain metro system recovery time.

1.3. Main contributions

Most of the previous studies have focused on severe emergencies or catastrophes, and the large-scale evacuation of passengers. The main objective of the bus-bridging service is to design a bus network and allocate the bus capacity among those affected metro stations. However, the common service disruptions, which account for a large percentage of the incidents in the metro systems, are seldom considered by the existing literature. In this paper, we propose a coordination strategy that allows the urban bus and metro system to evacuate the affected metro passengers, by considering the uncertainty of recovery time and the risk attitude of passengers. The main contributions include the following two aspects.

First, this study intends to establish a linkage between the risk-taking behavior of metro passengers and evacuation service design under metro disruption. The previous research always focused on severe emergencies, and naturally assumed that all the affected passengers leave the disrupted system. In common metro disruption, the abnormal behavior depends on the individual perceptions of the affected passengers. They value the metro disruption delay according to the disruption type, travel cost of the alternatives, and their own risk-taking behavior on the delay variance (Lo 2006). The proposed model enriches the studies on the quantitative analysis of abnormal passenger flows under common metro service disruptions.

Second, we propose an evacuation scheme that adopts the urban bus system during metro disruptions, in the line of the metro-bus coordination (Guo et al. 2018). The urban bus system is viewed as the potential bus-bridging service, given that it is expensive for the transportation agency itself to invest in a fleet and provide the bus-bridging service. Our scheme is quite simple, and equivalent to the optimal joint decision problem of selecting the efficient bus lines and corresponding frequencies from the existing urban bus network. The optimal scheme is dependent on the affected passenger flows, and the uncertainty characteristics of the disruptions and the urban bus system. Therefore, it is an adaptive and disruption-triggered strategy.

The remainder of the paper is organized as follows. Section 2 describes how to represent a metro-bus evacuation network, and how to formulate the heterogeneous evacuation behavior of passengers under common metro service disruptions. The mathematical model is proposed in the section discussing the problem of bus bridging, with service frequency determined by incorporating the heterogeneous risk-taking behavior of passengers. Section 3 investigates the properties of the optimal solution to the evacuation problem. A heuristic algorithm is presented in Section 4, to obtain the evacuation scheme. In Section 5, a numerical experiment based on the Wuhan metro and bus network is conducted, to demonstrate the proposed model and algorithm, as well as to provide some managerial insights. Section 6 concludes the paper and discusses directions for future research.

2. Notations, Assumptions and Problem Description

2.1. Metro-bus evacuation network

Consider a bi-modal urban public transport system comprising a metro network and public bus network. The metro network includes a set of metro stations connected by a set of metro services (lines), and the public bus network comprises a set of bus stops linked by a set of bus services (lines). We assume that when a common metro service disruption occurs, the affected passengers have two choices: (i) changing to the public bus services to reach their destination metro stations; and (ii) staying at their current metro stations to wait for the metro service's recovery. They make decisions by comparing the generalized travel costs of the above two alternatives. Note that the affected passengers would detour in the metro network to find alternative metro routes. We will explain how to incorporate this consideration at the end of the next subsection. It is also assumed that the recovery time of the metro service is uncertain. Furthermore, the perceived recovery time of a disrupted metro service is different among passengers due to their individual risk-taking behavior, which is captured by the weighting coefficient for the variance of the recovery time. In addition, it is reasonable to assume that passengers may take public bus services only if there exists a least one bus stop within an acceptable walking distance from his/her current metro station. The evacuation bus network can thus be constructed for all those viable bus stops near the metro stations.

To incorporate the bus frequency as the decision variable, we adopt the method proposed by De Cea and Fernandez (1993) to construct the bus-section network from the traditional bus-line network, to solve the common-line problem in the bus system (Chriqui and Robillard 1975). The bus-section network can be treated as the road network, and each link has a combined frequency determined by all bus lines crossing the segment. In addition, to explicitly capture the effect of the metro disruption, the metro network is represented by a directed graph with a set of OD pairs denoted by W. Each metro OD pair $w \in W$ is associated with a set of attractive bus lines. Therefore, the bus-section network is constructed according to the metro sub-network, and is the sub-graph of the urban bus network. The resultant network is referred to as the "metro-bus evacuation network", and can be constructed through the following steps.

The metro-bus evacuation network construction

Step 1. *Find all nearby bus stops for each metro station with a distance constraint, namely, that the distance between the bus stop and metro station is less than a predetermined threshold value.*

Step 2. *List all bus lines connecting at least two bus stops near to two different metro stations.*

Step 3. *Construct the bus-section network* via the bus lines obtained in Step 2, based on the method of De Cea and Fernandez (1993).

Step 4. Determine the attractive set of bus lines for each section according to the generalized cost, including walking distance, in-vehicle time and waiting time.

The construction of the bus-section network is dedicatedly designed for the aggregative transit assignment, and can be conducted with computer programs (De Cea and Fernandez 1993, Szeto and Jiang 2014). It involves several steps. Specifically, in Step 1, we select the related bus stops within the tolerance for walking distance, denoted by d_0, to one of the metro stations (for example, 500 meters). All the relevant bus stops are grouped into a set denoted by $N = \{n \in N_0, dis(n, W) \leq d_0\}$, where N_0 is the set of all bus stops in the urban area, and $dis(n, W)$ is the spherical walking distance between the bus stop n and the nearest metro station in W. For those bus stops, we choose the subset of the urban bus lines which connect any two bus stops via at most one transfer. In theory, we are interested in all the bus lines connecting two metro stations for each OD pair. However,

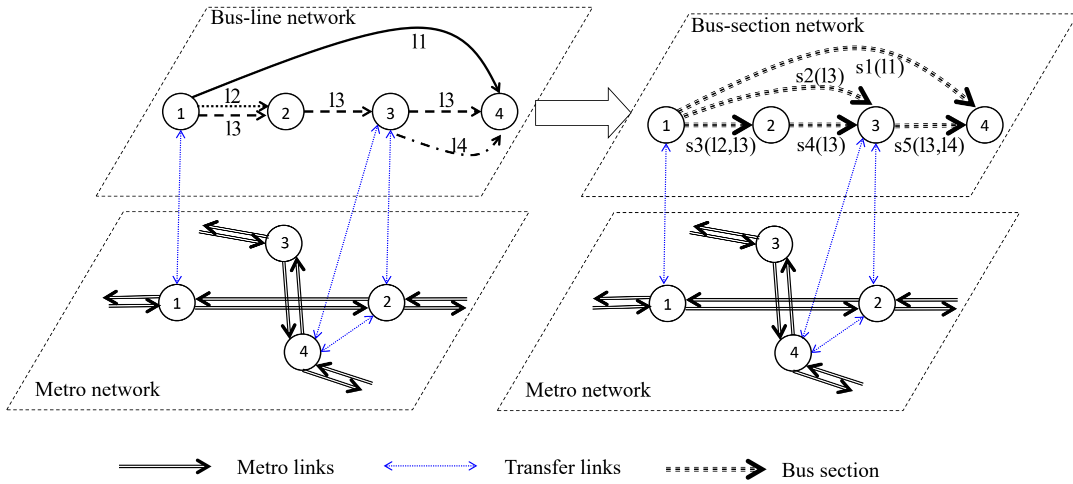


Figure 2 **A graphical explanation of the metro-bus evacuation network**

this is not practically necessary, because the bus routes with too many transfers are inefficient for evacuating metro passengers, and are seldom considered by those passengers.

Figure 2 intuitively illustrates the construction procedure of the bus-section network from the bus-line network of Step 3. For the bus-line network shown in the sub-figure on the left-hand side of Figure 2, there are four bus lines, named bus lines l_1 to l_4, and the metro network connects with the bus network at bus stops 1 and 3. We construct a unique section connecting any two nodes. The section is defined as a set of sections of all bus lines traversing the corresponding nodes, shown as the sub-figure on the right-hand side of Figure 2. For example, road section 3, denoted by s_3, is a set of line sections of bus line l_2 and l_3 from node 1 to node 2. Road section 5, denoted by s_5, is a set of line sections of bus line l_3 and l_4 from node 3 to node 4. The frequency of each section is the sum of the frequencies of all corresponding bus lines. The bus-section network is equivalent to the traditional road network, and it is convenient to apply the traffic assignment models.

2.2. Modeling passengers' evacuation behavior

In this subsection, we focus on modeling the passengers' choice behavior under the metro service disruption. We first explore how the passengers perceive the generalized travel cost of the disrupted metro system. For simplicity, we assume that all metro passengers start and end at the metro stations. Let c_w^M, $w \in W$ denote the generalized travel costs between metro OD pair w in the metro network without disruption. The recovery time between OD pair w, $w \in W$, is a random variable denoted by Δ_w. The mean and standard deviation of the recovery time for the service disruption are denoted by $E(\Delta_w)$ and $Std(\Delta_w)$ respectively. To capture the passengers' risk-taking behavior after the metro disruption, the uncertain generalized travel cost via disrupted metro is expressed as the travel time budget proposed by (Lo 2006) as

$$C_w^M = c_w^M + E(\Delta_w) + \lambda_w Std(\Delta_w) \tag{1}$$

where $\lambda_w \in [\lambda_w^m, \lambda_w^M]$ is the parameter to capture the risk attitude of the passengers between OD pair w to the uncertainty of recovery time. The larger the value of λ_w, the stronger the passengers' aversion to the disruption. This suggests that passengers with a stronger aversion to the disruption are more concerned about the variance of the recovery time. We further assume that λ_w follows a continuous distribution with density function $g_w(\lambda)$ and corresponding CDF $G_w(\lambda)$ and support set $\lambda \in [\lambda_w^m, \lambda_w^M]$.

As for the bus services, let us denote the bus-section network as a triple (N, L, S) obtained from the construction procedure described in Subsection 2.1, where N is the set of the interested bus stops related to all the metro stations, L is the set of all related bus lines, and S is the set of all sections in the bus-section network.

With the bus-section network, we now proceed to discuss the transit assignment method, which is the same as that of the road network (De Cea and Fernandez 1993, Szeto and Jiang 2014). Denote A_s as the set of the attractive lines for section s,$s \in S$. The frequency of the initial bus line is denoted by f_l^0,$l \in L$. Under service disruption, the bus operators choose the emergency frequency as f_l, $l \in L$, with corresponding vector $\mathbf{f}$, $\mathbf{f} = \{f_l, l \in L\}$. Considering the capacity constraint of a bus line, each bus line is associated with an upper bound of the frequency, denoted by $\hat{f}_l$, $l \in L$. The generalized travel cost of each section is the sum of the fixed travel time (including the average walking time), the fixed bus fare, and the frequency-dependent average waiting time, as follows:

$$c_s^B(\mathbf{f}) = t_s^B + \frac{\alpha}{\sum\limits_{l \in A_s} f_l} + \frac{\tau_s^B}{\beta}, f_l \in \left[f_l^0, \hat{f}_l\right] \tag{2}$$

where the first term is the fixed travel time of section s, the second term is the waiting time of section s, the last term is the bus fare of section s, and α is the model calibration parameter. Parameter β is the value-of-time parameter to convert the monetary cost to the time cost. During the disruption, as the time cost is of more concern to the affected passengers, we can select a high value of β to trade-off the monetary cost and time cost. Let R_w^B denote the set of all available bus routes between metro OD pair w, $w \in W$. Each generalized travel cost of the bus route can be calculated as

$$c_{wr}^B(\mathbf{f}) = \sum_{s \in S} c_s^B(\mathbf{f}) \, \delta_{wr}^s \tag{3}$$

where δ_{wr}^s equals 1 when section s is on the bus route r, and 0 otherwise.

We assume that the affected passengers make their choices according to their perceived generalized travel cost in the bus network. The multinomial Logit model is adopted to capture the passengers' choice behavior for bus routes (Sheffi 1985). In this case, the average generalized travel cost between the specified OD pair w, $w \in W$, can be expressed as

$$c_w^B(\mathbf{f}) = -\frac{1}{\eta_w} \ln\left(\sum_{r' \in R_w^B} \exp\left(-\eta_w c_{wr'}^B(\mathbf{f})\right)\right) \tag{4}$$

where η_w is the model parameter to capture the sensitivity of the passengers to the travel cost.

We now proceed to determine the number of passengers transferring to the bus network after the metro service is disrupted, denoted by $q_w(\mathbf{f})$ with given bus frequencies $\mathbf{f}$. Let Q_w denote the total number of passengers associated with OD pair w, $w \in W$. It is clear to see that all passengers with $C_w^M > c_w^B(\mathbf{f})$ will transfer to the urban bus network, while all passengers with $C_w^M < c_w^B(\mathbf{f})$ will wait for the recovery of the disrupted metro system. The passengers with $C_w^M = c_w^B(\mathbf{f})$ show no preference between the two choices. The number of passengers transferring to the bus network, i.e., q_w, $w \in W$, can be calculated by

$$\begin{aligned} q_w(\mathbf{f}) &= Q_w \Pr\left\{C_w^M \geq c_w^B(\mathbf{f})\right\} \\ &= Q_w \Pr\left\{c_w^M + E(\Delta_w) + \lambda_w Std(\Delta_w) \geq c_w^B(\mathbf{f})\right\} \\ &= Q_w \Pr\left\{\lambda_w \geq \frac{c_w^B(\mathbf{f}) - c_w^M - E(\Delta_w)}{Std(\Delta_w)}\right\} \\ &= Q_w \bar{G}_w\left(\frac{c_w^B(\mathbf{f}) - c_w^M - E(\Delta_w)}{Std(\Delta_w)}\right) \end{aligned} \tag{5}$$

where $\bar{G}_w(\lambda) = 1 - G_w(\lambda)$ is the survival function of $G_w(\lambda)$. Eq. (5) is the bus-based evacuation formulation under the service disruption of the metro network. For ease of presentation, we define:

$$\lambda_w(\mathbf{f}) = \frac{c_w^B(\mathbf{f}) - c_w^M - E(\Delta_w)}{Std(\Delta_w)} \tag{6}$$

Specifically, when $\lambda_w(\mathbf{f}) \leq \lambda_w^m$, $\bar{G}_w(\lambda) = 1$ and all affected passengers would transfer to the bus network after the disruption occurs; when $\lambda_w(\mathbf{f}) \geq \lambda_w^M$, $\bar{G}_w(\lambda) = 0$ and no affected passenger is willing to transfer to the bus system after the disruption occurs. The former case means that the bus service between the specified OD pair is not expensive in comparison with the corresponding disrupted metro service, and the affected passengers immediately evacuate to the bus system when the disruption occurs. The latter case implies that the bus service is more expensive than the corresponding disrupted metro service. In the intermediate case, there exists a separation equilibrium state, at which a portion of passengers leave the disrupted metro system, while the rest continue to wait for the recovery of the metro service.

It must be pointed out that the uncertainty characteristics of the recovery time are associated with the disruption types, as shown in Figure 1 of Section 1.1. Both the mean and variance of the recovery time caused by power failure are higher than those caused by door failure. Notably, the uncertainty characteristics of the recovery time have a significant effect on the abnormal behavior of the affected metro passengers. According to Eq. (6), the higher the mean and variance, the smaller is λ_w, and thus, more affected passengers tend to transfer to the bus system; and vice versa. It is straightforward to see that the more serious the disruption, the greater the delay and uncertainty about the system's recovery, and more affected metro passengers leave the system.

In practice, the affected metro passengers detour in the metro system and find alternative metro routes. This detouring behavior is quite commonly seen in many megacities with a complex metro network. The affected passengers can easily find alternative metro routes, to avoid time delay caused by the service disruption. In this case, we can define the generalized travel cost of the alternative routes as the average of the alternative bus routes and alternative metro routes. Then the number of affected passengers transferring to the bus system can be calculated as a probability via the Logit model. Consideration of the detouring behavior does not affect our analysis procedure. However, it has a significant effect on the calibration result, and the number of affected passengers transferring to the bus system is obviously decreasing.

2.3. Mathematical formulation of the evacuation scheme under metro disruption

With the consideration of the behavior of metro passengers under service disruption, we now model the evacuation problem of the transportation agency in the constructed metro-bus evacuation network. We first assume that the bus-section network has been predetermined in Section 2.1. The affected passengers make their choice between waiting in the disrupted metro system or transferring to the bus system according to their own generalized travel disutility. The objective of the agency is to minimize the total expected travel cost EC of all affected passengers by joint selection of the bus lines and corresponding frequencies, which can be formulated by

$$\min_{f_l^0 \le f_l \le \hat{f}_l, l \in L} \mathrm{EC}(\mathbf{f}) = \sum_{w \in W} \left(q_w(\mathbf{f}) c_w^B(\mathbf{f}) + (Q_w - q_w(\mathbf{f})) E\left(C_w^M\right) \right) \tag{7}$$

where $c_w^B(\mathbf{f})$ is the average travel cost via the bus services, given by Eq. (4); $q_w(\mathbf{f})$ is the number of the affected metro passengers leaving to the bus system, determined by Eq. (5).

According to the definition of λ_w given by Eq. (6), the objective function of problem (7) can be rewritten as

$$\begin{aligned} \mathbf{EC}(\mathbf{f}) &= \sum_{w \in W} \left[Q_w \bar{G}_w(\lambda_w(\mathbf{f})) c_w^B(\mathbf{f}) + \left(Q_w - Q_w \bar{G}_w(\lambda_w(\mathbf{f}))\right) E\left(C_w^M\right) \right] \\ &= \sum_{w \in W} \left[Q_w \left(c_w^M + E(\Delta_w)\right) + Q_w Std(\Delta_w) \lambda_w(\mathbf{f}) \bar{G}_w(\lambda_w(\mathbf{f})) \right]. \end{aligned} \tag{8}$$

Note that the first term in the square bracket of Eq. (8) is constant, as the service disruption is given. Therefore, problem (7) can now be reduced to

$$\min_{f_l^0 \le f_l \le \hat{f}_l, l \in L} \widetilde{\mathrm{EC}}(\mathbf{f}) = \sum_{w \in W} J_w(\mathbf{f}) = \sum_{w \in W} Q_w Std(\Delta_w) \lambda_w(\mathbf{f}) \bar{G}_w(\lambda_w(\mathbf{f})) \tag{9}$$

For the metro disruptions caused by serious emergencies, all the affected passengers should be transferred to the bus system or other metro lines. The urgent evacuation scheme is to design the most efficient dispatching network for the bus-bridging service or shuttle service, designed by Jin et al. (2016) and Wang et al. (2014). However, for common metro disruption, the bus-bridging service is seldom implemented by the agencies, because of its high operation cost. In this case, the passengers make their own decisions regarding leaving for the bus system or waiting for the recovery of the metro system. Model (9) provides the coordination scheme of urban buses and metros under the capacity constraint of the urban bus system. It is only necessary to adjust the frequencies of the urban bus lines by increasing the evacuation capacity during the metro disruption. Our method is an additional scheme of the bus-bridging service.

3. Theoretical analysis of the evacuation scheme

In this section, we first examine the choice behavior of the affected metro passengers under the metro service disruptions when the transportation agency does not change the bus frequency; namely, $f_l = f_l^0$ for all $l \in L$. The analysis can be used to estimate the number of passengers transferring to the bus system under metro disruption, with the given mean and variance of the recovery time. Then, according to the heterogeneous risk-taking behavior of passengers, we continue to investigate the optimal decision of the transportation agency, or the optimal solution to the problem (9).

3.1. Heterogeneous behavior of passengers under the metro disruption

Note that the survival function $\bar{G}_w()$ is strictly decreasing, and λ_w, given by Eq. (6) is strictly increasing with respect to $E(\Delta_w)$, $Std(\Delta_w)$ and $c_w^B(\mathbf{f}) - C_w^M$. The straightforward property of the number of passengers shifting to the bus system, $q_w(\mathbf{f})$, is helpful to investigate the optimal evacuation scheme and summarized as follow.

Property 1 *Given bus frequencies $f_l = f_l^0$ for all $l \in L$, $q_w(\mathbf{f})$ is not decreasing with respect to $E(\Delta_w)$, $Std(\Delta_w)$ and $c_w^B(\mathbf{f}) - C_w^M$ for all affected metro OD pairs $w \in W$.*

The property shows the properties of passengers' choice behaviour under metro service disruption. As the cumulative distribution function $G_w(\lambda)$ is increasing (the survival function $\bar{G}_w(\lambda) = 1 - G_w(\lambda)$ is decreasing), the number of passengers transferring to the bus system $q_w(\mathbf{f}^0)$, $w \in W$ is strictly increasing when the gap between the generalized travel cost via bus and metro increases without service disruption. In this section, we simplify $q_w(\mathbf{f}^0)$ and $c_w^B(\mathbf{f}^0)$ as q_w and c_w^B, since $\mathbf{f}^0$ is given unless otherwise specified. Generally, the metro system becomes more attractive to passengers than the bus when the journey distance increases. Therefore, the long-distance passengers have more incentive to wait for the recovery of the disrupted metro system, while the short-distance passengers tend to transfer to the bus system when they have the same risk-taking level. Similarly, the longer the expected recovery time and the larger the variance of the recovery time, the more

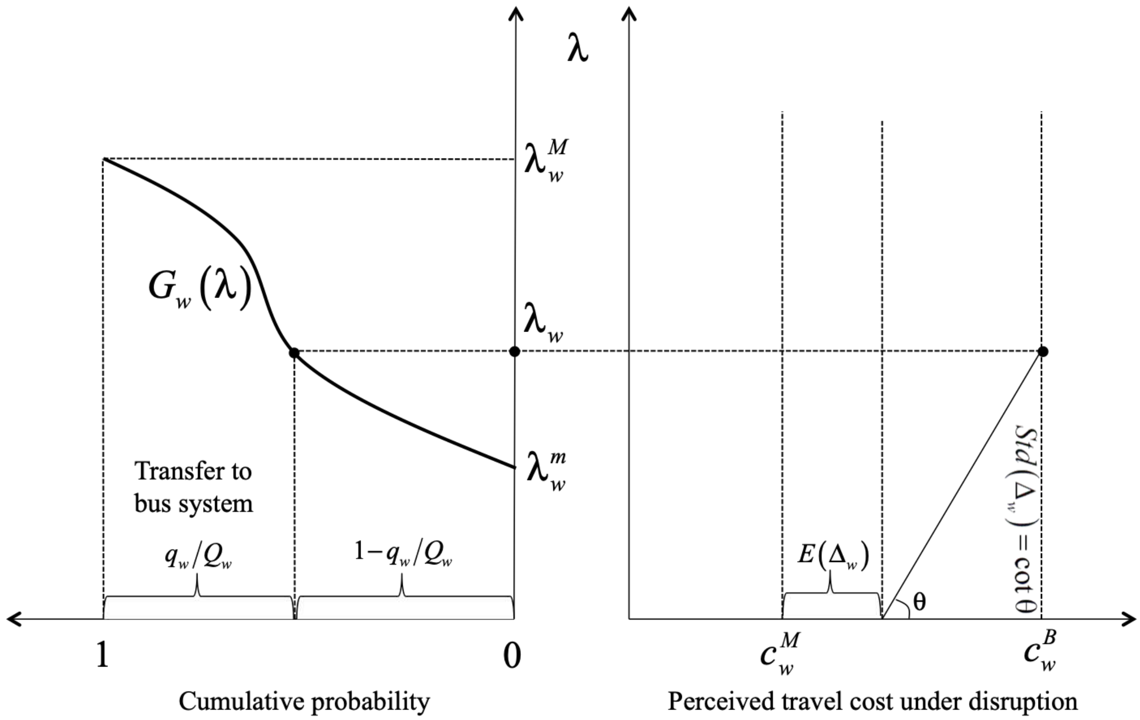


Figure 3 **Heterogenous risk-taking behavior with the urban bus system**

passengers transfer to the bus system for any given OD pair w. Therefore, there are more passengers transferring to the bus system under disruptions caused by power failure than those caused by door failure. The formulation given by Eq. (5) can be graphically explained in Figure 3.

Specially, if the perceived travel cost under service disruption C_w^M is sufficiently larger than that via a bus system, i.e., $C_w^M > c_w^B$ for all λ in support set $[\lambda_w^m, \lambda_w^M]$, all passengers will transfer to the bus system. The scenario corresponds to severe emergencies or catastrophes, such as fire emergency, rail track failure, etc. Conversely, if the perceived travel cost under service disruption C_w^M is sufficiently smaller than that via the bus system, i.e., $C_w^M < c_w^B$, for all λ in support set $[\lambda_w^m, \lambda_w^M]$, then all passengers will wait for the recovery of the disrupted metro system. In reality, no affected metro passenger would leave the system under the disruption, which results in less time delay. For example, the operational problem of buses stopping in a wrong place would cause about 2-3 minutes' delay by having to move the vehicle. In this case, no passenger would leave the system.

To explain the insight of Property 1, we consider the simplest case, with identical effect of the service disruption and identical distribution of the risk-taking heterogeneity among all OD pairs; namely, $\Delta_w \equiv \Delta$ and $G_w() \equiv G()$ for all OD pairs, $w \in W$, with identical support set $[\lambda^m, \lambda^M]$. We also assume that the gap of the travel costs between the metro system and the bus system increases as the journey length increases, meaning that $c_w^B - c_w^M$ is strictly increasing as the number of metro stations between the OD pair increases. With those assumptions, we can claim that $\lambda_w = (c_w^B - c_w^M - E(\Delta))/Std(\Delta)$ is strictly increasing with respect to the number of the metro stations or the journey length N_w. Therefore, we can adopt the following property to distinguish the behavior of the affected metro passengers.

Property 2 *When the bus frequencies $f_l = f_l^0$ for all $l \in L$, and λ_w is strictly increasing with the journey length of metro passengers,, then there exist N^- and N^+ with $N^- = \max\{N_w : \lambda_w \le \lambda^m\}$ and $N^+ = \min\{N_w : \lambda_w \ge \lambda^M\}$, such that*
(1) the passengers with a journey length less than N^- leave the metro system;
(2) the passengers with a journey length greater than N^+ wait for the recovery of the disrupted metro system;
(3) the passengers with a journey length between N^- and N^+ leave the metro system provided that their risk-taking parameter λ is larger than λ_w, and vice versa.

Property 2 depicts the affected metro passengers' choice behavior of leaving or waiting in the disrupted metro system. The heterogeneous behavior mainly depends on their journey length and their own risk-taking parameter λ_w. The passengers have more incentive to leave the metro system when their journey length is shorter. Similarly, the metro passengers with a larger risk-taking parameter λ_w dislike the uncertainty of the recovery time, and thus have more incentive to leave the system. The corollary provides a method for the transportation agency to estimate the number of affected passengers transferring to the bus system between each OD pair. Note that N^- could be zero, or no one will leave the metro system when the mean and variance of the recovery time are tiny. Also, N^+ could be larger than the longest journey, or all passengers could leave the metro system, such as in cases of power failure or severe catastrophes.

Taking a many-to-one metro line as an example, in this case it is reasonable to assume that $c_w^B - c_w^M$ is strictly increasing with the number of metro stations. In Figure 4, for OD pairs with a short journey length $N_w \le N^-$, all passengers shift to the bus system, shown in green color; for OD pairs with a long journey length $N_w \ge N^+$, all passengers choose to wait in the metro system, shown in red color; for other OD pairs with a journey length between N^- and N^+, the proportion of passengers shifting to bus system is calculated by the survival function, $\bar{G}_w(\lambda_w)$, shown in blue color. As the journey length increases, the proportion of passengers shifting to the bus system is decreasing. The results are consistent with the observation of the empirical study by Sun et al. (2016). Furthermore, as discussed in Section 2.2, the uncertainty characteristics of the recovery time have a significant effect on the abnormal behavior of the affected metro passengers. Therefore, with the seriousness of the disruption, both N^- and N^+ would increase, since λ_w increases.

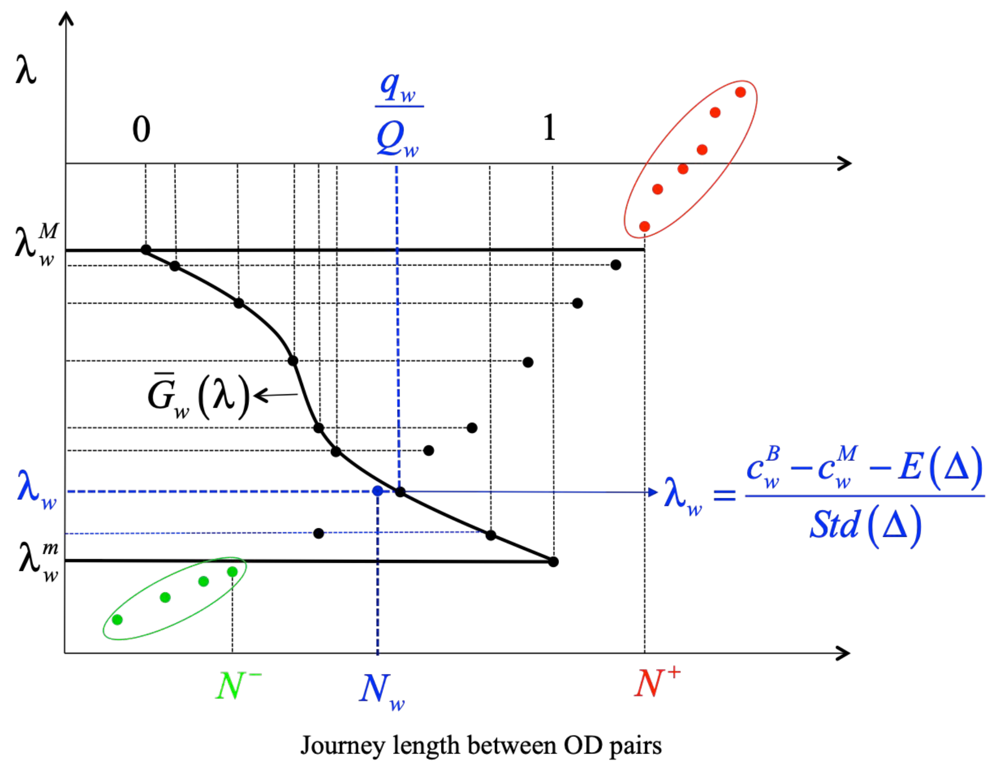


Figure 4 **Graphical explanation of Corollary 1**

3.2. Properties of the optimal evacuation scheme under metro disruption

In this subsection, we move to examine the optimal evacuation scheme under a given metro disruption, or the optimal solution of the minimization problem (9). We first assume that there is only one bus line connecting each metro OD pair, which is a unique bus route preferred by the affected passengers between the OD pair, $R_w^B = \{l_w\}$. The scenario corresponds to the bus-bridging problem studied by Jin et al. (2016). Note that, at optimum, not all frequencies of the bus lines are required to be adjusted, given the constraint of the available bus capacity. Therefore, in contrast to the traditional bus-bridging problem investigated by Jin et al. (2016), our scheme aims to determine the bus frequencies. The evacuation network consists of the bus lines, with a positive increase in their frequencies. We first make the following assumption, to investigate the optimal solution of problem (9).

Assumption 1 *The CDF $G_w()$, $w \in W$, has the increasing generalized failure rate function, namely, $\lambda g_w(\lambda)/\bar{G}_w(\lambda)$ is strictly increasing in λ, $\lambda \in [\lambda_w^m, \lambda_w^M]$.*

The assumption that CDF $G_w(\lambda)$ has a generalized failure rate function, introduced by Lariviere (1999), is commonly employed in operations research (Ziya et al. 2004). Tan and Yang (2012) introduced the property to capture the distribution of travelers' value-of-time heterogeneity. In fact, the commonly used distributions, such as the uniform, exponential, normal, truncated normal, Erlang, Weibull, Gamma, lognormal distributions and Pareto distributions, all have a generalized increasing failure-rate function. Under Assumption 1, we have the following property for the objective function in problem (9). The rigorous proof can be found in Ziya et al. (2004).

Property 3 *Under Assumption 1, J_w in problem (9) is a unimodal function of λ_w, namely, J_w first increases and then decreases* with respect to λ_w.

Property 3 shows the geometric property of the objective function. Note that, since there exists a unique bus line connecting each metro OD pair, the generalized travel cost c_w^B by bus can be strictly reduced by increasing the bus frequency f_l. However, the frequency for each bus line has an upper bound, because the bus line has a limited available capacity. Therefore, it is not always true to increase the bus frequency to reduce the total objective. Under Assumption 1, suppose there is a unique and separable bus line connecting each metro OD pair, we have the following scheme to select the optimal bus frequencies for each bus line.

Scheme I *Under Assumption 1, suppose there is a unique and separable bus line connecting each metro OD pair, for problem (9), the optimal bus frequency for any affected metro OD pair must be f_l^0 or $\hat{f}_l$.*

Scheme I illustrates that, for a given service disruption, the optimal evacuation scheme, in the sense of minimizing the total expected travel cost, would lead to two polar cases: either evacuating as many of the affected passengers as possible for some OD pairs, or encouraging the affected passengers to wait in the metro system for other OD pairs. For the former polar case, the agency can increase the frequency of the corresponding bus lines; whereas for the latter polar case, the agency should not increase the frequency of the corresponding bus lines. The conclusion can include that of Jin et al. (2016) as a special case, who considered severe disruption and all affected passengers should be evacuated. In fact, in this case, both the mean and variance of the recovery time are very large, and λ_w can be set to be infinite. As a result, the passengers between all the disrupted OD pairs must transfer to the bus system and the frequencies of bus lines connecting all OD pairs must be $\hat{f}_l$.

According to the proof of Scheme I, for any given service disruption, the key issue before designing the bus-bridging scheme for the agency is to partition the metro OD pairs into two groups, for evacuating affected passengers or not. Inequalities (19)-(21) in Appendix A show the conditions for partitioning the OD pairs. Notably, the partition of the OD pairs for evacuating or not depends on the stochastic characteristics of the recovery time and risk-taking behavior of the affected passengers.

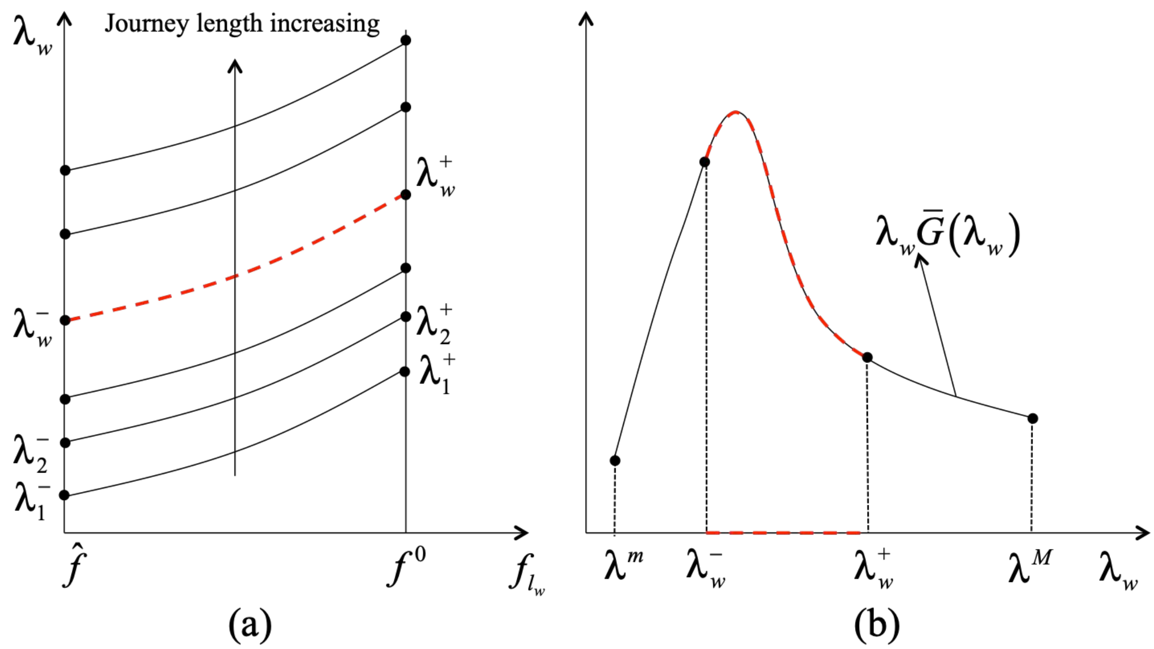


Figure 5 Graphical explanation of the proof of Scheme II

Consider the simplest case described in Property 2, namely, $\Delta_w \equiv \Delta$, $G_w() \equiv G()$ with identical support set $[\lambda^m, \lambda^M]$ for all OD pairs w, $w \in W$, and identical feasible domain of the frequency $\left[f_l^0, \hat{f}_l\right] = \left[f^0, \hat{f}\right]$ for all bus lines, $l \in L$. Scheme I can be clearly described as follow.

Scheme II *For the identical case, namely,* $\Delta_w \equiv \Delta$*,* $G_w() \equiv G()$ *with* $[\lambda_w^m, \lambda_w^M] = [\lambda^m, \lambda^M]$ *and* $\left[f_l^0, \hat{f}_l\right] = \left[f^0, \hat{f}\right]$ *for all* $w \in W$ *and* $l \in L$*, suppose the assumption of Property 2 holds, then, for problem (7) there exists* N_0*, such that: the optimal bus frequency must be* $\hat{f}_l$ *when the journey length is shorter than* N_0*, and* f_l^0 *when the journey length is larger than* N_0*.*

Note 1 Schemes I and II show the optimal choice of the bus frequencies for any given metro service disruption. At optimum, the transportation agency either adopts the maximal possible frequency or does nothing, based on the stochastic characteristics of the metro disruption. According to the optimal solution, in fact, the optimal bus network for evacuating the affected metro passengers is simultaneously designed, given that the evacuation bus network is constructed by the bus lines with maximal frequency. However, it must be pointed out that those theoretical results in this section depend on the assumption that each metro OD pair is connected by a unique bus line. This can be viewed as the special case of the bus-bridging service (7). Specifically, the critical value N_0 in Scheme II depends on the uncertain characteristics of the recovery time, caused by the different disruption types. For example, N_0 is smaller for the disruption caused by door failure than that caused by power failure, and for the latter, more bus lines would be set at their maximal frequency. For the general case, where each metro OD pair is connected by several bus lines, we can construct the bus evacuation network described in Section 2.1. For the general network, the optimal bus frequencies for all bus lines are hard to calculate, since the frequency on each road section is dependent on the whole bus system, and the metro OD pairs cannot be separately considered, as they are related to each other by the bus evacuation network. Hence, one of the heuristic algorithms will be proposed in the next section, to obtain the optimal solution.

Note 2 Consider the risk-taking behavior of the metro passengers and the characteristics of their journey length: Schemes I and II indicate that system delay is improved to a greater extent when additional bus service is provided for passengers with shorter journeys (since they have more incentive to transfer to the bus system) than when additional bus service is provided to passengers with longer journeys (since they have less incentive to transfer). However, the optimal strategy obtained from the mathematical model gives rise to the inequity issue among the metro passengers. At optimal strategy, some bus lines are selected by the agency to increase their frequencies, as a result, the affected passengers are encouraged to transfer to the urban bus since the system delay can be reduced. Nevertheless, the frequencies of other bus lines are not increased even though there are many available vehicles. And thus, the affected passengers are encouraged to wait for the discovery of the disrupted metro since the system delay would be increased if some affected passengers transfer to the urban bus system. Therefore, the inequity is that the optimal strategy does not provide the sufficient choices for all the affected passengers. The inequity is the natural conclusion resulted from the objective of the problem to minimize the total system delay caused by the metro service disruption.

It is important to mitigate the inequity among the passengers. One of the straightforward measures is to add the punishment on the ratio of the affected passengers waiting in the disrupted metro system, i.e., $\alpha G_w(\lambda_w)$, to the objective function. The penalty term forces the agency to consider the evacuation scheme for the affected passengers between all metro OD pairs. The agency can reduce the punishment cost by increasing the frequencies of bus lines between the metro OD pairs as more as possible. In this case, the penalty term changes the graphical property of the objective function. The calibration method and solution property would not be valid for the revised objective function. However, it is indeed an important research direction to design the new evacuation scheme with the equity issue.

4. Implementation of the bus evacuation scheme for a general network

In this section, we proceed to propose the calibration method to obtain the bus evacuation network and the corresponding bus frequencies for a general urban network. This is equivalent to selecting the bus lines from the urban bus system and determining the frequencies for those selected bus lines or sections. First, we assume that the metro-bus evacuation network is constructed according to the method described in Section 2.1. Secondly, we assume that the number of available buses or the maximal frequency of each bus line can be predetermined once the service disruption occurs. When we add the frequency of each bus line, the average travel costs of corresponding sections are reduced, and thus, the average travel cost of the corresponding bus routes is also reduced. The affected metro passengers make their individual choice according to the gap between the current bus system and metro system, the mean and variance of the disruption recovery time, and their own risk attitudes. As a result, the total system delay given by problem (7) would increase or reduce. Obviously, the selection of the frequency for each bus line is equivalent to allocating the available vehicles to the current bus lines, since we can calculate the frequency according the average headway of the bus line. In accordance with the spirit of the **steepest decent algorithm**, we will allocate the available vehicles to the most efficient bus line or section, such that the total system delay $\widetilde{EC}$ is reduced as rapidly as possible. The optimal solution is achieved until the objective function cannot be reduced.

4.1. Capacity allocation method among bus lines

Denote $\hat{N}_l$ and N_l^0 as the available number of vehicles for bus line l, and the initial number running along the line before the disruption, respectively. For the cycle time of bus line l, let N_l denote the number of vehicles allocated to bus line l, with the corresponding vector $\mathbf{N} = (N_l, l \in L)$ and $N_l \leq \hat{N}_l$. We simplify the relationship between the bus frequency and operating vehicle number as $f_l = \frac{N_l^0 + N_l}{T_l}$, where T_l is the cycle time of bus line l (including standing time at stops and maintenance time at the depot). Specifically, $f_l^0 = \frac{N_l^0}{T_l}$ and $\hat{f}_l = \frac{N_l^0 + \hat{N}_l}{T_l}$. Therefore, the frequency of bus line l can take the discrete values $f_l \in \{f_l^0, f_l^1, ..., \hat{f}_l\}$ when $N_l = 0, 1, ..., \hat{N}_l$. With those settings, problem (9) can now be expressed as the nonlinear integer programming problem

$$\min_{\mathbf{N}} \widetilde{\mathrm{EC}} = \sum_{w \in W} Q_w Std\left(\Delta_w\right) \lambda_w\left(\mathbf{f}(\mathbf{N})\right) \bar{G}_w\left(\lambda_w\left(\mathbf{N}\right)\right) \tag{10}$$

subject to

$$N_l \in \{0, 1, ..., \hat{N}_l\}, \forall l \in L. \tag{11}$$

The problem (10) with constraint (11) has $\prod_{l \in L} N_l$ feasible solutions. For the urban bus system, the number of the bus lines is too huge to obtain the optimal solution(s). GAMS/BARON can be adopted to obtain a global optimal solution. In our numerical example, we adopt the idea of the **steepest descent method** as follows.

Steepest descent allocation method

(1) Calculate $\mathcal{J}_l = \sum_{w \in \bar{W}} J_w$ *by setting* N_l *as* $N_l + 1$ *for each bus line* $l \in L$*, separately, at the current solution* $\{N_l : l \in L\}$.

(2) Allocate one available vehicle to bus line l^**, with* $l^* = \arg\min\{\mathcal{J}_l, l \in L\}$.

The **Steepest descent allocation method** loads the available vehicles to the bus line network one by one, and allocates each vehicle to the most efficient bus line, which reduces the total delay fastest. Note that each bus line is only related to a few bus sections, and thus, in the first step of the **Steepest descent allocation method**, we only need to calculate J_w for several metro OD pairs.

Note 3 The average cost c_w^B is decreasing in bus frequency f_l for all $l \in L$ and the number of allocated vehicles N_l. Therefore, λ_w is also decreasing in f_l and N_l for all $l \in L$. According to the proof of Scheme I, we can select the subset of bus lines in which the total objective is always decreasing, by increasing the bus frequency. And thus, we should allocate all the available vehicles to the bus lines in the subset. Also, we can determine another subset of the bus lines, in which the total objective is always increasing when increasing the bus frequency. Therefore, we should not allocate vehicles to those bus lines. Denote L_w as the set of the bus lines connecting metro OD pair w, such that each bus line in L_w must belong to A_s and $\delta_{wr}^s = 1$, $w \in W$. It is evident that λ_w^+ is achieved when $N_l = 0$ for all $l \in L_w$, and λ_w^- is achieved when $N_l = \hat{N}_l$ for all $l \in L_w$, according to Eqs. (2)-(4) and (6). Let $W_1 = \{w \in W : J_w(\lambda_w^-) < J_w(\lambda_w^+)\}$ and $W_2 = \{w \in W : J_w(\lambda_w^-) \geq J_w(\lambda_w^+)\}$. Denote that $\mathbb{L}_1 = \bigcup_{w \in W_1} L_w$ and $\mathbb{L}_2 = \bigcup_{w \in W_2} L_w$. Therefore, we can readily obtain a feasible solution to reduce the total delay via the following equation:

$$N_l = \begin{cases} \hat{N}_l, & \text{if} \phi_w^l = 0, \forall w \in W_2 \\ 0, & \text{if} \phi_w^l = 0, \forall w \in W_1 \end{cases} \tag{12}$$

where ϕ_w^l is the incident index of the bus line, with $\phi_w^l = 1$ when $l \in L_w$, otherwise $\phi_w^l = 0$. The Eq. (12) is equivalent to the **All-or-nothing allocation method**: allocate all available vehicles to the bus lines in the set $\mathbb{L}_1 \backslash \mathbb{L}_2$, but do not allocate any available vehicles to the bus lines in the set $\mathbb{L}_2 \backslash \mathbb{L}_1$.

Note 4 Furthermore, we can cluster the bus lines and metro OD pairs for the bus lines in set $\mathbb{L}_1 \cap \mathbb{L}_2$, because some specific metro OD pairs are only dependent on some specific bus lines. The nonlinear integer programming problem (10)-(11) can be separated into several small-scale sub-problems. Denote the incident matrix Φ with an element ϕ_w^l, $w \in W$ and $l \in \mathbb{L}_1 \cap \mathbb{L}_2$. The incident matrix Φ can be clustered into several sub-matrices. The sub-matrix has the following properties: $\forall l$ and l', there at least exists a w such that $\phi_w^l = 1$ and $\phi_w^{l'} = 1$; $\forall l$, there exist $w \in W_1$ and $w' \in W_2$, such that $\phi_w^l = 1$ and $\phi_{w'}^l = 1$; and all the elements beyond the sub-matrix are equal to 0. It is easy to determine the sub-matrices in the matrix Φ by an iterative search of bus lines and metro OD pairs, provided that the element equals one. Therefore, we only need to determine the bus frequencies among those lines specified by those sub-matrices, separately.

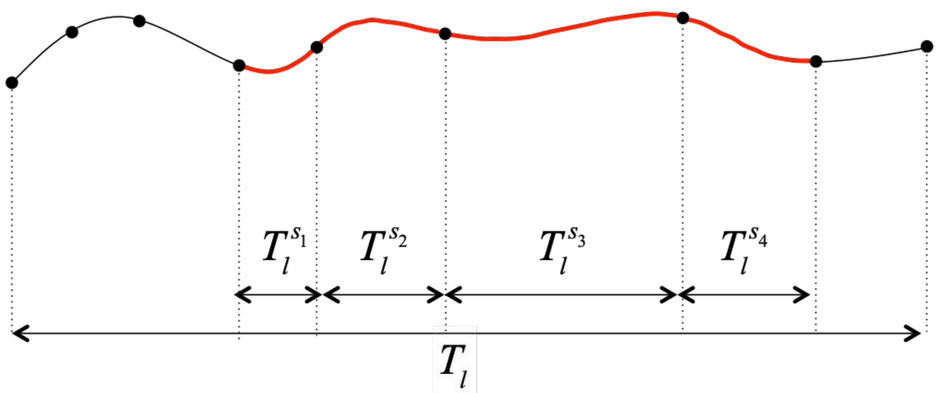


Figure 6 **An example of the relationship of bus sections and a bus line**

4.2. Capacity allocation method among bus sections

It must be pointed out that an inefficient time cost would result if the added buses run along the whole bus lines. Figure 6 shows an example of the relationship between the bus sections and a bus line. There are four bus sections in the bus-section network for bus line l, denoted by the line-section pairs $(l,s1)$-$(l,s4)$. Since the cycle time of bus line T_l is much longer than the sum of the four bus sections, we should allocate the available vehicles only to bus sections $(l,s1)$ to $(l,s4)$, to improve the evacuation efficiency.

Denote (l,s) as the line-section pair when $l \in A_s,\ s \in S$. The set of the line-section pairs is $\mathcal{L} = \{(l,s) : l \in A_s, s \in S\}$. Let f_l^s and N_l^s be the frequency and number of allocated available vehicles of the line-section pair $(l,s) \in \mathcal{L}$. It is clear to see that $f_l^s = \frac{N_l^s}{T_l^s}$, where T_l^s is the cycle time of line-section pair (l,s) (including standing time at corresponding stops), $(l,s) \in \mathcal{L}$. For convenience, let ψ_l^s denote the incident index and $\psi_l^s = 1$ if $l \in A_s$; otherwise, $\psi_l^s = 1$. With the above settings, we only need to determine the capacity allocation among the line-section pairs in $\mathcal{L}$ by keeping the frequency of each bus line in L unchanged. The capacity allocation among line-section pairs is similar to the nonlinear integer programming problem (10). The generalized travel cost of section (2) is revised as:

$$c_s^B(\mathbf{f}) = t_s^B + \frac{\alpha}{\sum\limits_{l \in A_s} (f_l^0 + f_l^s)} + \tau_s^B. \tag{13}$$

Furthermore, the total number of available vehicles for line-section pairs l with $\psi_l^s = 1$ must be less than $\hat{N}_l$. Therefore, the capacity allocation problem among bus sections can be modeled as the nonlinear integer programming problem (10), with the following constraints:

$$\sum_{(l,s) \in \mathcal{L}} N_l^s \leq \hat{N}_l, \forall l \in L \tag{14}$$

and

$$N_l^s \in \mathbb{Z}^+ \tag{15}$$

where $\mathbb{Z}^+$ is the set of the non-negative integers.

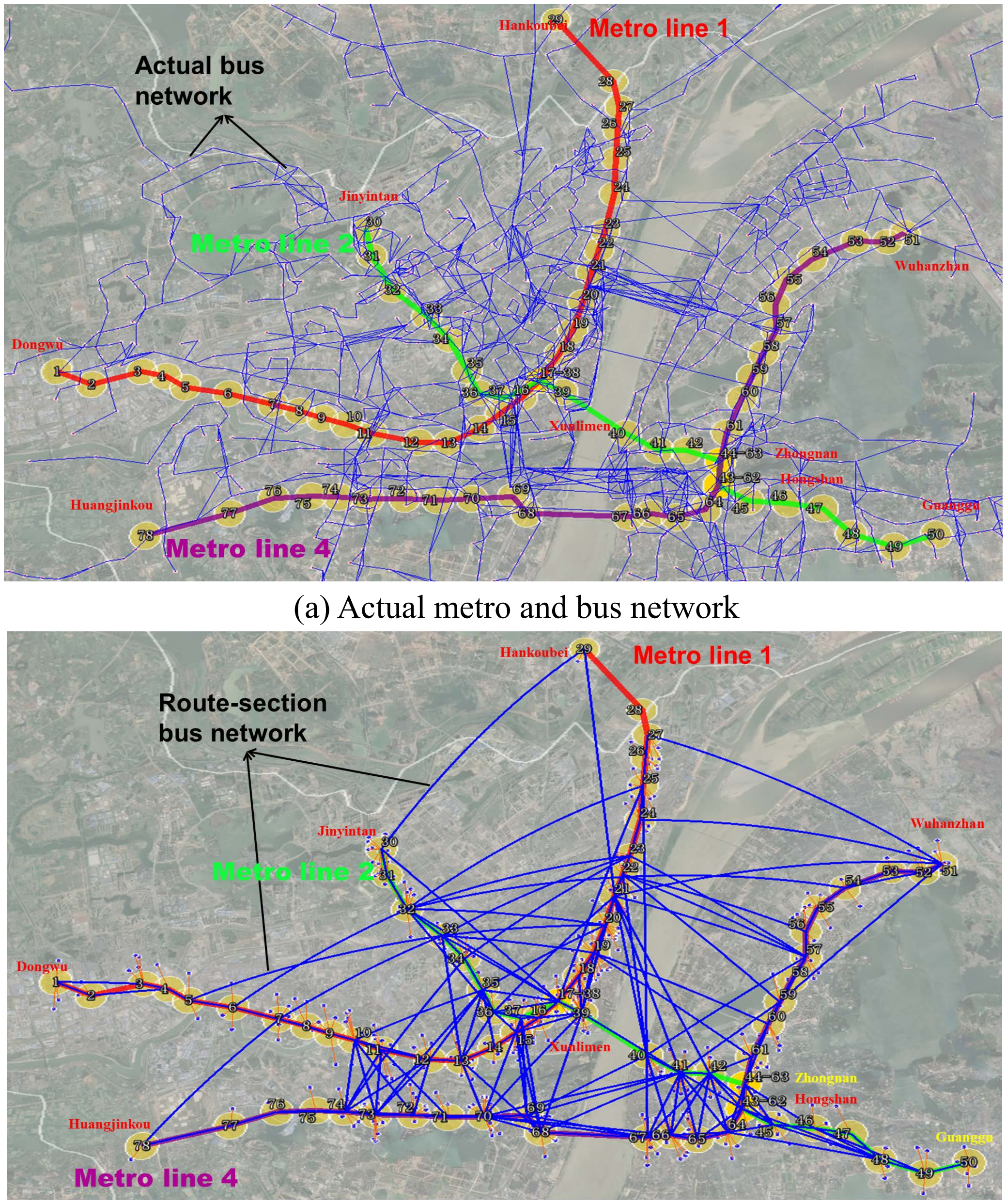
The nonlinear integer programming problem (10) with constraints (14) and (15) becomes more complicated in the presence of constraint (14). In this case, J_w^+ and J_w^- cannot be calculated according to the frequency of the individual bus line. However, the **steepest descent allocation method** is still valid when viewing each line-section pair as a new bus line. In each iteration, we need to calculate $\mathcal{J}_l^s = \sum_{w \in W} J_w$ according to Eq. (13), by setting N_l^s as $N_l^s + 1$ for each line-section pair (l, s), separately for the current solution $\{N_l^s : (l, s) \in \mathcal{L}$. Then, one available vehicle is allocated to the line-section pair (l^*, s^*) with $(l^*, s^*) = \arg\min\{\mathcal{J}_l^s, (l, s) \in \mathcal{L}\}$.

Note 5 The current paper mainly focuses on the common service disruptions, and proposes a coordination scheme between the urban bus system and metro system. It must be pointed out that the scheme should reduce the negative effects on the previous bus passengers during the metro disruption. In a serious emergency, the disruption causes great time delay; the passenger volume leaving the metro system is quite large. Hence, it is not efficient to schedule additional buses on the sections with the designed bus frequencies. In this case, the objective is to evacuate the affected passengers as quickly as possible. The transportation agencies should dispatch the available vehicles or modes to jointly evacuate the passengers, such as with urban buses, shuttle buses, taxis, sharing cars, etc. How to guide the affected metro passengers among multiple travel modes is one of the challenging issues, and is not covered by this paper.

5. Numerical experiment

5.1. Experiment description and metro-bus evacuation network construction

In this section, we illustrate the proposed method, and draw conclusions by taking the Wuhan Metro System as an example. The Wuhan Metro System in 2016 consists of three metro lines and seventy-eight stations, numbered from 1 to 78, shown in Figure 7(a). The local government in this city speeds up the building of the urban metro network, and is focusing on the response plan for emergencies that occur during the building period. The response plan for operational emergencies has not been designed and published. The metro lines 1 and 2 can be transferred at station Xunlimen, numbered 17 in line 1 or 38 in line 2. The metro lines 2 and 4 can be transferred at stations Zhongnanlu or Hongshan, numbered 44 and 43 in line 2 or 62 and 63 in line 4, respectively, shown in Figure 7(a). Suppose the passengers only select the nearby bus stops which are less than 500 meters from the metro station when considering alternative bus routes in case of service disruption. We collect the latitudes and longitudes of the metro stations and bus stops. By direct calibration of the spherical distance on the ground, we select 545 bus stops connected by 268 bus lines, from the 379 bus lines in Wuhan, China. For the metro evacuation issue, we consider only the bus sections between any two metro stations. Namely, we select the set



(a) Actual metro and bus network

(b) Route-section bus network

Figure 7 Metro-bus evacuation network of Wuhan, China

of sections of all bus lines connecting any given metro station pair. It must be pointed out that not all metro station pairs are connected by the bus section directly. In our case, we construct 213 bus sections associated with the three metro lines, as shown in Figure 7(b). Each bus section may consist of several bus lines. The generalized travel cost of each bus section can be calculated by Eq. (2).

The bus fare is assumed to be 1.6 Yuan for all bus lines, and the distance-based metro fare is calculated according to the rules posted by the city's transportation agency and the spherical distance between each metro OD pair. Note that in Wuhan city of China, the passengers pay a very low charge for their second and third transfer, if these are required, according to the bus discount policy. Therefore, the generalized travel cost on each bus section may not include the bus fare, since the bus fare can be simply subtracted from the metro fare by 1.6 Yuan with the value-of-time equal to 100, or $\beta = 100$. In case the bus fare is set according to the riding distance on the vehicle, such as in Singapore, we can incorporate the bus fare into the travel time by dividing the value-of-time parameter.

Furthermore, the average speeds of the bus and metro systems are assumed to be 20 and 60 km per hour, respectively. Walking speed is assumed to be 6.5 km per hour for each passenger. The headway for the bus system and metro system are assumed to be 10 and 2 minutes, respectively. The loading and unloading time at each bus stop is 2 minutes. The waiting time for both the bus stops and metro stations is half of the headway time. The generalized cost for each bus line in the attractive set of each section is the sum of the bus travel time , the stopping time at each middle bus stop, and the walking time. Hence, the travel disutility of each section is the average generalized travel cost among the attractive set plus the combined waiting time, as calculated by Eq. (2).

To obtain the bus evacuation routes for each OD pair, we adopt the K-shortest route (Yen 1971) to determine at most 10 bus routes, and then select the routes with a generalized cost not exceeding 10% of the shortest path of that OD pair. In this way, we have the bus route set for each OD pair in the metro network, and each route in the set is associated with a deterministic generalized travel cost.

5.2. Evacuation scheme with the urban bus system under different types of service disruption

We now proceed to analyze the evacuation behavior of the affected metro passengers according to different types of service disruption, incorporating the heterogeneous risk-taking behavior of passengers. We consider the service disruptions resulting from four types of failures, i.e. signal failure, vehicle failure, power failure, and door failure. The mean and variance of delay caused by each type of failure, calculated from the data for Beijing, are listed in Table 1. We assume that all four kinds of disruption happen on metro line 2 from Jinyintan to Guanggu stations, shown in Figure 6. All those disruptions are assumed to occur during peak hours. Since the service disruption occurs in the direction from Jinyintan to Guanggu along Metro line 2, the affected passengers include three categories: the passengers from metro line 1 to all stations after 38 (Xunlimen), the passengers along the disrupted direction of metro line 2, and the passengers from Stations 30-42 on metro line 2 to metro line 4 (except for stations 62 and 63). According to the bus-section network

Table 1 Mean and variance of disruptions resulting from different factors

Index	Factors	Mean (minutes)	Variance (minutes)	Occurrence location and direction
1	Door failure	17.6	12.9	Line 2 (Jinyintan to Guanggu)
2	Vehicle failure	18.1	18.0	Line 2 (Jinyintan to Guanggu)
3	Signal failure	29.1	20.6	Line 2 (Jinyintan to Guanggu)
4	Power failure	36.0	41.1	Line 2 (Jinyintan to Guanggu)

constructed in the previous section, we are interested in 238 bus lines, which jointly form 213 bus sections in total.

We further assume that there are 20 available vehicles for each bus line, with a total fleet size of 50 vehicles for each bus line. The number of passengers between each metro OD pair is determined by $Q_w = 500\varepsilon_w$, where $\varepsilon_w \sim U[0,1]$. The Weibull distribution is adopted to capture the heterogeneity of the passengers' risk-taking behavior: namely, for any OD pairs, $w \in W$, $G_w(\lambda) = 1 - \exp\left(-\left(\frac{\lambda}{\mu}\right)^{\kappa}\right)$, $x \in (0, +\infty)$ with $\mu = 1$ and $\kappa = 2$. The parameter $\eta_w = 0.1$ in Eq. (4) for all OD pairs $w \in W$. From Eq. (5), we calculate the ratio of the affected passengers transferring to the bus system for each OD pair by using the survival function $\bar{G}_w()$. The average generalized travel cost of the bus system can be calculated by Eq. (4) in the bus-section network obtained in Section 2.1.

We now examine the bus-bridging problem in the whole metro network under the power failure condition shown in Table 1, by considering the nonlinear integer programming problem (10)-(11). With the identical Weibull distribution for all OD pairs, by direct calibration, we know that without consideration of the frequency constraint, the optimal critical risk λ^* of problem (9) can be analytically calculated by $\lambda^* = \left(\frac{1}{\kappa-1}\right)^{1/\kappa}\mu = 1$ with $\kappa = 2$ and $\mu = 1$. According to Eqs. (2)-(4) and (6), we determine λ_w^- and λ_w^+ for all metro OD pairs with $N_l^s = 0$ or $N_l^s = N_l$. According to the description in Section 4.1, we can calculate λ_w^- and λ_w^+ by setting $N_l = 0$ and $N_l = \hat{N}_l$, respectively. Then, the set of the metro OD pairs is partitioned into W_1 and W_2 with $J_w(\lambda_w^-) \geq J_w(\lambda_w^+)$ or $J_w(\lambda_w^-) < J_w(\lambda_w^+)$. As shown in Figure 8, the metro OD pairs with red color have $J_w(\lambda_w^-) < J_w(\lambda_w^+)$, and metro OD pairs with blue color have $J_w(\lambda_w^-) \geq J_w(\lambda_w^+)$. It is clear that among all those cases, W_1 includes the most OD pairs under disruption caused by power failure, as the mean and variance of the recovery time resulting from the power failure are larger than the others. In fact, in Figure 8, we see that $W_1^{\text{door failure}} \subset W_1^{\text{vehicle failure}} \subset W_1^{\text{signal failure}} \subset W_1^{\text{power failure}}$. Note that in our case study, there exists no urban bus running through station 28, and thus, we cannot calculate λ_w^- and λ_w^+ related to that station. Furthermore, the passengers transfer between metro lines 2 and 4 through stations 62 and 44 or stations 63 and 43. And thus, we need not consider all metro OD pairs between 62 or 63 and the station on metro line 2.

Finally, for the capacity allocation among bus lines, we adopt the method proposed in Section 4.1 to obtain the vehicle allocation scheme under different disruption types. We only consider the case with power failure. Figure 9 shows the convergence of the heuristic algorithm. Two vehicles are

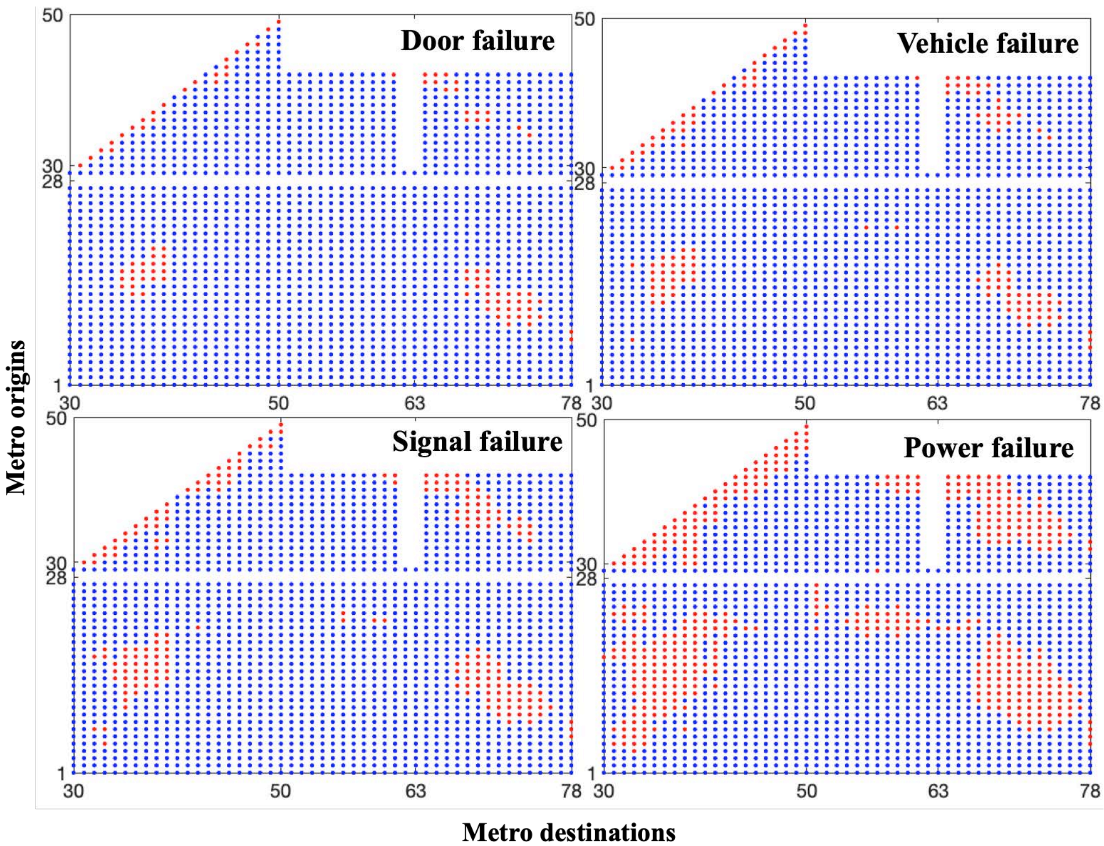


Figure 8 **The partitions of the OD pairs under different disruption types**

allocated at each iteration. The total expected travel delay reduces along with the vehicle allocation procedure, and the lowest value is achieved after 556 iterations. In this case, 1112 vehicles have been allocated among 69 bus lines. Figure 10 shows the bus lines and the corresponding allocated vehicle numbers to evacuate the affected metro passengers.

Figure 11 explicitly plots the urban bus lines which are allocated vehicles under different disruption types. The bus lines allocated with additional vehicles are depicted in green color, while the urban bus network is depicted in blue color. The green network shows the calibration results according to the mean and variance of each disruption. It is evident that the statistical characteristics of the recovery time have a significant effect on the results. For the disruption caused by door failure, fewer affected metro passengers between few OD metro pairs are considered to be evacuated, and thus, the green bus network is quite small. However, for serious emergencies such as power failure, both the mean and variance of the recovery time are very high, and more metro passengers between more metro OD pairs should be evacuated. As a result, the green bus network is very large.

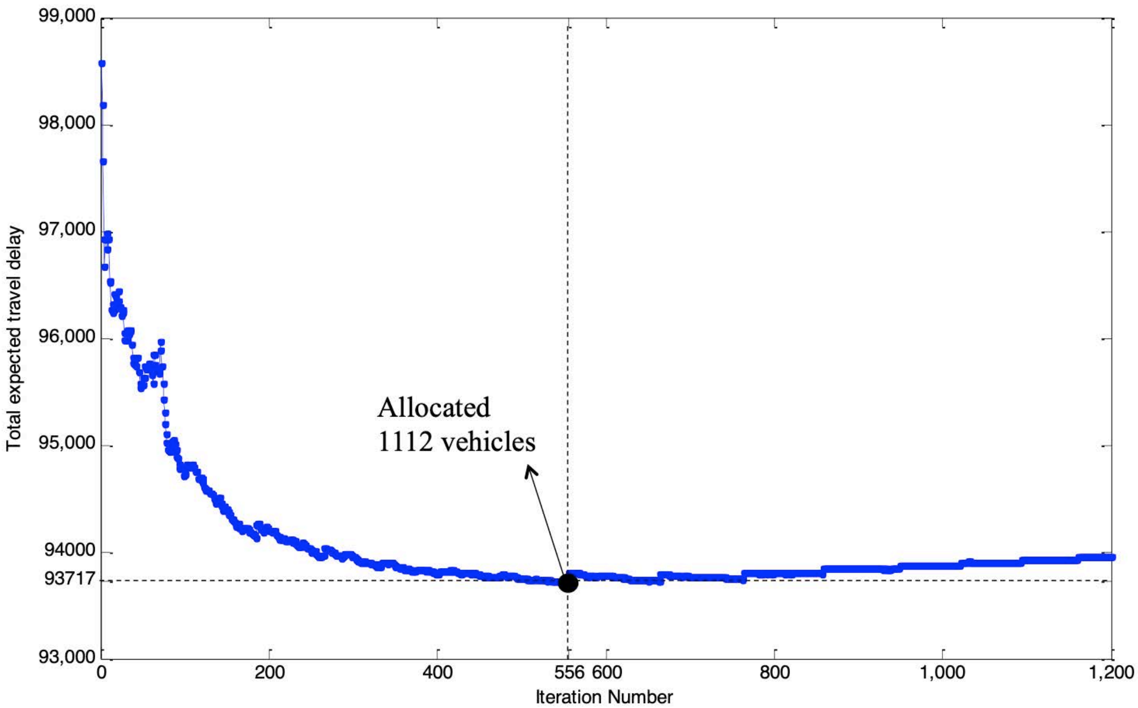


Figure 9 **The convergence of capacity allocation procedure**

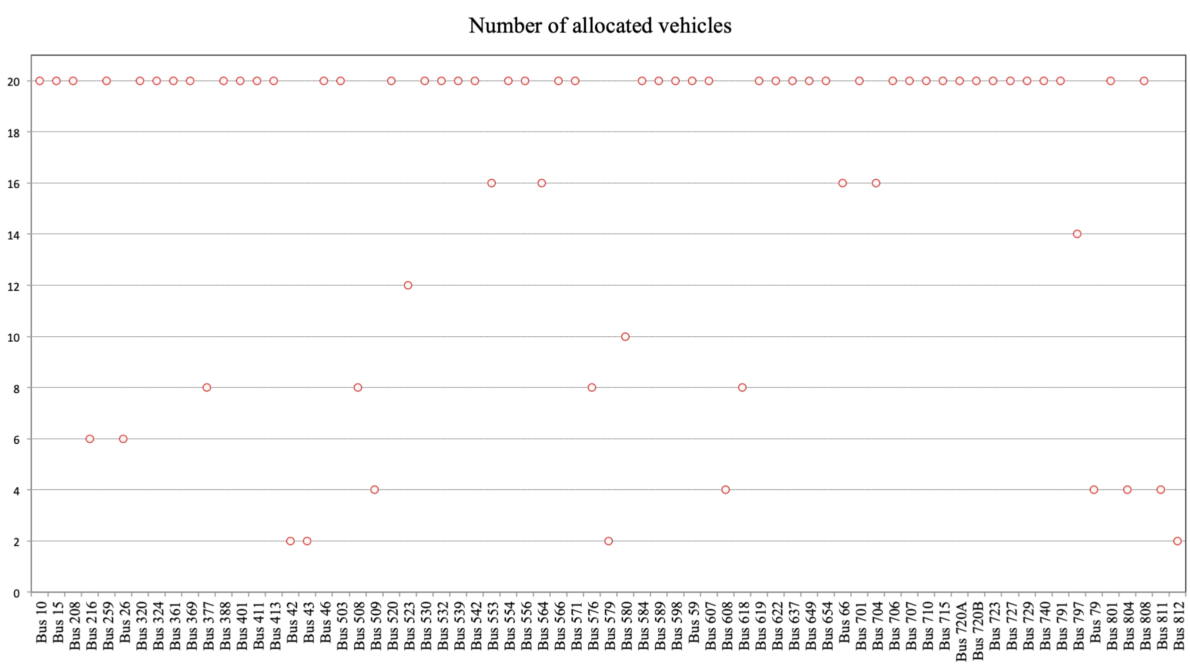


Figure 10 **The vehicle allocation among bus lines under metro disruption with door failure**

6. Conclusions

With excessive utilization by passengers, the metro system suffers from frequent service disruptions. Focusing on the common service disruptions, this paper has proposed an evacuation scheme

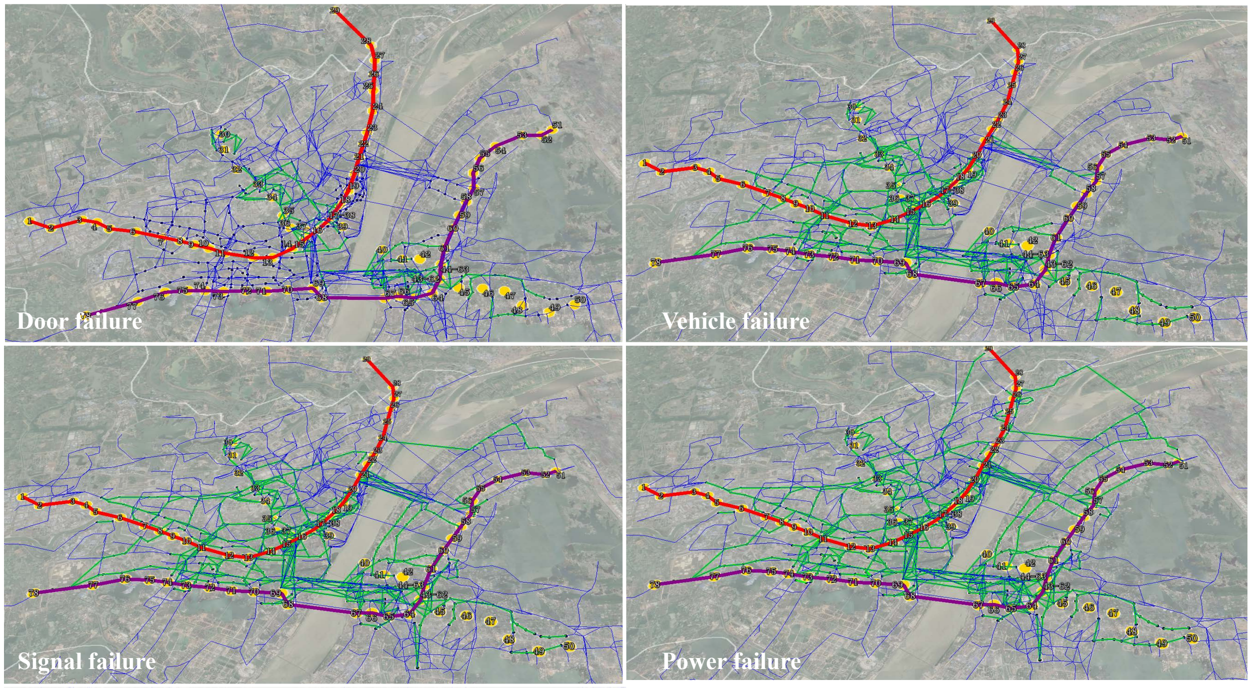


Figure 11 The bus-bridging network with capacity allocation under different disruption types

for the affected metro passengers, using the urban bus system. We first developed the disutility-based multi-modal traffic model to capture the abnormal passenger flows under metro disruptions. The model incorporated the effects of the uncertain characteristics of the disruption, and the heterogeneity of affected passengers' risk-taking behavior. The model explicitly estimated the number of affected metro passengers transferring to the urban bus system during the disruption. Then an evacuation scheme was proposed which would minimize the total expected cost for the affected metro passengers, by selecting the optimal bus frequencies. The proposed scheme considered the constraints of the network configuration and available capacity of the existing urban bus system. Under some mild conditions, we found that, the system delay is minimized when additional bus service is provided for passengers with shorter journeys since they have more incentive to transfer to the bus system.

For general cases, the evacuation scheme using the urban bus network can be formulated as a nonlinear integer programming problem. We proposed a heuristic algorithm based on the idea of the steepest descent method, by allocating the available vehicles to the most efficient bus lines, to reduce the total system delay. A numerical experiment using a real-world urban transit network was adopted to demonstrate the validity of our model and algorithm. Under some mild assumptions, we derived the optimal strategy in the sense of minimization of the total system delay caused by the metro service disruption. At optimum, the frequencies of some bus lines increase to encourage the affected passengers with short journey to transfer to the urban bus system, while other affected passengers are encouraged to wait for the discovery of the disrupted metro. However, the optimal strategy does not provide the sufficient choices for all the affected passengers, which bring the inequity issue for the evacuation scheme.

It should be pointed out that the inequity of the optimal evacuation scheme caused by the system objective is one of the important issues when considering the urban bus system as the bus-bridging scheme. As discussed in Note 2, we can add the penalty term on the inequity measurement to the objective function to mitigate the inequity among the passengers, such as the ratio of the affected passengers waiting in the disrupted metro system, i.e., $G_w(\lambda_w)$. In this case, the graphical property of the objective function will be changed with the penalty term. Therefore, the new model and solution algorithm must be developed to consider the total system delay and equity issue, simultaneously, which is the extension of the current paper. Furthermore, the detoured passengers who take alternative metro routes to their destinations are ignored in the current paper. And thus, a suggested future research direction is to extend the proposed model to incorporate all possible abnormal behavior of the affected metro passengers under common disruptions. Another direction is to conduct empirical studies to capture the heterogeneous risk-taking behavior of the affected metro passengers. Last but not least, it would be valuable and practically useful to design a smart coordinated evacuation scheme that incorporates multiple transportation modes, including trains, buses, taxis and sharing cars, etc.

Acknowledgments

The study was supported by the National Natural Science Foundation of China (71871037, 71471068,71831002), the research project "Public Transit Bus and Driver Scheduling" sponsored by the Academic Research Fund of the Ministry of Education of Singapore (R-302-000-172-114), and the Major Program of the National Social Science Foundation of China (13&ZD175).

Appendix

Appendix A: Proof of Scheme I

Viewing λ_w as the decision variable with feasible domain $[\lambda_w^m, \lambda_w^M]$, $w \in W$, and taking the derivative of $\widetilde{\mathrm{EC}}$ in Eq. (9) for each OD pair, we readily have

$$\frac{\partial \widetilde{\mathrm{EC}}}{\partial \lambda_w} = \frac{\partial J_w}{\partial \lambda_w} = Q_w Sta\left(\Delta_w\right) \bar{G}_w\left(\lambda_w\right) \left(1 - \frac{\lambda_w g_w\left(\lambda_w\right)}{\bar{G}_w\left(\lambda_w\right)}\right) \tag{16}$$

According to Property 1, that function J_w is first increasing and then strictly decreasing in domain $[\lambda_w^m, \lambda_w^M]$.

Since there is a unique and separable bus line connecting each metro OD pair, the average generalized travel cost between each OD pair via the bus system, given by Eq. (4), can be reduced to $c_w^B(\mathbf{f}) = c_w^B(f_{l_w})$ with $R_w^B = \{l_w\}$. Note that $c_w^B(f_{l_w})$ is a strictly decreasing function of the frequency. Let λ_w^- and λ_w^+ denote the lower and upper bounds when the bus frequency $f_{l_w} = \hat{f}_{l_w}$ and $f_{l_w}^0$, respectively. Namely,

$$\lambda_w^- = \frac{c_w^B\left(\hat{f}_{l_w}\right) - c_w^M - E\left(\Delta_w\right)}{Std\left(\Delta_w\right)} \text{and } \lambda_w^+ = \frac{c_w^B\left(f_{l_w}^0\right) - c_w^M - E\left(\Delta_w\right)}{Std\left(\Delta_w\right)} \tag{17}$$

It is clear that $\lambda_w^+ > \lambda_w^-$. Note that λ_w^+ and λ_w^- would not be feasible, i.e.,$\lambda_w^+, \lambda_w^- \notin [\lambda_w^m, \lambda_w^M]$. Therefore, the expected generalized travel cost J_w has the following three possible kinds of monotonicity with respect to λ_w in domain $[\lambda_w^-, \lambda_w^+]$: (I) strictly increasing with respect to λ_w in domain $[\lambda_w^-, \lambda_w^+]$, when the following condition holds:

$$\frac{\lambda_w^+ g_w\left(\lambda_w^+\right)}{\bar{G}_w\left(\lambda_w^+\right)} \leq 1. \tag{18}$$

(II) strictly decreasing with respect to λ_w in domain $[\lambda_w^-, \lambda_w^+]$, when the following condition holds:

$$\frac{\lambda_w^- g_w\left(\lambda_w^-\right)}{\bar{G}_w\left(\lambda_w^-\right)} \geq 1. \tag{19}$$

(III) first strictly increasing then decreasing with respect to λ_w in domain $[\lambda_w^-, \lambda_w^+]$ when both conditions (18) and (19) are violated, i.e.,

$$\frac{\lambda_w^- g_w\left(\lambda_w^-\right)}{\bar{G}_w\left(\lambda_w^-\right)} \leq 1 \leq \frac{\lambda_w^+ g_w\left(\lambda_w^+\right)}{\bar{G}_w\left(\lambda_w^+\right)}. \tag{20}$$

Since λ_w, given by Eq. (6), is decreasing in the bus frequency f_{l_w}, the monotonicity of J_w with respect to f_{l_w} is the opposite to that of J_w with respect to λ_w.

For scenario (I), J_w is strictly increasing in domain $[\lambda_w^m, \lambda_w^M] \bigcap [\lambda_w^-, \lambda_w^+]$, and the optimal frequency satisfies the following condition:

$$c_w^B\left(f_{l_w}^*\right) \leq c_w^M + E\left(\Delta_w\right) + \max\left\{\lambda_w^-, \lambda_w^m\right\} Std\left(\Delta_w\right). \tag{21}$$

Condition (21) implies that the transportation agency should evacuate as many of the affected passengers as possible. Specifically, $f_{l_w}^* = \hat{f}_{l_w}$ when $\lambda_w^- \geq \lambda_w^m$. Furthermore, when $\lambda_w^- < \lambda_w^m$, J_w given in problem (9) cannot be reduced by increasing bus frequency. However, the average travel cost via bus for the specific OD pair can be reduced by increasing the bus frequency, or the objective of problem (7) can still be improved. Therefore, the optimal bus frequency must also be $\hat{f}_{l_w}$.

For scenario (II), J_w is strictly decreasing in domain $[\lambda_w^m, \lambda_w^M] \bigcap [\lambda_w^-, \lambda_w^+]$. Therefore, the transportation agency should encourage the metro passengers to stay in the metro system. The optimal frequency satisfies the following condition:

$$c_w^B\left(f_{l_w}^*\right) \geq c_w^M + E\left(\Delta_w\right) + \min\left\{\lambda_w^+, \lambda_w^M\right\} Std\left(\Delta_w\right), \tag{22}$$

which implies that the transportation agency should not increase the bus frequency to evacuate the affected passengers, and $f_{l_w}^* = f_{l_w}^0$.

For scenario (III), J_w is first strictly increasing then decreasing in domain $[\lambda_w^m, \lambda_w^M] \bigcap [\lambda_w^-, \lambda_w^+]$. In this case, we must compare the value J_w at $\lambda_w = \max\{\lambda_w^m, \lambda_w^-\}$ and $\min\{\lambda_w^M, \lambda_w^+\}$. It follows the same discussion as one of the cases (I) and (II). The proof is completed. □

Appendix A: Proof of Scheme II

According to the proof of Scheme I and Property 2, since λ_w is monotonically increasing with respect to the journey length or the number of metro stations, both λ_w^- and λ_w^+ increase with respect to the journey length. Namely, $\lambda_w^- < \lambda_{w'}^-$ and $\lambda_w^+ < \lambda_{w'}^+$ for all $w, w' \in W$ with $N_w < N_{w'}$, as shown in Figure 5(a). In fact, the domain $[\lambda_w^-, \lambda_w^+]$ moves to the left along the λ_w-alex in Figure 5(b). Therefore, there exists a unique journey length N_0 (could be one), such that $J_w(\hat{f}_l) < J_w(^0_l)$ for all affected metro OD pairs with a journey length shorter than N_0, and $J_w(\hat{f}_l) \geq J_w(^0_l)$ for the other affected metro OD pairs with a journey length greater than N_0. According to Scheme I, for the former case, the optimal bus frequency must be $\hat{f}_l$, and for the latter case, the optimal bus frequency must be f_l^0. The proof is completed. □

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