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Optimal Deployment of Charging Stations Considering Path Deviation and Nonlinear Elastic Demand

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Abstract

This study aims to determine the optimal deployment of charging stations for battery electric vehicles (BEVs) by maximizing the covered path flows taking into account the path deviation and nonlinear elastic demand, referred to as DCSDE for short. Under the assumption that the travel demand between OD pairs follows a nonlinear inverse cost function with respect to the generalized travel cost, a BCAP-based (battery charging action-based path) model will be first formulated for DCSDE problem. A tailored branch-and-price (B&P) approach is proposed to solve the model. The pricing problem to determine an optimal path of BEV is not easily solvable by available algorithms due to the path-based nonlinear cost term in the objective function. We thus propose a customized two-phase method for the pricing problem. The model framework and solution method can easily be extended to incorporate other practical requirements in the context of e-mobility, such as the maximal allowable number of stops for charging and the asymmetric round trip. The numerical experiments in a benchmark 25-node network and a real-world California State road network are conducted to assess the efficiency of the proposed model and solution approach.

Keywords: Charging station location, branch-and-price, path deviation, nonlinear elastic demand

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1. Introduction

Battery electric vehicles (BEVs) have gained an increasing popularity over the past decade due to their environmental friendliness and high energy efficiency. The scarce charging infrastructure, however, is recognized as one of barriers for the mass adoption of BEVs (Egbue and Long, 2012; Xu et al., 2017b). In order to promote vehicle electrification, many governments across the world have made substantial investments in charging infrastructure (IEA, 2017). How to intelligently deploy these charging infrastructures in prospect of large-scale BEVs' uptake is one of the most prominent issues of local governments. Even excluding the expense for additional electrical or construction works, the simple procurement of one public charging station can easily cost up to USD 3,000 to 6,000 (Smith and Castellano, 2015). However, without any information regarding charging demand and charging behavior, early attempts to merely maximize the coverage of charging stations result in the low utilisation of some charging stations (Russo, 2015). For example, it has been found in Japan that some drivers may not have easy access to charging stations although the coverage of existing public charging stations on the whole is extensive (Xu et al., 2017c). These findings provide strong motivations for the investigation into optimal deployment of charging stations in this study.

The recovery of BEVs has brought in numerous studies over the past few decades, among which how to deploy charging infrastructure is one of the foremost topics (Arslan and Karaşan, 2016; Chen et al., 2016; Ghamami et al., 2016; He et al., 2013; He et al., 2015; Lee and Han, 2017; Li et al., 2016; Liu and Wang, 2017; Mak et al., 2013; Nie and Ghamami, 2013; Yıldız et al., 2019). In light of the similarities shared by BEVs and other alternative-fuel vehicles (e.g., hydrogen-gas vehicles, biofuel-based vehicles), we will also review the studies on the location of refueling stations for other alternative-fuel vehicles in the following subsection.

1.1 Literature review

Motivated by the well-studied facility location problem, early studies on location of refueling station generally assumed node-based demand, and *p*-median model was often used to locate a given number of refueling stations by minimizing the travel cost from the demand node, e.g., home, to the possible refueling facilities (Drezner and Hamacher, 2001; Owen and Daskin, 1998). Unlike other facilities that are generally visited on purpose as destinations, the refueling stations, however, are often utilized as mid-stops for further travel. In this regard, modeling the demand as the path flows between origin-destination (OD) pairs on a network more aligns with the reality. The idea of employing path-based demand was pioneered by Hodgson (1990) in a flow-capturing location model (FCLM). Based on the assumption that a flow would be captured if there exists at least one open refueling station along its path, FCLM aimed to locate *p* stations to capture as much flows as possible. This assumption, however, was restrictive for flows on longer paths, which may require multiple refueling stations along the path to ensure a successful trip. The limitation of FCLM was later overcome by Kuby and Lim (2005) in a flow refueling location model (FRLM). They defined a feasible "combination" of refueling facilities as a sequence of refueling stations along a path that enables a successful trip between an OD pair. The FRLM was later extended from several aspects, e.g., developing more efficient models and solution methods (Capar et al., 2013; Lim and Kuby, 2010; MirHassani and Ebrazi, 2012), or incorporating other aspects such as candidate site dispersion, station capacity and multi-period planning (Chung and Kwon, 2015; Kuby and Lim, 2007; Upchurch et al., 2009; Zhang et al., 2017).

The aforementioned studies, however, ignored the possible deviation that drivers are likely to make from the shortest paths for charging. The earliest research taking the path deviation into consideration was conducted by Kim and Kuby (2012), in which drivers were allowed to travel on the other paths with an additional travel distance. They also considered demand decay on the deviation paths, i.e., the flow on a deviation path would decrease with the increase of additional path deviation with respect to the shortest path. They developed a deviation-flow refuelling location model (DFRLM) based on pre-generated paths and combinations, which, however, is computational intractable for large networks. A few studies have also been conducted in consideration of path deviation. For example, Huang et al. (2015) extended the set covering model in Wang and Lin (2009) by incorporating the pre-generated paths. Yıldız et al. (2016) proposed a novel path-segment formulation to avoid the pre-generation of paths and combinations. In the same line of efforts, Zheng and Peeta (2017) considered station capacity and *p*-stops constraints, i.e., BEVs between an OD pair are allowed to stop at most *p* times for charging for deviation-flow refuelling location problem. Recently, Arslan et al. (2019) and Göpfert and Bock (2019) developed novel cut based formulations and customized branch-and-cut methods for DFRLM.

DFRLM in a set covering form has close parallel to the network design problem with relays (NDPR), which has been extensively examined in the context of telecommunication (Cabral, 2005; Cabral et al., 2007). Given the limited range that a signal can travel without replenishment, NDPR aims to locate the signal regenerating equipment, e.g., relays, and additional links in a telecommunicating network to ensure that a set of node pairs can communicate with each other and the construction cost is minimized. As recently reviewed by Leitner et al. (2017), however, the relevance of NDPR for deployment of charging stations has not been sufficiently acknowledged due to the additional constraints arising in the context of e-mobility, such as restricting the maximal allowable path deviation and number of stops for charging. They emphasized the need to incorporate these behavioural aspects for the deployment of charging stations. Moreover, though widely used in transportation network modelling, the demand elasticity is seldom considered for refueling stations deployment. The incorporation of demand elasticity is not trivial because the resulting model can easily become nonlinear or even implicit. Kim and Kuby (2012) have ever considered demand elasticity in a DFRLM. Regrettably, their approach entails the computationally intensive path and combination pre-generation. A more efficient model taking into account both the path deviation and elastic demand is thus highly anticipated.

1.2 Objective and contributions

To fill the aforementioned gaps, this study investigates the optimal deployment of charging stations considering path deviation and demand elasticity (DCSDE) without pre-generating the paths and combinations. In particular, BEVs are assumed to have a limited driving range. The travel demand between OD pairs follows a nonlinear inverse cost function with respect to the generalized travel cost, and the flow between an OD pair would travel on the shortest feasible path in terms of generalized travel cost between that OD pair. Our objective is to maximize the covered path flows by determining the deployment of charging stations subject to a limited budget. To achieve this objective, the battery charging action based path (BCAP) is first defined to facilitate the formulation of an integer programming model. A tailored branch-and-price (B&P) approach is subsequently developed to solve the model. The solution will be found by resorting to a column generation method to repeatedly solve the linear relaxation of integer programming model referred to as restricted master problem. The pricing problem to determine an optimal path of BEV is not easily solvable by available algorithms due to the path-based cost term in the objective function. We thus propose an improved label correcting method for solving the bi-objective shortest path problem (BSPP) on a meta-network. If the column generation method produces a non-integer optimal solution, a branch-and-bound method is used to repeatedly solve the column generation problems until an integer solution is found. The proposed B&P can yield the optimal deployment of charging stations. The model framework and solution method can be easily extended to incorporate other practical requirements in the context of e-mobility, such as the maximal allowable number of stops and the asymmetric round trip.

The remainder of this study is organized as follows. Assumptions, notations and problem statement are elaborated in Section 2. A BCAP-based model for the DCSDE problem is formulated in Section 3. The B&P approach for solving the BCAP-based model is presented in Section 4, in which a meta-network is constructed and an improved multi-label method is developed for solving the pricing problem within the B&P approach. Section 5 discusses some special applications and possible extensions of the proposed model and solution approach. The efficiency of the proposed model and algorithm is demonstrated by the numerical experiments in a 25-node network and the real-world California State road network in Section 6. Conclusions and future research are presented in Section 7.

2. Assumptions, Notations and Problem Statement

Without loss of generality, we consider the DCSDE problem in a bidirectional transportation network denoted by $\mathcal{G}=(\mathcal{N},\mathcal{A})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of links. The sets of origins and destinations are denoted by $\mathcal{R}\subseteq\mathcal{N}$ and $\mathcal{S}\subseteq\mathcal{N}$, respectively. A battery charging station can be located in a particular node, and all the candidate locations for battery charging stations are grouped into a set denoted by $\mathcal{I}\subseteq\mathcal{N}$. Each charging station $i\in\mathcal{I}$ is associated with the construction cost denoted by e_i. The total budget for charging station construction is represented by B. We consider one-way trips from origins to destinations, which are equivalent to symmetric round trips from the modeling point of view. Following the convention in the literature, we assume that a BEV will be fully charged at a charging station, and the

BEV will depart/arrive with initial/final state of charge (SOC) no larger/smaller than a known pre-specified threshold denoted by W_O / W_D, where W_O and W_D can be any value in the range [0, W] and W denotes the usable battery capacity of a BEV. Kindly note that this assumption is made merely for the ease of presentation, and the proposed model and algorithm can easily incorporate multiple BEVs with different initial SOC at origins and different final SOC at destinations. The one-way trip assumption will be relaxed in the latter of this study by considering the asymmetric round trips, i.e., the egress path is different from the ingress path.

It is commonly assumed in FRLM and/or DFRLM related studies that fuel consumption is merely determined by travel distance, and drivers' preference for a path depends on the path length. These assumptions imply that the link cost and weight are correlated, which provide great ease for path and combination generation. In this study, we relax them by assuming that (i) fuel consumption is determined by many other factors in addition to travel distance; and (ii) drivers' preference for a path depends on the generalized travel cost rather than the path distance. In particular, each link $a := (i, j) \in \mathcal{A}, i, j \in \mathcal{N}$ is associated with electricity consumption w_a, whose value may be obtained by regression analysis methods using the real or experimental data. Since how to estimate these values in practice is beyond the scope of this study, we assume for simplicity that the value of electricity consumption w_a is known a prior. Before we give a formal definition for the generalized travel cost, the battery charging action-based path (BCAP) will first be introduced in the following subsection for the ease of problem statement and model formulation.

2.1 Battery charging action-based path

The idea of battery charging action based path is inspired by Xu et al. (2017a), who defined battery swapping action based path (BSAP) to facilitate their model building for user equilibrium problem of mixed flow of BEVs and gasoline vehicles. They first introduced a special *physical path* between OD pairs in the network, which may contain cycles, but any cycle on the path is only allowed to appear at most once. They found that any physical path with several battery swapping stations along it can be formulated as several different paths with the battery swapping actions for BEV. Therefore, they defined BSAP as a physical path incorporating the battery swapping actions. All the BSAPs of BEVs can be generated from the physical paths according to whether a battery swapping action is taken by the BEV drivers at each battery swapping station. Specifically, a single physical path with L battery swapping stations can generate 2^L BSAPs in total by enumerating all the possible combinations of battery swapping actions along that physical path.

The BCAP proposed in this study is actually the BSAP in Xu et al. (2017a) except that we replace the battery swapping stations with battery charging stations. This idea is also coinciding with the novel definition of "combination of charging stations that can fuel a given path" by Kuby and Lim (2015) in the original FRLM. In particular, let $\mathcal{P}_{\text{physical}}^{\text{rs}}$ denote the set of aforementioned physical paths between an OD pair (r,s). Any physical path $p \in \mathcal{P}_{\text{physical}}^{\text{rs}}$ can be represented by a sequence of visiting nodes, i.e., $p := v_1 \to v_2 \to \cdots \to v_{l-1} \to v_l$, where l is the number of nodes on the path, and $v_1 = r$, $v_l = s$ and $v_i \in \mathcal{N}, i = 2,3,\cdots,l-1$. Let $v_{i_1}, v_{i_2}, \cdots, v_{i_{L_p}}$ be the sequence of charging stations along path p, where L_p denotes the number of charging stations. We can thus generate 2^{L_p} BCAPs in total. However, not all the BCAPs are feasible for BEVs. Based on aforementioned assumptions for one-way trips, a BCAP with h charging actions at the nodes denoted by $\hat{v}_{j_1}, \hat{v}_{j_2}, \cdots, \hat{v}_{j_h}$, where $\hat{v}_{j_k} \in \{v_{i_1}, v_{i_2}, \cdots, v_{i_{L_p}}\}, k = 1,2,\cdots,h$, is feasible for the BEV if and only if :

$$w[\sigma_p(r, \hat{v}_{j_1})] \le W_O \tag{1}$$

$$w[\sigma_p(\hat{v}_{j_k}, \hat{v}_{j_{k+1}})] \le W (k = 1,2,\cdots,h-1) \tag{2}$$

$$w[\sigma_p(\hat{v}_{j_h}, s)] \le W_D \tag{3}$$

where $\sigma_p(v_i, v_j)$ denote the sub-path of path p from nodes v_i to v_j, and $w[\sigma_p(v_i, v_j)]$ is the electricity consumption of a BEV on the sub-path $\sigma_p(v_i, v_j)$ calculated by $w[\sigma_p(v_i, v_j)] = \sum_{a \in \sigma_p(v_i, v_j)} w_a$. All the feasible BCAPs for OD pair (r,s) can be grouped into a set given by

$$\begin{aligned}\mathcal{P}_{\text{all}}^{\text{rs}} = \Big\{ & \sigma_p(r, \hat{v}_{i_1}) \oplus \sigma_p(\hat{v}_{i_j}, \hat{v}_{i_{j+1}}) \oplus \sigma_p(\hat{v}_{i_h}, s), \forall p \in \mathcal{P}_{\text{physical}}^{\text{rs}} \Big| w[\sigma_p(r, \hat{v}_{i_1})] \le \frac{1}{2} W \\ & \& w[\sigma_p(\hat{v}_{i_j}, \hat{v}_{i_{j+1}})] \le W (j = 1,2,\cdots,h-1) \& w[\sigma_p(\hat{v}_{i_h}, s)] \le \frac{1}{2} W \Big\}\end{aligned} \tag{4}$$

where operator $\oplus$ denotes the concatenation of two sub-paths.

We now use the example in Xu et al. (2017a) to illustrate how to pre-determine the set of BCAPs for BEVs. Part (a) of Figure 1 shows a physical path from origin r to destination s with three battery charging stations, i.e., nodes 1, 2 and 3. It can be seen that there are eight (2^3) possible combinations of battery charging actions, i.e., eight BCAPs, illustrated in the part (b) of Figure 1. Let path $p := r \to \hat{1} \to 2 \to \hat{3} \to s$ represent a BCAP on which the BEV drivers will charging their batteries at stations 1 and 3; then path p would be deemed as a feasible BSAP for BEVs between OD pair (r,s) if $w[\sigma_p(r,\hat{1})] \le \frac{1}{2}W$, $w[\sigma_p(\hat{1},\hat{3})] \le W$ and $w[\sigma_p(\hat{3},s)] \le \frac{1}{2}W$.

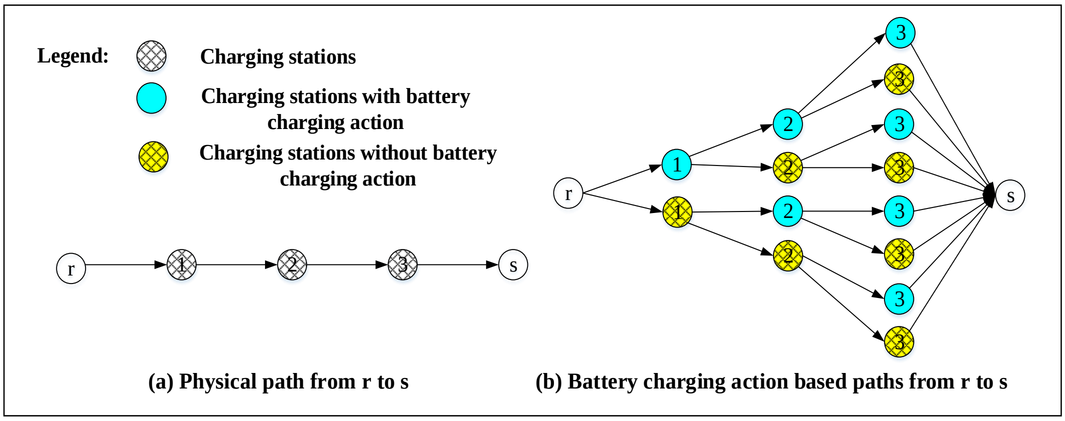

Figure 1. Illustration of the BCAP generation

Unlike the existing studies that define path and combination separately, BCAP can be viewed as a joint definition of path and combination, i.e., both the information of path and combination can be inferred from a BCAP. It implies that a BEV may traverse the charging station without a charging action. It would be seen later that this intuitive definition greatly facilitates our model building due to its implicit incorporation of charging logic.

2.2 Generalized travel cost and BCAP-based elastic demand

Let t_a denote the travel time of link $a := (i, j) \in \mathcal{A}$. For simplicity, it is assumed that both the battery charging cost and the dwell time of BEVs at charging station $i \in \mathcal{I}$ for a battery charging activity, denoted by λ_i and d_i respectively, are constant for all BEVs, and the drivers have a value of time (VOT) denoted by τ . We consider the generalized travel cost including three components for a feasible BCAP

$p \in \mathcal{P}_{\text{all}}^{\text{rs}}$ - travel time on the path, dwell time taken at the charging stations and the travel time converted from the battery charging cost using the VOT - denoted by c_p^{rs} with the expression:

$$c_p^{rs} = \sum_{a \in p} t_a + \sum_{i \in \mathcal{I}} d_i \delta_{i,p}^{rs} + \sum_{i \in \mathcal{I}} \lambda_i \delta_{i,p}^{rs} / \tau \tag{5}$$

where $a \in p$ denote any link traversed by the feasible BCAP p, and $\delta_{i,p}^{rs}$ is the BCAP-charging action incidence indicator which equals 1 if the feasible BCAP p traverses the charging station $i \in \mathcal{I}$ where a battery charging action is taken and 0 otherwise.

We further assume that drivers have a pre-specified tolerance for the path cost deviation. In other words, a feasible BCAP $p \in \mathcal{P}_{\text{all}}^{\text{rs}}$ has the potential to be chosen by a driver if and only if the deviation of its generalized travel cost with respect to the cost of the shortest path between that OD pair is no larger than a pre-specified value, i.e., $c_p^{rs} \le c_{p^*}^{rs} + \varepsilon$, where $c_{p^*}^{rs}$ denotes the generalized travel cost of the shortest path (in terms of generalized travel cost) in the network, and ε is a pre-specified tolerance for path deviation. Let f_p^{rs} denote the flow volume on a feasible BCAP $p \in \mathcal{P}_{\text{all}}^{\text{rs}}$ between OD pair (r, s). To capture the demand elasticity of traffic flow, we assume that f_p^{rs} is an inverse cost function with respect to the generalized travel cost of a BCAP $p \in \mathcal{P}_{\text{all}}^{\text{rs}}$, i.e.,

$$f_p^{rs} = F(c_p^{rs}) = f^{rs} e^{-\beta(c_p^{rs} - c_{p^*}^{rs})} \tag{6}$$

where f^{rs} is the flow volume between OD pair (r, s) when the travel cost is $c_{p^*}^{rs}$ whose value is assumed to be known a priori; β is a pre-specified indicator for the degree of demand elasticity. The inverse cost function is prevailing in the literature on transportation network modelling for the representation of elastic demand for transportation mode (Yang, 1997; Yang and Hai-Jun, 1997; Yang and Wong, 2000).

The objective of DCSDE is to deploy charging stations in the network without exceeding the budget B so that (i) the traffic flow between each OD pair travels on the shortest feasible BCAP satisfying $c_p^{rs} \le c_{p^*}^{rs} + \varepsilon$ if any; (ii) the flow volume on a path follows the elastic demand function with respect to the generalized travel cost of that path; and (iii) the total covered flow volume between all OD pairs is maximized.

3. A BCAP-based Model Building

The complexity of charging logic and nonlinearity of elastic demand function motivate us to formulate the DCSDE problem based on BCAP. Specifically, let $\mathcal{P}^{rs}$ denote the set of potential BCAPs among all the feasible BCAPs between OD pair (r,s), i.e., $\mathcal{P}^{rs} = \left\{ p \in \mathcal{P}_{\text{all}}^{\text{rs}} \middle| c_p^{rs} \le c_{p*}^{rs} + \varepsilon \right\}$. The proposed DCSDE problem can be formulated by the following model:

$$\max_{\mathbf{x},\mathbf{y}} \quad FLOW = \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{p\in\mathcal{P}^{rs}} f_p^{rs} x_p^{rs} \tag{7}$$

subject to

$$\sum_{p\in\mathcal{P}^{rs}} x_p^{rs} \le 1, \quad \forall r \in \mathcal{R}, s \in \mathcal{S} \tag{8}$$

$$\sum_{p\in\mathcal{P}^{rs}} \delta_{i,p}^{rs} x_p^{rs} \le y_i, \quad \forall r \in \mathcal{R}, s \in \mathcal{S}, \forall i \in \mathcal{I} \tag{9}$$

$$\sum_{i\in\mathcal{I}} e_i y_i \le B \tag{10}$$

$$x_p^{rs} \in \{0,1\}, \quad \forall r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}^{rs} \tag{11}$$

$$y_i \in \{0,1\}, \quad \forall i \in \mathcal{I} \tag{12}$$

where x_p^{rs}, $r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}^{rs}$ is the binary decision variable, and $x_p^{rs} = 1$ if the flow between OD pair (r,s) would travel on BCAP $p \in \mathcal{P}^{rs}$; y_i, $i \in \mathcal{I}$ is also the binary decision variable, and $y_i = 1$ if a charging station is built at location i.

The objective function expressed by Eq. (7) is to maximize the total covered path flows. Constraint (8) ensures that at most one BCAP is chosen to load flow between each OD pair[1]. Constraint (9) eliminates the BCAPs using unbuilt stations for battery

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[1] BEV drivers could travel on any range-feasible path between an OD pair. However, given the assumption that BEV drivers aim to minimize their generalized travel cost, the inverse relationship between travel demand and travel cost, and the objective to maximize the covered path flows, all BEVs

charging. Constraint (10) restricts the total budget for the deployment of charging stations. Constraints (11) and (12) define x_p^{rs} and y_i as binary variables, respectively.

3.1 Linear relaxation of path variables

Although both path variables x_p^{rs} and location variables y_i are defined as binary variables in the above model, it can be found in the following proposition that linear relaxation of path variables x_p^{rs} does not affect the optimality of the model.

Proposition 1: Let [DCSDE] denote the aforementioned BCAP-based model with path variables x_p^{rs} relaxed to be continuous variables. Then there always exists an optimal solution to model [DCSDE] in which all the path variables are integers.

Proof. Suppose that we have obtained an optimal solution to model [DCSDE] denoted by $\{x_p^{rs*}, y_i^*\}_{r\in\mathcal{R},s\in\mathcal{S},p\in\mathcal{P}^{rs},i\in\mathcal{I}}$, where there exists a fractional path variable, i.e., $0 < x_a^{mn*} < 1$. For this OD pair (m,n), there must exist at least one more feasible BCAP with a positive flow and the coefficient f_a^{mn} in the objective function, otherwise the objective function can be further increased by increasing the value of the path variable with the largest coefficient for OD pair (m,n) without validating any constraints of model [DCSDE], which is contrary to the optimality of the solution $\{x_p^{rs*}, y_i^*\}_{r\in\mathcal{R},s\in\mathcal{S},p\in\mathcal{P}^{rs},i\in\mathcal{I}}$. The sum of all those path variables would be 1. By letting $x_a^{mn*} = 1$ and all the other path variables be 0, and repeat the above procedure for any other fractional path variables between the other OD pairs, we can obtain an optimal solution to model [DCSDE] with integral path variables. □

Although BCAP-based model has a concise formulation, the huge number of feasible BCAPs makes it intractable to explicitly consider all of them, and an arbitrary subset of BCAPs may lead to a sub-optimal solution. Fortunately, the implicit consideration of all these BCAPs can be achieved by a column generation method introduced in the next section, which generates only "promising" BCAPs that have the potential to be included in the final optimal solution.

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between an OD pair will be "forced" to travel on a range-feasible path with the minimal generalized travel cost if any.

4. Branch-and-price Approach

The branch-and-price (B&P) approach is the same with branch and bound method except that the linear relaxation problems are solved by column generation. It enables us to find the optimal solution to an integer programming model especially with a huge number of columns/decision variables, such as the model [DCSDE] (Barnhart et al., 1998; Lübbecke and Desrosiers, 2005). The column generation method finds the optimal solution to the linear relaxation of [DCSDE], referred to as master problem (MP), by repeatedly solving a restricted master problem (RMP) with a subset of potential BCAPs $\bar{\mathcal{P}}^{rs} \subset \mathcal{P}^{rs}$, and a pricing problem for finding additional BCAPs with positive reduced cost (for maximization problem). The viability of B&P approach largely depends on whether we can find an effective method for solving the pricing problem.

4.1 Pricing problem

Let $\pi^{rs}, \forall r \in \mathcal{R}, s \in \mathcal{S}$ and $\mu_i^{rs}, \forall r \in \mathcal{R}, s \in \mathcal{S}, i \in \mathcal{I}$ denote the dual variables corresponding to C onstraints (8) and (9) of model [DCSDE], respectively. The pricing problem is essentially the problem of finding a nonbasic index associated with a positive reduced cost, a key step in simplex method for a linear programming model (Dantzig, 1963; Bertsimas and Tsitsiklis, 1997). According to the simplex method, the pricing problem for OD pair (r, s), named by [DCSDE-PP^{rs}], is presented as follows:

[DCSDE-PP^{rs}]

$$P^{rs*} = \max_{p \in \mathcal{P}^{rs} \setminus \bar{\mathcal{P}}^{rs}} F(c_p^{rs}) - \pi^{rs} - \sum_{i \in I} \delta_{i,p}^{rs} \mu_i^{rs} \tag{13}$$

where the objective function expresses the reduced cost of a BCAP among the set $\mathcal{P}^{rs} \setminus \bar{\mathcal{P}}^{rs}$.

The problem [DCSDE-PP^{rs}] aims to find a feasible BCAP with the largest traffic flow calculated from the elastic demand function plus an additional $(-\mu_i)$ flow volume for each battery charging actions along that BCAP. Since the travel demand is a nonlinear function of the path cost, it may not easily be split into link-based or node-based variables, thus making the pricing problem hard to be solved by the simple labeling method for conventional shortest path problem (SPP). As such, we have to solve the following bi-objective shortest path problem (BSPP):

[DCSDE-PP^{rs}-B]

$$\min_{p \in \mathcal{P}^{rs} \setminus \bar{\mathcal{P}}^{rs}} \begin{cases} c_p^{rs} \\ \mu_p^{rs} = \sum_{i \in \mathcal{I}} \delta_{i,p}^{rs} \mu_i^{rs} \end{cases} \tag{14}$$

where the two objectives are to find the shortest BCAP in terms of generalized travel cost and node-based dual values respectively. The following proposition demonstrates that an optimal solution to problem [DCSDE-PP^{rs}] can be identified by checking all non-dominated solutions to problem [DCSDE-PP^{rs}-B].

Proposition 2: Let $\mathcal{P}_{ndomit}^{rs}$ denote the set of all non-dominated solutions to problem [DCSDE-PP^{rs}-B]. Then we have

$$P^{rs*} = \max_{p \in \mathcal{P}_{ndomit}^{rs}} F(c_p^{rs}) - \pi^{rs} - \sum_{i \in \mathcal{I}} \delta_{i,p}^{rs} \mu_i \tag{15}$$

Proof. Suppose that we find an optimal solution to problem (13), denoted by p^*, that is not an non-dominated solution to problem (14). In other words, there would be a non-dominated solution to problem (14), denoted by $\hat{p}$, which will dominate solution p^*. It means we have

$$c_{p^*}^{rs} \geq c_{\hat{p}}^{rs} \tag{16}$$

$$\mu_{p^*}^{rs} \geq \mu_{\hat{p}}^{rs} \tag{17}$$

and at least one of the inequalities is strict. It follows that

$$F(c_{p^*}^{rs}) - \pi^{rs} - \mu_{p^*}^{rs} < F(c_{\hat{p}}^{rs}) - \pi^{rs} - \mu_{\hat{p}}^{rs} \tag{18}$$

which is contradictory to the optimality of solution p^* for problem (13).

Therefore we can conclude that the optimal solution to problem (13) must be one of the non-dominated solution to problem (14) associated with a maximal value of $F(c_p^{rs}) - \pi^{rs} - \sum_{i \in \mathcal{I}} \delta_{i,p}^{rs} \mu_i$. □

The multi-label method for BSPP (Brumbaugh-Smith and Shier, 1989; Raith and Ehrgott, 2009; Skriver and Andersen, 2000), however, cannot be directly applied to solve the problem [DCSDE-PP^{rs}-B] because it prohibits loops in the final non-dominated path, while the definition of BCAP allows existence of loops due to the BEVs' detours for charging. For example, consider the network in Figure 2, where each link is associated with two values in parenthesis, denoting the travel time and electricity consumption on that link, and each charging location is associated with a value beside it, representing the sum of the dwell time and travel time converted from the battery charging cost. There are two charging stations located at node 3 and 4. Assume that all the node-based dual values are zero. The usable battery capacity of BEV is 10 units and BEVs set out from node 1 with fully charged batteries. The optimal BCAP for BEV would be $1 \rightarrow 2 \rightarrow \hat{3} \rightarrow 4 \rightarrow 2 \rightarrow 5$ due to the limited driving range, which contains a loop, i.e., $2 \rightarrow \hat{3} \rightarrow 4 \rightarrow 2$.

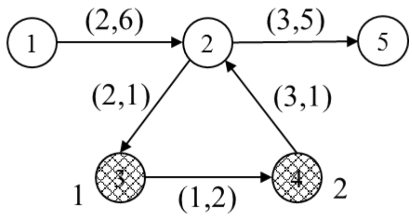


Figure 2. An illustrative example for existence of loops in BCAP

The above example suggests that the problem [DCSDE-PP^{rs}-B] calls for more refined algorithms due to the requirement of charging along BCAPs. Before we solve the bi-objective problem [DCSDE-PP^{rs}-B], we will first elaborate how to solve the problem $\min_{p \in \mathcal{P}^{rs} \setminus \bar{\mathcal{P}}^{rs}} c_p^{rs}$, i.e., the shortest battery charging action based path problem.

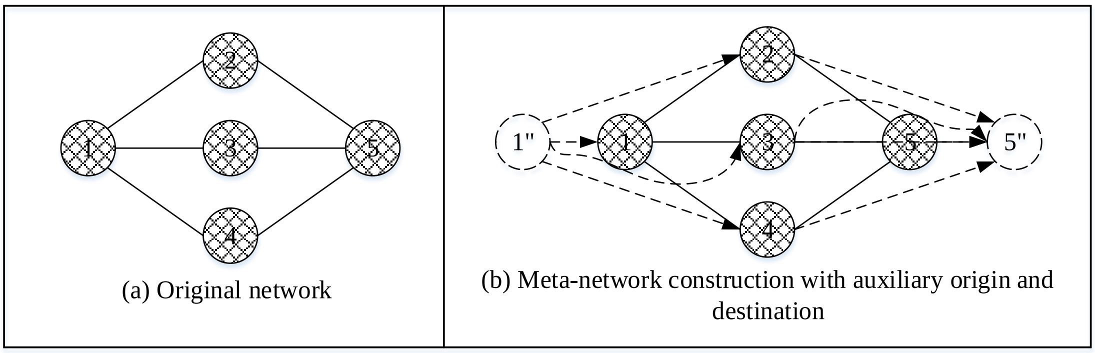
4.1.1 Shortest battery charging action based path problem $\min_{p \in \mathcal{P}^{rs} \setminus \bar{\mathcal{P}}^{rs}} c_p^{rs}$

As illustrated previously, the considered problem $\min_{p \in \mathcal{P}^{rs} \setminus \bar{\mathcal{P}}^{rs}} c_p^{rs}$ for BEVs is significantly different from the conventional SPP because it allows loops in the optimal path due to the detours for charging. It is also different from the weight constrained shortest path problem (WCSPP) because each battery charging action replenishes the battery of a BEV. Cabral (2005) and Laporte and Pascoal (2011) have considered replenishment at nodes in their effort to find the minimum cost path with relays (MCPPR) in a telecommunication network where all nodes are relay nodes. Cabral (2005) proposed three approaches for solving MCPPR including a two-phase method and two label correcting methods with different ways of storing the labels, and. The two-phase method exploits the structure of a feasible path formed by a sequence of sub-paths between two adjacent replenishments. After a higher level network is built in the first phase by connecting pairs of replenishment nodes with feasible shortest paths between them, MCPPR can be found readily by any available methods for conventional SPP in the resultant higher level network in the second phase. Although the two-phase method appears more intuitive, it was found far less efficient than the label correcting method (Cabral, 2005). Laporte and Pascoal (2011) implemented the multi-label method both in a label-setting and label-correcting ways and concluded that a label correcting variant performs best on average. Smith et al. (2012) later considered a similar variant of MCPPR in which the replenishments occur at links. They termed the higher-level network as meta-network and developed a series of improvements for the two-phase method. Although these improvements can significantly reinforce the two-phase method, Smith et al. (2012) found that its performance is still inferior to the multi-label methods. Xu et al. (2017a; 2018) recently made a few modifications to the multi-label method to suit their special needs in the context of e-mobility.

4.1.2 Comparison between multi-label and two-phase method for problem [DCSDE-PP^{rs}-B]

The shortest battery charging action-based path problem $\min_{p\in\mathcal{P}^{rs}\setminus\bar{\mathcal{P}}^{rs}} c_p^{rs}$ has close parallel to MCPPR. The existence of another objective function in problem [DCSDE-PP^{rs}-B], i.e., μ_p^{rs}, entails an additional attribute of a label when using the multi-label method. As for the two-phase method, after the meta-network is constructed, a BSPP instead of a SPP, should be solved in the second phase. Although the multi-label method shows a great advantage over the two-phase method in solving a single objective problem $\min_{p\in\mathcal{P}^{rs}\setminus\bar{\mathcal{P}}^{rs}} c_p^{rs}$, it may not be so for problem [DCSDE-PP^{rs}-B] because the multi-label method with augmented labels has been proven much more computational expensive than that with the original two dimensional labels (Laporte and Pascoal, 2011). In addition, the column generation would frequently invoke a solver for the pricing problem [DCSDE-PP^{rs}-B], and the meta-network, once constructed, needs few modifications to be used later for multiple times. The meta-network at the root node of B&P tree can be also easily modified for using in other nodes. In light of above reasons, we employ a customized two-phase method for solving [DCSDE-PP^{rs}-B] in B&P approach.

In the first phase, a copy of origin and destination nodes, referred to as auxiliary origins and destinations, should be first generated for meta-network construction. Figure 3 (b) illustrates the generation of links (the dotted links), referred to as meta-links, associated with auxiliary origin $1''$ and auxiliary destination $5''$ for OD pair (1, 5), from the original undirected network in Figure 3 (a). Each dotted link is a weight (i.e., driving range) constrained shortest path (Aneja et al., 1983; Dumitrescu and Boland, 2003). The generation of meta-links between the candidate location node pairs is similar to that of origin and destination nodes except that the meta-links are bidirectional. It can be found that the meta-network closely resembles the communication network proposed by Arslan et al. (2019), Göpfert and Bock (2019), and MirHassani and Ebrazi (2013) in the context of refueling station location problem. Therefore the proposed model and algorithm could also work on the communication network with slight modification. Instead of constructing the meta-network to facilitate the model formulation, the meta-network is built as a part of our algorithm design. The next subsection will elaborate how to solve the problem [DCSDE-PP^{rs}-B] in the resultant meta-network.



(a) Original network

(b) Meta-network construction with auxiliary origin and destination

Figure 3. Illustration for meta-network construction

4.1.3 Label correcting method for solving bi-objective shortest path problem [DCSDE-PP^{rs}-B] in a meta-network

Many approaches have been proposed for solving BSSP in the literature. For example, Brumbaugh-Smith and Shier (1989) made an empirical investigation of label correcting methods for BSSP with different strategies for handling list of labels. They concluded that the first-in-first-out (FIFO) principle for managing the labels is the most efficient implementation. Skriver and Andersen (2000) reinforced the label correcting method by imposing some domination conditions. Raith and Ehrgott (2009) compared different solution strategies for BSSP and found that the multi-label methods are preferable in light of their stable performance. In this study, we employ the framework of label correcting method with FIFO principle, and propose a series of improvements dedicated for problem [DCSDE-PP^{rs}], including an initialization procedure by shortest path algorithm, complete label elimination by reinforced pre-domination check, and general label elimination by the convexity of elastic demand function detailed as follows.

(1) Initialization by shortest path algorithm

Instead of solving problem [DCSDE-PP^{rs}] to optimality, the column generation allows the pre-termination of multi-label method once one or multiple feasible positive solutions are detected. Hence, an initialization procedure of finding the shortest path from an origin to a destination can provide rich information for solving problem [DCSDE-PP^{rs}]. For example, it may provide a promising feasible solution or a non-positive upper bound for the objective function that eliminates the necessity to invoke the labeling procedure or bounds information that is useful in the subsequent labeling procedure. Additionally, rather than being solved at every iteration, the shortest path problem in terms of generalized travel cost only needs to be solved once for use in later iterations of column generation.

In particular, let $(c_{\min}, \mu_{\max})$ and $(c_{\max}, \mu_{\min})$ denote the value of generalized travel cost and dual values corresponding to the shortest paths from origin r to destination s in terms of generalized travel cost and dual value, denote by path $p_{c_{\min}}$ and $p_{\mu_{\min}}$ respectively. If the reduced cost of path $p_{c_{\min}}$ or $p_{\mu_{\min}}$ is positive, then there is no need to invoke the multi-label method for BSPP because a feasible solution (i.e., a column) has been found. If, on the other hand, neither of them is positive, we can conclude that any BCAPs would have a non-positive reduced cost because the reduced cost of the most promising path associated with $(c_{\min}, \mu_{\min})$ is non-positive. If so, there is, again, no need to invoke the multi-label method for BSPP. In addition, $c_{\max}$ and $\mu_{\max}$ serve as upper bounds for the generalized travel cost and dual value, and they can be used to eliminate unpromising labels throughout the label correcting method. The initialization procedure is outlined in Algorithm 1, where *SPPMethd* (c) *and SPPMethod* (μ) are subroutines for solving SPP in meta-network $\mathcal{G}_{meta} := (\mathcal{N}_{meta}, \mathcal{A}_{meta})$ in term of generalized travel cost c and dual value μ , respectively, and *BSPPMethod* is the subroutine for solving BSPP to be discussed in the next subsections.

Algorithm 1: Pseudo-code of the initialization procedure

```
(p_{c_min}, c_min, μ_max) ← SPPMethod(c);
If c_min ≤ c^{rs}_{p*_global} + ε
    If F(c_min) − π^{rs} − μ_max > 0 Then
        p* ← p_{c_min}  // p* denotes a feasible path with a positive reduced cost
    Else (p_{μ_min}, c_max, μ_min) ← SPPMethod(μ);
        If c_max ≤ c^{rs}_{p*_global} + ε and F(c_max) − π^{rs} − μ_min > 0 Then
            p* ← p_{μ_min}
        Else if F(c_min) − π^{rs} − μ_min ≤ 0 Then
            p* ← nil
        Else UB_c = max{c_max, c^{rs}_{p*} + ε}; UB_μ = μ_max; p* ← BSPPMethod(UB_c, UB_μ)
        EndIf
    EndIf
    EndIf
Else p* ← nil
EndIf
```

(2) Complete label elimination by reinforced pre-domination check

Skriver and Andersen (2000) proposed a simple and efficient pre-domination check in the label updating process. Let us consider the example in Figure 4 to intuitively illustrate the pre-domination check in the label updating process from node i to node j. Suppose we have non-empty sets of labels $\mathcal{L}(i)$ and $\mathcal{L}(j)$ at node i and j respectively. Both sets of labels are sorted in an ascending order of the generalized travel cost, i.e., $\mathcal{L}(i)=\{(c_1^i,\mu_1^i),(c_2^i,\mu_2^i),...,(c_{n_i}^i,\mu_{n_i}^i)\}$, $c_1^i < c_2^i < ... < c_{n_i}^i$ and $\mathcal{L}(j)=\{(c_1^j,\mu_1^j),(c_2^j,\mu_2^j),...,(c_{n_j}^j,\mu_{n_j}^j)\}$, $c_1^j < c_2^j < ... < c_{n_j}^j$. Let c_{ij} be the cost of meta-link ij and μ_j be the dual value of node j. In principle, n_i new labels would be generated for node j from node i. Instead of performing the dominance check among the union set of existing labels and all new labels at node j, Skriver and Andersen (2000) suggested pre-checking whether all new labels are dominated by the existing first or the last labels at node j. In other words, if $c_1^i + c_{ij} \geq c_1^j$ and $\mu_{n_i}^i + \mu_j \geq \mu_1^j$, or $c_1^i + c_{ij} \geq c_{n_j}^j$ and $\mu_{n_i}^i + \mu_j \geq \mu_{n_j}^j$, the set of label $\mathcal{L}(j)$ would remain unchanged.

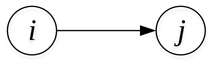


Figure 4. An illustrative example

We generalize the aforementioned pre-domination check by considering the possibility of complete domination by the existing labels other than the first and last labels at node j. Suppose $\mathcal{L}(j) = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$ and $c_1^i + c_{ij} = 3$, $\mu_{n_i}^i + \mu_j = 4$. It can be seen that neither the first nor the last label of node j dominates the newly generated labels. Nevertheless, these new labels are dominated by a label in the middle of $\mathcal{L}(j)$, i.e., label $(3,3)$. Although the pre-domination check considering the labels other than the first and last one would incur more computational time, our preliminary empirical tests have shown that this practice does provide a visible speed up for the label correcting method as a whole. In addition, complete label elimination is also possible if bounds information is available. For example, suppose $c_{\max}$ and $\mu_{\max}$ are known by the initialization procedure. If $c_1^i + c_{ij} > c_{\max}$ or $\mu_{n_i}^i + \mu_j > \mu_{\max}$, all the new labels would be eliminated by bounds and set $\mathcal{L}(j)$ again would remain unchanged.

(3) Label elimination by convexity-based dominance check

Once a new label is generated, the dominance test should be performed in the multi-label method for BSPP. This practice helps to eliminate the undesired intermediate paths that have no possibility to be extended to a final non-dominated path at an early stage. In addition to the conventional dominance test, we propose another tangible dominance test based on the convexity of elastic demand function, as elaborated in the following proposition.

Proposition 3: For any two un-dominated labels denoted by (c_1^i, μ_1^i) and (c_2^i, μ_2^i) with $c_1^i < c_2^i$, $\mu_1^i > \mu_2^i$ at node i, if $F(c_1^i) - \mu_1^i \le F(c_2^i) - \mu_2^i$, then the final path extended from label (c_1^i, μ_1^i) can never be better than that by (c_2^i, μ_2^i) in term of objective function (13), i.e., label (c_1^i, μ_1^i) is dominated by label (c_2^i, μ_2^i).

Proof. Let c_p^{is} and μ_p^{is} denote the generalized travel cost and dual value of any path p from node i to destination s. The objective function values of the final paths created by concatenating the partial path of label $(c_1^i, \mu_1^i)/(c_2^i, \mu_2^i)$ and path p are respectively given by

$$P_1^{rs} = F(c_1^i + c_p^{is}) - \pi^{rs} - (\mu_1^i + \mu_p^{is}) \quad (19)$$

$$P_2^{rs} = F(c_2^i + c_p^{is}) - \pi^{rs} - (\mu_2^i + \mu_p^{is}) \quad (20)$$

Given the values of c_1^i and c_2^i, let us define a function $G(x) = F(c_2^i + x) - F(c_1^i + x)$ on domain $x \in (0, \infty)$. It then follows from the convexity of function $F(\cdot)$ that

$$G'(x) = F'(c_2^i + x) - F'(c_1^i + x) > 0 \quad (21)$$

which indicates that $G(x)$ is an increasing function with respect to x. Therefore by substituting $x = c_p^{is} > 0$ and $x = 0$ into function $G(x)$, we have

$$G(c_p^{is}) - G(0) = [F(c_2^i + c_p^{is}) - F(c_1^i + c_p^{is})] - [F(c_2^i) - F(c_1^i)] > 0 \quad (22)$$

Hence, it follows that

$$P_2^{rs} - P_1^{rs} = F(c_2^i + c_p^{is}) - F(c_1^i + c_p^{is}) - (\mu_2^i - \mu_1^i) > F(c_2^i) - F(c_1^i) - (\mu_2^i - \mu_1^i) \geq 0 \quad (23)$$

which suggests that the final path extended from label (c_1^i, μ_1^i) can never be better than that by (c_2^i, μ_2^i) in term of objective function value of problem [DCSDE-PP^{rs}]. □

The new dominance test can further eliminate labels that are not dominated by the conventional dominance check. In addition, the bounds information provided by the initiation procedure also help to discard undesired labels. Note that the above improvements are dedicated for problem [DCSDE-PP^{rs}-B] by making use of its special characteristics, and they may not be applicable for a general BSPP. Let $\mathcal{X}$ be the list of nodes to be examined in the multi-label method. Algorithm 2 outlines the procedure of the multi-label method for finding a path with a positive reduced cost of OD pair (r, s), i.e., p^*, in the meta-network $\mathcal{G}_{meta}$ constructed at a particular node of B&P search tree.

Algorithm 2: Pseudo-code of multi-label method

```
Initialize p* ← nil; X ← {r}; L(r) ← {(0,0)} and L(i) ← ∅ for all i ∈ N_meta \ {r};
        flag=0//denote whether a solution has been found or not
While X ≠ ∅ Do
    If flag=1 Then
        break;
```

6 EndIf

7 $i \leftarrow$ top node of list $\mathcal{X}$; $\mathcal{X} - \{i\}$

8 For any $j \in \mathcal{N}_{meta}$ s.t. $(i, j) \in \mathcal{A}_{meta}$ Do

9 If ($c_1^i + c_{ij} > UB_c$) or ($\mu_{n_i}^i + \mu_j > UB_\mu$) or ($\exists l \in \mathcal{L}(j)$ s.t. $c_1^i + c_{ij} \ge c_l^j$ and $\mu_{n_i}^i + \mu_j \ge \mu_l^j$);

10 Else $\mathcal{L}(j) \leftarrow \mathcal{L}(j) \cup \{\mathcal{L}(i) + (c_{ij}, \mu_j)\}$

11 $\mathcal{L}(j) \leftarrow$ *BoundCheck*($\mathcal{L}(j)$);

12 $\mathcal{L}(j) \leftarrow$ *ConventionDomiCheck*($\mathcal{L}(j)$);

13 $\mathcal{L}(j) \leftarrow$ *ConvexDomiCheck*($\mathcal{L}(j)$);

14 If $\mathcal{L}(j)$ has been changed Then

15 If ($(j,s) \in \mathcal{A}_{meta}$) and ($\exists l \in \mathcal{L}(j)$ s.t. $c_l^j + c_{js} \le UB_c$ and $F(c_l^j + c_{js}) - \pi^{rs} - \mu_l^j > 0$)

16 $p^* \leftarrow$ path corresponding to label l ; flag=1;

17 break;

18 Else If $j \notin \mathcal{X}$ Then

19 append j on the bottom of list $\mathcal{X}$

20 EndIf

21 EndIf

22 EndIf

23 EndIf

24 EndFor

25 EndWhile

26 If flag=0 Then

27 $p^* \leftarrow nil$

28 EndIf

Note that *BoundCheck*, *ConventionDomiCheck*, and *ConvexDomiCheck* in above pseudo-code are three subfunctions to eliminate labels at a particular node that are impossible to be extended to a feasible or optimal path. Once the set of labels at a node has been updated, we will check whether the node is directly connected to the destination, and additionally whether it forms a feasible path with positive reduced cost in the meta-network. If so, the labeling process would be terminated immediately for the sake of time saving as indicated in line 15-17 in the pseudo-code.

4.2 Long tail effect

The slow convergence when the solution is near the optimum, referred to as long tail effect, has been recognized a major difficulty in column generation for solving MP (Ben Amor et al., 2006). In practical applications, it may be time-consuming to solve the MP to optimality, and we thus consider pre-terminate the column generation process once the gap between the incumbent value and the optimal value of the MP is within a pre-specified tolerance ε_1, as demonstrated by the following proposition.

Proposition 4: Suppose that in an iteration of the column generation process, the optimal objective value of the RMP is $LpObj$, and the corresponding largest reduced cost for each OD pair (r,s) satisfies $P^{rs*} \le \frac{LpObj \times \varepsilon_1}{M}$ where M is the number of OD pairs. Then $LpObj \times (1+\varepsilon_1)$ is an upper bound on MP.

Proof. Let $\left(\pi^{rs*}, \mu_i^{rs*}, \eta^*\right)_{r\in\mathcal{R}, s\in\mathcal{S}, i\in\mathcal{I}}$ be the optimal dual solution for incumbent RMP. It can be inferred that $\left(\pi^{rs*} + P^{rs*}, \mu_i^{rs*}, \eta^*\right)_{r\in\mathcal{R}, s\in\mathcal{S}, i\in\mathcal{I}}$ is a feasible solution to the dual problem of MP. Hence, the objective value of the dual problem with the feasible solution $\left(\pi^{rs*} + P^{rs*}, \mu_i^{rs*}, \eta^*\right)_{r\in\mathcal{R}, s\in\mathcal{S}, i\in\mathcal{I}}$ which equals $LpObj + \sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}} P^{rs*}$, is an upper bound on the optimal objective value of MP. Due to the strong duality theorem, the optimal objective value of MP equals the optimal objective value of its dual problem. Hence, $LpObj \times (1+\varepsilon_1)$, which is not less than $LpObj + \sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}} P^{rs*}$, is an upper bound on the dual problem of MP, and thereby the upper bound on MP. □

As a consequence of Proposition 4, the column generation process can be terminated faster without violating the relative optimality tolerance level ε_1.

4.3 Tailored Branch-and-Price Method

The initial subset of BCAPs for column generation can be constructed by assuming that there exists a dummy BCAP with no charging actions on it between each OD pair. We assign negative flow values to these dummy BCAPs to ensure that they are quickly removed from the solution. In addition, more initial columns can be created by assigning flow to the shortest path between each OD pair since these paths have already been found by the initialization process at the root node. To accelerate the column generation process, multiple columns may be added to the RMP in each iteration. In particular, a feasible BCAP for each OD pair can be added to the augmented subset of BCAPs as long as it satisfies $P^{rs*} > \frac{LpObj \times \varepsilon_1}{M}$.

Since only location variables y_i, $i \in \mathcal{I}$ are required to be binary variables, standard branching on y_i can be readily applied. In one branch, we have $y_i = 1$, suggesting that a charging station is built at location i. In the other branch, we have $y_i = 0$, indicating that there is no charging station built at location i. Moreover, we pre-specify a relative optimality tolerance $0 < \varepsilon_2 < 1$ associated with branching. Specifically, let LB denote the incumbent best lower bound of [DCSDE]. If the optimal objective value of linear relaxation of [DCSDE] in a branch is not larger than $(1+\varepsilon_2)LB$, this branch would be pruned. We employ the depth-first and back-tracking search rule to guide the node exploration.

The step-by-step procedure of the tailored B&P method is summarized as follows:

Step 0: (Initialization) Define the relative optimality tolerances associated with column generation and branching denoted by ε_1 and ε_2, respectively. The initial lower bound $LB = 0$. The binary tree $\mathcal{T}$ consists of only a root node n_0. The corresponding MP, denoted by MP(n_0) is associated with a set of initial columns denoted by $\bar{\mathcal{P}}(n_0) := \{p_0^{rs}, \forall (r,s) \mid \delta^{rs}_{i,p_0^{rs}} = 0 \wedge c^{rs}_{p_0^{rs}} = -1\} \bigcup \{p^{rs}_{c_{\min}}, \forall (r,s)\}$, a set of accepted charging station locations denoted by $\mathcal{SI}(n_0) := \varnothing$, a set of denied charging station locations denoted by $\mathcal{RI}(n_0) := \varnothing$. The upper bound for the root node is represented by $UB(n_0) := +\infty$. Node n_0 is marked as an active node. Initialize the incumbent feasible solution $x^{rs}_{incu}, y^{incu}_i := nil, \forall r,s,i$.

Step 1: (Node exploration) An incumbent node denoted by n is first selected from all the active nodes in the binary tree by the depth-first and back-tracking search rule.

Step 2: (Solve LP-DCSDE by column generation)

- ***Step 2.0:*** Let the iteration number $k := 1$ and denote the subset of BCAPs at iteration k by $\bar{\mathcal{P}}(n,k) := \bigcup_{r\in\mathcal{R}, s\in\mathcal{S}} \bar{\mathcal{P}}^{rs}(n,k)$. Initialize $\bar{\mathcal{P}}(n,1) := \bar{\mathcal{P}}(n)$.

- ***Step 2.1:*** Solve the RMP of node n at k^{th} iteration to optimality formulated by

[RMP (n,k)]:

$$\max_{\mathbf{x},\mathbf{y}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{p\in\bar{\mathcal{P}}^{rs}(n,k)} f_p^{rs} x_p^{rs} \tag{24}$$

subject to

$$\sum_{p\in\bar{\mathcal{P}}^{rs}(n,k)} x_p^{rs} \le 1, \quad \forall r\in\mathcal{R}, s\in\mathcal{S} \tag{25}$$

$$\sum_{p\in\bar{\mathcal{P}}^{rs}(n,k)} \delta_{i,p}^{rs} x_p^{rs} \le y_i, \quad \forall r\in\mathcal{R}, s\in\mathcal{S}, i\in\mathcal{I}\setminus(\mathcal{SI}\cup\mathcal{RI}) \tag{26}$$

$$\sum_{p\in\bar{\mathcal{P}}^{rs}(n,k)} \delta_{i,p}^{rs} x_p^{rs} \le 1, \quad \forall r\in\mathcal{R}, s\in\mathcal{S}, i\in\mathcal{SI} \tag{27}$$

$$\sum_{p\in\bar{\mathcal{P}}^{rs}(n,k)} \delta_{i,p}^{rs} x_p^{rs} \le 0, \quad \forall r\in\mathcal{R}, s\in\mathcal{S}, i\in\mathcal{RI} \tag{28}$$

$$\sum_{i\in\mathcal{I}\setminus(\mathcal{SI}\cup\mathcal{RI})} e_i y_i \le B - \sum_{i\in\mathcal{SI}} e_i \tag{29}$$

$$x_p^{rs} \ge 0, 1\ge y_i \ge 0, \quad \forall r\in\mathcal{R}, s\in\mathcal{S}, p\in\bar{\mathcal{P}}^{rs}(n,k), i\in\mathcal{I}\setminus(\mathcal{SI}\cup\mathcal{RI}) \tag{30}$$

Let $LpObj(n,k)$ be optimal objective value at the current iteration. Obtain the dual variables. Construct the meta-network for the set of charging stations $\mathcal{I}\setminus\mathcal{RI}$. For each OD pair (r,s), invoke the multi-label method to obtain a feasible or optimal BCAP denoted by $p^{rs*}(n,k)$ and its corresponding profit denoted by $P^{rs*}(n,k)$; if $P^{rs*}(n,k) > \frac{LpObj(n,k)\times\varepsilon_1}{M}$, then set $\bar{\mathcal{P}}^{rs}(n,k+1) := \bar{\mathcal{P}}^{rs}(n,k)\cup\{p^{rs*}(n,k)\}$. If there are new BCAPs added, then set $k := k+1$ and repeat Step 2.1; otherwise let $x_p^{rs*}(n), \forall r,s,p$ and $y_i^*(n), \forall i$ be the optimal solution to the model [RMP(n,k)] and $LpObj(n) := LpObj(n,k)$. Let $\mathcal{P}(n) := \bar{\mathcal{P}}(n,k)$ denote the final set of columns at node n and go to Step 3.

Step 3: (check integrality and update lower bound) If $y_i^*(n), \forall i$ are all integers and $LpObj(n) \le LB$, mark node n as inactive and go to Step 6; If $y_i^*(n), \forall i$ are all integers and $LpObj(n) > LB$, update the incumbent feasible solution $x_{p,incu}^{rs} := x_p^{rs*}(n), \forall r,s,p$ and $y_{i,incu} := y_i^*(n), \forall i$, set $LB = LpObj(n)$, search the whole tree and mark all active nodes n' satisfying $UB(n') \le (1+\varepsilon_2)\times LB$ as inactive (node n is also marked as inactive node) and go to Step 6. Otherwise, go to Step 4.

Step 4: (Node fathoming) If $LpObj(n) \le (1+\varepsilon_2)\times LB$, node n is marked as inactive and go to Step 6, otherwise go to Step 5.

Step 5: (Node branching)

Define location i^* such that

$$i^* := \arg \max_{i \in \mathcal{I} \setminus [\mathcal{SI}(n) \cup \mathcal{RI}(n)]} \{ y_i \mid y_i < 1 \} \tag{31}$$

then the node is branched into two child notes, denote by n_1 and n_2. Nodes n_1 and n_2 copy all the information from node n except that for node n_1 we have $\bar{\mathcal{P}}(n_1) := \mathcal{P}(n)$, $\mathcal{SI}(n_1) := \mathcal{SI}(n) \cup \{i^*\}$ and $UB(n_1) := LpObj(n)$; for node n_2 , we have $\bar{\mathcal{P}}(n_2) := \{ p \in \mathcal{P}(n) \mid \delta_{i^*,p}^{rs} = 0, \forall (r,s) \}$, $\mathcal{RI}(n_2) := \mathcal{RI}(n) \cup \{i^*\}$, $UB(n_2) := LpObj(n)$. Nodes n_1 and n_2 are marked as active nodes.

Step 6: (Stop criterion) If all the nodes in the binary tree are inactive, stop and output the incumbent feasible solution, i.e., $x_{p,incu}^{rs}, \forall r,s,p$ and $y_{i,incu}, \forall i$, and the corresponding lower bound LB. Otherwise, go to Step 1.

5. Special Cases and Extensions

This section discusses some special applications and possible extensions of the proposed model and solution approach for DCSDE problem.

5.1 Special cases

Our study generalizes the problems considered in many related literature by incorporating path deviation, nonlinear demand elasticity, and the independence between travel cost and electricity consumption. The proposed solution method can thus be easily tailored to solve those special cases. For example, Kim and Kuby (2012) defined both the drivers' preference for a path and the driving range of BEV based on path length without considering the independence between travel cost and electricity consumption. Their problem can be solved by the proposed B&P approach except that instead of applying the methods for WCSPP, the shortest path algorithm should be used to construct the meta-network. Yıldız et al. (2016) considered a fixed demand between each OD pair. This is equivalent to assuming a zero degree of elasticity in our model by specifying $\alpha = 1$ and $\beta = 0$ in the elastic demand function (6). In this way, the pricing problem would reduce to

$$P^{rs*} = \max_{p \in \mathcal{P}^{rs} \setminus \bar{\mathcal{P}}^{rs}} f^{rs} - \pi^{rs} - \sum_{i \in \mathcal{I}} \delta_{i,p}^{rs} \mu_i^{rs} \tag{32}$$

which can be solved readily by any available shortest path algorithms in the meta-network.

In addition, as a special case of nonlinear elastic demand, the linear elastic demand function also leads to a pricing problem readily solvable by algorithms for SPP. The great efficiency of shortest path algorithms for solving the pricing problem would significantly reduce the computational time of the proposed B&P approach. Under the assumption of zero degree of elasticity, our model can easily be written in a set covering form considered in Zheng and Peeta (2017) and Huang et al. (2015), by expressing the objective function as $\sum_{i\in\mathcal{I}} e_i y_i$, modifying constraint (8) to an equality, and removing constraint (10) in the model [DCSDE]. It is not difficult to find that the pricing problem would reduce to

$$P^{rs*} = \max_{p\in\mathcal{P}^{rs}\setminus\bar{\mathcal{P}}^{rs}} -\pi^{rs} - \sum_{i\in\mathcal{I}} \delta_{i,p}^{rs}\mu_i^{rs} \tag{33}$$

which, again, can be efficiently solved by any available algorithms for SPP.

Moreover, the existing studies considering a single shortest path between each OD pair are definitely special cases of our model. The pricing problem, which aims to find feasible or optimal charging combinations along a single shortest path, can be readily solved on a greatly reduced meta-network. If both the path deviation and elastic demand are not considered, the general solution framework still applies except that the pricing problem can be solved much more easily by some pseudo-polynomial algorithms for MCPPR (Cabral, 2005; Laporte and Pascoal, 2011; Smith et al., 2012). The simplest case in which both the path deviation, elastic demand and independence between travel cost and electricity consumption are not considered, is actually the basic FRLM. For this problem, the proposed B&P would be much more efficient because polynomial algorithms for the pricing problem are available (Adler et al., 2016).

5.2 Extensions

Although we assume inverse cost function as an expression for the elastic demand, the solution method may be also applicable for other forms of functions, such as the aforementioned linear elastic demand function. In particular, the multi-label method can be directly applied to the pricing problem with any other forms of convex elastic demand function because Proposition 3 still holds. If the elastic demand is concave function, such as the inverse distance function proposed by Kim and Kuby (2012), a concavity-based dominance check, similar to the convexity-based dominance check, can be applied because the following proposition holds by a similar argument for Proposition 3.

Proposition 5: Consider a concave elastic demand function $F(\cdot)$. For any two un-dominated labels denoted by (c_1^i, μ_1^i) and (c_2^i, μ_2^i) with $c_1^i < c_2^i$, $\mu_1^i > \mu_2^i$ at node i, if $F(c_1^i) - \mu_1^i \geq F(c_2^i) - \mu_2^i$, then the final path extended from label (c_2^i, μ_2^i) can never be better than that by (c_1^i, μ_1^i) in term of objective function (13), i.e., label (c_2^i, μ_2^i) is dominated by label (c_1^i, μ_1^i).

The current model can easily be extended to multiple types of BEVs in terms of the driving range, and the decision-makings on types of stations associated with different construction cost, charging efficiency, and charging cost at a candidate location. In addition, the current study assumes pre-specified and universal battery charging cost and dwell time of BEVs at charging stations for all BEVs. This assumption can be relaxed by allowing the charging cost and time vary according to the initial SOC before charging. The battery charging cost and dwell time should thus be obtained in the construction of meta-network, and their values at a particular station may be OD specific. Moreover, considering multiple scenarios of target SOC at the end of charging (instead of "full charge") is not impossible in theory. However, we caution that this will result in an augmented network and model, making the proposed solution method computationally intensive.

Other interesting and practically relevant extensions include the consideration of *p*-stops constraint and asymmetric round trips. Specifically, *p*-stops constraint may be incorporated by recording additional label information of how many stations the partial path has traversed in the multi-label method for the pricing problem. Let UB_{stops} denote the maximum number of stops a BEV is allowed to make for charging. The label with its third element exceeding UB_{stops} should be eliminated due to the infeasibility. The asymmetric round trips may be incorporated in an augmented network consisting of the original network and its mirror-symmetric counterpart. To illustrate, let us consider the generation of an augmented meta-network in Figure 5 for the same network example in Figure 3. The original meta-network construction for OD pair $(1,5)$ is also replicated in Figure 5 for ease of comparison. In addition to the meta-network in Figure 5 (b), the augmented meta-network requires the auxiliary counterparts for all candidate location nodes, and an additional copy of auxiliary origin node denoted by $1'''$. The generation of meta-links between candidate location node pairs is exclusive for each counterpart of original network, and they are the same with each other except that the nodes differ in their notations. Similar to meta-network construction, all the meta-links in the augmented meta-network are generated by the available methods for WCSPP.

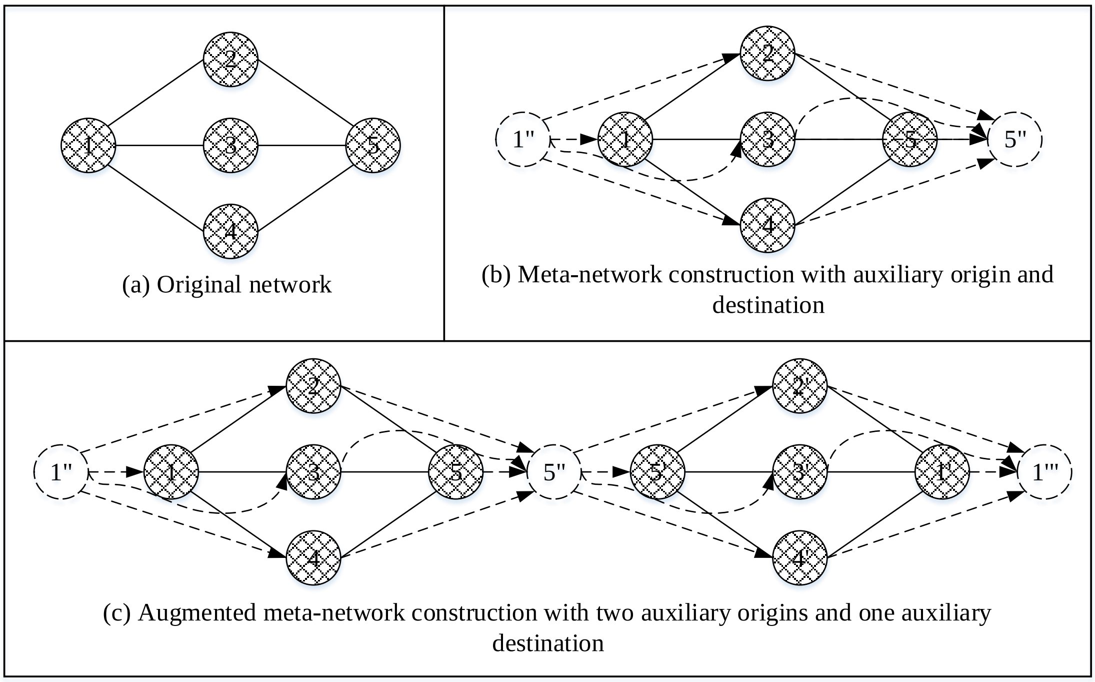


Figure 5. Illustration of augmented meta-network construction

Note that simply using two copies of auxiliary origins and destinations in the original network would not help, and a mirror-symmetric counterpart for the whole network is a must. One difference from the meta-network is that the augmented meta-network is OD specific, or put it more exactly, the destination specific. It is thus not recommended to put all the auxiliary destinations in one augmented meta-network, otherwise, the multi-label method may give out a wrong round trip from origin to the other destination and then return to the origin. The incorporations of *p*-stops constraint and asymmetric round trips bring more realism to the original DCSDE problem at the expense of additional computational complexity since the efficiency of multi-label method is heavily affected by the size of network and the dimension of labels.

6. Numerical Experiments

In this section, two network topologies have been used to evaluate the performance of the proposed model and B&P approach. The algorithm is coded in Matlab 2010b calling IBM ILOG CPEX 12.6 on a personal computer with Intel (R) Core (TM) Duo 3.4 GHz CPU. For simplicity, the construction cost of each station is assumed to be 1. As such, the value of budget indicates the maximum number of charging stations allowed to be built in the network. For computer implementation, the independence between the pricing problems for different OD pairs make them ideal to be parallelized. We thus populate 8 processors to solve the pricing problems in parallel. The relative optimality gap of the proposed B&P approach is controlled by ε_1 and ε_2. By setting $\varepsilon_1 = \varepsilon_5 = 0.0005$, the overall relative optimality gap is around 0.001.

6.1 25-node network

In order to test the proposed model and solution method, we first solve the DCSDE problem in a hypothetical network in Figure 6. This small network consists of 25 nodes and 86 links (43 undirected edges). It has been extensively used as a benchmark network in the literature for refuelling station location optimization (Kim and Kuby, 2012; MirHassani and Ebrazi, 2012; Yıldız et al., 2016). The link travel time is set to be the same with the link distance used by Kim and Kuby (2012), and its value is shown beside each edge in Figure 6. The electricity consumption of each link is chosen as a uniformly random integer from set $\{t_a - 2, t_a - 1, t_a, t_a + 1, t_a + 2\}$. For simplicity, we assume that the VOT is 1, and the sum of battery charging cost and dwell time of BEVs at each charging station equals 1. The initial and final state of charge (SOC) of the BEVs at their origins and destinations equal to half of the correspondent usable battery capacity, i.e., $\frac{1}{2}W$. All nodes are considered as origins, destinations, and candidate locations of charging stations. Hence, we have 300 OD pairs and 25 candidate locations in total. The traffic flow for each OD pair is estimated by the gravity model in Hodgson (1990). The parameter β in the elastic demand function is set to be 0.1.

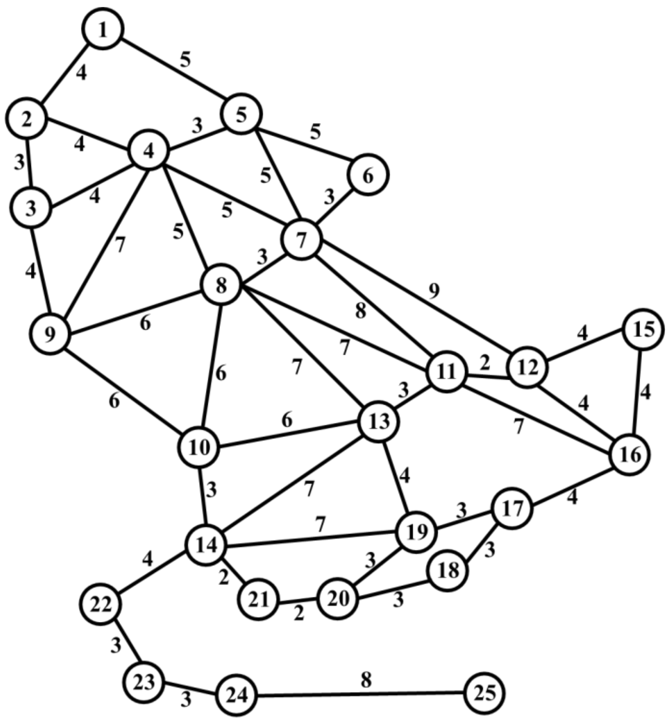


Figure 6. A hypothetical 25-node network

To evaluate the performance of B&P approach under different driving range, path cost deviation and budget, a total of 225 problem instances are generated by considering

- 3 BEV driving ranges: 6, 8 and 10;
- 3 levels of tolerance for path cost deviation: 0, 20% and 50% with respect to $c_{p^*}^{rs}$;
- 25 values of budget: 1, 2, …, 25.

Table 1 shows that the results of B&P approach in the 25-node network, and each row corresponds to the results of a particular problem instance indicated by the driving range of BEVs (D), budget (B), and level of tolerance for path cost deviation. Several output parameters are reported in the table, including the percentage of flow that can be refuelled by the optimal deployment of charging stations (Obj) and the solution of linear relaxation problem (LpObj) in %, the number of nodes traversed (#N), the number of priced out columns (#C), the maximal number of active nodes (#MaxN), and the maximal depth level (#MaxL) in the B&P search tree, and the runtime of B&P method to obtain the optimal solution (Time) in seconds.

According to the runtime in the table, we can see that the B&P approach can solve the DCSDE problem in a small network within 7 minutes. The total runtime increases obviously with the increase of path cost deviation and the driving range, whereas the impact of budget on the computational efficiency of B&P approach is somehow arbitrary. As expected, the runtime shows a positive correlation with the number of columns priced out, which also grows apparently with the increase of the driving range and path cost deviation. The number of nodes traversed and maximal depth level in the search tree generally measures the difficulty for solving a problem, while the maximal number of active nodes reflects the computer memory requirement for recording the information of a search tree. It can be seen from Table 1 that all these numbers are within a few dozens, demonstrating the efficiency of the proposed method for solving the DCSDE problem within a B&P tree.

In addition to the computational efficiency of B&P approach, Table 1 also reports the percentage of refuelled flow in these problem instances. On the whole, the refuelled flow grows with the increase of driving range, path cost deviation, and budget. Under a driving range of 6, a path cost deviation of 50%, and a budget of 25, the percentage of refuelled flow achieves its maximum value at 62.18%. Further increase of drive range or path cost deviation would result in more flow to be refuelled by the charging stations. Among the three influential factors, the driving range and budget show much more significant effects on the value of refuelled flow than the path cost deviation. Moreover, it appears that the effect of driving range decays nonlinearly with the increase of its value. For example, under a path cost deviation of 20% and a budget of 8, the percentage of refuelled flow has grown from 22.83% to 42.63%, i.e., almost doubles, when the driving range of BEV increases from 6 to 8. However, the increment of refuelled flow become much smaller, i.e., from 42.63% to 48.38%, when the driving range increases from 8 to 10. On the contrary, the effect of path cost deviation manifests with the increase of driving range of BEV. This can be seen from the results that the refuelled flow remains almost unchanged when the path cost deviation increases from 0 to 20% under budget of 6 and 8, while it shows a visible increase under the budget of 10.

To examine the effect of demand elasticity on the flow coverage, we compare the percentage of refuelled flow under the elastic demand (with $\beta = 0.1$) and the fixed demand (with $\beta = 0$) in %, denoted by Obj and Obj_F, respectively, in Table 2. The difference of Obj and Obj_F represented by Diff is also tabulated in the table. The results under 0 tolerance for path deviation is not present because there would be no demand decay on the considered shortest path. Overall, we can see that most instances are associated with a positive difference of Obj and Obj_F, and the difference can be up to 4.01%. As a demonstrable advantage over the traditional maximum flow model for refueling station deployment, the proposed model considering demand elasticity could capture demand loss resulted from travelers' mobility mode switch behavior due to additional travel cost on deviation path. The effect of demand elasticity, measured by the magnitude of difference, shows an upward trend with the increase of BEV driving range and path deviation tolerance. This is because the number of deviation paths increases with a larger BEV driving range and path deviation tolerance, making a journey on a deviation path, perhaps much longer than the shortest path, more likely to happen. We also find that for the same driving range, the largest differences under different tolerances are often associated with the same budget. For example, under the driving range of 8, the differences under 20% and 50%, reach the highest values at the same budget of 15. This also applies to the instances under the driving range of 10.

Table 1. Results of B&P approach for 25-node network

		No Tolerance							20% Tolerance							50% Tolerance						
D	B	Obj	LpObj	#N	#C	#MaxN	#MaxL	Time	Obj	LpObj	#N	#C	#MaxN	#MaxL	Time	Obj	LpObj	#N	#C	#MaxN	#MaxL	Time
6	1	0.15	3.11	25	4	2	12	3.35	0.15	3.11	25	15	2	12	2.70	0.15	3.14	25	54	2	12	4.18
	2	3.16	6.22	25	5	12	6	3.40	3.16	6.22	25	23	12	6	4.34	3.16	6.29	25	66	12	6	5.59
	3	6.99	9.33	7	6	2	3	1.65	6.99	9.33	7	22	2	3	2.79	6.99	9.43	7	62	2	3	4.09
	4	9.50	12.44	19	10	9	5	3.00	9.50	12.44	19	35	9	5	4.41	9.50	12.57	19	67	9	5	5.51
	5	13.72	15.54	7	6	2	3	2.02	13.72	15.54	7	15	2	3	1.96	13.72	15.72	7	48	2	3	2.49
	6	16.91	18.65	5	5	2	2	1.52	16.91	18.65	5	19	2	2	2.71	16.91	18.86	5	48	2	2	3.52
	7	17.94	21.76	27	11	9	6	4.41	17.94	21.76	27	45	9	6	6.33	17.94	22.01	27	92	9	6	8.01
	8	22.83	24.86	7	6	3	3	1.48	22.83	24.86	7	20	3	3	2.16	22.83	25.14	7	64	3	3	4.05
	9	24.36	27.96	17	12	6	5	2.54	24.39	27.96	17	34	6	5	4.04	25.19	28.27	9	71	5	3	3.95
	10	26.14	31.05	39	18	9	8	5.35	26.17	31.05	39	84	9	8	8.62	26.97	31.40	29	85	8	7	8.15
	11	29.83	34.15	25	6	5	8	2.90	29.83	34.15	25	44	5	8	5.62	29.83	34.53	37	96	9	9	9.65
	12	33.98	37.24	23	6	5	9	3.04	33.98	37.24	21	40	5	8	4.69	34.42	37.66	19	77	5	8	6.24
	13	39.30	40.33	9	6	3	4	1.65	39.30	40.33	9	21	3	4	2.34	39.82	40.79	7	53	3	3	4.74
	14	40.84	43.43	27	6	5	11	3.71	40.84	43.43	27	45	5	11	5.79	41.25	43.91	27	87	5	11	9.26
	15	46.52	46.52	1	2	1	0	0.35	46.52	46.52	1	6	1	0	0.58	47.04	47.04	1	25	1	0	1.31
	16	49.31	49.31	1	2	1	0	0.37	49.31	49.31	1	4	1	0	0.56	49.83	49.83	1	22	1	0	0.50
	17	51.83	51.98	3	2	2	1	0.66	51.83	51.98	3	8	2	1	1.12	51.83	52.25	5	80	2	2	5.28
	18	54.66	54.66	1	3	1	0	0.27	54.66	54.66	1	7	1	0	0.25	54.66	54.66	1	22	1	0	0.73
	19	56.30	56.99	3	2	2	1	0.53	56.30	56.99	3	6	2	1	0.71	56.82	56.99	3	21	2	1	1.08
	20	58.83	59.32	5	3	3	2	0.74	58.83	59.32	5	9	3	2	1.38	58.83	59.32	5	55	3	2	3.30
	21	61.65	61.65	1	2	1	0	0.17	61.65	61.65	1	3	1	0	0.16	61.65	61.65	1	3	1	0	0.16
	22	62.07	62.07	1	2	1	0	0.13	62.07	62.07	1	3	1	0	0.13	62.07	62.07	1	3	1	0	0.13
	23	62.18	62.18	1	2	1	0	0.11	62.18	62.18	1	2	1	0	0.13	62.18	62.18	1	2	1	0	0.12
	24	62.18	62.18	1	2	1	0	0.12	62.18	62.18	1	2	1	0	0.14	62.18	62.18	1	2	1	0	0.12
	25	62.18	62.18	1	0	1	0	0.10	62.18	62.18	1	0	1	0	0.11	62.18	62.18	1	0	1	0	0.11
8	1	2.71	5.48	9	26	2	4	16.98	2.71	5.48	9	278	2	4	38.36	2.71	5.49	9	556	2	4	63.76
	2	6.22	10.96	13	35	7	4	22.61	6.22	10.96	13	337	7	4	69.76	6.22	10.99	11	682	6	3	102.34
	3	14.91	16.45	5	23	2	2	11.93	14.91	16.45	5	302	2	2	49.62	14.91	16.48	5	541	2	2	72.13
	4	18.80	21.93	3	28	2	1	12.87	18.80	21.93	5	291	2	2	39.08	20.90	21.97	5	544	2	2	70.02
	5	23.87	27.41	11	34	5	4	21.38	23.87	27.41	9	336	5	3	76.06	24.28	27.47	15	749	7	4	106.83

	6	30.06	32.48	9	40	5	3	24.47	30.06	32.48	9	352	5	3	71.61	30.85	32.91	9	697	4	3	91.25
	7	35.29	37.56	9	32	3	4	27.04	35.29	37.56	9	323	3	4	65.43	36.03	38.35	7	649	3	3	93.09
	8	42.63	42.63	1	16	1	0	4.72	42.63	42.63	1	179	1	0	31.88	43.79	43.79	1	298	1	0	21.23
	9	46.38	47.37	9	24	4	3	23.49	46.38	47.37	9	334	4	3	85.93	48.14	48.72	7	433	2	3	82.29
	10	52.11	52.11	1	15	1	0	4.67	52.11	52.11	1	158	1	0	27.51	52.26	53.66	17	679	4	6	94.62
	11	56.05	56.64	5	30	2	2	11.53	56.47	56.80	5	229	2	2	44.00	58.59	58.59	1	300	1	0	27.50
	12	59.63	61.17	11	27	6	3	23.83	59.86	61.48	13	335	6	4	73.56	62.83	63.18	5	382	2	2	68.32
	13	65.70	65.70	1	15	1	0	4.27	66.17	66.17	1	136	1	0	12.46	67.34	67.77	3	370	2	1	59.29
	14	69.53	69.94	3	22	2	1	15.67	69.53	70.73	5	226	2	2	41.51	70.58	72.20	7	425	4	2	68.69
	15	72.37	74.18	5	18	2	2	10.76	73.50	75.18	5	203	3	2	39.22	74.78	76.51	7	454	3	3	90.78
	16	76.75	78.42	7	41	3	3	13.18	78.36	79.63	5	220	3	2	47.26	80.21	80.81	5	419	2	2	65.91
	17	80.66	82.66	15	50	5	6	23.65	82.57	84.08	11	273	3	5	77.23	84.78	85.12	5	439	2	2	64.51
	18	86.90	86.90	1	14	1	0	3.70	88.53	88.53	1	150	1	0	15.92	89.29	89.42	3	360	2	1	53.32
	19	90.93	90.93	1	13	1	0	2.34	92.87	92.87	1	173	1	0	16.36	93.72	93.72	1	259	1	0	14.22
	20	93.70	93.70	1	12	1	0	1.35	95.11	95.35	1	166	1	0	16.66	95.84	95.97	3	351	2	1	27.63
	21	95.76	95.76	1	10	1	0	1.16	97.39	97.39	1	118	1	0	7.76	97.66	97.66	1	221	1	0	7.05
	22	98.07	98.07	1	11	1	0	0.94	98.87	98.87	1	134	1	0	9.83	98.88	98.88	1	214	1	0	6.95
	23	99.22	99.22	1	8	1	0	0.65	99.36	99.36	1	138	1	0	10.36	99.37	99.37	1	226	1	0	7.40
	24	99.71	99.71	1	8	1	0	0.51	99.71	99.71	1	139	1	0	7.53	99.71	99.71	1	222	1	0	7.39
	25	100	100	1	0	1	0	0.49	100	100	1	0	1	0	6.47	100	100	1	0	1	0	7.23
10	1	8.32	8.32	1	31	1	0	10.79	8.32	8.32	1	272	1	0	27.84	8.32	8.32	1	692	1	0	77.45
	2	11.39	14.29	11	39	2	5	27.19	11.39	14.45	13	444	2	6	103.44	11.55	14.71	13	1066	2	6	222.77
	3	17.15	20.27	7	40	2	3	33.01	17.15	20.58	7	378	2	3	84.65	17.86	21.10	9	968	2	4	166.61
	4	20.74	26.23	11	44	4	4	46.13	21.03	26.71	21	592	7	5	246.92	22.46	27.48	13	1106	4	4	291.16
	5	27.53	32.18	11	46	5	4	40.15	27.82	32.84	9	439	5	3	125.98	29.25	33.87	15	1247	5	4	346.38
	6	33.68	37.76	11	49	3	5	47.03	34.53	38.66	7	409	3	3	116.81	35.83	39.78	11	1137	5	5	337.69
	7	40.53	43.33	11	42	5	4	45.94	41.81	44.47	9	452	3	4	146.84	43.20	45.68	9	1120	3	4	301.73
	8	47.54	48.91	5	38	3	2	29.52	48.38	50.29	9	481	3	4	124.67	49.03	51.59	11	1233	3	5	374.15
	9	54.49	54.49	1	32	1	0	16.13	56.10	56.10	1	367	1	0	58.61	57.49	57.49	1	807	1	0	133.03
	10	57.00	58.42	11	39	2	5	36.47	59.03	60.13	11	443	2	5	124.12	60.61	61.56	9	972	2	4	222.19
	11	60.40	62.36	5	32	2	2	24.17	62.79	64.16	5	370	2	2	93.29	64.92	65.65	3	634	2	1	103.96
	12	64.92	66.30	5	40	2	2	22.80	68.02	68.18	3	331	2	1	59.67	69.46	69.68	3	765	2	1	157.30
	13	68.08	70.23	19	67	5	7	48.23	71.83	72.21	5	357	3	2	83.06	73.43	73.74	5	836	3	2	168.81
	14	71.78	74.14	19	80	5	7	69.86	74.84	76.21	13	459	3	6	128.33	76.93	77.80	13	1055	3	6	233.71

15	76.35	78.05	11	63	3	5	44.78	79.53	80.21	11	454	5	6	112.14	81.64	81.87	9	872	3	4	193.64
16	81.20	81.96	7	49	3	3	30.46	83.71	84.21	11	460	4	4	127.89	85.93	85.93	1	499	1	0	41.23
17	85.87	85.87	1	30	1	0	9.58	88.21	88.21	1	252	1	0	40.77	89.69	89.69	3	603	2	1	93.76
18	88.91	88.91	1	28	1	0	11.54	91.25	91.25	1	169	1	0	20.45	92.40	92.40	1	364	1	0	25.69
19	91.20	91.20	1	23	1	0	5.12	93.17	93.30	3	244	2	1	39.53	94.29	94.29	1	359	1	0	34.67
20	94.96	94.96	5	36	4	2	16.68	95.83	95.83	3	244	2	1	49.60	95.88	95.91	5	555	3	2	77.40
21	97.08	97.08	3	33	2	1	13.05	97.95	97.95	1	162	1	0	19.21	98.10	98.10	1	342	1	0	26.79
22	98.30	98.30	1	22	1	0	4.31	99.28	99.28	1	146	1	0	6.10	99.33	99.33	1	287	1	0	11.66
23	99.63	99.63	1	15	1	0	1.33	99.66	99.66	1	141	1	0	8.00	99.69	99.69	1	267	1	0	7.48
24	99.92	99.92	1	14	1	0	0.63	99.95	99.95	1	148	1	0	6.76	99.99	99.99	1	294	1	0	13.80
25	100	100	1	0	1	0	0.51	100.00	100.00	1	0	1	0	6.63	100	100	1	0	1	0	0.68

Table 2. Comparison of results under elastic demand and fixed demand for 25-node network

B	D=6						D=8						D=10					
	20% Tolerance			50% Tolerance			20% Tolerance			50% Tolerance			20% Tolerance			50% Tolerance		
	Obj	Obj_F	Diff	Obj	Obj_F	Diff	Obj	Obj_F	Diff	Obj	Obj_F	Diff	Obj	Obj_F	Diff	Obj	Obj_F	Diff
1	0.15	0.15	0.00	0.15	0.15	0.00	2.71	2.71	0.00	2.71	2.71	0.00	8.32	8.32	0.00	8.32	8.32	0.00
2	3.16	3.16	0.00	3.16	3.16	0.00	6.22	6.22	0.00	6.22	6.88	0.66	11.39	11.39	0.00	11.55	11.68	0.13
3	6.99	6.99	0.00	6.99	6.99	0.00	14.91	14.91	0.00	14.91	14.91	0.00	17.15	17.79	0.64	17.86	18.52	0.66
4	9.5	9.50	0.00	9.5	9.50	0.00	18.8	18.80	0.00	20.9	22.36	1.46	21.03	21.78	0.75	22.46	23.43	0.97
5	13.72	13.72	0.00	13.72	13.72	0.00	23.87	23.87	0.00	24.28	25.68	1.40	27.82	28.56	0.74	29.25	31.33	2.08
6	16.91	16.91	0.00	16.91	16.91	0.00	30.06	30.06	0.00	30.85	31.35	0.50	34.53	34.64	0.11	35.83	38.75	2.92
7	17.94	17.94	0.00	17.94	17.94	0.00	35.29	35.29	0.00	36.03	36.52	0.49	41.81	42.55	0.74	43.2	44.02	0.82
8	22.83	22.83	0.00	22.83	22.83	0.00	42.63	42.63	0.00	43.79	44.53	0.74	48.38	48.44	0.06	49.03	52.63	3.60
9	24.39	24.42	0.03	25.19	26.03	0.84	46.38	46.38	0.00	48.14	50.38	2.24	56.1	56.85	0.75	57.49	58.31	0.82
10	26.17	26.20	0.03	26.97	27.81	0.84	52.11	52.11	0.00	52.26	54.80	2.54	59.03	59.97	0.94	60.61	62.08	1.47
11	29.83	29.83	0.00	29.83	29.83	0.00	56.47	56.70	0.23	58.59	59.95	1.36	62.79	63.97	1.18	64.92	66.83	1.91
12	33.98	33.98	0.00	34.42	34.93	0.51	59.86	60.09	0.23	62.83	64.60	1.77	68.02	69.11	1.09	69.46	71.07	1.61
13	39.3	39.34	0.04	39.82	40.35	0.53	66.17	66.40	0.23	67.34	69.49	2.15	71.83	73.11	1.28	73.43	75.65	2.22
14	40.84	40.84	0.00	41.25	41.78	0.53	69.53	69.53	0.00	70.58	74.19	3.61	74.84	76.39	1.55	76.93	80.94	4.01
15	46.52	46.56	0.04	47.04	47.57	0.53	73.5	74.80	1.30	74.78	78.54	3.76	79.53	80.27	0.74	81.64	85.25	3.61
16	49.31	49.35	0.04	49.83	50.36	0.53	78.36	79.12	0.76	80.21	82.06	1.85	83.71	84.40	0.69	85.93	89.42	3.49

17	51.83	51.83	0.00	51.83	51.83	0.00	82.57	82.97	0.40	84.78	86.92	2.14	88.21	88.44	0.23	89.69	92.73	3.04
18	54.66	54.66	0.00	54.66	54.66	0.00	88.53	88.79	0.26	89.29	91.44	2.15	91.25	91.48	0.23	92.4	95.06	2.66
19	56.3	56.35	0.05	56.82	57.35	0.53	92.87	93.28	0.41	93.72	94.89	1.17	93.17	93.28	0.11	94.29	97.37	3.08
20	58.83	58.83	0.00	58.83	58.83	0.00	95.11	95.91	0.80	95.84	97.21	1.37	95.83	96.22	0.39	95.88	97.90	2.02
21	61.65	61.65	0.00	61.65	61.65	0.00	97.39	98.03	0.64	97.66	98.24	0.58	97.95	98.54	0.59	98.1	98.78	0.68
22	62.07	62.07	0.00	62.07	62.07	0.00	98.87	98.92	0.05	98.88	98.92	0.04	99.28	99.36	0.08	99.33	99.41	0.08
23	62.18	62.18	0.00	62.18	62.18	0.00	99.36	99.41	0.05	99.37	99.41	0.04	99.66	99.66	0.00	99.69	99.71	0.02
24	62.18	62.18	0.00	62.18	62.18	0.00	99.71	99.71	0.00	99.71	99.71	0.00	99.95	99.96	0.01	99.99	100.00	0.01
25	62.18	62.18	0.00	62.18	62.18	0.00	100	100	0	100	100	0	100	100	0	100	100	0
Max			0.05			0.84			1.30			3.76			1.55			4.01

6.2 California State road network

To further examine its scalability to large networks, we implement the B&P approach in the California State (CA) road network in Figure 7 (Arslan et al., 2014, 2015; Yıldız et al., 2016). This network consists of 339 nodes and 1234 links, and has recently been used for location problem of charging stations by Yıldız et al. (2016). Considering all urban population centres in the California as origins or destinations would lead to a total of 1167 OD pairs. The traffic flow of each OD pair is again obtained by the gravity model. Since the incorporation of nonlinear elastic demand makes the pricing problem more computationally expensive, especially in large networks, we considered a subset of 320 OD pairs with the largest traffic flows in the numerical experiment. The sum of traffic flow of the 320 OD pairs accounts for more than 94% of total flow of all OD pairs. The first 180 nodes in terms of node weight in the gravity model are chosen as candidate charging station locations.

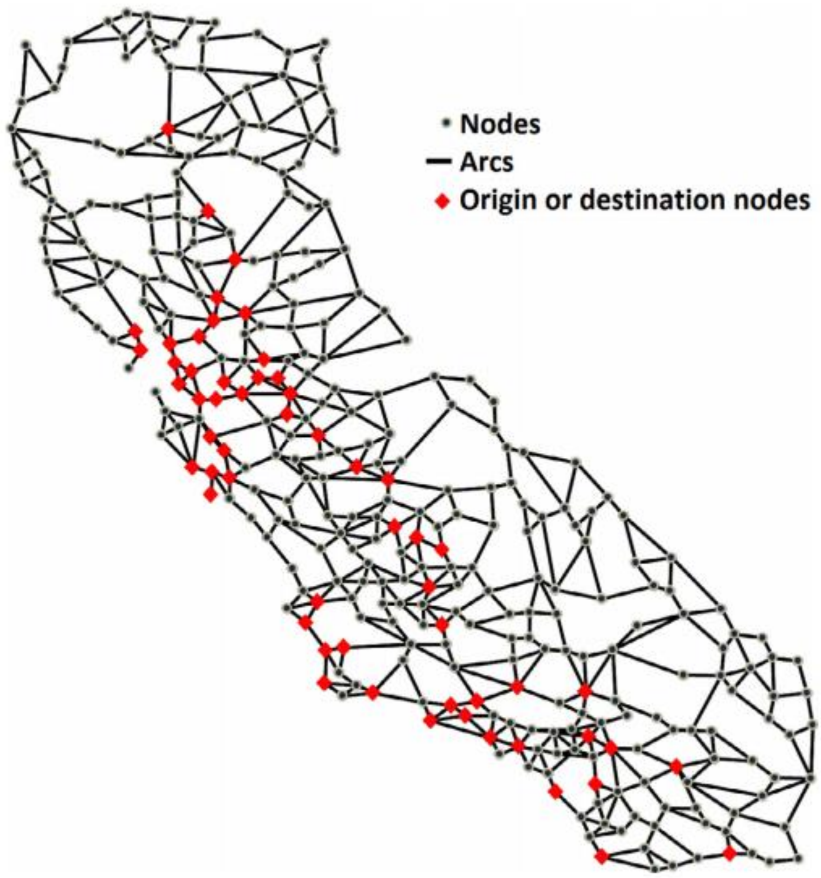


Figure 7. The California State road network (Yıldız et al., 2016)

All the BEVs are assumed to be Nissan Leaf 30 kWh (Nissan, 2017). We assume that the VOT is 1, and the sum of battery charging cost and dwell time of BEVs at each charging station is chosen as a uniformly random integer from set $\{5, 6, ..., 15\}$. The link travel time measured in minutes, is chosen as a uniformly random integer with a maximum of 5 minutes deviation from its mean value estimated by assuming an average travel speed of 60 km/h. The electricity consumption measured in kWh, is also chosen as a uniformly random integer with a maximum of 2 kWh deviation from its minimal value estimated by the particulars of Nissan Leaf 30 kWh. The parameter β of the elastic demand function is again set to be 0.1. The initial and final state of charge (SOC) of the BEVs at their origins and destinations equal to half of the correspondent usable battery capacity.

We create 30 problem instances by considering 3 levels of tolerance for the path cost deviation, i.e., 0, 0.1 and 0.2 with respect to $c_{p^*}^{rs}$, and 10 values of budget, i.e., 5, 10, …, 50. The results are shown in Table 2. In addition to the parameters in Table 1, the relative optimality gap calculated as the ratio of optimal objective value and the incumbent objective value achieved at 5 hours minus 1 (Gap) is reported for problem instances that are not solved to optimality within 5 hours in Table 2.

As the table shows, the runtime of B&P method has increased tremendously in the large network and more than half of the instances cannot be solved to optimality within one hour. This may be attributed to the more time required to solve the pricing problem, the larger number of columns to be generated, and the more nodes to be explored in the B&P tree. Though computationally intensive, it can be seen that the B&P method is able to solve more than 75% of problem instances within 5 hours. For the instances that are not solved to optimality, the optimality gap is no more than 0.0158, and five out of the seven instances obtain zero optimality gap, indicating that their optimal solutions have already been found within 5 hours. Similar to the results in the 25-node network, the runtime of B&P method increases obviously with the increase of path cost deviation, and it varies considerably with the budget. Under a specific path cost deviation, the most computationally intensive instances are always associated with a budget of 15 and 20. This result is consistent with the finding of Yıldız et al. (2016), which suggests that the problems with small or large budget are easier to solve and harder problems arise in between. Although the B&P approach takes much longer time for solving the DCSDE problem in a large network, the columns priced out are at most tens of thousands and the maximum number of active nodes is only a few dozens, indicating that the memory issue confronted by the path and charging combination pre-generation is not a big issue for B&P method.

Similar to Table 2, we also present the difference of flow coverage under elastic and fixed demand in Table 4. It can be found that the effect of demand elasticity on flow coverage becomes more significant in the CA network, with the difference of refueled flow percentage reaches up to 13.2%. Again, we can find that the effect of demand elasticity is amplified when the path deviation tolerance increases, and the maximum differences are often associated with the same budget.

Table 3. Results of B&P approach for CA network

Tolerance	B	Obj	#N	#C	#MaxN	#MaxL	Time	Gap
0	5	29.38	1	0	1	0	24	-
	10	37.02	5	5	2	2	137	-
	15	42.20	151	33	45	15	2190	-
	20	51.23	75	5	17	13	1393	-
	25	62.66	7	5	4	3	173	-
	30	70.32	9	5	6	4	219	-
	35	76.95	1	0	1	0	26	-
	40	84.40	1	0	1	0	25	-
	45	89.92	1	0	1	0	20	-
	50	92.02	1	0	1	0	16	-
10%	5	29.76	1	5483	1	0	7638	-
	10	37.56	11	7356	4	3	16334	-
	15	43.76	143	20452	48	11	18000	0
	20	53.17	99	19806	46	14	18000	0.0158
	25	64.71	13	4390	5	5	12151	-
	30	73.87	5	3475	2	2	7316	-
	35	82.71	1	2913	1	0	2859	-
	40	88.16	3	4013	2	1	7416	-
	45	91.76	1	1781	1	0	1925	-
	50	93.55	1	1862	1	0	2544	-
20%	5	29.76	1	10584	1	0	6235	-
	10	37.65	11	25370	4	3	18000	0
	15	43.95	143	73274	47	10	18000	0
	20	53.26	91	66633	43	14	18000	0.0157
	25	64.98	11	12672	4	4	18000	0
	30	74.25	5	10492	2	2	18000	0
	35	83.03	1	8280	1	0	9839	-
	40	88.55	1	5744	1	0	6166	-
	45	91.86	1	3546	1	0	4656	-
	50	93.65	1	2236	1	0	1978	-

Table 4. Comparison of results under elastic demand and fixed demand for CA network

B	10% Tolerance			20% Tolerance		
	Obj	Obj_F	Diff	Obj	Obj_F	Diff
5	29.76	29.76	0.00	29.76	29.76	0.00
10	37.73	37.56	0.17	38.47	37.65	0.82
15	45.79	43.76	2.03	46.81	43.95	2.86
20	58.09	53.17	4.92	58.59	53.26	5.33
25	73.04	64.71	8.33	73.94	64.98	8.96
30	82.99	73.87	9.12	87.45	74.25	13.20
35	88.89	82.71	6.18	91.61	83.03	8.58
40	92.04	88.16	3.88	93.78	88.55	5.23
45	93.84	91.76	2.08	95.29	91.86	3.43
50	95.46	93.55	1.91	96.80	93.65	3.15
Max			9.12			13.20

7. Conclusions and Future Research

This study investigates the optimal deployment of charging stations considering path deviation and nonlinear elastic demand without pre-generating paths and charging combinations. The battery charging action-based path is first proposed to facilitate model building. It is assumed that BEVs would travel on the shortest feasible path in terms of generalized travel cost between an OD pair, and the link travel time and electricity consumption are mutually independent. A BCAP-based model is formulated, and a tailored B&P approach is proposed to solve the model. The pricing problem is not easily solvable by available algorithms, and an improved label correcting method is proposed to solve a BSPP on a meta-network generated by the algorithm for WCSPP. Possible extensions of the proposed model and solution approach to incorporate the maximal allowable number of stops and the asymmetric round trips have also been discussed. Numerical experiments are conducted to evaluate the performance of the proposed B&P approach both in a hypothetical 25-node network and a real-world network, i.e., CA road network.

We find that the efficiency of the proposed solution method is largely affected by size of network, and the pricing problem is the most computational intensive part within the B&P approach. A future research direction is thus to further enhance the multi-label method and improve its efficiency for solving the pricing problem, and to develop promising heuristic methods for implementation in large-scale networks. In addition, the current study mainly focuses on the location of charging stations in highway networks. Another line of future studies may concern the optimal deployment of charging stations in an urban environment subject to more sophisticated constraints, such as road congestion, queue formation at charging stations due to limited capacities and long charging time. Simulation-based optimization approaches might be useful to incorporate these effects.

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Appendix: Notations

$\mathcal{N}$	Set of nodes
$\mathcal{A}$	Set of links
$\mathcal{R}$	Set of origins
$\mathcal{S}$	Set of destinations
$\mathcal{I}$	Set of candidate locations for battery charging stations
a	Index for link
i, j	Indices for node

e_i	Construction cost of charging station $i \in \mathcal{I}$
B	Total budget for charging station construction
W	Usable battery capacity of BEVs
w_a	electricity consumption of link a
$\mathcal{P}_{\text{physical}}^{\text{rs}}$	Set of physical paths between an OD pair (r,s)
p	Index for path
$\sigma_p(v_i,v_j)$	sub-path of path p from nodes v_i to v_j
$w[\sigma_p(v_i,v_j)]$	Electricity consumption of a BEV on the sub-path
$\mathcal{P}_{\text{all}}^{\text{rs}}$	Set of feasible BCAPs between an OD pair (r,s)
t_a	Travel time of link a
λ_i	Battery charging cost at charging station $i \in \mathcal{I}$
d_i	Dwell time for charging at charging station $i \in \mathcal{I}$
τ	Value of time
c_p^{rs}	Generalized travel cost on a feasible BCAP $p \in \mathcal{P}_{\text{all}}^{\text{rs}}$
$c_{p^*}^{rs}$	Generalized travel cost of the shortest BCAP (in terms of generalized travel cost) between the OD pair (r,s)
$\delta_{i,p}^{rs}$	BCAP-charging action incidence indicator which equals 1 if the feasible BCAP p traverses the charging station $i \in \mathcal{I}$ where a battery charging action is taken and 0 otherwise
ε	Pre-specified tolerance for path deviation
f_p^{rs}	Traffic flow volume on a feasible BCAP $p \in \mathcal{P}_{\text{all}}^{\text{rs}}$ between OD pair (r,s)
f^{rs}	Traffic flow volume between OD pair (r,s) when the generalized travel cost is $c_{p^*}^{rs}$
$\mathcal{P}^{rs}$	Set of potential BCAPs among all the feasible BCAPs between OD pair (r,s), i.e., $\mathcal{P}^{rs} = \left\{ p \in \mathcal{P}_{\text{all}}^{\text{rs}} \middle| c_p^{rs} \le c_{p*}^{rs} + \varepsilon \right\}$
x_p^{rs}	Binary decision variable indicating whether the flow between OD pair (r,s) would travel on BCAP $p \in \mathcal{P}^{rs}$
y_i	Binary decision variable indicating whether a charging station is built at location i.
$\bar{\mathcal{P}}^{rs}$	Subset of potential BCAPs in column generation method
π^{rs}	Dual variable corresponding to constraint (8)
μ_i^{rs}	Dual variable corresponding to constraint (9)
P^{rs*}	Optimal objective value of pricing problem for OD pair (r,s)
μ_p^{rs}	Sum of dual value μ_i^{rs} on a BCAP $p \in \mathcal{P}_{\text{all}}^{\text{rs}}$ between OD pair (r,s)
$\mathcal{P}_{ndomit}^{rs}$	Set of all the non-dominated solutions to problem [DCSDE-PP^{rs}-B]

$(c_{\min}, \mu_{\max})$	Value of generalized travel cost and dual values corresponding to the shortest paths from origin r to destination s in terms of generalized travel cost
$p_{c_{\min}}$	Shortest paths from origin r to destination s in terms of generalized travel cost
$(c_{\max}, \mu_{\min})$	Value of generalized travel cost and dual values corresponding to the shortest paths from origin r to destination s in terms of dual value
$p_{\mu_{\min}}$	Shortest paths from origin r to destination s in terms of dual value
$\mathcal{G}_{meta}$	Meta-network
$\mathcal{N}_{meta}$	Set of nodes in meta-network
$\mathcal{A}_{meta}$	Set of links in meta-network
p^*	A feasible path with a positive reduced cost in meta-network
$\mathcal{L}(i)$	Set of labels at node i , i.e., $\mathcal{L}(i) = \{(c_k^i, \mu_k^i)\}_{k=1,2,\dots,n_i}$ where c_k^i and μ_k^i denote the generalized travel cost and dual value of the k^{th} label of node i
$\mathcal{X}$	List of nodes to be examined in the multi-label method
UB_c	Upper bound of generalized travel cost in the multi-label method
UB_μ	Upper bound of dual value in the multi-label method
$LpObj$	Optimal objective value of the restricted master problem in the column generation method
M	Number of OD pairs
ε_1	Pre-specified tolerance in column generation method
ε_2	Pre-specified relative optimality tolerance associated with branching in B&P approach
LB	Incumbent best lower bound of model [DCSDE] in B&P approach
$\mathcal{T}$	Binary tree in B&P approach
n	Index for node in B&P search tree
$\mathcal{ST}(n)$	Set of constructed charging station locations at node n in B&P search tree
$\mathcal{RT}(n)$	Set of rejected charging station locations at node n in B&P search tree
$UB(n)$	Upper bound for MP at node n in B&P search tree
UB_{stops}	Maximum stops a BEV is allowed to make for charging

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