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On ride-pooling and traffic congestion

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Abstract:

Ridesourcing platforms, such as Uber, Lyft and Didi, are now launching commercial on-demand ride-pooling programs that enable their affiliated drivers to serve two or more passengers in one ride. It is generally expected that successful designs of ride-pooling programs can reduce the required vehicle fleet size, and achieve various societally beneficial objectives, such as alleviating traffic congestion. The reduction in traffic congestion can in turn save travel time for both ridesourcing passengers and normal private car users. However, it is still unclear to what extent the implementation of ride-pooling affects traffic congestion and riders' travel time. To this end, this paper establishes a model to describe the ridesourcing markets with congestion effects, which are explicitly characterized by a macroscopic fundamental diagram. We compare the time cost (sum of travel time and waiting time) of ridesourcing passengers and normal private car users (background traffic) in the ridesourcing markets without ride-pooling (each vehicle serves one passenger) and with ride-pooling (each vehicle serves one or more passengers). It is found that, a win-win situation can be achieved under some scenarios such that the implementation of on-demand ride-pooling reduces the time cost for both ridesourcing passengers and private car users. Furthermore, we find that the matching window is a key decision variable the platform leverages to affect the market equilibrium. As the matching window increases, passengers are expected to wait for a longer time, but the pool-matching probability (the proportion of passengers who are pool-matched) increases, which further alleviates traffic congestion and in turn reduces passengers' travel time. It is interesting to find that, there is a globally optimal matching window for achieving a minimum time cost for ridesourcing passengers in the normal flow regime.

Keywords: on-demand; ride-pooling; ridesourcing; traffic congestion

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1. Introduction

Enabled by advanced mobile internet-based technologies, recent years have witnessed the rapid growth of ridesourcing services, which use smartphone apps to connect passengers with dedicated drivers who provide services. The companies providing ridesourcing services are often termed as transportation network companies (TNCs); typical examples include Uber, Lyft, Grab and Didi. Ridesourcing services reshaped our mobility and attracted a lot of attentions from researchers, but also brought up some debatable issues. One of the most heated debates is the potential implications of ridesourcing services on traffic congestion. Advocates argue that ridesourcing services are complements to existing modes in transportation system, decrease car ownership and thus reduce traffic congestion. Moreover, TNCs can execute more efficient matching between drivers and passengers, and thus improve vehicle utilization (reducing searching time for customers on streets), which further reduces traffic congestion. Based on datasets from Uber and Urban Mobility Report, Li et al. (2016) find that the entry of Uber service significantly reduced traffic congestion in urban areas of the United States. On the other hand, critics claim that, by providing more convenient and comfortable ride services, TNCs add to vehicle traffic by shifting travelers from space-efficient modes like walking, transit or biking. A recent consulting report (Schaller, 2018) claims that TNCs add 5.7 billion miles of driving in metro areas of Boston, Chicago, Los Angeles, Miami, New York, Philadelphia, San Francisco, Seattle and Washington DC, in each year.

Recently, in order to make more efficient utilization of a limited vehicle fleet for serving more passengers, TNCs are now launching on-demand shared ridesourcing services which enable one vehicle to serve two or more passengers in each ride. These shared ride services, termed as ride-pooling services (Shaheen et al., 2016), are different from traditional ridesharing programs (such as carpooling and dial-a-ride) in the sense that the former are provided by dedicated (or for-hire) drivers while the latter are provided by non-dedicated drivers who have their own trip plans and do not expect to be profitable. Typical examples of ride-pooling services include UberPool, Lyft Line, GrabShare and DiDi Express Pool. It is reported that Lyft aimed to have 50 percent of rides being shared by 2022 (Schaller, 2018). It is generally expected that successful design of ridesharing programs (including both traditional ridesharing programs and ride-pooling programs) can reduce the required vehicle fleet size, and subsequently achieve various societally beneficial objectives, such as alleviating traffic congestion and air pollution. Alexander et al. (2015) use mobile phone records to investigate the influences of ridesharing services on the network-wide traffic congestion. By matching the origin-destination trips from both auto and non-auto travelers, they find that, when the number of ridesharing adopters from drivers is greater than from non-drivers, there will be a reduction in total vehicles. However, Schaller (2018) claims that on-demand ride-pooling services had not offset the traffic-clogging impacts brought by the normal non-ride-pooling services (named non-pooling services for short), in which one vehicle serves one passenger in one ride, of TNCs, such as UberX and Lyft. They show that, the normal non-pooling ridesourcing services put 2.8 new vehicle miles on the road for each mile of auto-driving taken off, and the inclusion of on-demand ride-pooling services leads to marginal reduction on mileage increase — 2.6 new vehicle miles for each mile of auto-driving removed. The possible reasons are: first, the pool-matching probability is too low such that many passengers opting for on-demand ride-pooling end up unpaired with others; second, the shared rides induce additional vehicle miles due to the necessary extra detours; third, the less expensive trip fare of ride-pooling services attract some passengers switching from public transit services. Due to these negative effects, it is still unclear to what extent the on-demand ride-pooling services reduce traffic congestion, and subsequently affect travel time of ridesourcing passengers and normal private car users.

The extent to which ride-pooling reduces congestion relies on a few key factors. One key factor for a successful ride-pooling program is passenger demand for ride-pooling services. Intuitively, with a higher passenger demand, the platform can pool-match more passengers, yielding a higher pool-matching probability. Therefore, fewer vehicles are required to serve a given number of passengers, thereby reducing traffic congestion and average travel time of both ridesourcing passengers and private car users. Clearly, if passenger demand is high, then implementing ride-pooling services is more economical (saving the travel time) and environmentally friendly (reducing traffic congestion) than non-pooling services. Particularly, if passenger demand is sufficiently high, replacing non-pooling services with ride-pooling services may even achieve a win-win situation in which both ridesourcing passengers and private car users are made better off. One interesting question here is to identify the critical passenger demand for achieving a win-win situation.

The other key factor for a successful ride-pooling program to reduce congestion is its pool-matching strategy, for example, the length of matching time window (or matching window). At the end of matching window, passengers opting for ride-pooling services can end up with being either matched or unmatched with another passenger. With no doubt, the matching window greatly affects pool-matching probability and time cost of passengers. If matching window is long, then the pool-matching probability is high, which further reduces traffic congestion and average travel time of passengers. However, a long matching window also directly increases passenger waiting time. Therefore, the length of matching window has both positive and negative effects on the time cost of passengers and thereby influence on passengers' mode choices between non-pooling and ride-pooling services. It is of immerse interest to determine the optimal matching window to minimize the time cost, and identify the range of matching window that leads to a win-win situation under given passenger demand.

This study is intended to investigate the impacts of ride-pooling on traffic congestion and subsequently the travel time of both ridesourcing passengers and normal private car users. To this end, we depict the ridesourcing system in the presence of traffic congestion, in which the relationship between density of vehicles and average vehicular speed is characterized by an aggregated speed-flow relationship in the spirit of macroscopic fundamental diagram (MFD). The major contributions and managerial findings of this paper are listed below.

- Our proposed model can well capture the complex relationships among network speed, vehicle fleet size required to serve a given passenger demand, and the background traffic in a ridesourcing system without ride-pooling. The model is further extended to model the ride-pooling system in a trackable way, where the relationships among pool-matching probability, passenger demand, and matching window are well captured.
- We explore the joint effects of the above mentioned two key variables, i.e. passenger demand and matching window on the time costs of users (ridesourcing passengers and private car users). By comparing the ridesourcing markets with and without ride-pooling service, we further identify the critical passenger demand and range of matching window for achieving a win-win situation.
- Our findings are useful for city managers and ridesourcing platform operators on when (i.e. at what level of passenger demand) and how (i.e. with what matching windows) to promote on-demand ride-pooling services for relieving traffic congestion and reducing travel time of different types of users. For example, it is recommended that the government should encourage ridesourcing platforms to promote ride-pooling services in regions with severe traffic congestion, using appropriate matching windows.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature and highlights our contributions. Taking into account the effects of traffic congestion, Section 3 first establishes an aggregate model to characterize the equilibrium state of a ridesourcing market without ride-pooling, and then extend the model to a ridesourcing market with ride-pooling. Section 4 gives closed-form solutions of the two market equilibriums by assuming a linear traffic flow model, and then examines the impacts of passenger demand and matching window on the time cost of both ridesourcing passengers and private car users. The conditions for existence of a win-win situation are also discussed. Section 5 provides numerical examples to verify our theoretical findings and offer insights on how to achieve a win-win situation by choosing a suitable matching window. Section 6 concludes the paper and offers potential directions for future studies.

2. Literature review

While empirical studies are available recently, to our best knowledge, this is the first theoretical study to look at market equilibrium in the presence of traffic congestion for emerging on-demand ride-pooling services provided by dedicated drivers affiliated with ridesourcing companies. As aforementioned, the ride-pooling service is distinguished from the traditional ridesharing programs in which individual non-dedicated drivers providing shared rides have their own trip plans. Here a brief review is outlined with regard to on the ridesharing programs provided by non-dedicated drivers and the ridesourcing services with and without ride-pooling.

Ridesharing has a long history, dating back to the Second World War, at which a car-sharing club is established by the US government to save fuel. Later, various forms of ridesharing programs, including carpooling, van pooling and dail-a-ride, were developed and studied for years (Ferguson et al., 1997; Yang and Huang, 1999; Huang et al., 2000; Cordeau et al., 2007; Ho et al., 2018). However, these types of ridesharing programs require participants to announce their requests in advance (for example, one day ahead), such that the intermediary agent can prearrange the matching between drivers and riders. Recently, thanks to the breakthroughs on mobile internet technologies, dynamic ridesharing programs become available, in which on-demand ride requests can be accommodated (Furuhata et al., 2013). The dynamic ridesharing programs do not need to collect requests from both drivers and riders in a whole day for prearrangement and but match drivers and riders on short notice or even en-route. Thus, the design of effective matching algorithms for dynamic ridesharing programs is an important and challenging task. Verified by a s imulation study in Atlanta metropolitan area, Agatz et al. (2011) show that the sophisticated optimization approaches can significantly outperform the greedy matching rules in dynamic ridesharing. Stiglic et al. (2015) find that introducing meeting points can further enhance the matching efficiencies by increasing the number of matched driver-rider pairs and reducing the total driving distance. Wang et al. (2017) further propose a stable matching algorithm for dynamic ridesharing that can make a trade-off between the maximization of total system efficiency and the optimization of each individual participant's benefit. Lee and Savelsbergh (2015) examine the benefits and costs of employing a small number of dedicated drivers to serve riders who would otherwise not be matched with a driver in a dynamic ridesharing program. Agatz et al. (2012) give a comprehensive review of the major concerns and challenges in designing sustainable dynamic ridesharing programs.

Recently, some primary efforts are directed towards the designs of cost-sharing strategies or trip fares in dynamic ridesharing programs. Xu et al. (2015) combine the classical Wardrop network equilibrium model with ridesharing passenger demand, and investigated the complex interactions among traffic congestion, passengers' route choice and trip fare, on a network. Di et al. (2018) reformulate the classical network design problem by considering the deployment of high-occupancy toll lanes for ridesharing. Wang et al. (2018) propose a user equilibrium model to capture the complex interactions among riders' and drivers' mode choices, cost sharing strategies between riders and drivers (how much a rider should pay for a driver when they are paired up), and the matching probability. They find that the cost-sharing strategies are very important for attracting sufficient number of drivers and riders to achieve a sustainable dynamic ridesharing program. In some cases, government subsidies are even required for a successful initialization of the ridesharing program. This viewpoint is also raised by Agatz et al., 2011, who claim that sufficient numbers of participants are important to enable dynamic ridesharing matches to be executed on short notice. They investigate whether a dynamic ridesharing program can be successfully initiated and sustained under different scenarios, and identify the critical mass (number of potential participants) for a successful ridesharing program. These studies all focus on dynamic ridesharing programs with individual non-dedicated drivers who have their own trip plans. In this case, the key point lies in the matching and cost-sharing between individual drivers and riders. However, in the on-demand ride-pooling programs provided by dedicated drivers examined in this study, drivers are dedicated service providers who serve passengers anytime and anywhere. Therefore, the pool-matching between two or more passengers sharing one vehicle is important but not considered in these studies.

The on-demand ride-pooling service provided by dedicated drivers is one type of ridesourcing services, available in many TNCs. Since their emergence in 2009, ridesourcing services have achieved huge success on popularity and provoke heated discussions by researchers. Previous studies mainly focus on pricing strategies (including trip fare collected from passengers and wage paid for drivers), policies and regulations (Bai et al., 2018; Taylor, 2018; Cachon et al., 2015; Zha et al., 2016; Xu et al., 2017; Zha et al., 2017; Zha et al., 2018; Ke et al., 2019a; Ke et al., 2020b). Particularly, surge pricing, commonly viewed as an efficient tool to coordinate supply and demand in real-time, has attracted much attention in research. For example, Castillo et al. (2017) find that "wild goose chase" occurs when the demand is excessively high, and the platform is depleted of vacant vehicles and forced to match passengers with distant vacant vehicles. They show that surge pricing on peak hours helps to alleviate the "wild goose chase" by suppressing the passenger demand. Some studies are also made on the matching and dispatching technologies (Xu et al., 2018; Ke et al., 2020a; Yang et al., 2020); real-time supply-demand forecasting (Ke et al., 2017; Ke et al., 2019b; Tong et al., 2017) and behavioral studies (Sun et al., 2011). A comprehensive review of the ridesourcing markets is given by Wang and Yang (2019). The modelling approaches of these studies can find their roots in the literature of conventional taxi market modelling (Yang and Yang, 2011; Yang et al., 2010). It is worth mentioning that Yang et al. (2005) analyze the taxi market equilibrium in the presence of traffic congestion.

A few recent studies are made on the pricing and matching strategies of on-demand ride-pooling services provided by dedicated drivers affiliated to TNCs. Jacob et al. (2019) design the optimal price-service menus, i.e. ride services (non-pooling or ride-pooling) and the corresponding prices, for TNCs. They find that offering both non-pooling and ride-pooling services to passengers is the optimal strategy when the congestion is less severe and passengers' preference type is not skewed. Yan et al. (2019) study the pool-matching scheme (a kind of ride-pooling) applied in Uber, in which passengers with similar origins and destinations can be served with one vehicle in a shared ride. They optimize the platform revenue and social welfare by jointly determining the pricing and matching strategies. However, none of these studies consider the effect of traffic congestion, which affects passengers' time costs. Also, they do not consider the critical passenger demand or density for achieving sensible on-demand ride-pooling programs which are aligned with various desirable objectives, such as reducing passengers' time cost and alleviating traffic congestion.

In addition, there is a sizeable body of literature on the simulations and optimization for ride-pooling services to investigate the potential implications of ride-pooling (or shared taxi) services on the system efficiency. Hosni et al. (2014) develop two heuristic approaches to solve the shared taxi problem, which can help minimize vacant seats in vehicles and reduce the costs of taxi operators. Jung et al. (2016) propose a dynamic shared-taxi dispatch algorithm with hybrid simulated annealing, and show that the sharing for taxicabs can increase productivity under different demand levels. Alonso-Mora et al. (2017) develop a dynamic trip-vehicle assignment algorithm for on-demand ridesharing of high capacity vehicles. They show that 98% of passenger demand can be served by 15% of taxi fleet size of capacity 10 (indicating one taxi can serve at most 10 passengers in each ride) at a cost of a mean waiting time of 2.8min and mean trip delay of 3.5min. Qian et al. (2017) show that over 47% of total taxi trip mileage can be saved if passenger trips with close origins and destinations and similar departure time are appropriately grouped into a single taxi ride. Simonetto et al. (2019) propose a linear assignment algorithm for large-scale ridesharing problems, and find that the real-time ridesharing offers clear benefits with respect to more traditional taxi fleets in terms of level of service. These studies only focus on the benefits of ride-pooling services on reducing the required taxi or ridesourcing vehicle mileages, and discuss the potentials to use a smaller vehicle fleet size to serve passenger trips by ride-pooling. However, they do not incorporate traffic congestion into the simulation system, and investigate how ride-pooling services affect passengers and other users' travel time costs. Recently, Beojone and Geroliminis (2020) establish a simulation integrated with a MFD that can simulate the dynamics of traffic congestion. Based on the simulation, they investigate the effects of expanding ridesourcing vehicle fleet size, passengers' inclination towards shared riders, and strategies to alleviate traffic congestion. Nevertheless, this simulation study cannot identify the critical passenger demand over which the implementation of ride-pooling service is able to achieve a win-win situation beneficial to both ridesourcing passengers and normal private car users. In addition, it does not discuss how to decide an optimal matching window to minimize the total time cost of ridesourcing passengers by balancing the trade-off between travel time and waiting time.

3. Model setting

This section will present an aggregate model to capture the complex interactions among network speed, passenger demand, vehicle fleet size required to serve the demand, background traffic in ridesourcing systems with and without ride-pooling. In the ridesourcing market with ride-pooling, the model further takes into account how passenger demand and matching window affect the pool-matching probability, and subsequently other system endogenous variables. For analytical tractability, certain assumptions are made.

First, our goal is to investigate the benefits brought up by ride-pooling under different levels of passenger demand and different pool-matching strategies. Therefore, passenger demand is not characterized by a demand function. Instead, it is treated as an input parameter, then the two ridesourcing markets with and without ride-pooling are compared under different levels of passenger demand through comparative statics. In particular, we try to identify the critical passenger demand, over which the use of ride-pooling will lead to a win-win situation that benefits all stakeholders. This is in the spirit of determination of a minimum mass of participants to sustain a traditional ridesharing program (Agatz et al., 2011).

Second, in actual operations, there may be excess or shortage of supply. In the former case, some vehicles are idle; in the latter case, some passenger requests are not met. Traffic congestion is governed by the vehicle fleet size (supply), which depends on many factors, such as the wage ridesourcing platforms pay to the drivers, drivers' reservation values, etc. For simplicity, this study focuses on the minimum vehicle fleet size that is required in order to serve a certain level of passenger demand. Therefore, we assume that the rate of supplied vehicles exactly clears passenger demand.

Third, we consider a simple pool-matching strategy, called *dynamic waiting*, which is first presented by Yan et al., (2019), and currently used in a product of Uber - Uber Express Pool (Uber, 2019a). This strategy pool-matches two passengers together if their origins and destinations are very close, namely, the distance between their pick-up locations and the distance between their drop-off locations are both within walkable range or walking radius. Two pool-matched passengers are required to walk to the middle point of their origins to be picked up, and are dropped off at the middle point of their destinations such that they walk equal distances. The walking radius is usually short (250m in Uber), thus the walking time of the passengers is negligible. This is a simplified ride-pooling strategy that is different from the other ride-pooling strategies, such as UberPool (Uber, 2019b), in which a passenger may be matched with a second passenger en-route (rather than up font) and thus experience additional pick-ups and/or drop-offs along his/her trip. It is worth mentioning that ride-pooling strategies like UberPool are very hard to model in an analytical way due to the complex interactions among average detour distance, pool-matching probability, passenger demand and matching strategies. Therefore, in what follows, we simply adopt the dynamic waiting strategy to facilitate our theoretical analyses.

Based on the above mentioned major assumptions, we consider an urban road network that is used by ridesourcing passengers (with a demand q_r and average trip distance l_r) and normal private car users (with a demand q_n and average trip distance l_n). Demand q_r and q_n are defined as the arrival rates of two types of users. Let $\bar{L}$ denote the total length of the road network, and let k denote the average density of vehicles and v the average vehicular speed. An aggregate speed-density) is utilized to characterize the relationship between the speed and density: $v = V(k)$ with $\partial v / \partial k < 0$. For simplicity, two extreme scenarios in a stationary state are first examined and compared: (a) all ridesourcing passenger requests are served without ride-pooling (denoted by "non-pooling market"); (b) all ridesourcing passenger requests are served through ride-pooling (denoted by "ride-pooling market"). It is assumed that the speed-density relationships of the two markets follow the aggregate traffic flow model. We then compare the time cost of both ridesourcing passengers and normal private car users under these two extreme scenarios, and show that, under certain conditions, ride-pooling programs can bring benefits to all users in terms of time cost.

3.1 Non-pooling market

In the non-pooling market or ridesourcing market without ride-pooling, each ridesourcing vehicle is occupied by one passenger (assuming one request by one passenger), thus arrival rate of ridesourcing vehicles is identical to the demand of ridesourcing passengers. At any instant of the stationary state, the number of ridesourcing vehicles and normal vehicles (private cars), denoted as N_r and N_n, equals the product of their arrival rate, q_r and q_n, and the average trip time, l_r/v and l_n/v, respectively. Then, the density of all vehicles in the road network equals $(N_r + N_n)/\bar{L}$. Therefore, the equilibrium in non-pooling market can be characterized by the following system of simultaneous nonlinear equations

$$N_r = q_r \cdot \left(\frac{l_r}{v}\right) \tag{1}$$

$$N_n = q_n \cdot \left(\frac{l_n}{v}\right) \tag{2}$$

$$k = \frac{N_r + N_n}{\bar{L}} \tag{3}$$

$$v = V(k) \tag{4}$$

where Eq. (4) represents the traffic flow model that characterizes the relationship between speed and density. Then speed and density at equilibrium of the non-pooling market, denoted as v^{NP}, k^{NP}, can be spelled out, where 'NP' stands for 'non-pooling'. Furthermore, the average travel time of ridesourcing passengers T_r^{NP} and normal private car users T_n^{NP} are given by

$$T_r^{NP} = \frac{l_r}{v^{NP}} \tag{5}$$

$$T_n^{NP} = \frac{l_n}{v^{NP}} \tag{6}$$

3.2 Ride-pooling market

We now consider the ride-pooling market (the ridesourcing market with all passengers opting for ride-pooling). As aforementioned, we assume that the platform adopts the dynamic waiting strategy to pool-match two passengers with similar origins and destinations up font, and vehicles only need to drive to the middle points to pick-up and drop-off passengers. In the batch matching setting, each passenger is required to wait for a certain time until the end of matching window (denoted as ϕ), at which the platform looks through all requests in the current batch and tries to match as many requests as possible. Then the platform dispatches nearby idle vehicles to serve both the matched and unmatched requests. Let p denote the probability that a passenger is pool-matched (termed as pool-matching probability), which is dominated by two major factors: passenger demand for ride-pooling q_r, and matching window ϕ. Intuitively, for a given q_r, the larger the ϕ, the larger the p (more passengers can be pool-matched with longer matching window). Meanwhile, for a given ϕ, the larger the q_r, the larger the p (higher density of passengers in the matching pool increases the probability that a passenger is pool-matched). Moreover, as ϕ or q_r approaches zero, the matching pool is almost empty (with almost no passenger requests to be pool-matched), then the pool-matching probability p becomes zero. Conversely, as ϕ or q_r approaches infinity, the requests that can be pool-matched in the matching pool are sufficiently dense, and the pool-matching probability p becomes almost 1. Yan et al., (2019) proposed a specific function to describe the pool-matching probability in the batch matching setting, which depends on ϕ or q_r, as below:

$$p(q_r, \phi) = 1 - \exp\left(-\gamma q_r \phi\right) \tag{7}$$

where γ is a parameter that may depend on the topological property of the city and the walking radius. This expression is said to well fit the relationship between the pool-matching probability and the two major factors, i.e. passenger demand and matching window in Uber's historical data. More recently, through extensive experiments with the ridesourcing data in one US city (Manhattan, New York) and two Chinese mainland cities (Hakou and Chengdu), Ke et al. (2020c) find that pool-matching probability p exhibits a negative-exponential increasing saturation curve with respect to the number of passengers in the matching pool. Formally, $p = 1 - \exp(-\gamma N)$, where N is the number of passengers in the matching pool. It is also shown that this formula fits the data quite well with reasonably high goodness-of-fit under various walking radii and cities of different sizes. It is evident that this formula is essentially the same as Eq. (7), since $q_r\phi$ can be viewed as the number of accumulated passengers at the end of each matching window in the batch matching setting. Based on the formula in Eq. (7), we can obtain the following properties:

Lemma 1. *The pool-matching probability $p(q_r, \phi)$ satisfies:*

1. *For all $q_r \geq 0$, $p(q_r, \phi)$ increases with ϕ and $p(q_r, 0) = 0$ and $\lim_{\phi \to \infty} p(q_r, \phi) = 1$;*
2. *For all $\phi \geq 0$, $p(q_r, \phi)$ increases with q_r and $p(0, \phi) = 0$ and $\lim_{q_r \to \infty} p(q_r, \phi) = 1$.*

Now we look at the equilibrium state in the ride-pooling market. At any instant of the stationary state, the number of normal vehicles running on the road can still be represented by Eq. (2), and the traffic flow model in Eqs. (3) and (4) also hold for the ride-pooling market. However, unlike the non-pooling market, there are two types of ridesourcing vehicles at each instant in the ride-pooling market: 1) vehicles serving a shared ride (with two pool-matched passengers); 2) vehicles serving an unshared ride (with only one passenger). Hence, given the pool-matching probability p, the expected number of ridesourcing vehicles consumed by a single passenger is:

$$f(q_r, \phi) = \frac{1}{2} p(q_r, \phi) + [1 - p(q_r, \phi)] = 1 - \frac{1}{2} p(q_r, \phi) \tag{8}$$

where the first term indicates that each passenger request occupies half of a ridesourcing vehicle in a shared ride (passengers are pool-matched pair by pair), while the second term shows that each passenger request occupies one ridesourcing vehicle in an unshared ride (passengers are not pool-matched). Thus, the number of ridesourcing vehicles at any instant of the stationary state is

$$N_r = q_r \cdot f(q_r, \phi) \cdot \left(\frac{l_r}{v}\right) \tag{9}$$

where $q_r \cdot f(q_r, \phi)$ is the number of ridesourcing vehicles per unit time for serving all passenger requests. In view of the fact that $0 \leq p(q_r, \phi) \leq 1$, we have $1/2 \leq f(q_r, \phi) \leq 1, \forall q_r, \phi \geq 0$. From Lemma 1, we can easily find that $f(q_r, \phi)$ is a decreasing function of both ϕ and q_r and satisfies the following boundary conditions:$f(q_r, 0) = 1$, $f(0, \phi) = 1$, $\lim_{\phi\to\infty} f(q_r, \phi) = 1/2$, $\lim_{q_r\to\infty} p(q_r, \phi) = 1/2$.

Moreover, on the basis of Eq. (7), the following property can be obtained:

Lemma 2. *For all $\phi \geq 0$, the number of ridesourcing vehicles required to satisfy all requests per unit time $q_r \cdot f(q_r, \phi)$ is increasing with q_r.*

Proof. Taking the partial derivative of $q_r \cdot f(q_r, \phi)$ with respect to q_r yields

$$\frac{\partial\big(q_r \cdot f(q_r, \phi)\big)}{\partial q_r} = \frac{1}{2}[1 + \exp(-\gamma q_r \phi) - \gamma q_r \phi \exp(-\gamma q_r \phi)]$$

Let $f(x) = 1 + \exp(-x) - x\exp(-x)$, where $x > 0$. Clearly, $f'(x) = \exp(-x)\,(x-2)$ which implies that $f'(x) > 0$ if $x > 2$ and $f'(x) < 0$ if $x < 2$. Therefore, we shall have $f(x) \geq f(2) > 0$, and $f(x) \leq \max\big(f(0), f(\infty)\big) = 2$ if $x > 0$. In view of the fact that $\gamma q_r \phi > 0$, we can prove that

$$\frac{\partial\big(q_r \cdot f(q_r, \phi)\big)}{\partial q_r} > 0$$

This completes the proof. ■

This lemma indicates that, as passenger demand increases, additional ridesourcing vehicles required for new riders cannot be offset by the reduction in vehicle fleet size due to the increased pool-matching probability, and therefore, more vehicles are required to serve passenger demand on the whole.

Then the resulting speed and density at equilibrium in the ride-pooling market, denoted as v^{RP} and k^{RP}, where 'RP' stands for 'ride-pooling', can be solved by a system of simultaneous nonlinear equations consisting of Eqs. (2)-(4) and (8)-(9). Without accounting for monetary cost, the average cost for private car users is given as the average travel time (average travel distance divided by the vehicular speed); the average time cost for ridesourcing passengers is given as the sum of average travel time and expected waiting time (a passenger waits to be pool-matched). As already mentioned, in the batch matching, each passenger waits until the end of each batch time window and then ends up with being either pool-matched or not. The expected waiting time of a passenger regardless of whether being pool-matched or not can be written as a function $H(\phi)$ of the matching window ϕ. Clearly, $H(\phi)$ should be strictly increasing with ϕ. Particularly, in the batch matching with uniform arrival of passengers over time, $H(\phi)$ is on average half of the matching window, i.e. $\phi/2$.The average time cost of ridesourcing passengers T_r^{RP} and normal private car users T_n^{RP} are then given by

$$T_r^{RP} = \frac{l_r}{v^{RP}} + H(\phi) \tag{10}$$

$$T_n^{RP} = \frac{l_n}{v^{RP}} \tag{11}$$

4. Equilibrium solution and properties

In this section, we examine the equilibrium solutions of the two markets and compare the average time costs of both ridesourcing passengers and normal private car users in the two markets under different levels of passenger demand and matching windows. As mentioned above, the equilibrium state of the non-pooling market is given by a system of simultaneous Eqs. (1)-(4), which yields

$$v = V\left(\frac{q_r l_r + q_n l_n}{\bar{L}v}\right) \tag{12}$$

which is an implicit function of speed v. Similarly, the system of simultaneous Eqs. (2)-(4) and (8)-(9) that depict the equilibrium state of the ride-pooling market give rise to

$$v = V\left(\frac{q_r f(q_r, \phi) l_r + q_n l_n}{\bar{L}v}\right) \tag{13}$$

which is also an implicit function of speed v. Clearly, without a specific traffic flow model that depicts the relationship between v and k, namely, the function $V(\cdot)$, neither Eq. (12) nor Eq. (13) can yield a close-form solution. For analytical tractability, we now consider a linear traffic flow model: $v = A - Bk$, where A and B are two parameters in both markets.

4.1 Equilibrium speed and density

With the linear traffic flow model, the following two closed-form equilibrium solutions of the vehicular speed v to the system of simultaneous Eqs. (1)-(4) in the non-pooling market are obtained

$$v^{NP} = \begin{cases} \dfrac{A}{2} + \sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}(q_n l_n + q_r l_r)} & \text{(normal flow regime)} \\ \dfrac{A}{2} - \sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}(q_n l_n + q_r l_r)} & \text{(hyper} - \text{congested flow regime)} \end{cases} \tag{14}$$

where the first equation gives the equilibrium speed in the normal flow regime ($0 < \mathrm{A}/2 \le v \le A$), while the second equation gives the equilibrium speed in the hyper-congested flow regime ($0 \le v \le \mathrm{A}/2$). The resulting equilibrium density is given by $k^{NP} = (A - v^{NP})/B$ for both regimes. Notice that the term $(q_n l_n + q_r l_r)/\bar{L}$ can be viewed as a proxy rate of traffic flow.

It should be noted that the normal flow regime under our consideration does imply that speed decreases with flow rate due to congestion effect; it is not a free-flow condition where speed is a constant independent of flow rate. This implies that, as the passenger demand for ridesourcing service increases, the proxy rate of traffic flow increases, which will decrease the network speed by increasing the level of traffic congestion. In contrast, the hyper-congested flow regime is generally regarded as a system failure where the speed is extremely low, the density is extremely high, while the speed exhibits an increasing trend with the proxy rate of traffic flow.

Our consideration is consistent with the overwhelming assumption adopted in transportation network analysis and modeling (e.g., network design and road pricing) that travel time on a link is assumed to be a monotonically increasing function of traffic flow. This assumption becomes the norm in traffic equilibrium analysis. First, it allows for analysis of traffic equilibrium properties such as existence and uniqueness of equilibria. Second, when designing a road network or facilities, transportation planners focus on the normal flow regime but implicitly assume that the system failure (i.e. hyper-congested flow regime) should be avoided in a medium to long term planning. In the same spirit, our goal is to investigate the effects of ride-pooling and its associated operating strategies (the matching window) on the level of congestion and system efficiency, and try to obtain managerial insights in the normal flow regime (in Section 4.4); discussion of its properties and operations in the hyper-congested flow regime is presented in a separate session (in Section 4.5).

Similarly, from Eqs. (2)-(4) and (8)-(9), we can obtain the following two closed-form equilibrium solutions of the vehicular speed in the ride-pooling market

$$v^{RP} = \begin{cases} \dfrac{A}{2} + \sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}[q_n l_n + q_r \cdot f(q_r, \phi) l_r]} & \text{(normal flow regime)} \\ \dfrac{A}{2} - \sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}[q_n l_n + q_r \cdot f(q_r, \phi) l_r]} & \text{(hyper} - \text{congested flow regime)} \end{cases} \tag{15}$$

Again the equilibrium densities in both of the two regimes are given by $k^{RP} = (A - v^{RP})/B$ and the term $q_n l_n + q_r \cdot f(q_r, \phi) l_r/\bar{L}$ can be viewed as a proxy rate of traffic flow in a fundamental diagram in the ride-pooling market.

4.2 Maximum service rate of passenger demand

In traffic flow theory, there is a maximum rate of flow due to limited road capacity, and the traffic flow on the road never exceeds the maximum rate of flow. In the same vein, for a given and fixed background traffic (normal private car) demand, there is a maximum level of ridesourcing passenger demand (or arrival rate) that can be served in the non-pooling and ride-pooling markets. We denote this maximum serviceable level of ridesourcing passenger demand as maximum passenger service rate. Combining Eq. (12) and the linear traffic flow model, we have

$$q_r = \frac{(A-v)v\bar{L}}{Bl_r} - \frac{q_n l_n}{l_r} \le q_{max}^{NP} = \frac{1}{l_r}\left(\frac{A^2\bar{L}}{4B} - q_n l_n\right) \tag{16}$$

where $q_{\max}^{NP}$ is the maximum passenger service rate in the non-pooling market, which is achieved when the derivative of q_r with respect to v is zero, or equivalently, $v^{NS} = A/2$.

To obtain the maximum passenger service rate in the ride-pooling market, we combine Eq. (13) and the linear traffic flow model, which yields

$$q_r f(q_r, \phi) = \frac{(A-v)v\bar{L}}{Bl_r} - \frac{q_n l_n}{l_r} \le \frac{1}{l_r}\left(\frac{A^2\bar{L}}{4B} - q_n l_n\right) \tag{17}$$

which indicates that, the maximum passenger service rate in the ride-pooling market, denoted by $q_{\max}^{RP}$, can be obtained by solving the following implicit equation

$$q_r f(q_r, \phi) = \frac{1}{l_r}\left(\frac{A^2\bar{L}}{4B} - q_n l_n\right) \tag{18}$$

Define

$$\mathrm{S}(q_r, \phi) = q_r f(q_r, \phi) \tag{19}$$

then $q_{\max}^{RP}$ can be obtained as the solution of the following implicit equation

$$\mathrm{S}(q_r, \phi) = \frac{1}{l_r}\left(\frac{A^2\bar{L}}{4B} - q_n l_n\right) \tag{20}$$

From Lemma 2, we know $\mathrm{S}(q_r, \phi)$ is strictly increasing with q_r, and in view of the fact that $1/2 \le f(q_r, \phi) \le 1$, we have $\mathrm{S}(0, \phi)$=0 and $\lim_{q_r \to +\infty} \mathrm{S}(q_r, \phi) = +\infty$. This indicates that $\mathrm{S}(q_r, \phi)$ monotonically increases from zero to infinity as q_r increases from zero to infinity, and thus, the nonlinear Eq. (20) has one and only one positive solution in terms of q_r. Therefore, we have the following finding:

Lemma 3. *There is at most one maximum passenger service rate in the ride-pooling market, q_{max}^{RP}, which can be uniquely determined by Eq. (18) or q_{max}^{RP} can be written as*

$$q_{\max}^{RP} = S^{-1}\left(\frac{1}{l_r}\left(\frac{A^2\bar{L}}{4B} - q_n l_n\right), \phi\right) \tag{21}$$

Of interest here is a comparison in maximum passenger service rates between non-pooling and ride-pooling markets, i.e., a comparison between $q_{\max}^{NP}$ and $q_{\max}^{RP}$. Combining Eq. (16) and (18), we have $q_{\max}^{RP} f(q_{\max}^{RP}, \phi) = q_{\max}^{NP}$. Notice that $f(q_r, \phi) \leq 1$, thus we can prove that $q_{\max}^{RP} \geq q_{\max}^{NP}$. Furthermore, it is also worth exploring the impacts of the external factors (such as the matching window ϕ and the arrival rate of private car users q_n) on the maximum passenger service rate $q_{\max}^{RP}$.

First, since $f(q_r, \phi)$ is strictly decreasing with ϕ, then $S(q_r, \phi)$ is also strictly decreasing in ϕ. For any two matching windows ϕ_1 and ϕ_2 ($\phi_1 > \phi_2$), suppose their resulting maximum passenger service rates are $q_{\max}^{RP1}$ and $q_{\max}^{RP2}$, respectively, then we have $S(q_{\max}^{RP1}, \phi_1) = S(q_{\max}^{RP2}, \phi_2) = q_{\max}^{NP}$. Yet, since $\phi_1 > \phi_2$ and $S(q_r, \phi)$ is decreasing with ϕ, we have $S(q_{\max}^{RP1}, \phi_1) < S(q_{\max}^{RP1}, \phi_2)$, and thus $S(q_{\max}^{RP2}, \phi_2) < S(q_{\max}^{RP1}, \phi_2)$. Notice that $S(q_r, \phi)$ is increasing with q_r (Lemma 2), then we can easily see that $q_{\max}^{RP2} < q_{\max}^{RP1}$. It shows that the maximum passenger service rate the ride-pooling market monotonically increases with the matching window ϕ. This is simply due to the fact using a longer matching window can increase the pool-matching probability and thus serve more passengers through ride-pooling.

Second, since function $S(\cdot, \phi)$ is a strictly increasing function for a given ϕ, its inverse function $S^{-1}(\cdot, \phi)$ is also a strictly increasing function. This indicates that $q_{\max}^{RP}$ is strictly increasing with the term

$\frac{1}{l_r}\left(\frac{A^2\bar{L}}{4B} - q_n l_n\right)$, and therefore, it is not hard to find that $q_{\max}^{RP}$ strictly decreases with the arrival rate of private car users q_n. These findings are summarized in the following proposition.

Proposition 1. *The maximum passenger service rate in the ride-pooling market $q_{\max}^{RP}$ satisfies the following:*

1) for any matching window $\phi \geq 0$, $q_{\max}^{RP}$ is larger than or equal to the maximum passenger service rate in the non-pooling market q_{max}^{NP};

2) q_{max}^{RP} monotonically increases with the matching window ϕ and decreases with the demand of private car users q_n.

4.3 Marginal effects of passenger demand

Next, we look at how the equilibrium vehicular speed varies with the ridesourcing passenger demand in the two markets considered. Taking the partial derivatives of v^{NP} and v^{RP} with respect to q_r leads to

$$\frac{\partial v^{NP}}{\partial q_r} = \begin{cases} \dfrac{-Bl_r}{2\bar{L}\sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}(q_n l_n + q_r l_r)}} < 0 & \text{(normal flow regime)} \\ \dfrac{Bl_r}{2\bar{L}\sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}(q_n l_n + q_r l_r)}} > 0 & \text{(hyper} - \text{congested flow regime)} \end{cases} \tag{22}$$

$$\frac{\partial v^{RP}}{\partial q_r} = \begin{cases} \dfrac{-Bl_r}{2\bar{L}\sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}[q_n l_n + q_r \cdot f(q_r,\phi) l_r]}} \dfrac{\partial q_r f(q_r,\phi)}{\partial q_r} < 0 & \text{(normal flow regime)} \\ \dfrac{Bl_r}{2\bar{L}\sqrt{\dfrac{A^2}{4} - \dfrac{B}{\bar{L}}[q_n l_n + q_r \cdot f(q_r,\phi) l_r]}} \dfrac{\partial q_r f(q_r,\phi)}{\partial q_r} > 0 & \text{(hyper} - \text{congested flow regime)} \end{cases} \tag{23}$$

where $q_r f(q_r,\phi)$ strictly increases with q_r by Lemma 2. Clearly, the equilibrium speed decreases with q_r in the normal flow regime, and increases with q_r in the hyper-congested flow regime, in both the non-pooling and ride-pooling markets. Now we compare $\left|\frac{\partial v^{RP}}{\partial q_r}\right|$ and $\left|\frac{\partial v^{NP}}{\partial q_r}\right|$, the absolute marginal change in equilibrium speed by one unit increase in passenger demand q_r within the two flow regimes in the two markets. Note that, from definition, we have $f(q_r, \phi) < 1$, thus $q_r f(q_r, \phi) l_r < q_r l_r$, and thus the denominator of the second equation of Eq. (23) (in absolute value) is greater than that of Eq. (22). Meanwhile, we have

$$\frac{\partial q_r f(q_r, \phi)}{\partial q_r} = f(q_r, \phi) + q_r \frac{\partial f(q_r, \phi)}{\partial q_r} < 1 \tag{24}$$

where $f(q_r, \phi) \leq 1$ and $\partial f(q_r, \phi)/\partial q_r < 0$ by definitions. It indicates that the numerator of the second equation of Eq. (23) (in absolute value) is less than that of Eq. (22). In summary of the comparisons, it is easily found that $\left|\frac{\partial v^{RP}}{\partial q_r}\right| < \left|\frac{\partial v^{NP}}{\partial q_r}\right|$, leading to the following proposition:

Proposition 2. *In the normal flow regime (hyper-congested flow regime), the marginal decrease (increase) in equilibrium vehicular speed caused by a unit increase in ridesourcing passenger demand in the ride-pooling market is less than that in the non-pooling market.*

This proposition tells us that a unit increase of passenger demand brings less effect on the equilibrium vehicular speed in a ride-pooling market than in a non-pooling market. Particularly, in the normal flow regime, this is because ride-pooling programs can accommodate a given number of passengers with fewer vehicles, and thus make less congestion.

4.4 Properties in the normal flow regime

In this section, we discuss properties of the model in the normal flow regime, which will help us understand the impacts of the critical factors (passenger demand and matching window) on the time costs of both ridesourcing passengers and normal private car users under normal traffic situations in the medium or longer term.

The matching window is one critical decision variable that the platform can leverage to affect the equilibrium state of the ride-pooling market. Intuitively, as the platform utilizes a larger matching window, the pool-matching probability increases and thus the road network becomes less congested (pointing to a higher speed), which further reduces the travel time of both ridesourcing passengers and private car users. On the other hand, a long matching window also increases the waiting time of ridesourcing passengers. It is thus of interest to understand how the length of matching window affects the time cost of ridesourcing passengers and private car users, and to determine a matching window to minimize the time cost.

We first look at the marginal effects of matching window on the equilibrium speed v^{RP} (in the normal flow regime of the ride-pooling market). Taking the first- and second-order partial derivatives of v^{RP} in matching window ϕ leads to

$$\frac{\partial v^{RP}}{\partial \phi}=\frac{Bq_r l_r}{4\bar{L}\sqrt{\frac{A^2}{4}-\frac{B}{\bar{L}}[q_n l_n+q_r f(q_r,\phi)l_r]}}\frac{\partial p(q_r,\phi)}{\partial \phi} \tag{25}$$

$$\frac{\partial^2 v^{RP}}{\partial \phi^2}=\frac{Bq_r l_r}{4\bar{L}}\left[\frac{A^2}{4}-\frac{B}{\bar{L}}[q_n l_n+q_r f(q_r,\phi)l_r]\right]^{-\frac{1}{2}}\left\{\frac{\partial^2 p(q_r,\phi)}{\partial \phi^2}-\frac{Bq_r l_r\left[\frac{\partial p(q_r,\phi)}{\partial \phi}\right]^2}{4\bar{L}\left[\frac{A^2}{4}-\frac{B}{\bar{L}}[q_n l_n+q_r f(q_r,\phi)l_r]\right]}\right\} \tag{26}$$

where $\partial p(q_r,\phi)/\partial \phi>0$, and thus we have $\partial v^{RP}/\partial \phi>0$. This indicates that the equilibrium speed strictly increases with the matching window. This is because, prolonging the matching window increases the pool-matching probability, and thus reduces the number of vehicles, which further alleviates traffic congestion and increases the equilibrium speed. In addition, from Eq. (7), we can obtain

Lemma 4. *The pool-matching probability $p(q_r, \phi)$ is concave with respect to the matching window ϕ.*

Proof. Taking the first- and second-order partial derivatives of $p(q_r, \phi)$ with respect to ϕ gives rise to

$$\frac{\partial p(q_r, \phi)}{\partial \phi} = \gamma q_r \exp(-\gamma q_r \phi) > 0 \tag{27}$$

$$\frac{\partial^2 p(q_r, \phi)}{\partial \phi^2} = -(\gamma q_r)^2 \exp(-\gamma q_r \phi) < 0 \tag{28}$$

This completes the proof. ■

This lemma is intuitive: as ϕ increases from 0 to infinity, the pool-matching probability $p(q_r, \phi)$ increases from 0 to 1. As ϕ is small, $p(q_r, \phi)$ is also low (nearly zero), and a unit increase of ϕ can lead to more sharp increase in $p(q_r, \phi)$; as ϕ is sufficiently large, $p(q_r, \phi)$ approaches 1 (nearly all passenger requests can be pool-matched), and thus a unit increase of ϕ brings very limited change in $p(q_r, \phi)$. This means the probability function $p(q_r, \phi)$ exhibits an increasing saturation curve property, i.e. concave with respect to ϕ.

Combining Lemma 4 and Eq. (26), it can be easily proved that $\partial^2 v^{RP}/\partial\phi^2 < 0$, which implies that the equilibrium speed is concave with respect to the matching window. Then, taking the partial derivative of the average travel time of private car users T_n^{RP} in matching window ϕ yields

$$\frac{\partial T_n^{RP}}{\partial \phi} = -\frac{l_n}{(v^{RP})^2}\frac{\partial v^{RP}}{\partial \phi} < 0 \tag{29}$$

which indicates that T_n^{RP} always decreases with ϕ; in other words, prolonging the matching window is always beneficial to the private car users.

Next, we study the marginal effects of matching window on the average time cost of each ridesourcing passenger T_r^{RP}. The partial derivative of T_r^{RP} in ϕ is given below.

$$\frac{\partial T_r^{RP}}{\partial \phi} = -\frac{l_r}{(v^{RP})^2}\frac{\partial v^{RP}}{\partial \phi} + \frac{\partial H(\phi)}{\partial \phi} \tag{30}$$

The first term of the right-hand-side (which is always positive) shows that, with everything else being equal, an increase in matching window increases the network equilibrium speed and thus reduces the average travel time of ridesourcing passengers. The second term represents the direct effect of increasing matching window on passengers' waiting time. Note that the whole value of $\partial T_r^{RP}/\partial \phi$ can be either positive or negative, depending on the relative magnitude of these two opposite effects. To further examine the existence of the optimal matching window, we now take the second-order partial derivative of T_r^{RP} with respect to ϕ as follows,

$$\frac{\partial^2 T_r^{RP}}{\partial \phi^2} = -\frac{l_r}{(v^{RP})^2}\frac{\partial^2 v^{RP}}{\partial \phi^2} + \frac{\partial^2 H(\phi)}{\partial \phi^2} \tag{31}$$

As aforementioned, by assuming a uniform arrival of ridesourcing passengers over time in the batching matching, the average waiting time of ridesourcing passengers can be given by $H(\phi) = \phi/2$. In this case, $\partial H(\phi)/\partial \phi = 1/2$ and $\partial^2 H(\phi)/\partial \phi^2 = 0$. Recall that we already prove $\partial^2 v^{RP}/\partial \phi^2 < 0$, then we have $\partial^2 T_r^{RP}/\partial \phi^2 > 0$, which indicates that T_r^{RP} is convex with respect to ϕ. Then one can easily find that there exists one and only one optimal matching window ϕ^* that lead to the minimum of the average time cost of passengers, which first decreases and then increases with matching window. The optimal matching window ϕ^* is obtained by setting $\partial T_r^{RP}/\partial\phi = 0$, giving rise to

$$\left.\frac{\partial v^{RP}}{\partial \phi}\right|_{\phi=\phi^*} = \frac{(v^{RP})^2}{2l_r} \tag{32}$$

where v^{RP} is also a function of ϕ. The condition in Eq. (32) shows that the marginal impact of matching window on the equilibrium speed is proportional to the square of the current speed at optimal matching window. These findings are summarized below.

Proposition 3. *In the normal flow regime, there exists a unique optimal matching window that gives the globally minimum average time cost of ridesourcing passengers, which is given by the implicit Eq. (32).*

This proposition tells us that the platform can always set an optimal matching window for minimizing the average time cost of ridesourcing passengers, for any given arrival rates of ridesourcing passengers q_r and normal private car users q_n, in the normal flow regime.

Now we look into the impacts of implementation of ride-pooling on the average time cost of ridesourcing passengers and normal private car users. We say that a win-win situation emerges if the time cost of both ridesourcing passengers and normal private car users in the ride-pooling market is less than that in the non-pooling market. Of particular interest here is the existence of win-win situation and under what situations a win-win situation can be found.

First, the difference in travel time of private car users between the two markets of non-pooling and ride-pooling is given by

$$\Delta T_n = T_n^{NP} - T_n^{RP} = \frac{l_n}{v^{NP} v^{RP}} (v^{RP} - v^{NP}) \tag{33}$$

Comparing Eqs. (14) and (15), and in view of $f(q_r, \phi) \leq 1$, one can easily find that $v^{RP} > v^{NP}$ in the normal flow regime, implying that $\Delta T_n > 0$ or $T_n^{NP} > T_n^{RP}$. Namely, the implementation of ride-pooling program can always reduce the travel time of normal private car users. This is because the private car users benefit from the reduction of traffic congestion arising from ride-pooling.

Similarly, the difference in average time cost of ridesourcing passengers between the two markets is given by

$$\Delta T_r = T_r^{NP} - T_r^{RP} = \frac{l_r}{v^{NP} v^{RP}} (v^{RP} - v^{NP}) - H(\phi) \tag{34}$$

where the first term and the second-term of the right-hand-side are positive and negative, respectively. The positive term describes that ride-pooling alleviates traffic congestion and further reduces the travel time of ridesourcing passengers, and the negative term indicates that the extra waiting time arising from matching two rides. Clearly, ΔT_r can be either positive or negative, depending on the relative magnitude of these two terms, which are jointly determined by the matching window ϕ and passenger demand q_r.

To look at the above time cost difference of ridesourcing passenger further, we first note that, for any given q_r, the maximum ΔT_r can be obtained when $\phi = \phi^*$, since T_r^{NP} is independent of ϕ. Second, taking the partial derivative of ΔT_r with respect to q_r yields

$$\frac{\partial \Delta T_r}{\partial q_r} = \frac{l_r}{(v^{NP})^2 (v^{RP})^2} \left[(v^{RP})^2 \left| \frac{\partial v^{NP}}{\partial q_r} \right| - (v^{NP})^2 \left| \frac{\partial v^{RP}}{\partial q_r} \right| \right] \tag{35}$$

where $|\partial v^{RP} / \partial q_r| = -\partial v^{RP} / \partial q_r$ and $|\partial v^{NP} / \partial q_r| = -\partial v^{NP} / \partial q_r$ in the normal flow regime. As shown in Proposition 2, $|\partial v^{RP} / \partial q_r| < |\partial v^{NP} / \partial q_r|$ and $v^{RP} > v^{NP}$ in the normal flow regime. Thus, $\partial \Delta T_r / \partial q_r > 0$, which shows that ΔT_r monotonically increases with q_r, for any given ϕ. Note that, to compare the non-pooling and the ride-pooling markets, q_r should satisfy $q_r < \min(q_{\max}^{NP}, q_{\max}^{RP}) = q_{\max}^{NP}$. These findings are summarized in the following proposition.

Proposition 4. *In the normal flow regime, the difference, ΔT_r, in the average time cost of ridesourcing passengers between the non-pooling market and the ride-pooling market satisfies:*

1. *For any given passenger demand q_r, ΔT_r is concave in ϕ, and a globally maximum value of ΔT_r is always given at $\phi = \phi^*$;*
2. *For any given matching window ϕ, ΔT_r monotonically increases as q_r increases from zero to $q_{\max}^{NP}$.*

It tells us that, for any given passenger demand, the platform can always determine a globally optimal matching window to maximize ΔT_r. In addition, for any given matching window, ΔT_r increases with passenger demand. This is because, with increase in passenger demand, the ride-pooling market has higher pool-matching probability and thus requires smaller fleet size of vehicles for all ridesourcing passengers. As a result, the advantages (alleviating traffic congestion and further reducing travel time) of ride-pooling over non-pooling becomes more obvious.

The win-win situation can be achieved if $\Delta T_n > 0$ and $\Delta T_r > 0$. Given a matching window ϕ, let $\bar{q}_r$ denote the critical passenger demand for $\Delta T_r = 0$. In view of that ΔT_r strictly increases with q_r, the win-win situation is achieved when $q_r > \bar{q}_r$, since $\Delta T_n > 0$ is always true. However, the win-win situation can never appear or the solution of $\Delta T_r = 0$ does not exist for any feasible q_r ($q_r < q_{\max}^{NP}$) when $\bar{q}_r > q_{\max}^{NP}$ (out of the feasible range of q_r), namely, $\Delta T_r < 0$ for any feasible value of q_r. Because ΔT_r increases with q_r, from Eq. (34), occurrence of win-win situation or $\Delta T_r > 0$ requires that

$$\frac{l_r}{\frac{A}{2}+\sqrt{\frac{A^2}{4}-\frac{B}{\bar{L}}[q_n l_n + q_{\max}^{NP} l_r]}} - \frac{l_r}{\frac{A}{2}+\sqrt{\frac{A^2}{4}-\frac{B}{\bar{L}}[q_n l_n + q_{max}^{NP} f(q_r,\phi) l_r]}} - H(\phi) > 0 \tag{36}$$

Substituting Eq. (16) into Eq. (36) gives rise to:

$$p(q_{\max}^{NP},\phi) > \frac{A^4 H^2(\phi)}{2[AH(\phi)-2l_r]^2\left(\frac{A^2}{4}-\frac{B}{\bar{L}}q_n l_n\right)} \tag{37}$$

in which $p(q_{\max}^{NP},\phi)$ represents the resulting pool-matching probability associated with the maximum passenger demand $q_{\max}^{NP}$ and the matching window ϕ, and the right-hand-side is a term dependent on ϕ but independent of q_r. This indicates that there exists a win-win situation if $p(q_{\max}^{NP},\phi)$ is larger than a certain critical value. To sum up, we have

Proposition 5. *In the normal flow regime, for a given matching window ϕ, if the inequality in Eq.* (37) *holds, a win-win situation exists when the passenger demand q_r is greater than a critical value $\bar{q}_r$ (or equivalently, the passenger density is larger than a critical density).*

4.5 Properties in the hyper-congested flow regime

In this section, we discuss the properties of the model in the hyper-congested flow regime. As mentioned above, the hyper-congested flow regime is generally not incorporated in standard transportation systems analysis and modeling since it is in essence a system failure that should be avoided in traffic planning and operations. Nonetheless, for completeness, our analysis is extended to discuss how passenger demand affects the system performances and how the platform chooses appropriate operating strategies in a hyper-congested flow regime.

To this end, we take the first-order partial derivative of speed v^{RP} of the hyper-congested flow regime with respect to matching window ϕ, giving rise to,

$$\frac{\partial v^{RP}}{\partial \phi} = \frac{-Bq_r l_r}{4\bar{L}\sqrt{\frac{A^2}{4} - \frac{B}{\bar{L}}[q_n l_n + q_r f(q_r, \phi) l_r]}} \frac{\partial p(q_r, \phi)}{\partial \phi} < 0 \tag{38}$$

which indicates that network speed in the hyper-congested flow regime decreases with the matching window. This is opposite to the trend of speed in the normal flow regime, due to the fact that a long matching window increases the pool-matching probability and reduces the rate of required vehicles, which however, reduces the equilibrium speed in the hyper-congested flow regime. Then, taking the partial derivative of the average time cost of private car users T_n^{RP} and ridesourcing passenger T_r^{RP} with respect to matching window ϕ yields

$$\frac{\partial T_n^{RP}}{\partial \phi} = -\frac{l_n}{(v^{RP})^2} \frac{\partial v^{RP}}{\partial \phi} > 0 \tag{39}$$

$$\frac{\partial T_r^{RP}}{\partial \phi} = -\frac{l_r}{(v^{RP})^2} \frac{\partial v^{RP}}{\partial \phi} + \frac{\partial H(\phi)}{\partial \phi} > 0 \tag{40}$$

which indicate that both T_n^{RP} and T_r^{RP} increase with ϕ; in other words, prolonging the matching window is always harmful to both private car users and ridesourcing passengers in the hyper-congested flow regime. Therefore, if the traffic situation collapses into a hyper-congested flow regime, the optimal strategy of the platform is to set the matching window to be zero, which is beneficial to both private car users and ridesourcing passengers. The reason is that, in the hyper-congested flow regime, the speed increases with rate of flows, which is governed by the arrival rate of vehicles required for serving a given ridesourcing passenger demand. In this case, a zero matching window entertains almost a zero ride-pooling probability and thus leads to a maximum arrival rate of ridesourcing vehicles. As a result, the maximum speed and minimum time cost for both private car users and ridesourcing passengers are obtained. This in turn implies that implementation of ride-pooling does not create a win-win situation when the system is trapped in the hyper-congested flow regime.

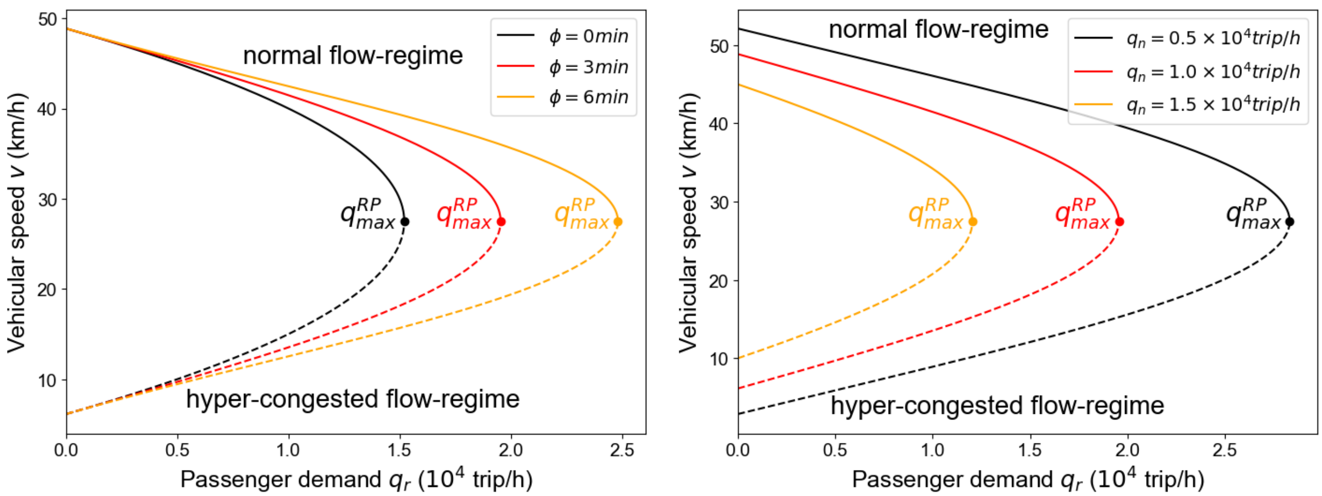
5. A numerical example

To elucidate the theoretical findings obtained so far, we provide a numerical example based on a linear traffic flow model with parameters A=55km/h and $\mathrm{B} = 0.3\mathrm{km}^2/\mathrm{h}$. The average trip distance of both ridesourcing passengers and private car users, i.e. l_r and l_n, is set to be 10km, the total length of the road network $\bar{L}$ is assumed to be 100km. Based on Uber's historical data and a walking radius of 250m, Yan et al. (2019) obtained an estimation of γ =0.0006 in Eq. (7) with a reasonably good goodness-of-fit. This estimated value is adopted in our numerical example.

5.1 Fundamental diagrams

In this sub-section, we depict the fundamental diagrams that describe the relationships between vehicular speed and passenger demand in different scenarios. First, Figure 1(a) portrays the fundamental diagrams in the ride-pooling market (given a fixed q_n and three different matching windows), with the solid line representing the normal flow regime and the dashed line representing the hyper-congested flow regime. Note that the ride-pooling market with ϕ=0 min becomes the non-pooling market, since the pool-matching probability $p(q_r, \phi) = 0$ and the number of vehicles consumed by each passenger $f(q_r, \phi) = 1$, and thus Eqs. (14) and (15) become identical. As seen from the figure, the maximum passenger service rate $q_{\max}^{RP}$ increases with the matching window. This implies that the maximum passenger service rate in the ride-pooling market is always larger than or equal to that in the non-pooling market. Second, Figure 1(b) displays the fundamental diagrams in the ride-pooling market for a given matching window ϕ=3 min and three given different values of q_n. It is found that the maximum passenger service rate in the ride-pooling market decreases with q_n. This is intuitive since the ridesourcing passengers and private car users share common road resources. These findings verify Proposition 1.

From Figure 1(a), one can find that, at a given value of passenger demand, the slope of the change of vehicular speed with passenger demand (representing the marginal effect of passenger demand on vehicular speed) decreases with matching window. It also implies that, the marginal effect (decreasing effect in the normal flow regime and increasing effect in the hyper-congested flow regime) of passenger demand on vehicular speed in the ride-pooling market is smaller than or equal to that in the non-pooling market. These observations are consistent with Proposition 2.



(a) $q_n = 1.0 \times 10^4$ trip/h

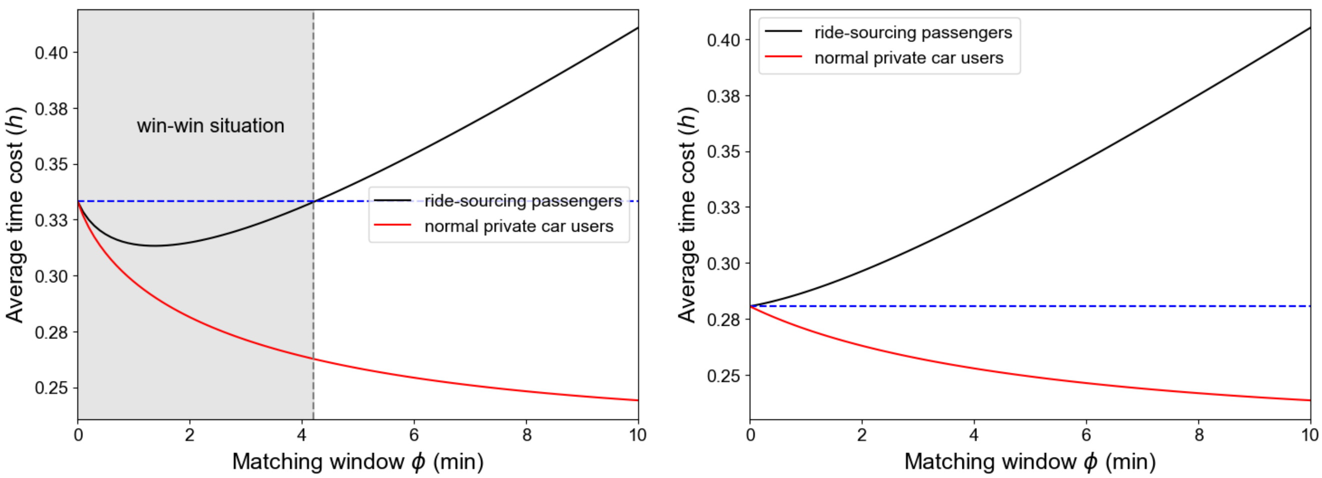
(b) $\phi = 3$ min

Figure 1. Fundamental diagrams of the ride-pooling market

5.2 Effects of demand and matching window in normal flow regime

This section will investigate the joint impacts of matching window and passenger demand on time cost of both ridesourcing passengers and normal private car users in the normal flow regime. In addition, the combinations of matching window and passenger demand that lead to a win-win situation are discussed.

Figure 2a and 2b illustrate how the average time cost of ridesourcing passengers and private car users change with matching window with a high and low level of passenger demand, respectively. It can be seen that the average time cost of private car users always decreases with matching window. This is because increasing matching window increases the pool-matching probability, which further decreases traffic congestion or increases vehicular speed, and thus reduces the time cost of private car users. Figure 2a shows that the average time cost of ridesourcing passengers first decreases and then increases with matching window with a high level of passenger demand. In this case, there exists a win-win situation (shadowed area) in which the ride-pooling market has lower time cost than the non-pooling market. From the convex curve, one can easily find an optimal matching window for minimizing the average time cost of ridesourcing passengers. However, Figure 2b shows that the average time cost of ridesourcing passengers strictly increase with matching window with a low level of passenger demand. In this case, there is no win-win situation and the minimum time cost of ridesourcing passengers is achieved at zero matching window.

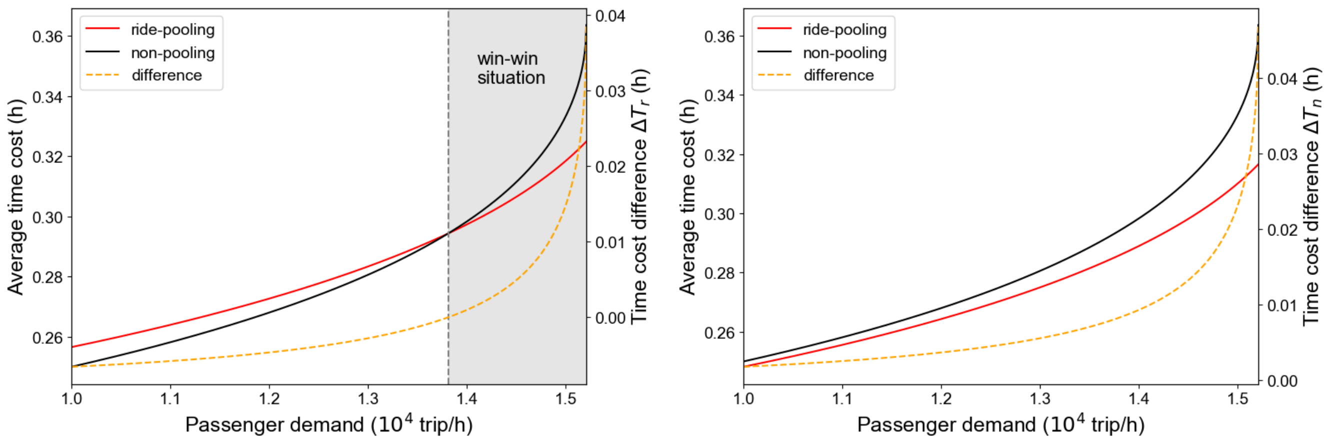


(a) $q_r = 1.5 \times 10^4$ trip/h

(b) $q_r = 1.3 \times 10^4$ trip/h

Figure 2. The impacts of matching window on the time cost of ridesourcing passengers and private car users with a high and low level of passenger demand in the normal flow regime

Figure 3 and 4 display the impacts of passenger demand on the time cost of ridesourcing passengers and private car users, with a short and long matching window, respectively. As seen from Figure 3a-b (with a short matching window of 0.5 min), the time cost of both ridesourcing passengers (T_r^{RS} and T_r^{NS}) and private car users (T_n^{RS} and T_n^{NS}) increase with passenger demand in both the ride-pooling and non-pooling market. It is also found that the differences, ΔT_r and T_n, in the average time costs of ridesourcing passengers (Figure 3a) and private car users (Figure 3b) between the two markets increase with passenger demand. This implies that, with increase in passenger demand, the advantage of ride-pooling over non-pooling become more significant or on-demand ride-pooling program is more desirable when passenger demand is high. Obviously, a win-win situation emerges when the passenger demand exceeds a certain threshold.

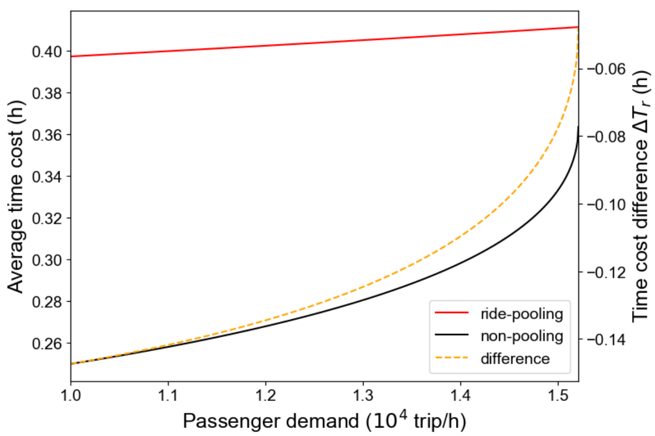


(a) time cost of ridesourcing passengers

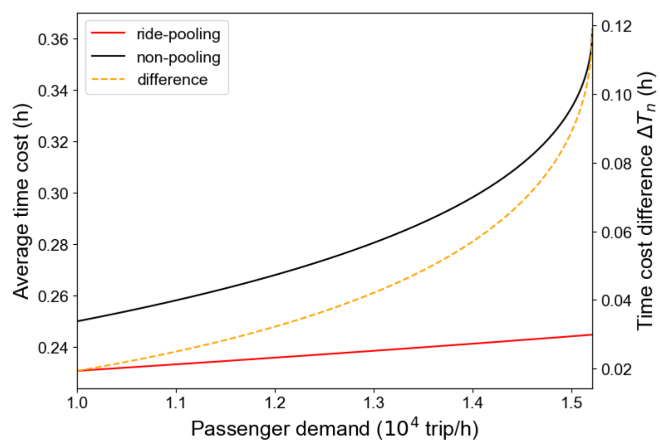
(b) time cost of private car users

Figure 3. The impacts of passenger demand on the time cost of ridesourcing passengers and private car users (with a short matching window $\phi = 0.5$ min) in the normal flow regime

Figure 4a-b demonstrate the trend of change of the time cost of both ridesourcing passengers and private car users with passenger demand in the two markets with a long matching window of 10 min. In this case, the differences in the time costs between the two markets also increase with passenger demand. However, the win-win situation does not arise as passenger demand increases from zero to its maximum. This is because ridesourcing passengers' average waiting time is too long due to a long matching window. As a result, the travel time saving due to increased vehicular speed is not enough to offset the long extra waiting time for pool-matching. These observations are consistent with Proposition 4.



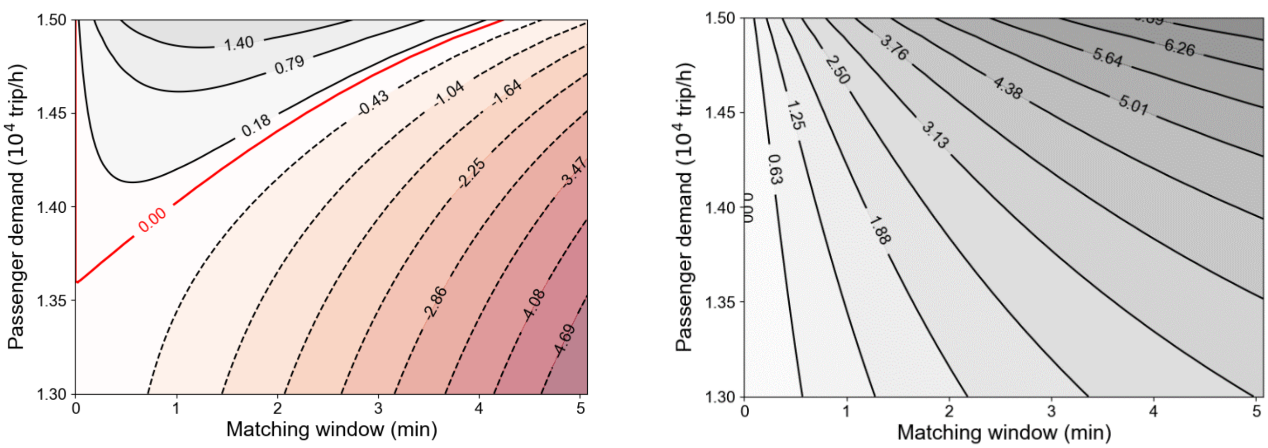
(a) time cost of ridesourcing passengers



(b) time cost of private car users

Figure 4. The impacts of passenger demand on the time cost of ridesourcing passengers and private car users (with a long matching window $\phi = 10$ min) in the normal flow regime

Figure 5 further portrays the difference in the average time costs, ΔT_r and ΔT_n, of ridesourcing passengers (Figure 5a) and private car users (Figure 5b) between the two markets within the two-dimensional space of passenger demand and matching window. As seen from Figure 5b, ΔT_n is always positive and increases with both matching window and passenger demand. This is because increasing either matching window or passenger demand increases pool-matching probability and further increases vehicular speed, bringing up travel time savings to private car users. From Figure 5a, we can find an area of win-win situation (located in the upper left of the red zero-line), where the passenger demand is sufficiently large and the matching window is within a suitable range. This implies that, when the passenger demand is very high, implementation of a ride-pooling program with an appropriate matching window can benefit both ridesourcing passengers and private car users.



(a) ΔT_r

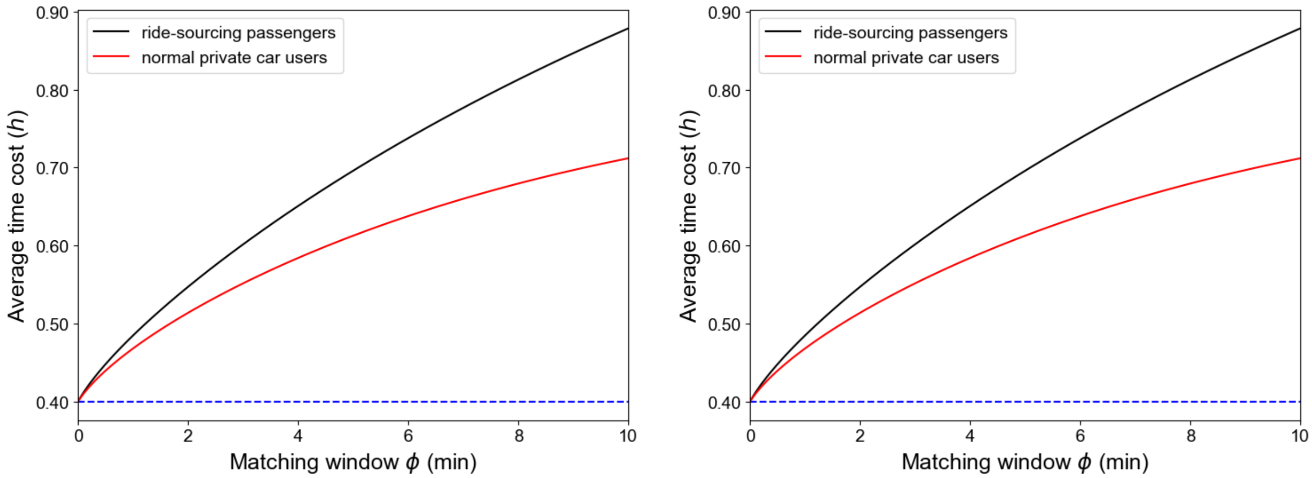
(b) ΔT_n

Figure 5. ΔT_r and ΔT_n in the two-dimensional space of passenger demand and matching window in the normal flow regime

5.3 Effects of demand and matching window in hyper-congested flow regime

This section investigates the joint impacts of matching window and passenger demand on time cost of both ridesourcing passengers and normal private car users in the hyper-congested flow regime. To be cautious, we iterate again that the hyper-congested flow regime is generally regarded as a system failure, where speed increases with flow rate as indicated by the dashed line in Figure 1.

Figure 6(a) and Figure 6(b) illustrate how the average time cost of ridesourcing passengers and private car users change with matching window with a high and low level of passenger demand, respectively, in the hyper-congested flow regime. It can be found that the average time cost of ridesourcing passengers and normal private car users both increase with matching window. This counter-intuitive phenomenon is due to the properties of the hyper-congested flow regime, in which the speed is an increasing function of the steady flow rate. Therefore, a prolonged matching window reduces the proxy traffic flow rate and then decreases the speed and increases travel time for both ridesourcing passengers and normal private car users. This theoretically indicates that, if an area unfortunately collapsed to a hyper-congested flow regime, one should first set a zero matching window to help turn the hyper-congested flow regime back to the normal one.

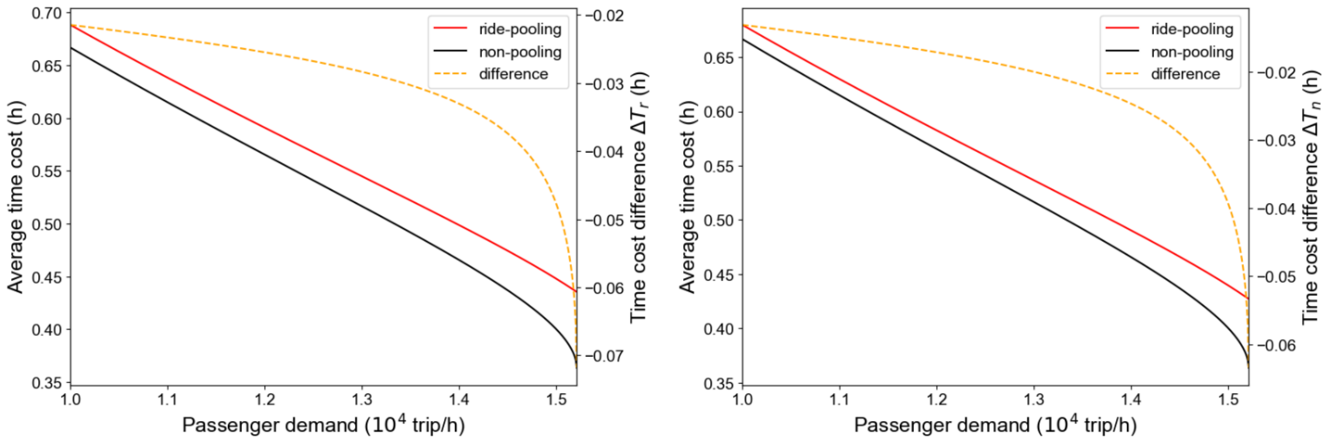


(a) $q_r = 1.5 \times 10^4$ trip/h

(b) $q_r = 1.3 \times 10^4$ trip/h

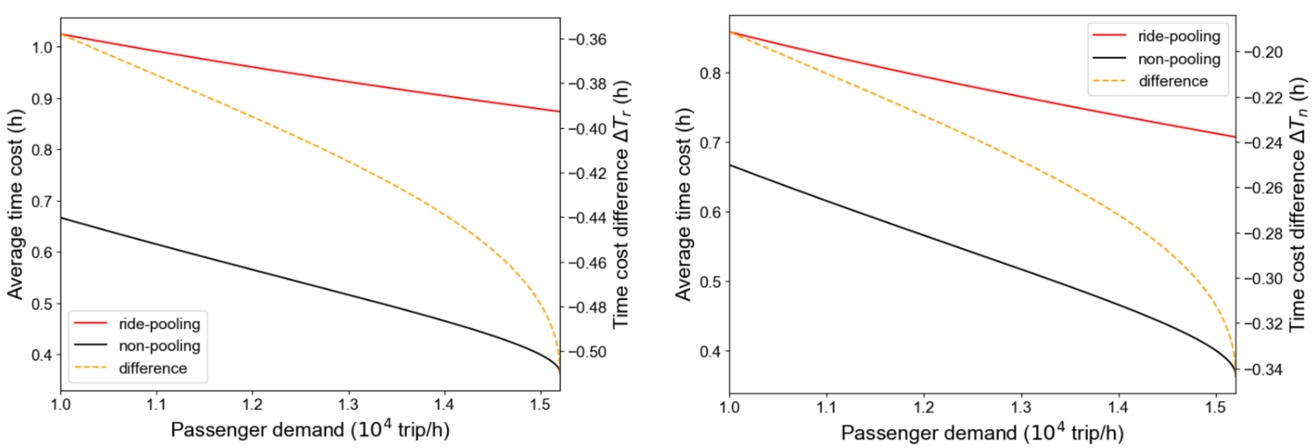
Figure 6. The impacts of matching window on the time cost of ridesourcing passengers and private car users with a high and low level of passenger demand in the hyper-congested flow regime

Figure 7 and Figure 8 demonstrate the impacts of passenger demand on the time cost of ridesourcing passengers and private car users, with a short and long matching window, respectively, in the hyper-congested flow regime. In both figures, the average time cost of ridesourcing passengers and normal private car users decrease with passenger demand, which stems from the fact that network speed increases with passenger demand in the hyper-congested flow regime (as illustrated in Figure 1). We can also see that both ridesourcing passengers and normal private car users have larger average time cost in the ride-pooling market than they would have in the non-pooling market in the hyper-congested flow regime. This implies that implementation of ride-pooling programs has negative impacts on both ridesourcing passengers and normal private car users in the hyper-congested flow regime. As expected, this is opposite to the findings in the normal flow regime, since the speed-flow relationship exhibits opposite trends in the two regimes. In addition, the difference between the average time costs of ridesourcing passengers and normal private car users in the non-pooling market and those differences in the ride-pooling market decrease with rise-sourcing passenger demand. This implies that, the increase of passenger demand will amplify the disadvantages of ride-pooling over non-pooling in the hyper-congested flow regime. It also implies that the platform should not promote ride-pooling by extending the matching window, as the system is unfortunately trapped in the hyper-congested flow regime.



(a) time cost of ridesourcing passengers (b) time cost of private car users

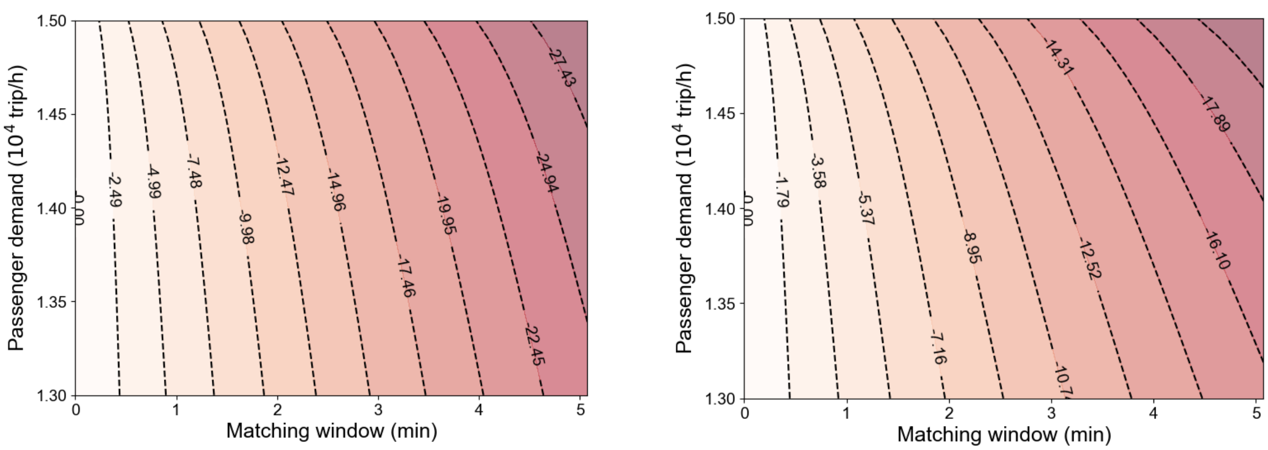
Figure 7. The impacts of passenger demand on the time cost of ridesourcing passengers and private car users (with a short matching window $\phi = 0.5$ min) in the hyper-congested flow regime



(a) time cost of ridesourcing passengers (b) time cost of private car users

Figure 8. The impacts of passenger demand on the time cost of ridesourcing passengers and private car users (with a long matching window $\phi = 10$ min) in the normal flow regime

We then illustrate in Figure 9 the difference in the average time costs, ΔT_r and ΔT_n, of ridesourcing passengers (Figure 9a) and private car users (Figure 9b) between the two markets under different combinations of matching window and passenger demand. It can be found that both ΔT_r and ΔT_n are always negative, implying the implementation of ride-pooling makes negative impact to both ridesourcing passengers and normal private car users in the hyper-congested flow regime. This also indicates that, ride-pooling does not lead to a win-win situation in the hyper-congested flow regime.



(a) ΔT_r (b) ΔT_n

Figure 9. ΔT_r and ΔT_n in the two-dimensional space of passenger demand and matching window in the hyper-congested flow regime

6 Conclusion

This study investigates the on-demand ride-pooling services provided by dedicated drivers affiliated to TNCs in the presence of traffic congestion. Equilibrium states of two markets, a ride-pooling market (with all ridesourcing passengers opting for on-demand ride-pooling) and a non-pooling market (with all ridesourcing passengers choosing non-pooling services), are modelled with the traffic congestion effects depicted by an aggregate traffic flow model like MFD. By assuming a linear traffic flow model, we obtain the following major results both analytically and numerically:

First, with a given pool-matching mechanism, we show that the maximum passenger service rate in the ride-pooling market is always larger than or equal to that in the non-pooling market. Furthermore, the maximum passenger service rate in the ride-pooling market strictly increases with matching window.

Second, the marginal change in vehicular speed (decrease in the normal flow regime and increase in the hyper-congested flow regime) with a unit increase in passenger demand in the ride-pooling market is smaller than that in the non-pooling market. This implies that the change of speed is less sensitive to passenger demand in a market with ride-pooling services. Third, in the normal flow regime, we prove that the time cost of ridesourcing passengers is a convex function of matching window, and thus one can always identify a unique and optimal matching window for minimizing the time cost of ridesourcing passengers. Moreover, through numerical studies, we demonstrate the joint effects of passenger demand and matching window on the time cost of ridesourcing passengers and private car users, and ascertain their combinations that lead to a win-win situation.

Our study opens up several avenues for future extensions. To name a few, (1) incorporating elasticity of passenger demand into the model, in which passenger demand is affected by the travel time and waiting time; (2) examining passengers' mode choices (both external choice between ridesourcing service and public transit service and internal choice between ride-pooling and non-pooling) and how ride-pooling services influences public transit ridership and traffic congestion.

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