A Combined Finite Element-Domain Elimination Method for Minimizing Torque Ripples in Inverter-Fed AC Motor Drive Systems

S. L. Ho, Shiyou Yang, Ping Zhou, H. C. Wong, and M. A. Rahman

***Abstract*—This paper presents a new method for minimizing the torque ripple of inverter-fed induction motor drives. The method is based on the combination of the time-stepping (complex) finite element method coupled to the external circuit model and the domain elimination method. The emphasis of this paper is on the development of a fast and accurate finite element method for the computation of the steady state performance of inverter-fed motor drives with periodic nonsinusoidal time-varying voltages. Improvements on the domain elimination optimization algorithm will also be considered. The proposed method is validated by a numerical example.**

***Index Terms*—Domain elimination method, finite element method, inverter-fed induction motor drives, steady state performance.**

I. Introduction

THE role of inverter-fed AC motor drives in industrial and domestic applications has grown dramatically due to the high efficiency with which electrical power can be manipulated. As power electronics become cheaper, it is envisaged that virtually all electrical machines will be equipped with power electronics in the front end. Unfortunately, the high-speed switching operations of the power electronic elements in inverters always result in skin effects and magnetic saturation. These complications are making it very difficult for the designers to realize the optimal design based on traditional analytical methods such as the Park's transformation, because some fictitious d-q flux linkages and currents are involved in those analyzes. Moreover, all the nonlinearity and space harmonics other than the fundamental in the winding mmfs and in the flux density waveforms are normally neglected in most of the analysis. Therefore, there is a need for a robust optimization design tool, using numerical methods, to take into consideration those factors such as saturation, skin effects, the influences of the external electric circuit as well as the relative movement between the stator and rotor.

However, the major drawback of most numerical methods is their requirements for excessive computational resources and time. In studying inverter-fed motor drives with periodic nonsinusoidal time-varying voltages, the limitations of the numerical methods are even more serious since:

(1) The finite element formulation must be coupled to its external circuit model when dealing with nonsinusoidal sources because of the iteration between the electric and magnetic models;
(2) There is no fast solution method to study the steady state performances of the drive since the resultant governing equation is nonlinear when one considers the high saturation and the thin skin effects. Hence the complex finite element method is not applicable. On the other hand, the transient time-step finite element method coupled with the circuit model is not suitable for the analysis of the steady state performance of inverter-fed motor drives with periodic complex time-varying voltage sources, since the transients will persist at least several cycles before reaching its steady state. In other words, several hundred integral time steps are needed before one obtains the steady state drive performance;
(3) There might be multi-optimal points within the feasible region in the objective functions.

This paper presents a combined finite element-domain elimination method for the minimization of the torque ripples of inverter-fed motor drives. The attention focuses on the development of fast numerical methods for computing the steady state performance of inverter-fed motor drive systems with periodic nonsinusoidal voltage sources accurately. The development of a robust and efficient global optimization method is also described.

II. Steady State Performance Computation

It is commonly known that the steady state performance of inverter-fed motor drives with periodic nonsinusoidal voltage sources can neither be analyzed using the transient time-step finite element method nor the complex finite element method in view of the excessive computation time required for inverse problems. Thus Demerdash and Baldasari [1] propose to use the finite element-state space modeling and the traditional d-q transformation to estimate the initial conditions with the assumption that saturation and skin effect are negligible. In this paper, a new approach is introduced. The proposed approach as described below is iterative in nature:

Step 1. Estimate the initial solution of the stator current profiles and the magnetic vector potential at some

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S. L. Ho is with the EE Department, the Hong Kong Polytechnic University.

S. Y. Yang was with the Zhejiang University, Hangzhou, China. He is now with the EE Department, Hong Kong Polytechnic University, Hong Kong.

P. Zhou is with Ansoft Corporation, USA.

H. C. Wong is with the Industrial Centre, the Hong Kong Polytechnic University, Hong Kong.

M. A. Rahman is with the Faculty of Engineering and Applied Science, Memorial University of Newfoundland, Canada.

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specific positions using the complex finite element method coupled to the external circuit model with some coarse consideration of saturation, skin effects, etc.

Step 2. According to the initial value of the magnetic vector potential at the specific position, obtain both the steady state current profiles and the value of the magnetic vector potential at the same position using transient time-stepping finite element methods coupled to the external circuit model. In this step, the computation is needed to cover at least one total time period so as to include factors such as the saturation of iron materials, the skin effect and the relative movement between the rotor and stator.

Step 3. Compare the values of the computed current profiles and the magnetic vector potential with those of the initial ones. If significant differences exist, change the initial values and return to Step 2; Otherwise, one can assume that the steady state performances of the drive system with some specific geometry and switching conditions have been found.

It should be pointed out that the steady state performances obtained with the proposed approach are exactly the steady-state solutions of the nonlinear partial differential equation.

A. Estimation of Initial Condition

By assuming the permeability of the iron being constant, the system can be taken as linear and the principle of superposition is applicable. Hence the periodic complex input voltage is firstly decomposed into its harmonic components. Then the steady state performances of the system are obtained by superimposing each frequency terms which have just been computed by using the complex finite element method. Since the above procedure requires as many matrix manipulations as the number of the harmonics, the high order frequency sensitivity approach are used in this paper to reduce the computation burden [2]. Thus one should compute the drive performance of the fundamental component and then the sensitivities of the harmonics once and only once. From these information one should be able to obtain the steady state performances of the system with periodic nonsinusoidal voltage sources using the Taylor series.

B. Exact Determination of the Steady State Performances

After estimating the initial conditions approximately, one needs to determine the exact solution with due considerations of the iron saturation, thin skin effects and the relative movement between the rotor and stator. For this purpose, the time step finite element method coupled with the external circuit model is used to account for both the nonlinearity of iron materials and skin effects. Due to the high switching frequency of the output voltages of PWM inverters, especially for those with IGBT elements, the length of the time step must be sufficiently small in order to capture the variations in the output voltage. The interpolating moving boundary is used in this paper to deal with such problem. The detailed discussions about the time step finite element method and the moving interpolating boundary are reported in [3] and [4].

C. Termination Criteria

The criteria to determine the convergence in this paper is:

$$\begin{aligned} |[I_0]_i - [I_1]_i| &\le \varepsilon (i = 1, \cdots, N_f) \\ |[A_0]_i - [A_1]_i| &\le \varepsilon (i = 1, \cdots, N_A) \end{aligned} \tag{1}$$

here, $[I_0]_i, [I_1]_i$ are, respectively, the computed and initial values of the ith frequency current component; $[A_0]_i, [A_1]_i$ are, respectively, the computed and initial values of magnetic vector potential at node i at the specific position.

III. Improved Domain Elimination Algorithm

An improved domain elimination method is used in this paper. Basically the domain elimination method is a stochastic one [5] and it has a global phase where the cost function is evaluated at a number of randomly sampled points within the feasible domain. It also has a local phase where the sample points are manipulated to yield a global candidate point. For the following optimization condition

$$\min f(x) \colon R'' \to R \tag{2}$$

where $x = [x_1 \; x_2 \cdots x_n]^T$, with the following side conditions

$$R'' = \{x | a_i \le x_i \le b_i, \qquad i = 1, 2, \cdots, n\} \tag{3}$$

which are subject to N equality constraints and M inequality constraints

$$\begin{aligned} F_i(x) &= 0 \qquad (i = 1, 2, \cdots, N) \\ G_j(x) &\le 0 \qquad (j = 1, 2, \cdots, M) \end{aligned} \tag{4}$$

The basic iteration process of this method is

(a) Generate a random point x^0 that satisfies the bound constraints (3) of the explicit design variable;

(b) Check some rejection criteria based on the proximity of x^0 with respect to sets comprising of the previous starting points or the local minimum points or the rejected points. If a rejection criterion is satisfied, go to (a); Otherwise go to (c);

(c) Add x^0 to the set of starting points. Start a local minimization procedure from x^0 to yield a local minimum x^* of the cost function subject to constraints (4), and then go to (d);

(d) Check if x^* is a new local minimum; If so, add it to the set of local optimum points; Otherwise, add it to the set of rejected points; then go to (a).

This algorithm can thus be viewed as a modification of multistart procedures that have the ability to learn as the search progresses by means of the introduction of some sets such as the starting point set X_0, the local minimum set X_l, the trajectory set X_t etc. Hence it is expected to be more efficient.

According to this general procedure, the following part of this section gives the detail discussion of the improved domain elimination method.

A. Some Typical Sets

In order to enhance the searching efficiency of the algorithm, some typical sets are also introduced in the improved domain elimination method. These sets include:

1) Starting point set:

$$X_0 = \{x | x = x^0; \text{ a starting point for local optimizations}\};$$

2) Local optimum set:

$$X_1 = \{x | x = x^l; \text{ a local optimum point}\};$$

3) Trajectory set:

$$X_t = \{Y^i | Y^i = (x^{i1}, x^{i2}, \cdots, x^{im})\}; \quad x^{ik}(k = 1, 2, \cdots m) \text{ is the } k\text{th sample point in the } i\text{th trajectory from the start point } x^{i0} \text{ to the } i\text{th local optimum point } x^{il}.$$

Here a trajectory is the searching history from a starting point to the corresponding local optimum point in the local phase. There could be many different trajectories meeting at one local optimum point. Since some rejected points may be included within or near to the three aforementioned sets, the Rejected Point Set used in conventional domain elimination methods is being excluded purposely in order to reduce the storage requirement in the proposed method.

After generating a random point, one needs to determine if it will yield a new local optimum. For this purpose, the random point, denoted as x^r, is compared with the points which are stored in the three sets X_0, X_l, and X_t. If it is within a certain critical distance from any point, it is discarded and a new random point will be generated again. Only if all of these aforementioned conditions are satisfied, the algorithm would allow the point to begin a new cycle of iterations. This operation would prevent unnecessary minimization steps that would lead to already known local optima. The same procedure is also used for a local optimum point and an intermediate point x^{IM}. Considering the fact that generating a random point and evaluating it or evaluating the local point or x^{IM} are much cheaper than beginning a new iterative cycle with this kind of points, especially for electromagnetic inverse problems, this operation will significantly increase the computation efficiency as well as the chance of finding new local optimums in the unexplored region.

In order to reduce the storage requirement, the lengths of set X_0, set X_l, and set X_t are finite in the present domain elimination method. Hence some specified points, say x^0, x^l and x^t, are only effective for certain iteration cycles. This approach is particularly useful since it does not only prevent the search to revisit previously searched subspaces too often, it would also explore the entire feasible domain uniformly.

B. Trajectory Approximation and Distance Computation

The trajectories are approximated by passing straight-line segments through the selected consecutive or adjacent points along the trajectories in this paper. In order to reduce the storage requirements, the procedure would store up the points after every M points. For checking the proximity of two points, a hyper-prism around the specified point x^s (a point belonging to one of the three sets mentioned above) is constructed. The proposed point x^r is rejected if it lies inside the hyper-prism just as mentioned in [5]. For checking the distance between a proposed point and a trajectory, the distance of the point to the nearest line section of the trajectory is used. The proposed point is rejected if this distance is smaller than the critical distance D_{cr}.

C. Termination Criterion

The search process will be terminated in the present domain elimination method if the number of consecutive moves with no improvements in the best objective function being searched so far is larger than a threshold K_{hold}.

D. Algorithm Description

From the above description, it is natural to derive the following improved domain elimination method:

Step 0 (initialization)
Select a value for the parameter IM that specifies the maximum number of local search iterations required to determine an intermediate point x^{IM}; Empty the sets X_0, X_l, and X_t; Set initial values for other iteration parameters; Generate an initial feasible point x^0 and compute the cost function $f(x^0)$; Let $x^i = x^0, f(x^i) = f(x^0)$;

Step 1
Starting from point x^i, generate a random point x^{i0};

Step 2
If the point x^{i0} is within or near some point of X_0 or X_l in or near a trajectory in X_t, then go to Step 1; Else add x^{i0} to set X_0, and go to Step 3;

Step 3
Execute a local minimization procedure up to IM iterations; If the local minimization procedure yields a new local optimum point x^{i*}, then (1) store the trajectory from point x^{i0} to point x^{i*} after every M points, (2) add x^{i*} to set X_l, and go to Step 4; Else go to Step 4 directly;

Step 4
Set $x^i = x^{i*}$ or $x^i = x^{IM}$;

Step 5
If $f(x^{i*})$ or $f(x^{IM}) < f_{\text{opt}}$, then set $f_{\text{opt}} = f(x^{i*})$ or $f_{\text{opt}} < f(x^{IM})$, and let $x_{\text{opt}} = x^{i*}$ or $x_{\text{opt}} = x^{IM}$;

Step 6
Check the terminating criterion. If test is passed, go to Step 7; Else go to Step 1;

Step 7
Output $x_{\text{opt}}, f_{\text{opt}}$, program terminates.

IV. Numerical Results

The proposed method is used to minimize the torque ripple of a mathematical model of squirrel cage induction motor drive

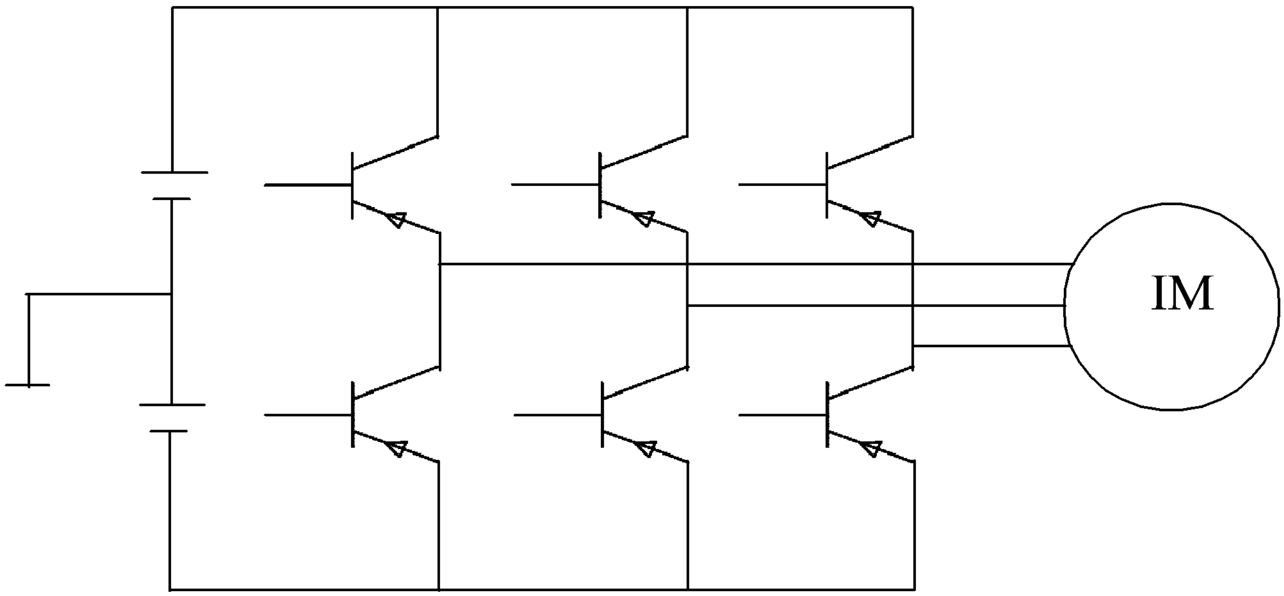

Fig. 1. Schematic diagram of a typical PWM inverter-fed a cage induction motor open-loop system.

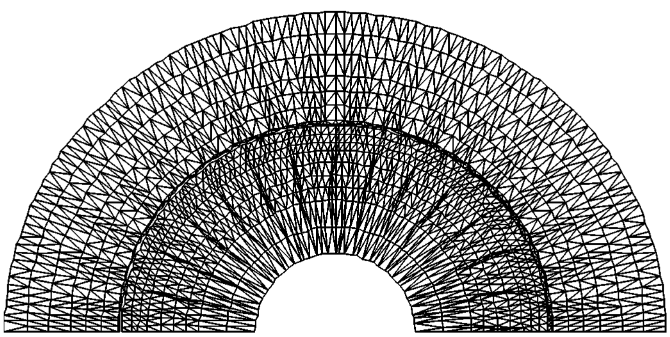

Fig. 2. The mesh generation for the computation.

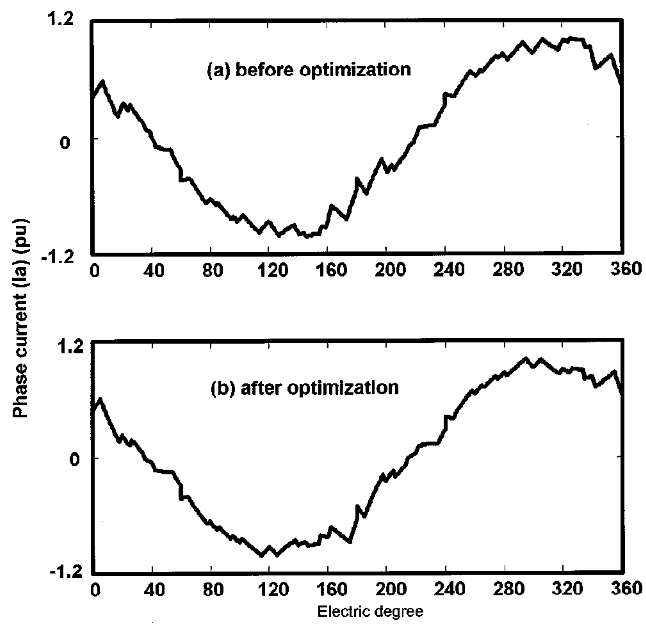

Fig. 3. The stator current profiles of the machine being tested.

fed from a PWM inverter as shown in Fig. 1 at rated operating conditions. The optimization model is:

$$\min \quad f = (1/T_0)\sqrt{\sum_{i=0}^{N_f} T_i^2(\alpha)}$$

$$\text{subject to} \quad \begin{cases} \alpha_{i-1} + \Delta_{\min} \le \alpha_i \le \alpha_{i+1} \\ \qquad -\Delta_{\min}(i = 2, \cdots, N-1) \\ \alpha_1 \ge \alpha_{\min}, \alpha_n \le \alpha_{\max} \end{cases} \tag{5}$$

here N is the number of switching angles within one quarter period of the PWM inverter, α_i is the ith switching angle, T_i is the ith frequency component of the electromagnetic torque, $\Delta_{\min}, \alpha_{\min}$, and $\alpha_{\max}$ are related to the switching frequency of the power electronic elements and the operating conditions.

Fig. 2 shows the mesh generation used in the numerical computation. Figs. 3 and 4 show, respectively, the computed results of the steady state performances of the machine, i.e., the profile of stator currents and the electromagnetic torque. Table I presents the optimized results in the case of a PWM inverter having five switching angles in a quarter period. In the numerical computation, $\Delta_{\min}, \alpha_{\min}$, and $\alpha_{\max}$ are assumed to be 1, 4 and 36 (electric degrees), respectively. In order to reduce the computation burden, the initial values of these parameters are taken from the optimized results obtained using traditional analytical methods. Moreover, by beginning with different initial conditions, the procedure can still find the final solution, but with a much higher function evaluation number. These results show that the objective function f, i.e., the ratio of the square root of the sum of the square of the different frequency components of the electromagnetic torque to its direct component, is reduced from 1.03 to 1.01 by using the proposed algorithm. The CPU time of 5.2 hours is also considered to be reasonable for such complex investigations.

V. Conclusion

This paper presents an efficient algorithm on a combined finite element and domain elimination method. The emphasis of

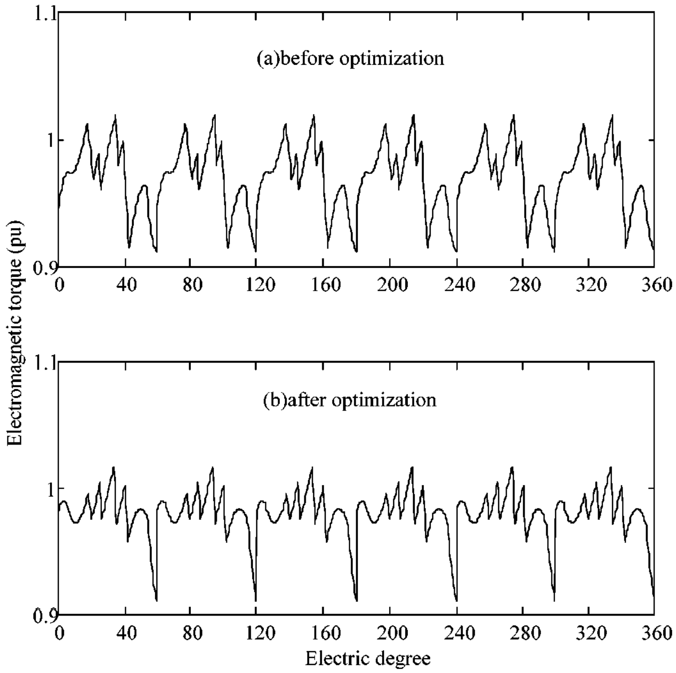

Fig. 4. Steady state electromagnetic torque of the machine being tested.

TABLE I
OPTIMIZATION RESULTS

	α_1 (deg)	α_2 (deg)	α_3 (deg)	α_4 (deg)	α_5(deg)	T_0(pu)	F	CPU time
Initial	6.79	17.30	21.03	34.66	35.98	0.997	1.03	/
Optimized	5.20	18.07	19.88	33.70	35.11	0.998	1.01	5.2 (hour)*

this paper is on the development of a fast algorithm for computing the steady state performances of a typical inverter drive system. Minimization of the torque ripples of the PWM inverter-fed induction motor has been achieved. In short, the numerical example given has fully demonstrated the high efficiency of the proposed approach both on computing the steady state performances of inverter-fed motor drives with periodic nonsinusoidal time-varying voltage sources as well as on the minimization of the torque ripples of the system. The authors are also confident that the proposed algorithm can be applicable to other optimization problems in general.

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