Schedule Design for Liner Services under Vessel Speed Reduction Incentive Programs

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Abstract

Gas and particulate emissions from ship transportation have been increasing in recent years. In order to mitigate ship emissions near coastal areas, voluntary Vessel Speed Reduction Incentive Programs (VSRIPs) were put in place by a number of ports. This paper studies a schedule design problem faced by liner shipping companies under VSRIPs. It proposes a mixed-integer non-linear mathematical model for the minimization of the total cost, consisting of fuel cost, as well as operating cost, minus dockage refunds. The model balances three determinants, i.e., the compliance of VSRIPs, the speed limit (the maximum physical speed of ships and the upper speed limit imposed by VSRIPs), and the limited number of ships. An enumerative algorithm and a piecewise-linear approximation algorithm are developed, based on some properties of the non-linear model. The efficiency of the proposed algorithms is validated through extensive computational experiments.

Keywords: Liner shipping; schedule design; VSRIP; speed limit; enumerative algorithm; piecewise-linear approximation

1. Introduction

The increase in ship transportation has a significant impact on air pollution. Nikopoulou et al. (2013) pointed out that more than 95% of the world's vessels have been traditionally powered by diesel engines, and even the newest marine engines produce higher pollutant emissions per power output than the regulated on-road diesel engines due to their low fuel quality. Ships emit exhaust gases and particulates, including sulfur dioxide (SO_2)[1], nitrogen oxides (NO_x), carbon dioxide (CO_2) and particulate matter (PM), from their operations at sea or in port areas. The three pollutants SO_2, NO_x and PM are major concerns since they may cause acid rain, photochemical smog and some serious diseases, such as respiratory and cardiovascular problems (Cullinane and Edwards, 2010). It is well known that CO_2 is the dominant greenhouse gas. The United Nations Conference on Trade and Development (UNCTAD, 2009, 2015) estimated shipping emitted 3.3% the global emission of CO_2 in 2007 and 2.2% in 2012, and the estimation from Third International Maritime Organization (IMO) Greenhouse Gas (GHG) Study (IMO, 2014) is a bit higher with 3.5% in 2007 and 2.6% in 2012. Slow steaming is an effective way of reducing gas and particulate emissions. It is revealed that lowering speed among some containership routes can lead to reductions in carbon emissions by up to 70% (Corbett et al., 2009), and speed reduction is also effective in reducing SO_2, NO_x and PM emissions (Chang et al., 2014). In addition, a shipping line may reduce its total cost by taking advantage of slow steaming (Chang and Wang, 2014).

In order to reduce ship emissions near coastal areas, the Port of Los Angeles (LSA) proposed a voluntary Vessel Speed Reduction Program in 2001. With some improvements in the following years, the program recently offers financial incentives and thus is named Vessel Speed Reduction Incentive Program (VSRIP), which has two Vessel Speed Reduction Zones (VSRZs) with radii of 20 nautical miles (nm) and 40 nm, respectively (see Fig. 1). The ships that voluntarily participate in the program must reduce their speed to 12 nm/hour (knots) in the VSRZs. A shipping company can receive dockage refunds that are 15% and 30% of the first day of dockage for each vessel visit in a calendar year if during that year its ship visits achieve a 90% or higher compliance rate in the VSRZs with radii of 20 nm and 40 nm, respectively. It should be noted that the two refunds cannot be applied at the same time. Statistically, out of 3,360 ships entering and leaving LSA in 2018, 91% slowed to 12 knots within 20 nm, and 85% did so within 20 to 40 nm (LSA, 2019). The Port of LSA is not the only one adopting a voluntary VSRIP. For example, the Port of Long Beach (LGB) designed a similar program in 2001 (the same year as LSA), followed by the Port of San Diego (SDI) in 2009 and the Port of New York and New Jersey in 2010. In addition, the Ports of Seattle, Tacoma, Oakland and Houston, as well as the coastal areas in Korea are projected to adopt programs for

[1]The table of abbreviations in the Electronic Companion provides a list of the main abbreviations used in this paper.

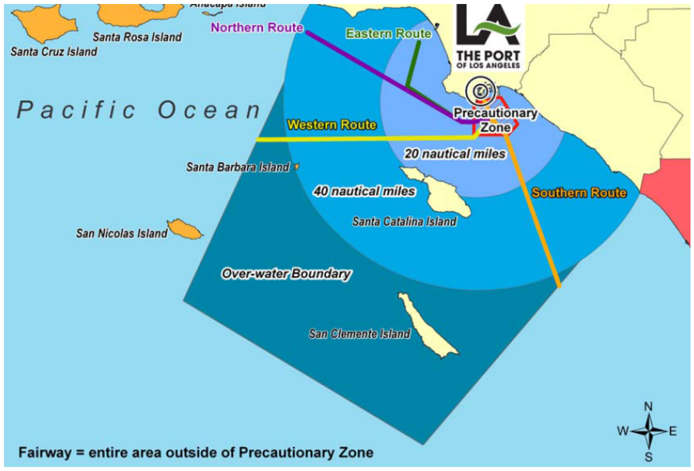

Figure 1: The VSRZs of LSA (Source: LSA, 2009)

reducing the sailing speed of ships.

Although the VSRIP contributes to reducing ship sailing speeds in VSRZs, the speed outside VSRZs may be increased in order to complete the on-time delivery of containers. Therefore, the total fuel consumption and the fuel cost may increase due to the implementation of the VSRIP, while the total dockage rate may decrease as a result of dockage refund. Focusing on a schedule design problem of a liner shipping company with a limited number of ships, this research aims to minimize the total shipping cost, consisting of fuel cost, as well as the operating cost of ships, minus dockage refunds.

All VSRIPs are voluntary activities, which means that shipping companies can determine whether to participate in a VSRIP or not, and to comply with the rules that apply in a VSRZ (e.g., 20 nm VSRZ or 40 nm VSRZ). Since all routes provide weekly service frequencies, the rotation time of each route is dependent on the number of deployed ships on the route. For a given route, the increase in the number of deployed ships yields a higher operating cost as well as a lower fuel consumption if the compliance of VSRIPs at visited ports is unchanged. We will present a mixed-integer non-linear mathematical model to minimize the total cost considering three determinants: the compliance of VSRIPs, the speed limit (determined by the maximum physical speed of ships and by the upper speed limit imposed by VSRIPs), and the limited number of ships. Based on the properties of the proposed model, we will then put forward an exact algorithm and a piecewise-linear approximation scheme to solve the model.

The remainder of this paper is organized as follows. The related works are reviewed in Section 2. Section 3 provides some definitions and proposes a mixed-integer non-linear programming model. Two algorithms, an enumerative algorithm and a piecewise-linear approximation scheme, are developed in Section 4 to solve the model. Section 5 conducts extensive computational experiments based on real data, whose results yield a number of managerial implications. Concluding remarks are presented in Section 6. The proofs of lemmas and propositions are in the Electronic Companion.

2. Literature review

There has been considerable research on maritime transportation in recent years, such as Lee et al. (2012) and Hennig et al. (2012), which is also witnessed by the surveys of Christiansen et al. (2007), Fransoo and Lee (2013), Christiansen et al. (2013), Meng et al. (2014) and Lee and Song (2017). Here we concentrate on the contributions devoted to speed limits and speed reduction.

Speed limits have been recommended as an effective way of enforcing speed reduction (Lindstad et al., 2011). Speed limits and speed reduction (slow steaming) are two concepts having common characteristics, but differing in several respects. In contrast to speed limits applied in some regional zones, speed reduction is usually implemented in a global context. Achieving the dockage refund by complying with VSRIPs can be treated as an extra benefit of the speed limit. Another difference is that slow steaming is a voluntary response of shipping companies, while speed limits are an imposed measure. If the speed limit is implemented on a local or on a regional basis and is below the optimal slow steaming speed, it may distort the market or the total shipping cost (Psaraftis and Kontovas, 2013). Nevertheless, this difference can be ignored since the VSRIPs studied in our paper are voluntary programs.

The motivations for implementing speed limits or speed reduction have been studied from different perspectives. Cariou (2011) found that speed reductions can only be sustained when the fuel price is sufficiently high for the main container trades. Kontovas and Psaraftis (2011) considered five factors, including higher fuel price, higher fuel cost, saving in other costs (e.g., local taxes), lower freight rates, and mandatory or voluntary regulations, as the main incentives for slow steaming. Linder (2014) divided the available incentives into three types, including social pressure, regulatory drivers, and economic motivations. According to the investigation described in this paper, over 90% of the surveyed operators in shipping companies felt they would be influenced by local public opinion on the air pollution issue. Social pressure is deemed as a key incentive for emissions reduction due to the importance of public image for each company. Regulatory pressure comes from the signed programs and regulations. Operators may feel their operations are controlled by regulatory agencies in an environment with high regulatory pressure to reduce emissions. The VSRIP at LSA can be viewed as an economic motivation. Some companies will participate in the program in order to gain dockage refunds which will reduce their costs.

An extensive literature focuses on the impact of speed reduction on cost and emissions. Meyer et al. (2012) and Lee et al. (2015) have confirmed that lowering speed is an effective fuel cost saver. Maloni et al. (2013) reported that extra slow steaming (18 knots) is optimal for saving the total cost, i.e., carrier cost (fuel and vessel) and shipper cost (pipeline inventory), and for reducing CO_2 under different container volumes and fuel prices. They further explored how to achieve equity in terms of financial savings between carriers and shippers through contractual-based agreements of slow steaming. Chang and Wang (2014) showed that speed reduction may lead to less CO_2 emissions and lower total cost, including opportunity cost, fuel cost and operating cost, and the optimal speed reduction could be a dynamic process related to fuel prices and charter rates. On the other hand, Psaraftis and Kontovas (2010) found that lowering speed may not bring economic performance since it could have a negative influence on in-transit inventory and non-fuel operational costs, possibly exceeding the fuel cost savings. In a study on the cost efficiency of speed reduction, Chang and Chang (2013) claimed that speed reduction can lead to less fuel consumption and lower CO_2 emissions, but larger ship operating costs.

There is also some literature on shipping considering speed limits. Investigating a global mandatory speed limit, Corbett et al. (2009) reported that a speed reduction mandate can lead to the reduction of CO_2 emissions in a container fleet and showed how the cost of this fleet increases when ships are required to sail at a non-optimal speed. Cariou and Cheaitou (2012) claimed that a regional speed limit zone may increase emissions considering the acceleration of the ships outside the zone. Chang et al. (2014) investigated a local speed reduction zone with a compulsory speed limit and confirmed that the emissions of SO_2, NO_x and PM from ships will be reduced in this zone. Zis et al. (2015) studied a similar local speed limit zone, whose results indicated that this zone will result in the increase of the total fuel consumption and operating cost and contribute to the reduction of NO_x emissions near the port but the increase of CO_2 emissions in the whole trip.

Vessel speed reduction programs have been explored in some extant literature. Zis et al. (2014) and Zis (2015) considered emission reduction actions, including VSRIP and cold ironing, and investigated their impacts on ship emissions near and at ports and their economic implications for ship operator and the studied port. There also exist two previous works (Chang and Jhang, 2016; Ahl et al., 2017) focusing on the voluntary Green Flag Incentive Program (GFIP) at LGB, which is similar to the program at LSA. Referring to the 20 nm GFIP at LGB, Chang and Jhang (2016) designed a 15% dockage waiver for a ship whose speed decreases to 12 knots in a VSRZ with a radius of 20 nm. This enabled them to assess fuel consumption and emissions for bulk and container ships entering the Kaohsiung Port. The CO_2 emissions and SO_2 emissions were calculated in their study. It was found that small ships are less environmentally friendly than large ships since small ships emit more CO_2 and SO_2 per ton·nm. Their results also showed that both bulk ships and container carriers can benefit from lowering speed. Ahl et al. (2017) focused on the impact of financial incentives, which bear some similarities to our research, but they only considered the 40 nm GFIP at LGB. The value of time for the operators of different ship types was estimated and reported. Their results also revealed that the financial incentive is indeed effective but the effects vary by ship type, and differentiated dockage discounts for different ship types may be more effective in improving compliance. In contrast to the above research, our paper explores how to balance the compliance of each VSRIP, the speed limit and the limited number of ships for minimizing the total cost (i.e., fuel cost and operating cost, minus dockage refunds) of all liner routes for a shipping company. In contexts where there are different VSRIPs at ports, our study aims to determine the compliance of VSRIPs, the optimal speeds within and outside VSRZs, and the numbers of ships deployed on routes.

3. Model formulation

We now present a mathematical program for the schedule design problem under VSRIPs.

3.1. Definitions

We first define the terms used in the paper. A route is composed of several ports of call in a given sequence. Each route is a round trip (see Fig. 2), which means that a ship departs from its first port of call, visits the following ports of call according to the sequence, and returns to its first port of call. The total sailing time and time spent at all ports of call on a route is called rotation time. For simplicity, we use the term “VSRIP port” to denote a port adopting a VSRIP, e.g., LSA, and “non-VSRIP port” to denote a port without any VSRIP, such as the Port of Shanghai. “γ nm VSRZ” means a VSRZ with a radius of γ nm, and a γ nm VSRZ is also called as “non-zero VSRZ” when γ is greater than 0. “Chosen VSRZ” means that the shipping company under consideration complies with the rules in the VSRZ.

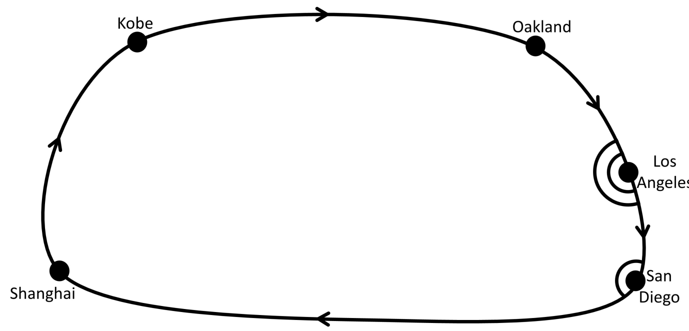

Figure 2: A round-trip route

As implemented at LSA, the shape of each non-zero VSRZ is similar to a sector and the port is the circle center, that is, the distances from the port to all points of the VSRZ boundary are the same. For a voyage from one port to the next with a VSRIP port, changing from the shortest sailing path to another will increase the distance outside a VSRZ without any decrease for the distance within the VSRZ, meaning the shortest path is still optimal after considering VSRIPs. Therefore, we consider ships always sail along the shortest path between each two consecutive ports of call. To simplify the model, we add a dummy port at the middle of the shortest sailing path between two consecutive ports of call if both ports have VSRIPs, and the dummy port is regarded as a non-VSRIP port. For example, a dummy port is added at the middle point of the shortest path from LSA to SDI, which are both VSRIP ports, and thus the voyages from LSA to the dummy port and from the dummy port to SDI with equal lengths each includes a VSRIP port (see Fig. 3).

There are three types of voyages between two consecutive ports of call: from a non-VSRIP port to another non-VSRIP port (type-1 voyage), e.g., a sailing from Shanghai to Kobe; from a VSRIP port to a non-VSRIP port (type-2 voyage), e.g., a sailing from SDI to Shanghai; from a non-VSRIP port to a VSRIP port (type-3 voyage), e.g., a sailing from Oakland to LSA. A type-1 voyage is defined as a “non-VSRZ leg”, and a non-VSRZ leg

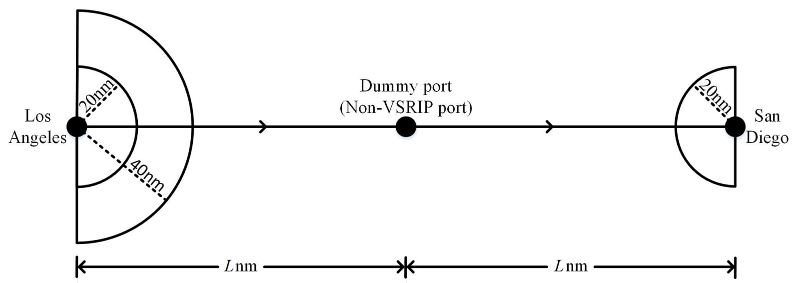

Figure 3: An illustration on adding a dummy port between two consecutive VSRIP ports of call

$l \in L$ on route $r \in R$ has a fixed sailing distance, denoted by d_{rl}. A type-2 or type-3 voyage is deemed as two legs, i.e., the sailing within the chosen VSRZ is a leg, defined as an "in-VSRZ leg", and the sailing outside the zone is another leg, called an "out-VSRZ leg", and we will use a pair of in-VSRZ and out-VSRZ legs to denote the two legs. Note that if 0 nm VSRZ is chosen, the 0 nm voyage is also regarded as an "in-VSRZ leg". A pair of in-VSRZ and out-VSRZ legs is numbered as follows: for a type-2 voyage including an in-VSRZ leg followed by an out-VSRZ leg, we define that if the ID of the in-VSRZ leg is l, then the ID of the out-VSRZ leg is $l+1$; for a type-3 voyage including an out-VSRZ leg followed by an in-VSRZ leg, we define that if the ID of the in-VSRZ leg is l, then the ID of the out-VSRZ leg is $l+1$ for notational convenience. The total sailing distance of a pair of in-VSRZ leg l and out-VSRZ leg $l+1$ is fixed, denoted by $d_{r,l,l+1}$, and the sailing distances of the two legs are also fixed for a chosen VSRZ. For a chosen VSRZ j at VSRIP port p with a radius of d_{pj}, we define in-VSRZ leg l including port p with the sailing distance d_{pj} as leg option j of the in-VSRZ leg, and out-VSRZ leg $l+1$ with the sailing distance $d_{r,l,l+1} - d_{pj}$ as leg option j of the out-VSRZ leg. It should be noted that an in-VSRZ leg and its following out-VSRZ leg must choose the same leg option that is related to the chosen VSRZ. As shown in Fig. 4, a pair of in-VSRZ and out-VSRZ legs between the Port of Oakland and LSA each has three leg options. For instance, if leg option 2 with a 20 nm VSRZ at LSA is chosen, the pair of in-VSRZ and out-VSRZ legs also chooses option 2 with the sailing distances of 20 nm and $L - 20$ nm, respectively. Similarly, a pair of in-VSRZ and out-VSRZ legs between two consecutive ports of call with a VSRIP port adopting only one non-zero VSRZ each has two leg options, and a non-VSRZ leg has only one leg option since its sailing distance is fixed. In-VSRZ legs refer to the voyages within VSRZs, and non-VSRZ and out-VSRZ legs are the voyages outside VSRZs. Recall that a non-VSRZ leg is a type-1 voyage, and an out-VSRZ leg is a sailing outside the chosen VSRZ in a type-2 or type-3 voyage.

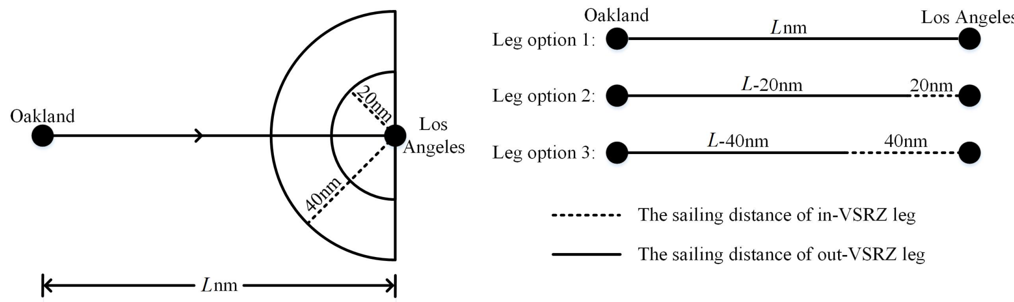

Figure 4: An illustration of leg options for a pair of in-VSRZ and out-VSRZ legs

We take the route in Fig. 2 as an example. In a round trip, a ship will first visit Shanghai which is considered as the first port of call, then Kobe, Oakland, LSA, SDI, and finally return to Shanghai. Referring to Fig. 3, a dummy port is added at the middle of the shortest path between two consecutive VSRIP ports of call LSA and SDI. The type-1 voyages from Shanghai to Kobe and from Kobe to Oakland are two non-VSRZ legs with the IDs of 1 and 2, respectively. There is an out-VSRZ leg and an in-VSRZ leg included in the type-3 voyage from Oakland to LSA, i.e., the voyage outside the chosen VSRZ at LSA is an out-VSRZ leg, whose ID is defined as 4, and the voyage within the chosen VSRZ, even if the 0 nm VSRZ is chosen, is regarded as an in-VSRZ leg, whose ID is 3. The in-VSRZ and out-VSRZ legs included in the type-2 voyage from LSA to the dummy port have the IDs of 5 and 6, respectively, while the IDs of out-VSRZ and in-VSRZ legs in the type-3 voyage from the dummy port to SDI are 8 and 7, respectively. The final voyage from SDI to Shanghai is a type-2 voyage including a pair of in-VSRZ and out-VSRZ legs with the IDs of 9 and 10, respectively. Hence, the route shown in Fig. 2 has 10 legs, including two non-VSRZ legs, four in-VSRZ legs and four out-VSRZ legs.

3.2. Problem description and notation

We classify the ships into a set of types, denoted by I, according to ship sizes since the fuel consumption of a ship is dependent on its size; the total number of ships of type i is Q_i, $i \in I$. In our study, a route can only deploy ships of a given type. The set of routes on which ships of type i will be deployed is denoted by R_i. Route r provides a weekly service frequency; non-VSRZ legs define the set L_r^N, in-VSRZ legs define the set $L_r^{S,in}$, and out-VSRZ legs define the set $L_r^{S,out}$. Suppose that some ports located on these routes adopt VSRIPs. The rules on radii, speed limits and refunds for different VSRZs of different VSRIP ports may not be identical. The refunds of different VSRZs at the same VSRIP port cannot be gained at the same time. Thus, a shipping company can choose only one VSRZ at a VSRIP port. A 100% compliance rate for each VSRIP is assumed in our paper, and all ships in the company visiting a VSRIP port should comply with the rules in the chosen VSRZ in order to obtain the refund. The objective of the shipping company is to minimize the total cost for all routes, including fuel cost and operating cost, minus dockage refunds, considering the compliance of VSRIPs, the speed limit, and the limited number of ships. We will first optimize the sailing speed of only one route in Section 3.3, by assuming that the VSRZs are chosen and the ship number is given, and then in Section 3.4, we will develop a mixed-integer non-linear programming model of all routes to optimize the decision on which VSRZ is chosen at each VSRIP port, the speed of each leg, and the number of deployed ships on each route.

We provide the notation of parameters and variables frequently used in this section as follows:

Sets

I	Set of ship types
K	Set of VSRZ plans; a VSRZ plan includes the total chosen VSRZs of all VSRIP ports
$\mathbb{N}$	The set of natural numbers
P^S	Set of VSRIP ports
J_p	Set of VSRZs with different radii (including a radius equal to zero) at VSRIP port $p \in P^S$; index $j = 0 \in J_p$ refers to the VSRZ with a radius of zero, meaning the rules of a VSRIP at port p have not been complied with; note that leg options of in-VSRZ leg $l \in L_r^{S,in}$ and out-VSRZ leg $l+1 \in L_r^{S,out}$ on route $r \in R$ where in-VSRZ leg l includes VSRIP port p are dependent on the chosen VSRZ $j \in J_p$
R_i	Set of routes where ships of type $i \in I$ can be deployed
R	Set of routes; $R = \bigcup_{i \in I} R_i$
L_r^N	Set of non-VSRZ legs on route $r \in R$
$\hat{L}_{kr}^N$	Set of legs on route $r \in R$ for plan $k \in K$ obtained by Algorithm 1 whose optimal sailing speeds in model [P0] are the same and are lower than their speed limits
$L_r^{S,in}$	Set of in-VSRZ legs on route $r \in R$
$L_r^{S,out}$	Set of out-VSRZ legs on route $r \in R$

$\hat{L}_{kr}^{S}$ Set of legs on route $r \in R$ for plan $k \in K$ obtained by Algorithm 1 whose optimal sailing speeds in model [P0] are equal to their speed limits; $\hat{L}_{kr}^{N} \cup \hat{L}_{kr}^{S} = L_r$ and $\hat{L}_{kr}^{N} \cap \hat{L}_{kr}^{S} = \varnothing$

L_r Set of all non-VSRZ, in-VSRZ and out-VSRZ legs on route $r \in R$; $L_r = L_r^N \cup L_r^{S,in} \cup L_r^{S,out}$

Parameters

i_r Type of ship that will be deployed on route $r \in R$; $i_r \in I$

α_i, β_i Conversion factors between fuel consumption per unit distance and sailing speed for ships of type $i \in I$: fuel consumption per unit distance of a ship of type $i \in I$ is $\alpha_i \cdot \text{speed}^{\beta_i}$ (tons/nm)

$\overline{c}_i$ Operating cost (USD) of each ship of type $i \in I$ per week

$\tilde{c}_{ipj}$ Refund (USD) of each visit of a ship of type $i \in I$ when the rules in VSRZ $j \in J_p$ of port $p \in P^S$ are complied with; $\tilde{c}_{ipj}$ is equal to zero when the radius of the VSRZ is zero

d_{krl} Sailing distance (nm) for leg $l \in L_r$ of route $r \in R$ in plan $k \in K$

d_{pj} Sailing distance (nm) within VSRZ $j \in J_p$ of port $p \in P^S$

d_{rl} Sailing distance (nm) on non-VSRZ leg $l \in L_r^N$ of route $r \in R$

$d_{r,l,l+1}$ Total sailing distance (nm) of a pair of in-VSRZ leg $l \in L_r^{S,in}$ and out-VSRZ leg $l+1 \in L_r^{S,out}$ of route $r \in R$

p_{rl}^{S} The port on leg $l \in L_r^{S,in}$ of route $r \in R$ that has a VSRIP

$q_{kr}^{\min}$ Minimum integer number of deployed ships on route $r \in R$ for plan $k \in K$ such that model [P0] is feasible

$\hat{q}_{kr}$ Optimal integer number of deployed ships on route $r \in R$ for plan $k \in K$ to gain the minimum fuel and operating cost

Q_i Total number of ships of type $i \in I$ in the fleet of the liner shipping company

$\hat{t}_{krl}$ Optimal sailing time (hours) on leg $l \in L_r$ of route $r \in R$ for plan $k \in K$ to obtain the optimal solution of model [P0] without considering the speed limit

T Number of hours in a week, $T = 168$

T_r Total time (hours) spent at all ports of call on route $r \in R$

V_{rl} Upper speed limit (knots) of each VSRZ of port $p \in P^S$ for in-VSRZ leg $l \in L_r^{S,in}$ of route $r \in R$ if the leg includes VSRIP port $p \in P^S$; otherwise, maximum physical speed (knots) of ships of type $i \in I$ for non-VSRZ leg $l \in L_r^N$ or out-VSRZ leg $l \in L_r^{S,out}$ of route $r \in R$ if ships of type $i \in I$ are deployed on the route

w	Unit fuel price (USD/ton)

Decision variables

q_r	Integer number of ships deployed on route $r \in R$
t_{rl}	Sailing time (hours) for leg $l \in L_r$ of route $r \in R$
$\boldsymbol{t}_r$	All vectors of $(t_{rl}, l \in L_r)$
x_{pj}	Binary variable, equal to one if the rules in VSRZ $j \in J_p$ of port $p \in P^S$ are complied with, and zero otherwise
y_{rl}	Sailing distance (nm) of leg $l \in L_r$ on route $r \in R$.

3.3. Basic model considering only one route with chosen VSRZs

This paper studies a schedule design problem for the sailing routes of a liner shipping company considering VSRIPs established at some ports. As mentioned above, a liner shipping company can choose only one VSRZ at a VSRIP port. The total chosen VSRZs of all VSRIP ports constitute a VSRZ plan. Changing a chosen VSRZ of a VSRIP port, we will gain another VSRZ plan. The total number of VSRZ plans is $\Pi_{p\in P^S}|J_p|$. Given a VSRZ plan k and the number of deployed ships for each route, the sailing speed of each route can be optimized independently. Therefore, for a given plan, we first analyze the schedule design problem of one route with a fixed number of ships deployed.

We define $q_{kr}^{\min}$ as the minimum integer number of ships deployed on route r in plan k, which can be computed by letting ships sail at V_{rl} on leg l for all $l \in L_r$:

$$q_{kr}^{\min} = \left\lceil \frac{\sum_{l\in L_r} d_{krl}/V_{rl} + T_r}{T} \right\rceil, \tag{1}$$

where $\lceil z \rceil$ is the smallest integer greater than or equal to z. The minimum sum of operating cost and fuel cost of route r for plan k with a given number of deployed ships q_r ($q_r \geq q_{kr}^{\min}$) can be written as

$$\text{[P0]} \quad f_{kr}(q_r) := \bar{c}_{i_r} \cdot q_r + \underset{\boldsymbol{t}_r}{\text{minimize}} \sum_{l\in L_r} w \cdot d_{krl} \cdot \alpha_{i_r} (\frac{d_{krl}}{t_{rl}})^{\beta_{i_r}} \tag{2}$$

subject to

$$\sum_{l\in L_r} t_{rl} = T \cdot q_r - T_r \tag{3}$$

$$\frac{d_{krl}}{t_{rl}} \leq V_{rl}, \forall l \in L_r \tag{4}$$

$$t_{rl} \geq 0, \forall l \in L_r. \quad (5)$$

The objective function of model [P0] consists of the operating cost of ships deployed on route r and the fuel cost of all legs (including all non-VSRZ, in-VSRZ and out-VSRZ legs). The operating cost per ship is determined by its type, meaning that the operating cost of ships in the same type is identical. The total operating cost of route r is the sum of operating cost of ships deployed on the route, which is calculated as the operating cost per ship of a certain type multiplied by the number of ships in this type deployed on the route. We calculate the fuel cost of a leg as unit fuel price multiplied by the fuel consumption of the leg, and the fuel consumption of a leg is related to the sailing distance and time of the leg, and the type of ships deployed on the leg. We define $0/0 = 0$ whenever $d_{krl} = 0$ and $t_{rl} = 0$ for all $k \in K$, $r \in R$ and $l \in L_r^{S,in}$. Constraint (3) shows that the sailing time for all legs of route r is equal to the total sailing time of the route, calculated as the rotation time minus the total time spent at all ports of call. The rotation time of a route is equal to the number of ships deployed on the route multiplied by the number of hours in a week since the route provides a weekly service frequency, which means that each port of call on the route will be visited once in a week. Constraints (4) state the speed of each leg should not exceed its speed limit, i.e., the speed of each ship on non-VSRZ and out-VSRZ legs should not exceed the maximum physical speed of the ship, and the speed of each ship sailing through in-VSRZ legs should not exceed the upper speed limit specified by the VSRIP port. The domain of decision variable t_{rl} is defined in Constraints (5).

Lemma 1. *In [P0], if there are no speed limit constraints (4), then in the optimal solution, denoted by* $(\hat{t}_{krl}, l \in L_r)$*, all legs have the same speed, i.e.,* $d_{krl}/\hat{t}_{krl}$ *is the same for all* $l \in L_r$.

The finding in Lemma 1 makes sense. In [P0], the sailing distance and time of route r in plan k are fixed, and due to the approximate cubic relationship between sailing speed and fuel consumption, it is easy to derive that different sailing speeds on a route will lead to higher fuel cost than an average speed.

Lemma 2. *In [P0], the optimal sailing times for legs on route* r *will not decrease with the number of deployed ships* q_r *on the route. That is, if* $q_r^1 < q_r^2$*, and* $(\hat{t}_{krl}^1, l \in L_r)$ *and* $(\hat{t}_{krl}^2, l \in L_r)$ *are the optimal solutions to* $f_{kr}(q_r^1)$ *and* $f_{kr}(q_r^2)$*, respectively, then* $\hat{t}_{krl}^1 \leq \hat{t}_{krl}^2$ *for all* $l \in L_r$.

The approximate cubic relationship between vessel speed and fuel consumption can also be used to explain the rationality of Lemma 2, that is, with the increase of the sailing time, the reduced amount of fuel consumption for each leg caused by adding one unit time is decreased considering the approximate cubic relationship. When the number of deployed ships is q_r^1 in [P0], the optimal solution means that except for the minimum sailing time for guaranteeing the sailing speeds of all legs no higher than their speed limits, the remaining sailing time has been distributed effectively to minimize the amount of fuel consumption. Similarly, when q_r^1 is increased to q_r^2, the total amount of fuel consumption will increase if we reduce the sailing time of some legs from the optimal solution $(\hat{t}_{krl}^1, l \in L_r)$ and add the reduced time to other legs, and the increased sailing time (i.e., $(q_r^2 - q_r^1) \cdot T$) will be distributed to some legs based on $(\hat{t}_{krl}^1, l \in L_r)$.

Proposition 1. *In [P0], the optimal sailing speeds (equivalently, the optimal sailing time) can be derived by Algorithm 1 and have the following structure: the set of legs L_r can be partitioned into two mutually exclusive and collectively exhaustive sets $\hat{L}_{kr}^S$ and $\hat{L}_{kr}^N$; the speed limits (i.e., V_{rl}) of all legs in $\hat{L}_{kr}^S$ are strictly lower than those of legs in $\hat{L}_{kr}^N$; the optimal sailing speed of a leg in $\hat{L}_{kr}^S$ is equal to its speed limit; the optimal sailing speeds for legs in $\hat{L}_{kr}^N$ are the same and are lower than their speed limits.*

It is proved in Lemma 1 that all legs of a route have the same sailing speed in the optimal solution if we do not consider the speed limit. Therefore, in [P0], it is reasonable that we should reduce the difference of sailing speeds among all legs of route r for minimizing the fuel cost. When the speed limit is considered in [P0], the average speed may be higher than the speed limits of some legs on route r, and thus the speeds of these legs have to be reduced to their speed limits in the optimal solution. To minimize the fuel cost of the route, the optimal speeds of the remaining legs should be the same, which are lower than their speed limits.

In the spirit of Wang (2016), we analyze the function $f_{kr}(q_r)$ and state Proposition 2.

Proposition 2. *$f_{kr}(q_r)$ is a strictly integrally convex function in q_r.*

The convexity of $f_{kr}(q_r)$ implies that $f_{kr}(q_r)$ generally first decreases and then increases with q_r. Therefore, with the increase of q_r, the decrease in fuel cost outweighs the increase in operating cost when q_r is small and the increased operating cost is higher than the decreased fuel cost when q_r is large in the general case.

Algorithm 1 Compute $f_{kr}(q_r)$

Input q_r. Output two sets of legs $\hat{L}^S_{kr}$, $\hat{L}^N_{kr}$, optimal sailing times $(t^*_{krl}, l \in L_r)$ and optimal objective function value of [P0], i.e., $f_{kr}(q_r)$.

Step 1. Initialize two temporary sets of legs L^S_{kr} and L^N_{kr}: set $L^S_{kr} \leftarrow \varnothing$, $L^N_{kr} \leftarrow L_r$.

Step 2. Calculate the average speed $\overline{v}^N_{kr}$ in L^N_{kr} by

$$\overline{v}^N_{kr} = \frac{\sum_{l \in L^N_{kr}} d_{krl}}{T \cdot q_r - T_r}. \tag{6}$$

Step 3. If $\overline{v}^N_{kr} < V_{rl}$ for all $l \in L^N_{kr}$, set $\hat{L}^S_{kr} \leftarrow L^S_{kr}$, where the optimal sailing speeds of these legs are equal to their speed limits, set $\hat{L}^N_{kr} \leftarrow L^N_{kr}$, where the optimal speeds of these legs are the same and lower than their speed limits, and then calculate the optimal sailing time of leg l (denoted by t^*_{krl}) for all $l \in L_r$ in plan k by (7) and (8) in order to gain $f_{kr}(q_r)$,

$$t^*_{krl} = \frac{d_{krl}}{V_{rl}}, \forall l \in \hat{L}^S_{kr} \tag{7}$$

$$t^*_{krl} = \frac{d_{krl}}{\overline{v}^N_{kr}}, \forall l \in \hat{L}^N_{kr} \tag{8}$$

and stop.

Step 4. Find out the legs with the lowest speed limit $V^{\min}_r$ from L^N_{kr}:

$$V^{\min}_r = \underset{l \in L^N_{kr}}{\text{minimize}}\, V_{rl}. \tag{9}$$

Set $L^S_{kr} \leftarrow L^S_{kr} \cup \{l \in L^N_{kr} | V_{rl} = V^{\min}_r\}$ and $L^N_{kr} \leftarrow L^N_{kr} \setminus \{l \in L^N_{kr} | V_{rl} = V^{\min}_r\}$. The optimal sailing speeds for legs in L^S_{kr} should be equal to their speed limits. Calculate the optimal total sailing time T^S_{kr} for legs in L^S_{kr}:

$$T^S_{kr} = \sum_{l \in L^S_{kr}} \frac{d_{krl}}{V_{rl}}. \tag{10}$$

Compute the average sailing speed for legs in L^N_{kr}:

$$\overline{v}^N_{kr} = \frac{\sum_{l \in L^N_{kr}} d_{krl}}{T \cdot q_r - T_r - T^S_{kr}}. \tag{11}$$

Go to **Step 3**.

We define Q_{kr} and $\hat{q}_{kr}$ as follows:

$$Q_{kr} = Q_{i_r} - \sum_{r' \in R_{i_r} \setminus \{r\}} q_{kr'}^{\min} \tag{12}$$

$$\hat{q}_{kr} = \operatorname*{arg\,min}_{q_r = q_{kr}^{\min}, q_{kr}^{\min}+1, \ldots, Q_{kr}} f_{kr}(q_r). \tag{13}$$

The value of $\hat{q}_{kr}$ is computed by Algorithm 2. If there are two different integer numbers of deployed ships on route r leading to the minimum cost, we take the smaller one as $\hat{q}_{kr}$. As $f_{kr}(q_r)$ is a strictly integrally convex function, there are three relationships between $q_{kr}^{\min}$ and $\hat{q}_{kr}$, as shown in Fig. 5. If $\hat{q}_{kr} < Q_{kr}$, $\hat{q}_{kr}$ is the value from which $f_{kr}(q_r)$ starts to increase with q_r; otherwise, $\hat{q}_{kr}$ is equal to Q_{kr}. Therefore, we enumerate the number of deployed ships q_r from $q_{kr}^{\min}$ to $\hat{q}_{kr} + 1$ or Q_{kr} and compare $f_{kr}(q_r)$ with $f_{kr}(q_r - 1)$ in Algorithm 2 to compute $\hat{q}_{kr}$.

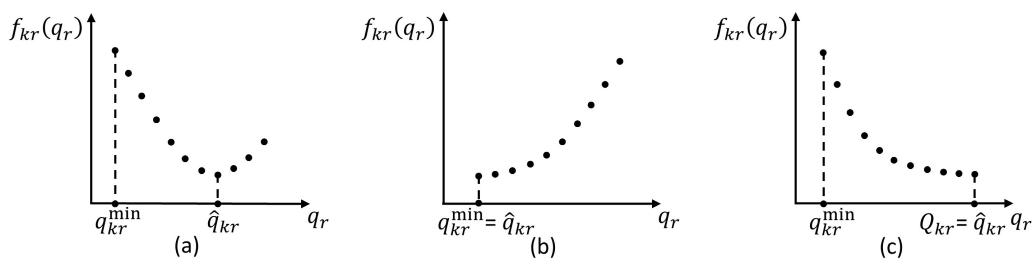

Figure 5: The relationships between $q_{kr}^{\min}$ and $\hat{q}_{kr}$

Algorithm 2 Calculate $\hat{q}_{kr}$

Set $q_r \leftarrow q_{kr}^{\min}$. Compute $f_{kr}(q_r)$ by Algorithm 1.
  do
    Set $q_r \leftarrow q_r + 1$. Compute $f_{kr}(q_r)$.
  while $f_{kr}(q_r) < f_{kr}(q_r - 1)$ and $q_r + 1 \leq Q_{kr}$.
  if $f_{kr}(q_r) < f_{kr}(q_r - 1)$ **then**
    Set $\hat{q}_{kr} \leftarrow q_r$.
  else
    Set $\hat{q}_{kr} \leftarrow q_r - 1$.
  end if

3.4. Integrated model

We develop a mixed-integer non-linear programming model on the schedule design problem for a liner shipping company considering VSRIPs in this section, which can be formulated as:

[P1]

$$\text{minimize} \sum_{i\in I}\sum_{r\in R_i}\sum_{l\in L_r} w\cdot y_{rl}\cdot \alpha_i \left(\frac{y_{rl}}{t_{rl}}\right)^{\beta_i} + \sum_{i\in I}\sum_{r\in R_i}\bar{c}_i\cdot q_r - \sum_{i\in I}\sum_{r\in R_i}\sum_{l\in L_r^{S,in}, p=p_{rl}^S}\sum_{j\in J_p}\frac{1}{2}\tilde{c}_{ipj}\cdot x_{pj} \tag{14}$$

subject to

$$\sum_{l\in L_r} t_{rl} = T\cdot q_r - T_r, \forall r\in R \tag{15}$$

$$\sum_{r\in R_i} q_r \le Q_i, \forall i\in I \tag{16}$$

$$\sum_{j\in J_p} x_{pj} = 1, \forall p\in P^S \tag{17}$$

$$y_{rl} = d_{rl}, \forall r\in R, \forall l\in L_r^N \tag{18}$$

$$y_{rl} = \sum_{j\in J_p} d_{pj}\cdot x_{pj}, \forall r\in R, \forall l\in L_r^{S,in}, p = p_{rl}^S \tag{19}$$

$$y_{rl} + y_{r,l+1} = d_{r,l,l+1}, \forall r\in R, \forall l\in L_r^{S,in} \tag{20}$$

$$\frac{y_{rl}}{t_{rl}} \le V_{rl}, \forall r\in R, \forall l\in L_r \tag{21}$$

$$q_r\in\mathbb{N}, \forall r\in R \tag{22}$$

$$t_{rl}\ge 0, \forall r\in R, \forall l\in L_r \tag{23}$$

$$x_{pj}\in\{0,1\}, \forall p\in P^S, \forall j\in J_p \tag{24}$$

$$y_{rl}\ge 0, \forall r\in R, \forall l\in L_r. \tag{25}$$

The objective function of model [P1] contains three terms. The first term is the total fuel cost, consisting of fuel costs of non-VSRZ, in-VSRZ and out-VSRZ legs. We define $0/0 = 0$ whenever $y_{rl} = 0$ and $t_{rl} = 0$ for all $r \in R$ and $l \in L_r^{S,in}$. The second term is the total operating cost of ships, i.e., the sum of operating cost of ships in each type. The third term is the total refund from VSRIPs. Since a visit consists of a ship entering and leaving the port, which means that two in-VSRZ legs related to the port are involved in a visit, the chosen VSRZ at a VSRIP port is the same for the two legs. Therefore, regarding the third term, each of the two in-VSRZ legs should correspond to one half of the refund. Constraints (15) state that the sailing time for all legs of a route should be equal to the rotation time minus the total time spent at all ports of call. Constraints (16) ensure that for each ship type, the number of deployed ships does not exceed the total number of ships in the fleet. Constraints (17) state that all ships visiting a port choose the same VSRZ (recall that a VSRZ with radius 0 is defined to account for the option of not slowing down near the port). Constraints (18) confirm the sailing distances of non-VSRZ legs. Constraints (19) establish the connection between the chosen VSRZ at each VSRIP port and the sailing distances of in-VSRZ legs. Since the VSRZs are chosen in model [P0], the sailing distances of all non-VSRZ, in-VSRZ and out-VSRZ legs are fixed. However, the sailing distances of in-VSRZ and out-VSRZ legs are related to the decision on the choice of VSRZs in model [P1] and will be determined by Constraints (19). Constraints (20) indicate that the total sailing distance of each pair of in-VSRZ and out-VSRZ legs is fixed. Constraints (21) state the speed limit for each leg. Constraints (22)–(25) define the domains of the decision variables.

We now find that the problem defined by [P1] is NP-hard:

Proposition 3. *The schedule design problem under VSRIPs for a liner shipping company formulated in model [P1] is NP-hard.*

4. Algorithms

Based on some properties of the proposed model, we put forward two algorithms in this section. The first algorithm enumerates each chosen VSRZ at each VSRIP port and the number of deployed ships on each route to determine the optimal solution. This is an exact algorithm which is efficient for the solution of instances with a large number of routes and a small number of VSRIP ports. The second algorithm transforms the non-linear objective function and constraints into linear ones. Its computation time is less than that of the first algorithm for instances with a large number of VSRIP ports and a small number of routes. In addition, the maximum approximation error of the second algorithm can be controlled within a prespecified tolerance level.

4.1. Enumerative algorithm based on properties

Since there are only several VSRIP ports in reality, it is easy to enumerate all VSRZ plans. In each plan, the dockage refunds as well as the sailing distances of all non-VSRZ, in-VSRZ and out-VSRZ legs are fixed. Without considering the limited number of ships of each type in the fleet, the total cost function of all routes in each plan can be further decomposed into several functions, each for one route. Note that the objective function of model [P0] (i.e., $f_{kr}(q_r)$) is the minimum sum of operating cost and fuel cost of route r for plan k with a given number of deployed ships q_r. Therefore, we can obtain the optimal solution of each route with a given plan and a fixed number of deployed ships by referring to the properties of $f_{kr}(q_r)$.

Based on the above analysis, we develop Algorithm 3, which is an enumerative algorithm, to solve model [P1].

The above lemmas and propositions have focused on the analysis of the properties of model [P1] with a fixed VRSZ plan. Next, we analyze properties of two special cases (special case 1 and special case 2) of model [P1] from a new perspective, i.e., how the compliance of a VSRIP at a port will change with the number of deployed ships. These properties are summarized in Propositions 4 and 5. Recall that the rules of a VSRIP are considered to be complied with only when a non-zero VSRZ of the program is chosen. In special case 1, we consider only one route $r = 1$ ($|R| = 1$), only one port is a VSRIP port ($|P^S| = 1$), denoted by port p, and port p is called only once on route r. Suppose that we analyze the case with the increase of the number of deployed ships q_r on the route from $q_r^{\min}$, where $q_r^{\min}$ is the minimum integer number of deployed ships calculated by letting ships sail at the maximum physical speed on route r, and for a given q_r, we compute the optimal VSRZ to choose at port p and the optimal speed on each leg that minimize the total cost of the route, including fuel cost and operating cost minus dockage refund. In special case 2, there is only one route r, which includes $|P^S|$ ($|P^S| \geq 2$) VSRIP ports; each VSRIP port has only one non-zero VSRZ and is called only once; all the VSRZs have the same radius and speed limit; the dockage refunds at the $|P^S|$ VSRIP ports are all different. Denote by $p^{(1)}, ..., p^{(|P^S|)}$ the VSRIP ports in the decreasing order of dockage refund. Moreover, we consider two situations in special case 2, case 2.1 and case 2.2, with the only difference on dockage refunds of the $|P^S|$ VSRIPs. Suppose that the dockage refunds of VSRIP port $p \in P^S$ in the two special cases are identical, and we have $p = p^{(1)}$ in case 2.1 and $p = p^{(n)}, n \geq 2$ in case 2.2.

Algorithm 3 Enumerative algorithm

Step 1. Define K as the set of all VSRZ plans, where $|K| = \Pi_{p \in P^S} |J_p|$, and $\tilde{C}_k$ as the total dockage refunds of plan $k \in K$. Recall that a VSRZ plan includes the total chosen VSRZs of all VSRIP ports.

Step 2. For each $k \in K$

Step 2.1. For each $r \in R$, obtain $\hat{q}_{kr}$ and $f_{kr}(q_r), q_r = q_{kr}^{\min}, q_{kr}^{\min} + 1, ..., \hat{q}_{kr}$ by Algorithm 2.

Step 2.2. Solve the following model by the pseudo-polynomial-time Algorithm EC.7.1 in the Electronic Companion.

$$[\text{P2}] \quad TotalCost_k = -\tilde{C}_k + \underset{(q_r, r \in R)}{\text{minimize}} \sum_{r \in R} f_{kr}(q_r) \tag{26}$$

subject to

$$\sum_{r \in R_i} q_r \leq Q_i, \forall i \in I \tag{27}$$

$$q_r \geq q_{kr}^{\min}, \forall r \in R \tag{28}$$

$$q_r \leq \hat{q}_{kr}, \forall r \in R \tag{29}$$

$$q_r \in \mathbb{N}, \forall r \in R. \tag{30}$$

Step 3. Obtain the optimal plan k^* with the minimum total cost by $k^* = \arg\min_{k \in K} TotalCost_k$.

Proposition 4. *The compliance of the VSRIP in special case 1 with the increase of the number of deployed ships q_r on route r is organized as follows. If there is only one non-zero VSRZ in the VSRIP, then there exists a threshold of the number of deployed ships, denoted by q_{rp}^{comp1}, such that ships on the route will participate in the program if and only if $q_r \geq q_{rp}^{\text{comp1}}$. If there are two non-zero VSRZs, then there exist two thresholds, denoted by q_{rp}^{comp1} and q_{rp}^{comp2}, $q_{rp}^{\text{comp1}} \leq q_{rp}^{\text{comp2}}$, such that ships will choose the non-zero VSRZ with a longer radius if and only if $q_r \geq q_{rp}^{\text{comp2}}$ and the non-zero VSRZ with a shorter radius if and only if $q_{rp}^{\text{comp1}} \leq q_r \leq q_{rp}^{\text{comp2}} - 1$ (note that if $q_{rp}^{\text{comp1}} = q_{rp}^{\text{comp2}}$, then the non-zero VSRZ with a shorter radius will never be chosen).*

Proposition 4 indicates that the company tends to comply with VSRIPs when more ships are deployed. To the best of our knowledge, when the shipping market is at the trough, the operating cost of a ship will decrease and hence more ships will be deployed on liner routes, meaning more shipping companies will comply with VSRIPs.

Proposition 5. *The compliance of VSRIPs in special case 2 with the increase of the number of deployed ships q_r on route r is organized as follows. (i) There exist $|P^S|$ thresholds, denoted by $q_{rp^{(1)}}^{\text{comp}}, ..., q_{rp^{(|P^S|)}}^{\text{comp}}$, $q_{rp^{(1)}}^{\text{comp}} \leq q_{rp^{(2)}}^{\text{comp}} \leq ... \leq q_{rp^{(|P^S|)}}^{\text{comp}}$, such that ships will participate in the VSRIP at port $p^{(n)}, n = 1, ..., |P^S|$ if and only if $q_r \geq q_{rp^{(n)}}^{\text{comp}}$. (ii) For VSRIP port $p \in P^S$ with the same dockage refund in special cases 2.1 and 2.2, there exist two thresholds of port p in the two cases, denoted by $q_{rp^{(1)}}^{\text{comp}\prime}$ and $q_{rp^{(n)}}^{\text{comp}\prime}$, $q_{rp^{(1)}}^{\text{comp}\prime} \leq q_{rp^{(n)}}^{\text{comp}\prime}$, such that ships will participate in the VSRIP at port p in cases 2.1 and 2.2 if and only if $q_r \geq q_{rp^{(1)}}^{\text{comp}\prime}$ and $q_r \geq q_{rp^{(n)}}^{\text{comp}\prime}$, respectively.*

Proposition 5 shows that different VSRIP ports have competition to attract ships to slow down when they serve the same route. We can obtain the following managerial implication: for instance, if only one VSRIP port is located in Asia and mainly serves intra-Asia shipping routes, then the VSRIP will be very effective because there are no other ports on the routes that adopt VSRIPs.

4.2. Piecewise-linear approximation algorithm

We now describe an approximation algorithm to solve model [P1] directly without enumerating the compliance decisions of VSRIPs and the number of deployed ships. We first rewrite constraints (21) as

$$t_{rl} \geq \frac{d_{rl}}{V_{rl}}, \forall r \in R, \forall l \in L_r^N \tag{31}$$

$$t_{rl} \geq \frac{y_{rl}}{V_{rl}}, \forall r \in R, \forall l \in L_r^{S,in} \tag{32}$$

$$t_{r,l+1} \geq \frac{d_{r,l,l+1} - y_{rl}}{V_{r,l+1}}, \forall r \in R, \forall l+1 \in L_r^{S,out}. \tag{33}$$

Since the objective function of model [P1] is non-linear, we linearize the model to take advantage of the CPLEX solver. For the linearization, the following variables are added.

Decision variables

- u_{rl} Fuel consumption for non-VSRZ leg $l \in L_r^N$ on route $r \in R$
- u_{rl}^{in} Fuel consumption for in-VSRZ leg $l \in L_r^{S,in}$ on route $r \in R$
- u_{rl}^{out} Fuel consumption for out-VSRZ leg $l \in L_r^{S,out}$ on route $r \in R$.

There are two types of decision variables (i.e., t_{rl} and y_{rl}) in the non-linear part of the objective function in [P1], which are replaced by three types of alternative variables u_{rl}, u_{rl}^{in} and u_{rl}^{out} in the following model [P3]:

$$\text{[P3]} \quad \underset{q_r, t_{rl}, u_{rl}, u_{rl}^{in}, u_{rl}^{out}, x_{pj}, y_{rl}}{\text{minimize}} \sum_{r \in R} w \left[\sum_{l \in L_r^N} u_{rl} + \sum_{l \in L_r^{S,in}} u_{rl}^{in} + \sum_{l \in L_r^{S,out}} u_{rl}^{out} \right] + \sum_{i \in I} \sum_{r \in R_i} \overline{c}_i \cdot q_r$$

$$- \sum_{i \in I} \sum_{r \in R_i} \sum_{l \in L_r^{S,in}, p = p_{rl}^S} \sum_{j \in J_p} \frac{1}{2} \tilde{c}_{ipj} \cdot x_{pj} \tag{34}$$

subject to (15)–(20), (22)–(25), (31)–(33) and

$$u_{rl} \geq d_{rl} \cdot \alpha_i \left(\frac{d_{rl}}{t_{rl}} \right)^{\beta_i}, \forall i \in I, \forall r \in R_i, \forall l \in L_r^N \tag{35}$$

$$u_{rl}^{in} \geq y_{rl} \cdot \alpha_i \left(\frac{y_{rl}}{t_{rl}} \right)^{\beta_i}, \forall i \in I, \forall r \in R_i, \forall l \in L_r^{S,in} \tag{36}$$

$$u_{r,l+1}^{out} \geq (d_{r,l,l+1} - y_{rl}) \cdot \alpha_i \left(\frac{d_{r,l,l+1} - y_{rl}}{t_{r,l+1}} \right)^{\beta_i}, \forall i \in I, \forall r \in R_i, \forall l+1 \in L_r^{S,out} \tag{37}$$

$$u_{rl} \geq 0, \forall r \in R, \forall l \in L_r^N \tag{38}$$

$$u_{rl}^{in} \geq 0, \forall r \in R, \forall l \in L_r^{S,in} \tag{39}$$

$$u_{rl}^{out} \geq 0, \forall r \in R, \forall l \in L_r^{S,out}. \tag{40}$$

It is easy to enumerate all leg options for each leg since there are not many (see Fig. 4).

The sailing distances of in-VSRZ leg $l \in L_r^{S,in}$ (i.e., y_{rl}) including VSRIP port p and out-VSRZ leg $l+1 \in L_r^{S,out}$ (i.e., $d_{r,l,l+1} - y_{rl}$) are fixed in an option j of this pair of in-VSRZ leg l and out-VSRZ leg $l+1$, i.e., d_{pj} and $d_{r,l,l+1} - d_{pj}$. To linearize the three constraints (35), (36) and (37), we define three new functions:

$$F_{rl}(t_{rl}) = d_{rl} \cdot \alpha_i \left(\frac{d_{rl}}{t_{rl}} \right)^{\beta_i}, \forall i \in I, \forall r \in R_i, \forall l \in L_r^N \tag{41}$$

$$F_{rlj}^{in}(t_{rl}) = d_{pj} \cdot \alpha_i \left(\frac{d_{pj}}{t_{rl}} \right)^{\beta_i}, \forall i \in I, \forall r \in R_i, \forall l \in L_r^{S,in}, p = p_{rl}^S, \forall j \in J_p \tag{42}$$

$$\begin{aligned} F_{r,l+1,j}^{out}(t_{r,l+1}) = (d_{r,l,l+1} - d_{pj}) \cdot \alpha_i \left(\frac{d_{r,l,l+1} - d_{pj}}{t_{r,l+1}} \right)^{\beta_i}, \forall i \in I, \forall r \in R_i, \\ \forall l+1 \in L_r^{S,out}, p = p_{rl}^S, \forall j \in J_p, \end{aligned} \tag{43}$$

where the minimum value of t_{rl} (denoted by $\underline{t}_{rl}$) for non-VSRZ leg $l \in L_r^N$ is d_{rl}/V_{rl}, $\underline{t}_{rl}$ for option j of in-VSRZ leg $l \in L_r^{S,in}$ including VSRIP port p is d_{pj}/V_{rl}, and $\underline{t}_{r,l+1}$ for option j of out-VSRZ leg $l+1 \in L_r^{S,out}$ is $(d_{r,l,l+1} - d_{pj})/V_{r,l+1}$ due to constraints (31), (32) and (33). It is easy to prove that $F_{rl}(t_{rl})$, $F_{rlj}^{in}(t_{rl})$ and $F_{rlj}^{out}(t_{rl})$ are all convex functions. Hence, we can use some piecewise-linear functions to approximate each convex function and control the approximation error within a tolerance level by employing a suitable underestimation approximation scheme. The piecewise-linear approximating functions must also be convex. As a result, each of the three functions can be transformed into the sum of the piecewise-linear approximating functions. Since the three functions are similar to each other, we will illustrate the approximating algorithm of $F_{rl}(t_{rl})$, and then $F_{rlj}^{in}(t_{rl})$ and $F_{rlj}^{out}(t_{rl})$ can be approximated by the same scheme. Referring to Wang and Meng (2012), a piecewise-linear approximation algorithm of $F_{rl}(t_{rl})$ with the approximation error ϵ_{rl} is presented in Algorithm 4, and we can obtain some approximation lines of $F_{rl}(t_{rl})$, including line $F_{rl} = 0$.

Proposition 6. *The number of lines B_{rl} in Algorithm 4 is the smallest among all piecewise-linear underestimation approximation schemes when approximation errors are not greater than ϵ_{rl}. Furthermore, Algorithm 4 can achieve the unique solution of approximation lines when $\epsilon_{rl}^0 = \epsilon_{rl}$ (ϵ_{rl}^0 is the error between $F_{rl} = 0$ and $F_{rl}(\bar{t}_{rl})$ where $\bar{t}_{rl}$ is the zero point of line B_{rl}).*

According to Proposition 6, define the smallest number of approximation lines as B_{rl}^*

Algorithm 4 Piecewise-linear approximation algorithm

Step 0. Define a set of lines Ψ including line $F_{rl} = 0$ initially. Set $b = 1$, $\epsilon_{rl}^b = \epsilon_{rl}$, $t_{rl}^b = \underline{t}_{rl}$, $F_{rl}^b = F_{rl}(t_{rl}^b) - \epsilon_{rl}$. If $F_{rl}^b \leq 0$, stop; else go to **Step 1**.

Step 1. Let line b pass the point (t_{rl}^b, F_{rl}^b) and support the epigraph of $F_{rl}(t_{rl})$. Suppose that line b supports the epigraph of $F_{rl}(t_{rl})$ at the point $(\hat{t}_{rl}^b, \hat{F}_{rl}^b)$, see Fig. 6a. Then

$$\frac{\hat{F}_{rl}^b - F_{rl}^b}{\hat{t}_{rl}^b - t_{rl}^b} = \frac{dF_{rl}(\hat{t}_{rl}^b)}{d\hat{t}_{rl}^b} = -\alpha_i \cdot \beta_i \cdot (d_{rl})^{\beta_i+1} \cdot (\hat{t}_{rl}^b)^{-(\beta_i+1)}. \tag{44}$$

According to (41), we have

$$\hat{F}_{rl}^b = d_{rl} \cdot \alpha_i \cdot (\frac{d_{rl}}{\hat{t}_{rl}^b})^{\beta_i}. \tag{45}$$

Combining the two equations above, $\hat{t}_{rl}^b$ and $\hat{F}_{rl}^b$ can be estimated by a bisection search method. Line b is written as

$$F_{rl} - F_{rl}^b = \frac{\hat{F}_{rl}^b - F_{rl}^b}{\hat{t}_{rl}^b - t_{rl}^b} \cdot (t_{rl} - t_{rl}^b). \tag{46}$$

Calculate the slope_{rl}^b and the intercept_{rl}^b for tangent line b and add it to Ψ.

Step 2. Set $b = b+1$. We find a point (t_{rl}^b, F_{rl}^b) on line $b-1$ by a bisection search method such that $F_{rl}^b = F_{rl}(t_{rl}^b) - \epsilon_{rl}$. (i) If $F_{rl}^b \leq 0$, find out the zero point of line $b-1$, denoted by $\bar{t}_{rl}$, and set $\epsilon_{rl}^0 = F_{rl}(\bar{t}_{rl})$ (when $t_{rl} \geq \bar{t}_{rl}$, the gap between $F_{rl} = 0$ and $F_{rl}(t_{rl})$ is not greater than ϵ_{rl}^0, see Fig. 6a). (ii) Else, set $\epsilon_{rl}^b = \epsilon_{rl}$ and go to **Step 1**.

Step 3. Let B_{rl} be the number of lines in Ψ except line $F_{rl} = 0$, i.e., $B_{rl} = b - 1$. These lines are expressed by the generic form

$$F_{rl} = \text{slope}_{rl}^b \cdot t_{rl} + \text{intercept}_{rl}^b, b = 1, 2, ..., B_{rl}. \tag{47}$$

The piecewise-linear approximating function of $F_{rl}(t_{rl})$ is

$$\overline{F}_{rl}(t_{rl}) = \begin{cases} \text{maximize}\left\{\text{slope}_{rl}^b \cdot t_{rl} + \text{intercept}_{rl}^b, b = 1, 2, ..., B_{rl}\right\} & \text{if } \underline{t}_{rl} \leq t_{rl} < \bar{t}_{rl} \\ 0 & \text{if } t_{rl} \geq \bar{t}_{rl} \end{cases}, \tag{48}$$

which is shown by the thickest solid line in Fig. 6b.

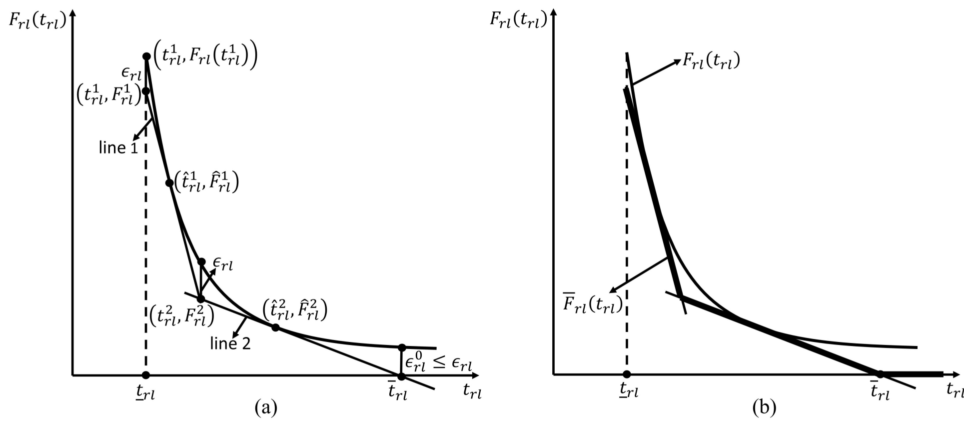

Figure 6: A piecewise-linear approximating function illustration

when the approximation error is ϵ_{rl}. The piecewise-linear approximating function $\overline{F}_{rl}(t_{rl})$ with the error of ϵ_{rl} is convex because $F_{rl}(t_{rl})$ is a convex function. Hence we infer that

$$F_{rl}(t_{rl}) - \epsilon_{rl} \leq \overline{F}_{rl}(t_{rl}) \leq F_{rl}(t_{rl}), \forall r \in R, \forall l \in L_r^N, \forall t_{rl} \geq \underline{t}_{rl}. \tag{49}$$

To formulate $\overline{F}_{rl}(t_{rl})$ by the CPLEX solver, we define new decision variables F_{rl}, and the non-linear constraints (35) can be transformed into linear constraints (50) and (51):

$$F_{rl} \geq \text{slope}_{rl}^b \cdot t_{rl} + \text{intercept}_{rl}^b, \forall r \in R, \forall l \in L_r^N, \forall b = 1, 2, ..., B_{rl}^* \tag{50}$$

$$u_{rl} \geq F_{rl}, \forall r \in R, \forall l \in L_r^N. \tag{51}$$

Using the same underestimation approximation scheme, we obtain the approximating functions of $F_{rlj}^{in}(t_{rl})$ and $F_{rlj}^{out}(t_{rl})$ as well as the smallest numbers of lines $(B_{rlj}^{in})^*$ and $(B_{rlj}^{out})^*$. Constraints (36) and (37) are linearized as follows:

$$F_{rlj}^{in} \geq (\text{slope}^{in})_{rlj}^b \cdot t_{rl} + (\text{intercept}^{in})_{rlj}^b - M_1 \cdot (1 - x_{pj}), \forall r \in R, \forall l \in L_r^{S,in}, p = p_{rl}^S, \forall j \in J_p, \forall b = 1, 2, ..., (B_{rlj}^{in})^* \tag{52}$$

$$u_{rl}^{in} \geq F_{rlj}^{in}, \forall r \in R, \forall l \in L_r^{S,in}, p = p_{rl}^S, \forall j \in J_p \tag{53}$$

$$F_{r,l+1,j}^{out} \geq (\text{slope}^{out})_{r,l+1,j}^b \cdot t_{r,l+1} + (\text{intercept}^{out})_{r,l+1,j}^b - M_2 \cdot (1 - x_{pj}), \forall r \in R,$$

$$\forall l+1 \in L_r^{S,out}, p = p_{rl}^S, \forall j \in J_p, \forall b = 1,2,...,(B_{r,l+1,j}^{out})^* \tag{54}$$

$$u_{r,l+1}^{out} \geq F_{r,l+1,j}^{out}, \forall r \in R, \forall l+1 \in L_r^{S,out}, p = p_{rl}^S, \forall j \in J_p, \tag{55}$$

where $M_1 = \text{maximize}\left\{(\text{intercept}^{in})_{rlj}^b, \forall r \in R, \forall l \in L_r^{S,in}, p = p_{rl}^S, \forall j \in J_p, \forall b = 1,2,...,(B_{rlj}^{in})^*\right\}$ and $M_2 = \text{maximize}\left\{(\text{intercept}^{out})_{r,l+1,j}^b, \forall r \in R, \forall l+1 \in L_r^{S,out}, p = p_{rl}^S, \forall j \in J_p, \forall b = 1,2,...,(B_{r,l+1,j}^{out})^*\right\}$.

We derive from Proposition 1 that the optimal speeds outside VSRZs on one route should be identical. Hence, we add two new constraints (56) and (57) regarding the sailing speed outside VSRZs for route $r \in \hat{R}$, where $\hat{R}$ is the set of all routes with at least one non-VSRZ leg, to optimize the solution designed by the piecewise-linear approximation algorithm.

$$\frac{t_{rl_1}}{d_{rl_1}} = \frac{t_{rl_2}}{d_{rl_2}}, \forall r \in \hat{R}, \forall l_1, l_2 \in L_r^N, l_1 < l_2 \tag{56}$$

$$\frac{t_{rl_1}}{d_{rl_1}} - M_3(1-x_{pj}) \leq \frac{t_{r,l_2+1}}{d_{r,l_2,l_2+1} - d_{pj}} \leq \frac{t_{rl_1}}{d_{rl_1}} + M_3(1-x_{pj}), \forall r \in \hat{R}, \forall l_1 \in L_r^N,$$
$$\forall l_2+1 \in L_r^{S,out}, p = p_{rl_2}^S, \forall j \in J_p, \tag{57}$$

where M_3 is a sufficiently large positive number. We also present constraints (58) for route $r \in \tilde{R}$, where $\tilde{R}$ is the set of all routes without non-VSRZ legs.

$$\frac{t_{r,l_1+1}}{d_{r,l_1,l_1+1} - d_{p_1j_1}} - M_3(2 - x_{p_1j_1} - x_{p_2j_2}) \leq \frac{t_{r,l_2+1}}{d_{r,l_2,l_2+1} - d_{p_2j_2}}$$
$$\leq \frac{t_{r,l_1+1}}{d_{r,l_1,l_1+1} - d_{p_1j_1}} + M_3(2 - x_{p_1j_1} - x_{p_2j_2}),$$
$$\forall r \in \tilde{R}, \forall l_1+1, l_2+1 \in L_r^{S,out}, l_1 < l_2, p_1 = p_{rl_1}^S, \forall j_1 \in J_{p_1}, p_2 = p_{rl_2}^S, \forall j_2 \in J_{p_2}. \tag{58}$$

Therefore, we can derive a new linear model [P4], which employs the same objective function as [P3] and is subject to constraints (15)–(20), (22)–(25), (31)–(33), (38)–(40) and (50)–(58) with decision variables q_r, t_{rl}, u_{rl}, u_{rl}^{in}, u_{rl}^{out}, x_{pj} and y_{rl}.

Let $\text{OBJ}_{[\text{P1}]}$ denote the optimal objective value of [P1]. We solve [P4] by the CPLEX and the optimal objective value of [P4] is a lower bound (denoted by LB) of $\text{OBJ}_{[\text{P1}]}$ because [P1] is transformed into [P4] by a piecewise-linear underestimation approximation algorithm. Suppose the optimal solution of [P4] is q_r^*, t_{rl}^*, u_{rl}^*, $(u_{rl}^{in})^*$, $(u_{rl}^{out})^*$, x_{pj}^* and y_{rl}^*, which is also the feasible solution but may not be the optimal one of [P1]. We plug the optimal solution of [P4] into the objective function of [P1], and therefore the result, recorded as an upper bound (UB), should be no less than $\text{OBJ}_{[\text{P1}]}$. The UB is calculated below:

$$\text{UB} = \sum_{i\in I}\sum_{r\in R_i}\sum_{l\in L_r} w \cdot y_{rl}^* \cdot \alpha_i \left(\frac{y_{rl}^*}{t_{rl}^*}\right)^{\beta_i} + \sum_{i\in I}\sum_{r\in R_i} \overline{c}_i \cdot q_r^* - \sum_{i\in I}\sum_{r\in R_i}\sum_{l\in L_r^{S,in},p=p_{rl}^S}\sum_{j\in J_p} \frac{1}{2}\tilde{c}_{ipj} \cdot x_{pj}^*. \tag{59}$$

The maximum approximation error ϵ of model [P4] is related to the approximation error of $F_{rl}(t_{rl})$ (i.e., ϵ_{rl}), $F_{rlj}^{in}(t_{rl})$ (denoted by ϵ_{rl}^{in}) and $F_{rlj}^{out}(t_{rl})$ (denoted by ϵ_{rl}^{out}). Define

$$\epsilon = \sum_{r\in R} w \left[\sum_{l\in L_r^N} \epsilon_{rl} + \sum_{l\in L_r^{S,in}} \epsilon_{rl}^{in} + \sum_{l\in L_r^{S,out}} \epsilon_{rl}^{out}\right]. \tag{60}$$

According to the property of the proposed piecewise-linear approximation algorithm, the gap between UB and LB should be no more than the maximum approximation error ϵ. Hence, we conclude that

$$\text{LB} \leq \text{OBJ}_{[\text{P1}]} \leq \text{UB} \leq \text{LB} + \epsilon. \tag{61}$$

5. Computational study

To validate the efficiency of the two proposed algorithms, we use some routes, which include existing VSRIP ports of LSA, LGB and the Port of New York (NYK), provided by American President Lines (APL). Extensive numerical experiments are conducted by using a personal computer (Intel Core i7, 2.5GHz; Memory, 8G) and model [P4] can be solved by CPLEX12.5.1 with the technology of C# (Visual Studio 2012).

Some parameter settings of the case study are as follows. APL operates a modern fleet of 86 owned and chartered ships, which are divided into four types, 2,000-TEU, 6,000-TEU, 10,000-TEU and 14,000-TEU, in our study (see Table 1). The operating cost of a ship is the sum of manning cost, stores and lubricants, repairs and maintenance, insurance, interest, debt repayment and some general costs. The conversion factors of fuel consumption are devised by referring to Wang and Meng (2012). We selected 12 routes, each of which includes one VSRIP port, from the official website of APL and designed the type of deployed ships for each route, as shown in Table 2. We use the shortest sailing distance between each two consecutive ports of call in the experiment. The radii of non-zero VSRZs, the proportion of dockage refund and the speed limit for VSRZs of the three VSRIP ports are shown in Table 3. The estimated refund for a ship visit in a non-zero VSRZ is determined by the published daily dockage rate of the ship at the related port and the proportion of dockage refund in the VSRZ (see Table 4). According to the price of bunker fuel in January 2018, we set the average fuel price at 410 USD/ton. All approximation errors (ϵ_{rl}, ϵ_{rl}^{in} and ϵ_{rl}^{out}) are designed as 1 ton.

Table 1: Ship fleet profile

Ship type	1	2	3	4
TEU capacity	2,000	6,000	10,000	14,000
Number of ships	21	30	29	6
Max. speed (knots)	20.5	25	23.5	23.5
Operating cost (USD/(week·ship))	77,000	301,000	399,000	483,000
Conversion factors (α_i, β_i)	$(4.5 \times 10^{-4}, 2)$	$(2 \times 10^{-4}, 2.3)$	$(5 \times 10^{-4}, 2.1)$	$(3.5 \times 10^{-4}, 2.2)$

Table 2: Ship route information

No.	Ship type	Portcalls (port time, hours)
1	10,000-TEU	Tianjin(48)→Qingdao(24)→Shanghai(24)→Prince Rupert(24)→LGB(72)→Oakland(24) →Tianjin
2	10,000-TEU	Dalian(12)→Lianyungang(24)→Shanghai(48)→Ningbo(24)→LGB(96)→Seattle(24) →Dalian
3	6,000-TEU	Qingdao(24)→Shanghai(48)→Ningbo(48)→LSA(72)→Oakland(24)→Tokyo(12) →Qingdao
4	6,000-TEU	Ningbo(24)→Shanghai(48)→Busan(24)→LGB(96)→Busan(24)→Ningbo
5	2,000-TEU	Naha(12)→Qingdao(24)→Shanghai(48)→Busan(24)→LSA(72)→Oakland(24) →Dutch Harbour(24)→Yokohama(24)→Busan(24)→Naha
6	2,000-TEU	Kobe(24)→Nagoya(12)→Tokyo(12)→Sendai(12)→LSA(48)→Oakland(24)→Tokyo(12) →Nagoya(12)→Kobe
7	14,000-TEU	Fuqing(24)→Nansha(24)→Hong Kong(24)→Yantian(24)→Xiamen(24)→LSA(96) →Oakland(48)→Fuqing
8	10,000-TEU	Cai Mep(12)→Shekou(12)→Hong Kong(24)→Yantian(24)→Kaohsiung(12)→LGB(96) →Kaohsiung(24)→Cai Mep
9	2,000-TEU	Yantian(24)→Hong Kong(24)→Kaohsiung(24)→Taipei(24)→LSA(72)→Oakland(24) →Tacoma(24)→Kaohsiung(24)→Yantian
10	10,000-TEU	Xiamen(24)→Kaohsiung(24)→Hong Kong(24)→Yantian(24)→Colon(24)→NYK(48) →Baltimore(24)→Norfolk(24)→Xiamen
11	6,000-TEU	Qingdao(48)→Ningbo(24)→Shanghai(24)→Busan(24)→Colon(24)→Savannah(24) →Charleston(24)→NYK(24)→Colon(24)→Qingdao
12	6,000-TEU	Hong Kong(12)→Yantian(24)→Ningbo(24)→Shanghai(24)→NYK(24)→Norfolk(24) →Savannah(48)→Charleston(24)→Hong Kong

Table 3: Information on VSRIP ports

VSRIP port	Radius of non-zero VSRZ (nm) & Proportion of refund	Speed limit (knots)
LSA	20 & 15%; 40 & 30%	12
LGB	20 & 15%; 40 & 25%	12
NYK	20 & 15%	10

Table 4: Dockage refund (USD)

Ship type	LSA		LGB		NYK
	20 nm	40 nm	20 nm	40 nm	20 nm
1	438	877	432	721	479
2	971	1,942	957	1,595	1,062
3	1,441	2,881	1,420	2,367	1,388
4	1,572	3,144	1,551	2,584	1,522

5.1. Performances of the enumerative and piecewise-linear approximation algorithms for the APL case

Using the proposed enumerative algorithm, we have 18 VSRZ plans and obtain an optimal solution for the 12 routes operated by APL, as shown in Table 5. The table shows the results regarding the number of deployed ships on each route, the choice of VSRZ for each VSRIP port, and the sailing speeds within and outside VSRZs. APL participates in the programs at LSA and LGB with 40 and 20 nm VSRZs, respectively. The optimal sailing speeds in the two chosen VSRZs are 12 knots. The optimal speeds for non-VSRZ and out-VSRZ legs on one route are the same, but for these legs on different routes are different. The minimum total cost of model [P1] achieved by the enumerative algorithm is 35,589,249 USD.

Table 5: Computational results by enumerative or piecewise-linear approximation algorithm

No.	Number of deployed ships	VSRIP port (Chosen VSRZ, nm)	Optimal speed outside VSRZ (knots)	Optimal speed within VSRZ (knots)
1	6	LGB (20)	16.23	12
2	6	LGB (20)	16.48	12
3	6	LSA (40)	15.67	12
4	5	LGB (20)	18.97	12
5	7	LSA (40)	14.54	12
6	6	LSA (40)	12.88	12
7	6	LSA (40)	17.40	12
8	7	LGB (20)	15.13	12
9	8	LSA (40)	12.41	12
10	10	NYK (0)	15.79	N.A.
11	10	NYK (0)	15.91	N.A.
12	9	NYK (0)	17.87	N.A.

The computational results on the number of deployed ships on each route, the choice of VSRZs, and the optimal speed of each leg by using the piecewise-linear approximation algorithm are identical to those of the enumerative algorithm (See Table 5). The objective value of model [P4] by the obtained solution is 35,569,363 USD, which is an LB of the objective value of [P1], i.e., 35,569,363 USD < 35,589,249 USD. Since the two algorithms obtain the same solution, the UB is equal to $\text{OBJ}_{[\text{P1}]}$. The maximum approximation error ϵ is calculated as 45,510 USD, and hence we have $LB + \epsilon = 35,614,873$ USD. Therefore, (61) is confirmed in this experiment.

5.2. Comparing the enumerative and piecewise-linear approximation algorithms on randomly generated cases

Some instances were randomly generated to assess the efficiency of the enumerative algorithm and of the piecewise-linear approximation algorithm. A part of parameter settings on APL case, including ship types, the parameters related to ship types (maximum speed, operating cost and conversion factors), fuel price, and approximation errors involved in the piecewise-linear approximation algorithm, are kept in this computational experiment. The scale of each experimental instance is determined by three factors: the numbers of VSRIP ports, non-VSRIP ports and routes. The voyage distance between two VSRIP ports, two non-VSRIP ports, or a VSRIP port and a non-VSRIP port is uniformly generated between 100 and 6,000 nm. The number of non-zero VSRZs at each VSRIP port could be one or two. If a VSRIP port has only one non-zero VSRZ, the radius of the VSRZ is designed as 20 nm; if a VSRIP port has two non-zero VSRZs, the radii are 20 and 40 nm, respectively. The upper speed limit of each VSRIP is chosen from 10 and 12 knots. Referring to the refunds at LSA and LGB, we randomly generate the refunds for the four ship types in each non-zero VSRZ of each VSRIP port. Each route randomly chooses a type of deployed ships from the four types, and uniformly generates the number of ports of call between 4 and 12. Each port of call is selected from all VSRIP and non-VSRIP ports, and the consecutive ports of call cannot be the same port. A dummy port is added at the middle point of the shortest sailing path between two consecutive VSRIP ports of call (see the example in Fig. 3), and each of the obtained two voyages is regarded as two legs: an in-VSRZ leg and an out-VSRZ leg. The total time spent at all ports of call on route r follows a continuous uniform distribution $(12 \cdot |L_r^N \cup L_r^{S,in}|,\ 72 \cdot |L_r^N \cup L_r^{S,in}|)$ hours. We set the total number of available ships of type i as $Q_i = \left\lceil 1.3 \cdot \sum_{r \in R_i} (\sum_{l \in L_r^N} d_{rl}/V_i^{\max} + \sum_{l \in L_r^{S,in}} d_{r,l,l+1}/V_i^{\max} + T_r)/T \right\rceil$, where

$V_i^{\max}$ is the maximum physical speed of ships of type $i \in I$.

The numbers of VSRIP ports and routes are two important factors on the computation time of each instance. For the enumerative algorithm, the increase of the number of VSRIP ports will significantly increase the number of VSRZ plans, and the increase of the number of routes will increase the computation time of each plan. For the piecewise-linear approximation algorithm, more VSRIP ports lead to a more complex decision on the compliance of VSRIPs for an instance, and more routes usually mean that more legs with linearized fuel consumption functions need to be considered and optimized. By contrast, the number of non-VSRIP ports has little impact on the computation time.

Numerical experiments on the increase of the number of routes were preformed in two groups each including the same numbers of non-VSRIP and VSRIP ports. There are five cases in each group with different numbers of routes, that is, 10, 30, 50, 100 and 500, and three instances are generated in each case. The optimal result can be obtained by the enumerative algorithm. Table 6 reveals that model [P4] derived by the piecewise-linear approximation algorithm and solved by CPLEX generates an LB of the optimal result. Plugging the solution from model [P4] into the objective function of model [P1], we obtain a UB of the optimal result. The gaps between the optimal results and the UBs and between the LBs and the UBs do not exceed 0.014% and 0.069%, respectively. However, the computation time of the enumerative algorithm is shorter than that of the piecewise-linear approximation algorithm when solving instances with a small number of routes, and is significantly shorter when the instances contain more routes. Therefore, the enumerative algorithm is more efficient in solving instances with a large number of routes. The computational results also show that an instance with 10 VSRIP ports and 500 routes can be solved within 200s by the enumerative algorithm. Since the world's largest container shipping company, Maersk, has a fleet of 639 ships, we consider that no more than 500 routes will be operated by a single liner shipping company. Hence, the enumerative algorithm can solve the schedule design problem under VSRIPs for any liner shipping company whose service routes involve no more than 10 VSRIP ports.

Table 6: Comparing the efficiency of two algorithms with increase of the number of routes

Instances	Enumerative algorithm		Piecewise-linear approximation			$\mathrm{Gap_{UO}}$	$\mathrm{Gap_{UL}}$
	$\mathrm{OBJ_{[P1]}}$	Time	LB	Time	UB		
10-5-10-1	40,248,863	0.1s	40,227,366	2s	40,250,942	0.005%	0.059%
10-5-10-2	50,519,212	0.1s	50,495,837	2s	50,521,998	0.006%	0.052%
10-5-10-3	48,077,609	0.2s	48,056,822	3s	48,082,608	0.010%	0.054%
10-5-30-1	133,618,059	0.3s	133,555,766	93s	133,618,059	0	0.047%
10-5-30-2	147,475,173	0.3s	147,426,265	45s	147,475,173	0	0.033%
10-5-30-3	133,178,500	0.3s	133,120,606	94s	133,181,041	0.002%	0.045%
10-5-50-1	270,870,690	0.5s	270,792,057	764s	270,870,690	0	0.029%
10-5-50-2	225,073,098	0.4s	224,950,481	2,386s	225,073,098	0	0.055%
10-5-50-3	226,125,504	0.3s	226,048,347	1,661s	226,125,504	0	0.034%
10-5-100-1	446,301,008	0.9s	N.A.	>3h	N.A.	N.A.	N.A.
10-5-100-2	467,712,287	1.2s	N.A.	>3h	N.A.	N.A.	N.A.
10-5-100-3	454,200,489	1s	N.A.	>3h	N.A.	N.A.	N.A.
10-5-500-1	2,307,863,270	4s	N.A.	>3h	N.A.	N.A.	N.A.
10-5-500-2	2,323,872,297	5s	N.A.	>3h	N.A.	N.A.	N.A.
10-5-500-3	2,305,225,791	5s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-10-1	46,051,081	2s	46,029,952	25s	46,057,457	0.014%	0.060%
20-10-10-2	49,586,570	1s	49,566,137	5s	49,587,307	0.001%	0.043%
20-10-10-3	41,972,722	1s	41,952,102	32s	41,976,168	0.008%	0.057%
20-10-30-1	131,178,949	3s	131,121,974	1,080s	131,186,090	0.005%	0.049%
20-10-30-2	132,245,881	3s	132,192,227	1,469s	132,250,648	0.004%	0.044%
20-10-30-3	119,537,863	3s	119,469,821	1,429s	119,551,970	0.012%	0.069%
20-10-50-1	237,569,164	5s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-50-2	238,197,644	10s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-50-3	231,230,792	15s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-100-1	447,129,586	22s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-100-2	458,747,465	14s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-100-3	457,930,701	29s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-500-1	2,242,511,832	86s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-500-2	2,365,651,242	183s	N.A.	>3h	N.A.	N.A.	N.A.
20-10-500-3	2,209,553,419	120s	N.A.	>3h	N.A.	N.A.	N.A.

Notes: (1) Instance "10-5-10-1" means the first instance in the case of 10 non-VSRIP ports, 5 VSRIP ports and 10 routes. (2) "$\mathrm{Gap_{UO}}$" is calculated as $(\mathrm{UB} - \mathrm{OBJ})/\mathrm{OBJ}$ and "$\mathrm{Gap_{UL}}$" is calculated as $(\mathrm{UB} - \mathrm{LB})/\mathrm{LB}$.

Similarly, we conducted two groups of experiments on the increase of the number of VSRIP ports by keeping the number of non-VSRIP ports and routes unchanged in each group. Each group contains five cases with the given numbers of VSRIP ports (i.e., 5, 10, 15, 20 and 30) and each case consists of three instances. The computational results in Table 7 show that the solution of the UB is near-optimal since the maximum gap between the optimal result and the UB for the instances with no more than 15 VSRIP ports is only 0.022% and the maximum gap between the LB and the UB for all instances is only 0.085%. When the number of VSRIP ports is no less than 20, the computation time of the enumerative algorithm is more than three hours, but the piecewise-linear approximation algorithm can solve the problem with 30 VSRIP ports and 10 routes within an acceptable computation time. We therefore conclude that the piecewise-linear approximation algorithm is superior to the enumerative algorithm on instances with a large number of VSRIP ports.

Table 7: Comparing the efficiency of two algorithms with increase of the number of VSRIP ports

Instances	Enumerative algorithm		Piecewise-linear approximation			Gap_{UO}	Gap_{UL}
	$\text{OBJ}_{[\text{P1}]}$	Time	LB	Time	UB		
20-5-10-1	45,685,623	0.1s	45,655,197	2s	45,689,894	0.009%	0.076%
20-5-10-2	46,845,951	0.2s	46,827,863	3s	46,846,337	0.001%	0.039%
20-5-10-3	46,139,073	0.2s	46,108,284	3s	46,139,985	0.002%	0.069%
20-10-10-1	46,051,081	2s	46,029,952	25s	46,057,457	0.014%	0.060%
20-10-10-2	49,586,570	1s	49,566,137	5s	49,587,307	0.001%	0.043%
20-10-10-3	41,972,722	1s	41,952,102	32s	41,976,168	0.008%	0.057%
20-15-10-1	43,970,907	221s	43,943,258	155s	43,973,575	0.006%	0.069%
20-15-10-2	43,183,869	155s	43,163,530	517s	43,193,218	0.022%	0.069%
20-15-10-3	46,067,072	142s	46,044,961	351s	46,069,983	0.006%	0.054%
20-20-10-1	N.A.	>3h	41,257,422	3,705s	41,283,389	N.A.	0.063%
20-20-10-2	N.A.	>3h	50,499,889	641s	50,529,349	N.A.	0.058%
20-20-10-3	N.A.	>3h	55,675,849	4,287s	55,703,751	N.A.	0.050%
20-30-10-1	N.A.	>3h	49,811,956	1,004s	49,843,645	N.A.	0.064%
20-30-10-2	N.A.	>3h	45,784,876	10,241s	45,813,454	N.A.	0.062%
20-30-10-3	N.A.	>3h	55,548,250	8,918s	55,579,352	N.A.	0.056%
30-5-10-1	41,881,761	0.2s	41,859,778	3s	41,890,743	0.021%	0.074%
30-5-10-2	44,245,166	0.1s	44,219,619	2s	44,248,513	0.008%	0.065%
30-5-10-3	39,611,601	0.2s	39,577,835	4s	39,611,614	0.00003%	0.085%
30-10-10-1	50,538,139	2s	50,512,819	6s	50,539,185	0.002%	0.052%
30-10-10-2	51,862,235	1s	51,833,022	10s	51,864,522	0.004%	0.061%
30-10-10-3	35,974,629	2s	35,954,086	48s	35,977,039	0.007%	0.064%
30-15-10-1	43,829,647	250s	43,805,208	201s	43,831,233	0.004%	0.059%
30-15-10-2	45,211,900	278s	45,190,554	476s	45,214,012	0.005%	0.052%
30-15-10-3	50,119,133	124s	50,094,593	292s	50,125,362	0.012%	0.061%
30-20-10-1	N.A.	>3h	46,242,028	3,897s	46,266,954	N.A.	0.054%
30-20-10-2	N.A.	>3h	50,185,866	1,876s	50,216,262	N.A.	0.061%
30-20-10-3	N.A.	>3h	50,482,128	970s	50,515,311	N.A.	0.066%
30-30-10-1	N.A.	>3h	51,368,683	3,461s	51,401,297	N.A.	0.063%
30-30-10-2	N.A.	>3h	53,089,484	9,686s	53,124,225	N.A.	0.065%
30-30-10-3	N.A.	>3h	34,630,414	8,202s	34,655,548	N.A.	0.073%

6. Conclusions

We have studied a schedule design problem of a liner shipping company under VSRIPs. A mixed-integer non-linear mathematical model was proposed for the problem, considering the compliance of VSRIPs, the speed limit and the limited number of ships. An enumerative algorithm and a piecewise-linear approximation algorithm were developed to solve the proposed model. Extensive numerical experiments were performed to assess the efficiency of the proposed algorithms.

The major contribution in this paper is threefold. First, to the best of our knowledge, the schedule design problem regarding how to balance the compliance of each VSRIP, the speed limit and the limited number of ships simultaneously is new. Second, our comparison of the two algorithms has demonstrated that the enumerative algorithm is more efficient in solving the problems with a large number of routes and a small number of VSRIP ports, while the piecewise-linear approximation algorithm is suitable for the problems with a small number of routes and a large number of VSRIP ports. Finally, our study can generate the optimal or near-optimal solution on the compliance of VSRIPs and the schedule design of ships for shipping companies, which will contribute to cost savings.

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