An Efficient Multiobjective Optimizer Based on Genetic Algorithm and Approximation Techniques for Electromagnetic Design

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To provide an efficient multiobjective optimizer, an approximation technique based on the moving least squares approximation is integrated into an improved genetic algorithm. In order to use fully, both the *a posteriori* information gathered from the latest searched nondominated solutions and the *a priori* knowledge about the search space and individuals, in guiding the search towards more and better Pareto solutions, a gradient direction based perturbation search strategy and a preference function based fitness penalization scheme are proposed. Numerical results are reported to validate the proposed work.

***Index Terms*—Approximation technique, evolutionary computation, genetic algorithm (GA), multiobjective optimization.**

I. Introduction

To MEET THE ever increasing demands in the design automations of electromagnetic devices, a considerable amount of efforts are dedicated to the development of multiobjective or vector optimizers. A wealth of vector algorithms, such as genetic algorithm (GA) or evolutionary algorithm (EA) [1], [2], simulated annealing algorithm (SA) [3], tabu search method [4] and particle swarm optimization (PSO) method [5], to name but a few, are proposed and used successfully to solve typical electromagnetic design problems. However, a multiobjective problem is characterized by the need to optimize several incommensurable and conflicting objectives simultaneously. In general, the solution of a multiobjective optimal problem is not a single point but is a set of optimal solutions called Pareto optimal or nondominated solutions.

For a multiobjective solver, the following two issues must be addressed carefully: 1) means to accomplish the fitness assignment and selection in order to guide the search towards the Pareto optimal front; 2) means to maintain the diversity in the searched Pareto optimal front. To meet these two goals, most of the reported efforts are focusing on techniques to extend the available scalar optimal methods to obtain some nondominated points with the prescribed diversities in both parameter and objective function spaces.

In spite of significant progresses being realized in the development of multiobjective optimal algorithms, the robustness and efficiency of available vector optimal methods are still unsatisfactory, hence there are still many problems yet to be addressed [6]. Indeed, very few studies have been devoted, hitherto, to the development of approximating techniques of nondominated sets in continuous multiobjective optimization problems [7]. However, a proper approximation of the non-dominated set could provide a wealth of useful information to guide the search towards the finding of more and better Pareto solutions with enhanced convergence performances. In this regard, an approximation technique is proposed and then integrated into an improved vector genetic algorithm (IVGA) in the design of an efficient vector optimizer.

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II. An Efficient Multiobjective Optimizer

A. An Improved Vector Genetic Algorithm

Both well established and newly proposed techniques are integrated in the proposed IVGA to help finding the Pareto solutions efficiently and to distribute them uniformly along the Pareto optimal front.

1) Reporting of Nondominated Solutions: As similar to the global repository of [8], a Pareto set, S_{Pareto}, is introduced to report the so far searched non-dominated solutions in the proposed algorithm. Also, the solutions of S_{Pareto} will be used in the assignment of the fitness value to an individual in the iteration process.

2) Assigning Fitness Value for New Individuals: To assess the quality of an individual in a Pareto optimal sense, some scalar techniques must firstly be designed for multiobjective function optimizations. To favor the selection of individuals near the Pareto optimal front, and also to distribute them uniformly along the tradeoff surface, other things being equal, the fitness assignment mechanism as proposed by Zitzler and Thiele [9] is extended and used in the proposed algorithm. The fitness value of an individual in the current population is assigned in accordance to a two-stage process as follows.

Step 1) Determine the strength of the solutions of S_{Pareto}

For each solution i in S_{Pareto}, its strength s_i is proportional to the number of individuals in the current population which are dominated by it, i.e.,

$$s_i = n_i/(N+1) \tag{1}$$

where n_i is the number of individuals in the current population which are dominated by the solution I; N is the size of the population. The fitness value of the solution i is the inverse of its strength.

Step 2) Fitness assignment of individuals in population P

The fitness value of an individual j in the population is computed by summing the strengths of all Pareto set members which dominate individual j. Mathematically

$$(f_{\text{fit}})_j = \frac{1}{1 + \sum\limits_{i(i \succ j)} s_i} \tag{2}$$

where $i \in S_{\text{Pareto}}, j \in P, k \succ l$ means that solution k dominates solution l.

3) A Priori Penalization of the Fitness Value: In most of the available multiobjective optimizers, only the information gathered on the so far searched solutions, the *a posteriori* information, is used to guide the search towards more promising solutions. However, if some *a priori* knowledge about the search space and individuals is available and could be used, the search efficiency and quality of the solution of the optimizer could be improved. Hence a preference function, that includes the *a priori* information about an individual as proposed in [10], is used to penalize the fitness value to guide the search to find better Pareto solutions. For the paper to be self-contained, the concept of the preference function will be described firstly.

For illustrative simplicity, one considers the minimization of an optimal problem involving two decision parameters $\overline{x} = (x_1, x_2)$ and two objective $(f_1(\overline{x}), f_2(\overline{x}))$ functions. Mathematically, if the contour lines of f_1 and f_2 in the parameter space exhibiting different convexities are intersecting at a feasible point P as shown in Fig. 1(a), there will be at least one point Q in the feasible space which is better than point P for both objective functions. By repeatedly and symmetrically reducing the values of both functions f_1 and f_2, it is possible to locate a Pareto optimal point at the tangential point between the contour lines of the two objectives, as shown in Fig. 1(b). In order to characterize the feature of an intersection point of the contour lines of the two objectives in the parameter space, one defines the preference function as

$$p(\overline{x}) = \frac{\nabla f_1(\overline{x})}{\|\nabla f_1(\overline{x})\|} \bullet \frac{\nabla f_2(\overline{x})}{\|\nabla f_2(\overline{x})\|} + 1 \tag{3}$$

where $\| \bullet \|$ is the Euclidean norm.

The preference function $p(\overline{x})$ reaches its minimum function value of zero at all Pareto optimal points. In other words, the preference function provides a "gauge" to measure the "closeness" of a feasible point to the nondominated solutions of the optimal problems. Therefore, to intensify searches around the points which are "close" to a Pareto solution, the fitness value of individuals in the current population is penalized by using

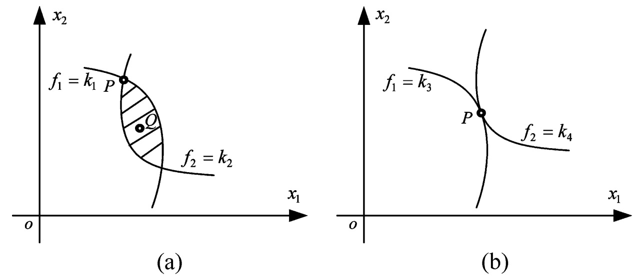

Fig. 1. Illustration of intersection points for a design problem comprising of two decision parameters and two objective functions.

$$f_{\text{fit}}\left(\overline{x}^{(j)}\right) = \frac{f_{\text{fit}}\left(\overline{x}^{(j)}\right)}{(1+\alpha)^{p\left(\overline{x}^{(j)}\right)}} \tag{4}$$

where $\overline{x}^{(j)}$ is the jth individual; $\alpha \in [0, 0.1]$ is a constant predefined by the user.

When the number of objective functions is more than 2, the preference functions are defined in a "pairwise" sense for every two objectives, and the power of the penalized function of (4), i.e., $p(\overline{x}^{(j)})$, is the weighted sum of the values of all possible preference functions pairwisely defined. From (4), it is clear that the smaller the preference function, the smaller is the fitness being devalued. Consequently, the proposed penalization scheme of the fitness value of an individual will force the search towards more and better Pareto solutions.

4) Fitness Sharing: To produce a uniform distribution of the searched Pareto solutions, not only in the objective but also in the parameter spaces, the fitness sharing concept is introduced. In order to reduce the implementation complexity, a simple fitness sharing function is proposed. Mathematically, the fitness sharing function is defined as

$$f_{\text{share}}\left(\overline{x}^{(i)}\right) = \frac{1/d_f\left(\overline{x}^{(i)}\right)}{\sum\limits_{j=1}^{N_T} 1/d_f\left(\overline{x}^{(j)}\right)} + \frac{1/d_X\left(\overline{x}^{(i)}\right)}{\sum\limits_{j=1}^{N_T} 1/d_X\left(\overline{x}^{(j)}\right)} \tag{5}$$

where $d_u(\overline{x}^{(k)})(u = f, X)$ is the point density around the specified point $\overline{x}^{(i)}$ in the u-space; N_T is the number of the total solutions which are available in the current generation; f and X are, respectively, the objective and parameter spaces.

To compute the point density of a specified point, a hypersphere with the specific point as the centre is constructed and the number of the solutions which fall inside this sphere is used as a measure of its fitness sharing function. The fitness value of an individual, $\overline{x}^{(i)}$, is the weighted sum of its fitness and fitness sharing function values, i.e.,

$$f_{\text{fit}}\left(\overline{x}^{(i)}\right) = w_1 f_{\text{fit}}\left(\overline{x}^{(i)}\right) + w_2 f_{\text{share}}\left(\overline{x}^{(i)}\right) \tag{6}$$

where w_1 and w_2 are two weighting constants.

5) Selection of New Generation: At the end of every generation, a new population is selected from all solutions which are available in the current generation using a Roulette wheel selection mechanism according to the fitness values of the solutions. The new populations are generally composed of three different individuals, i.e., those generated by means of selection, crossover, and mutation operations; those that come from the current population; and those that are stored in the external Pareto optimal set.

B. Approximation and Utilization of Nondominated Set

To fully use the information gathered from the so far searched discrete nondominated solutions, a continuous approximation of these discrete points at the end of each generation is constructed using a moving least squares approximation based response surface model [11]. As better nondominated points may be found, in the gradient direction of this approximated presentation from a geometrical perspective, an intensifying searching phase is activated by perturbing a specific point selected from this approximation on its gradient direction, in order to obtain such solutions. Essentially, the proposed intensifying searching phase is:

1) Select a point from the approximated presentation, using a Roulette wheel selection rule according to the point densities of the searched discrete nondominated solutions, in the objective function space.
2) Perturb the specific point on its gradient direction, and determine the objective function values of the newly perturbed point using

$$f_{\mathrm{obj}}^{i} = f_{\mathrm{obj}}^{i} - \frac{\partial g}{\partial f_{\mathrm{obj}}^{i}} \Delta_{\mathrm{pert}}^{i} \tag{7}$$

where f_{obj}^{i} is the value of the ith objective function at the specific point; g is the approximated continuous contour of the so far searched nondominated solutions in the objective space; $\Delta_{\mathrm{pert}}^{i}$ is a small perturbation value of the ith predefined by the user.
3) Calculate the decision parameter values of the new perturbation point according to their objective function values. As i) the number of the objective functions is generally smaller than that of the decision parameters, and ii) one image in the objective space may relate to multiple points in the decision parameter space, the values of the decision parameters of the new perturbation point is determined from

$$\min \quad \sum_{i=1}^{k} \left(\left(f_{\mathrm{obj}}^{i}\right)^{\mathrm{perturb}} - \left(f_{\mathrm{obj}}^{i}\right)^{\mathrm{compute}} \right)^2 \tag{8}$$

where $(f_{\mathrm{obj}}^{i})^{\mathrm{perturb}}$ and $(f_{\mathrm{obj}}^{i})^{\mathrm{compute}}$ are the values of the ith objective function obtained, respectively, in step (2) and from a response surface of the corresponding objective function as proposed in this paper.
4) Check if the newly calculated decision parameters are in the feasible space. If the answer is "Yes," go to next step; Otherwise, reduce the perturbation value and return to step 2.
5) Determine if the stop criterion is satisfied. If the answer is "Yes," stop this search phase; Otherwise, go to step 1.

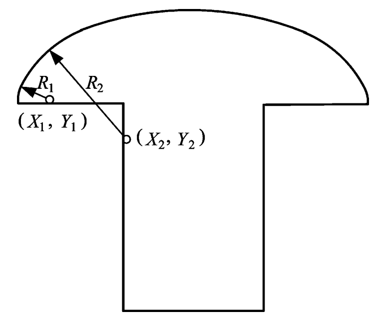

Fig. 2. Schematic diagram of the multisectional pole shoes.

The initial value of the perturbation value, $\Delta_{\mathrm{pert}}^{i}$, is automatically adjusted during the iterative process in such a way that it jumps to the maximum value at the beginning of the search and reaches the minimum value at the end of the search. Typically, the maximum and minimum values of the perturbation parameter of $\Delta_{\mathrm{pert}}^{i}$ are set, respectively, to 10% and 5% of the "averaged" value of the ith objective function over some sampling points. Moreover, from the aforementioned description it is clear that: 1) since the selection probability for a point in the approximated presentation of the so far searched nondominated solutions is inversely proportional to the point density, the sparser the points in the objective space are, the higher are the probabilities for the points are to be selected. Therefore, the diversity of the nondominated solutions is guaranteed; 2) compared with a pure stochastic perturbation method, the proposed gradient based one can reach a promising point in the maximum descending direction, resulting in an enhancement in search efficiency. Consequently, a high performance multiobjective optimizer could be expected.

III. Numerical Example

To critically compare the performances of the proposed algorithm with other vector optimal techniques, the geometrical design of the multisectional pole arcs of a large hydrogenerator [3] is used as a case study. Mathematically, this multiobjective design problem is formulated as

$$\begin{aligned} &\max \quad B_{f1}(X) \\ &\min \quad (e_v, \mathrm{THF}) \\ &\mathrm{s.t} \quad\ \mathrm{SCR} - \mathrm{SCR}_0 \geq 0 \\ &\qquad\quad X'_d - X'_{d0} \leq 0 \end{aligned} \tag{9}$$

where B_{f1} is the amplitude of the fundamental component of the flux density in the air gap, e_v is the distortion factor of a sinusoidal voltage of the machine at the no-load, telephone harmonic factor (THF) X'_d is the direct axis transient reactance of the short circuit ratio (SCR) generator.

The decision parameters of this problem are the center positions and radii of the multisectional arcs of the pole shoes (Fig. 2). In the numerical implementation, B_{f1} is directly computed from the finite element solution of the no-load electromagnetic field of the machine, and the other performance parameters of (9) are derived based on these finite element simulations.

TABLE I
ALGORITHM PARAMETERS USED BY THE PROPOSED AND A STANDARD GENETIC ALGORITHM

Population size:	80
Crossover probability:	0.8
Mutation probability:	0.001
Max generation:	50
Chromosome length	10+10

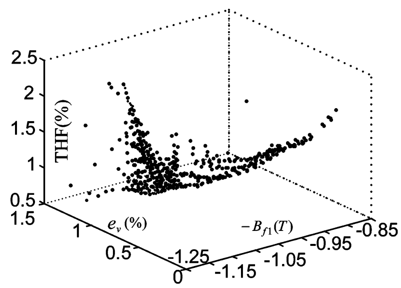

Fig. 3. Searched Pareto solutions by a SA based method.

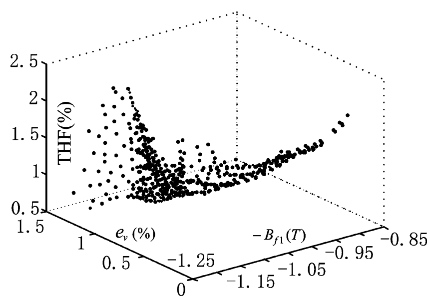

Fig. 4. Searched Pareto solutions by the proposed algorithm.

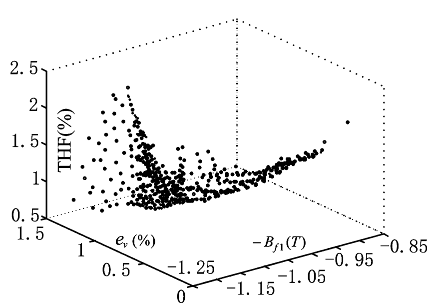

Fig. 5. Searched Pareto solutions by a standard generic algorithm.

This problem is solved by using, respectively, the proposed IVGA, a standard GA and a SA algorithm. The parameters used by the proposed and the standard GA are given in Table I. The parameters of the SA algorithm used in this paper are the same as those of [3].

In the numerical experiments, for the geometrical optimal design of a 300 MW hydro-generator, every algorithm is run independently three times, and the "averaged" iterative number of the three runs for the proposed, the SA, and the standard GA algorithms are, respectively, 1576, 1784, and 2045. The searched Pareto fronts using three different algorithms are shown in Figs. 3–5. By comparing these searched Pareto fronts, it is obvious that the standard GA algorithm cannot find some parts of the Pareto solutions, i.e., those in the back-right corner and in the front-right sub-regions, which can however be found by either the proposed or the SA algorithms because the exploitation search ability of a standard GA is not as strong as its exploration searching ability. In summary, the performances of the proposed algorithm are superior to its ancestor, the standard GA algorithm, in terms of search efficiency and solution quality; and it is comparable to the SA based algorithm, as demonstrated by this primary numerical experiment as reported in this paper.

IV. CONCLUSION

An efficient multiobjective optimizer is proposed and its performances are evaluated by numerical experiments on a practical inverse vector optimal problem. The primary numerical results have demonstrated the robustness and feasibility of the proposed algorithm in the study of engineering applications. Therefore, the proposed work offers an alternative algorithm for multiobjective optimizations of electromagnetic devices.

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