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# Using DGCM to predict transient flow in plastic pipe

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**Abstract.** Transient flows with cavitation are commonly existent and observed in pressurized conveyance systems. The occurrence of large cavitation caverns accompanying hydraulic impacts in the pressurized pipes is particularly dangerous. There may then be interference of pressure waves traveling inside the pipe (especially between initial and secondary pressure waves). The maximum instantaneous pressures can rise in the most dangerous scenario to – magnitude that are twice as large as those calculated with a classic formula of Joukowsky. To protect these systems at the design stage it is advisable to estimate large pressure pulsations using appropriate numerical models. In this work transient cavitating flow in the plastic pipe was modeled using the modified discrete gas cavity model (DGCM). Retarded strain occurring in the studied plastic pipe was modeled with the aid of a convolutional integral of stress history and a derivative of the creep compliance function of the plastic pipe wall. In the simulation, the unsteady wall shear stress was determined using the recent Urbanowicz's computationally effective method.

## 1. Introduction

At present, unsteady pipe flow is still not fully understood, especially under some complicated situations (transient flow with cavitation). The key issues for this problem may include: single versus multiple phase flow; laminar versus turbulent flow; elastic versus viscoelastic strain behavior; gaseous versus vaporous cavitation; accelerated versus pulsatile flow; Newtonian versus non-Newtonian flow; rigid versus flexible pipe walls; fast (impulsive) versus slow-transient flow. Consideration of the above-mentioned issues should often include selected accompanying phenomena: mechanical energy dissipation due to fluid friction; mechanical energy dissipation due to the occurrence of viscoelastic retarded deformations of the pipe walls; liquid column separation resulting from cavitation; fluid structure interaction. In this paper, a modified form of the unsteady flow model coupled with liquid stream column separation (cavitation) and taking into account the unsteady hydraulic resistance is used for the investigation. In view of the fact that there are at least four different approaches to modeling gas areas of cavitation flows in the literature [1–3], below the focus is mainly put on the discrete gas cavitation model (DGCM). With the help of this model, two flow parameters (flow rate Q and piezometric head H) can be calculated for both elastic and viscoelastic pipes [4]. In this work the output system of equations will be based on changes in pressure and average flow velocity - therefore these two parameters will be determined with the help of the presented model of unsteady flows in a plastic pipe. In order to further improve model compliance, unsteady flow resistance will also be taken into account using the improved calculation algorithm [5]. The simulation results will used to demonstrate the accuracy and validity of the modified model.

## 2. Discrete gas cavity model (DGCM)

In the DGCM a quantity of free air is assumed to be concentrated at each computational section [3]. The pressure in a cavity satisfies the ideal gas law. For tiny bubbles the free gas is assumed to behave isothermally, while large bubbles (column separations) tend to behave adiabatically. It is believed that Brown [6] was the precursor of this model. In a rigorous mathematical treatise of the DGCM, Liou [7], applying Von Neumann analysis to a linearized set of equations, showed that the numerical wave speed converges to the theoretical (physical) one, and explained why the DGCM exhibits nonlinear variable wave-speed features. The main assumption lays in focus the cavities at the grid points. Between the grid points pure liquid is assumed for which the basic water-hammer equations remain valid. This means that the pressure wave speed c is maintained (and convective terms neglected) between grid points in distributed cavitation regions. The next assumption of DGCM is that the cavities do not moves, which is consistent with the acoustic approximation.

The unsteady flow of liquids in horizontal plastic pipes can be described by the system of equations as follows [8]:

$$\begin{cases} \frac{1}{\rho c^2}\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} + 2\frac{\partial \varepsilon_r}{\partial t} & = & 0, \\ \rho\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{4}{D}\tau & = & 0, \end{cases} \tag{1}$$

where $p$ - pressure $[Pa]$, $v$ - mean cross section velocity $[m/s]$, $t$ - time $[s]$, $x$ - distance along the pipe $[m]$, $\rho$ - liquid density $[kg/m^3]$, $c$ - pressure wave speed $[m/s]$, $D$ - inner pipe diameter $[m]$, $\tau$ - wall shear stress $[Pa]$, $\varepsilon_r$ - retarded strain $[-]$.

### *2.1. Numerical solution for inner grid nodes*

According to [3], the formula for the volume of gas is as follows:

$$\overline{V}_g = \frac{p_0^* \alpha_0 \overline{V}}{p - p_v}, \tag{2}$$

where $\overline{V} = A \cdot \Delta x$ - inner volume of pipe between two corresponding grid nodes in method of characteristics $[m^3]$, $p_0^*$ - a reference pressure (in this work equal to atmospheric pressure: $p_0^* = p_a$) $[Pa]$, $\alpha_0$ - void fraction $[-]$, $p_v$ – vapour pressure $[Pa]$.

Using the method of characteristics, the system of equations (1) can be transformed to the following numerical form:

$$\begin{cases} \frac{1}{c\rho}\left(p_{i(t+\Delta t)} - p_{i-1,(t)}\right) + \left(v_{L,i(t+\Delta t)} - v_{R,i-1(t)}\right) + \frac{4\Delta t}{\rho D}\tau_{i-1(t)} + 2c\Delta t\frac{\partial \varepsilon_{i(t+\Delta t)}}{\partial t} & = & 0, \\ -\frac{1}{c\rho}\left(p_{i(t+\Delta t)} - p_{i+1,(t)}\right) + \left(v_{R,i(t+\Delta t)} - v_{L,i+1(t)}\right) + \frac{4\Delta t}{\rho D}\tau_{i+1(t)} - 2c\Delta t\frac{\partial \varepsilon_{i(t+\Delta t)}}{\partial t} & = & 0, \end{cases} \tag{3}$$

where $\Delta t$ – numerical time step $[s]$.

In the above system of equations, the derivative of retarded strain (expressed by a convolutional integral of local pressure change over time with derivative of creep function) can be calculated using the following formula [9]:

$$\frac{\partial \varepsilon_r}{\partial t}(t+\Delta t) = \frac{\Xi}{2}\int_0^t \frac{\partial p(u)}{\partial t}\cdot w_J(t-u) \approx \frac{\Xi}{2}\sum_{i=1}^{k}\underbrace{\left(z_i(t)\cdot e^{-\frac{\Delta t}{T_i}} + \frac{J_i}{\Delta t}\left[1-e^{-\frac{\Delta t}{T_i}}\right]\left(p_{(t+\Delta t)}-p_{(t)}\right)\right)}_{z_i(t+\Delta t)}, \tag{4}$$

where $w_J(t-u)$ - time derivative of pipe material creep compliance function $[s^{-1}Pa^{-1}]$, $\Xi = \frac{D}{e}\xi$ - enhanced $\xi$ parameter $[-]$, $e$ - thickness of pipe wall $[m]$, $\xi$ - parameter describing support condition of the pipe $[-]$, $z_i(t+\Delta t)$ - time dependent coefficient $[s^{-1}]$, $J_i$ - the creep-compliance of the spring of the Kelvin-Voigt $k$-element $[Pa^{-1}]$, $T_i$ – the retardation time of the dashpot of k-element $[s]$.

$$\frac{\partial \varepsilon_r}{\partial t}(t+\Delta t) = p_{(t+\Delta t)}F - V_{(t)}, \tag{5}$$

$$F = \frac{\Xi}{2}\sum_{i=1}^{k} M_i,\ V_{(t)} = \frac{\Xi}{2}\sum_{i=1}^{k}\left(M_i p_{(t)} - z_i(t)\cdot O_i\right),\ M_i = \frac{J_i}{\Delta t}\left[1-e^{-\frac{\Delta t}{T_i}}\right],\ O_i = e^{-\frac{\Delta t}{T_i}}. \tag{6}$$

The formulas for the left $L$ and right $R$ sides of the gas area will be obtained:

$$\begin{cases} v_{L,i(t+\Delta t)} &= C_A - p_{i(t+\Delta t)}C_P, \\ v_{R,i(t+\Delta t)} &= p_{i(t+\Delta t)}C_P + C_B, \end{cases} \tag{7}$$

where

$$\begin{cases} C_A &= \frac{1}{c\rho}p_{i-1,(t)} + v_{R,i-1(t)} - \frac{4\Delta t}{\rho D}\tau_{i-1(t)} + 2c\Delta t V_{i(t)}, \\ C_B &= -\frac{1}{c\rho}p_{i+1,(t)} + v_{L,i+1(t)} - \frac{4\Delta t}{\rho D}\tau_{i+1(t)} - 2c\Delta t V_{i(t)}, \\ C_P &= \frac{1}{c\rho} + 2cF\Delta t \end{cases} \tag{8}$$

The equation of continuity of the gas area for the rectangular mesh of the characteristics is:

$$\overline{V}_{g,i(t+\Delta t)} = \overline{V}_{g,i(t)} + \left[\psi\left(v_{R,i(t+\Delta t)} - v_{L,i(t+\Delta t)}\right) + (1-\psi)\left(v_{R,i(t)} - v_{L,i(t)}\right)\right]A\Delta t. \tag{9}$$

In the above equation:

$$\overline{V}_{g,i(t)} = \frac{p_0^*\alpha_0 A\Delta x}{p_{i(t)} - p_v} = \frac{C_1}{p_{i(t)} - p_v} = C_2 \text{ and } \overline{V}_{g,i(t+\Delta t)} = \frac{p_0^*\alpha_0 A\Delta x}{p_{i(t+\Delta t)} - p_v} = \frac{C_1}{p_{i(t+\Delta t)} - p_v} \tag{10}$$

After using the dependences from equations (7) and (10) to the condition of the continuity of the cavitation region in the form of the formula (9), then followed by some rearrangements and grouping of constants one gets:

$$\frac{C_1}{p_{i(t+\Delta t)} - p_v} = C_3(p_{i(t+\Delta t)} - p_v) + C_4. \tag{11}$$

The above equation is in essence a quadratic equation. Details on the solution of the above quadratic equation can be found in [3]. After calculating the pressure from equation (11), it is necessary to determine the velocity values from the left and right sides of the vapour area, using the formula (7) and the volume of the vapour area in a given time step, the formula (9). At a boundary, one of the compatibility equations is replaced by boundary equation(s).

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### 2.2. Modeling of unsteady friction

In addition to the cavitation phenomenon and the influence of retarded strains of pipe walls, this study also considers unsteady friction losses during transient flow process in pipes. The wall shear stress contained in the motion equation (1) during transient flows can be modeled using the formula [10]:

$$\tau = \tau_q + \tau_u = \frac{\rho v|v|}{8} f + \frac{4\mu}{D} \int_0^t \frac{\partial v(t-u)}{\partial t} \cdot w(u)du, \tag{12}$$

where $f$ - Darcy-Weisbach friction coefficient $[-]$, $\mu$ - dynamic viscosity of liquid $[Pa \cdot s]$, $w(t-u)$ - weighting function $[-]$.

The convolutional integral found in the above solution was solved in an efficient way using the method in the details described in [5]:

$$\tau(t+\Delta t) \approx \frac{\rho f v_{(t)}|v_{(t)}|}{8} + \frac{4\mu}{D} \sum_{i=1}^{3} \underbrace{\left[A_i y_i(t) + \eta B_i \left[v_{(t)} - v_{(t-\Delta t)}\right] + [1-\eta] C_i \left[v_{(t-\Delta t)} - v_{(t-2\Delta t)}\right]\right]}_{y_i(t+\Delta t)} \tag{13}$$

where

$$\eta = \frac{\int_0^{\Delta \hat{t}} w_{class.}(u)du}{\int_0^{\Delta \hat{t}} w_{eff.}(u)du}, \; A_i = e^{-n_i \Delta \hat{t}}, \; B_i = \frac{m_i}{n_i \Delta \hat{t}} \left[1 - A_i\right], \; C_i = A_i B_i. \tag{14}$$

The application of the above approximation form may provide a compromise between the efficiency of calculations and the qualitative and quantitative compatibility of the modeled pressure runs.

## 3. Simulation examples and analysis

In the literature, only a limited number of experimental studies on unsteady flows in this type of system is available, such as Güney's research [8] and Soares [4]. In this paper, the measurements obtained by Güney will be used for the investigation. The test stand was composed of 43.1 meters long (pipe inner diameter $D = 0.0416m$ and wall thickness $e = 0.0042m$) LDPE pipe. Two cases of turbulent flow are retrieved from that system for analysis, in which the main difference will be the temperature of the flowing liquid, with $T_1 = 25°C$ for Case 1 and $T_2 = 35°C$ for Case 2. It is known that the temperature significantly changes the mechanical properties of the pipe material and consequently affects the damping of pressure oscillations.

**Table 1.** Details of Güney's research

| Case | $T$ | $\nu$ | $c$ | $p_v$ | $\rho$ | $v_0$ | $p_{Rr}$ |
|---|---|---|---|---|---|---|---|
| 1 | 25 | $0.892 \cdot 10^{-6}$ | 260 | $3.17 \cdot 10^3$ | 997.1 | 1.37 | $1.3056 \cdot 10^5$ |
| 2 | 35 | $0.723 \cdot 10^{-6}$ | 240 | $5.62 \cdot 10^3$ | 994.1 | 1.37 | $1.3038 \cdot 10^5$ |

where $T$ - temperature $[°C]$, $\nu$ - kinematic viscosity of water $[m^2/s]$, $p_v$ - vapour pressure $[Pa]$, $v_0$ - initial velocity $[m/s]$, $p_{Rr}$ - reservoir pressure $[Pa]$

From the simulation results in Fig. 1, the following conclusions can be obtained:

- taking into account retarded strains (Fig. 1a and 1c) of the pipe walls instead of the classical Hook's law approach (elastic deformation - see Fig. 1b and 1d) has contributed to a significant improvement in simulation compliance with experimental results;

**Table 2.** Creep compliance function coefficients

| Case | $T$ | $J_0$ | $J_1$ | $J_2$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|
| 1 | 25 | $1.5 \cdot 10^{-9}$ | $1.046 \cdot 10^{-9}$ | $1.237 \cdot 10^{-9}$ | 0.0222 | 1.864 |
| 2 | 35 | $1.77 \cdot 10^{-9}$ | $1.797 \cdot 10^{-9}$ | $2.349 \cdot 10^{-9}$ | 0.0265 | 2.392 |

where $J_i$ - creep-compliance coefficients $[Pa^{-1}]$, $T_i$ - retardation times $[s]$

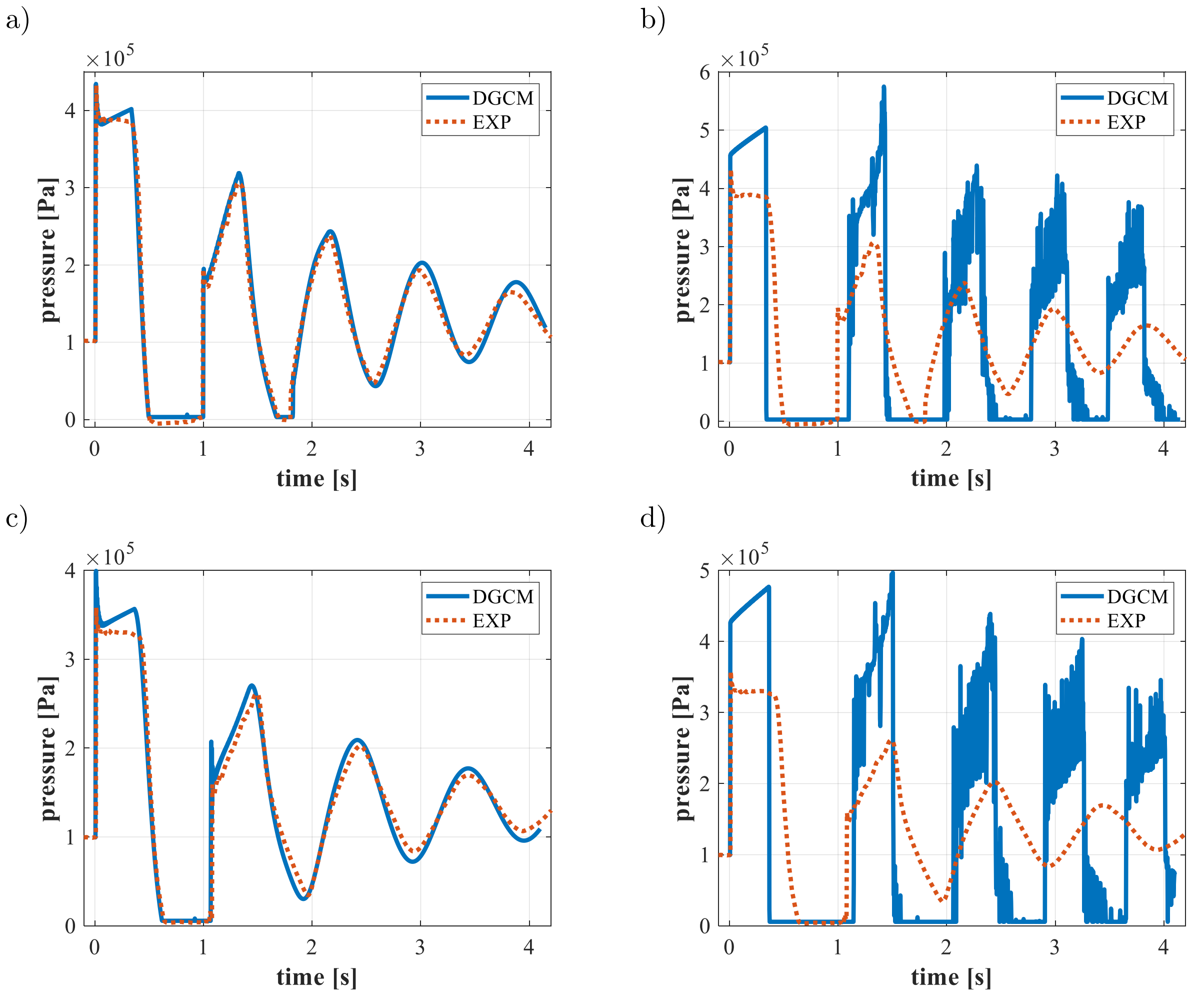


**Figure 1.** Simulation results a) CASE 1 ($T = 25°C$) Viscoelastic DGCM, b) CASE 1 ($T = 25°$) Elastic DGCM, c) CASE 2 ($T = 35°C$) Viscoelastic DGCM, d) CASE 2 ($T = 35°C$) Elastic DGCM

- the use of experimental creep function, which has been filtered in accordance with the principle discussed in [9], guarantees sufficient model compliance. Therefore, complicated calibration procedures have been avoided;
- As noticed in both Case 1 and Case 2 at the top of the first modeled amplitudes, a significant pressure increase, not covered by experimental results, suggests incorrect conditions of the initial pressure distribution along the pipe. However in this work the initial condition was

assumed, according to the Güney's PhD work [8], and there it appeared that the pressure drop over the length of the pipe was around 50% larger (CASE 1: $h_L = 2.99\ m$, CASE 2: $h_L = 2.98\ m$) than theoretically calculated from the well-known formula:

$$h_L = f \cdot \frac{L}{D} \cdot \frac{v_0^2}{2g} = 0.02 \cdot \frac{43.1}{0.0416} \cdot \frac{1.37^2}{19.62} = 1.98\ [m]$$

This indicates that the pressure distribution in pre-transient state of analyzed pipe can play an important role on modeling the shape of the first amplitude.

## 4. Conclusions

The work presents a modified version of DGCM that provides potential to analyze unsteady flows with cavitation in a plastic pipe. The use of computationally effective methods of modeling two convolutional integrals, sequentially describing unsteady friction and retarded strain, allows transient analysis to be done in close to real time. With a noticeable increase in the speed of computers in the near future it will be possible to change settings of selected hydraulic devices of the real systems in follow-up way. This will be significant to increase the safety of these systems, and thereby to reduce their costs. The presented results show that the way of modeling retarded strain proposed in this study, in particular the creep compliance weighting function that is a derivative of the creep function, has a significant effect on improving the simulation process. In the analyzed cases, it is found that the role of unsteady friction was much less significant than the retarded strains. It is also noted that only the DGCM is applied in this study, and further research will be necessary in future work to comparative analysis of different models for transient cavitation flows in plastic pipes, such as DVCM (discrete vapour cavity model), DACM (discrete Adamkowski cavity model) and DBCM (discrete bubble cavity model).

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