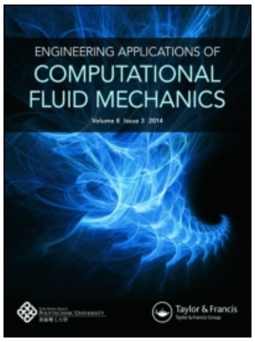

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Flutter speed estimation using presented differential quadrature method formulation

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ABSTRACT
In this paper the flutter behavior of a typical wing is investigated. The study is performed by presented Deferential Quadrature Method (DQM). The aerodynamic part adopted Wagner functions to model subsonic regime. Quasi steady and unsteady aerodynamics are considered to estimate the instability speed of the structure. Based on the presented model, a code is developed, for an arbitrary typical section beam. The obtained results validated the existing methods in the literature. The proposed method provides the advantage of finding the modes of oscillation and other dynamic parameters with less than 0.2% difference.

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aero-elasticity; subsonic regime; deferential quadrature method (DQM); typical section beam; aerodynamic; wing design; airfoil; computational fluid dynamics (CFD); flutter speed prediction

1. Introduction

The flutter behavior of coupled two- and three-dimensional systems have been investigated both numerically and analytically in a vast amount of research. Some simplifications have been conducted to approximate this type of instability condition. Estimation of aerodynamics behavior established steady, quasi-steady, and nonsteady theories. Each of the aerodynamic theories has been presented in both time and frequency domains. In the time domain, Wagner (1925) introduced the indicial function to study the lift response of a 2D flat plate in incompressible flow. Jones (1960), using the Laplace transformation method, studied the dynamic behavior of an airplane. Sears (1940) applied this method to assess the non-uniform motion of airfoil. Later, perusing unsteady aerodynamics modeling was developed by such new techniques as Finite state induced flow models and reduced order model (ROM). The new type of finite state induced flow was suggested by Peters, Karunamoorthy, and Cao (1995). However, many authors (Behbahani-Nejad, Haddadpour, & Esfahanian, 2005; Dowell, Hall, & Romanowski, 1997; Garrick, 1939; Hall, 1994; Marzocca, Librescu, & Chiocchia, 2001; Peters et al., 1995) introduced, developed, and used the time domain methods to describe flows about airfoils, cascades, and wings, but most of the aerodynamic theories were presented in the frequency domain. Theodorsen and Mutchler (1935) introduced this theory to describe the lift response of structures in unsteady flows. In this area, some semi-analytical schemes have been suggested to determine instability condition. The first method is called P method and is usually applied to the steady and quasi-steady models. Most of the unsteady aerodynamic theories could not be determined by this technique. The K method (Bisplinghoff, Ashley, & Halfman, 2013; Hodges & Pierce, 2011) is another approach to studying structural response in an unsteady flow. This method introduces artificial structural damping to the system and determines only the instability point. Another scheme in this region was introduced by Hassig (1971). His technique is based on the oscillatory aerodynamic force and generalized P method. This scheme, known as P-K method later, was modified by Rodden et al. (Rodden & Johnson, 1994; Rodden, Harder, & Bellinger, 1979). The P-P method (Haddadpour & Firouz-Abadi, 2009) in the Laplace domain is another technique to investigate damping and frequency quantity of an aero-elastic system.

Among some numerical approaches (Ghalandari, Mirzadeh Koohshahi, Mohamadian, Shamshirband, &

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Chau, 2019; Mou, He, Zhao, & Chau, 2017), another technique which is known as Differential Quadrature Method (DQM) was introduced in 1971 by Bellman and Casti (1971). Low computation cost and non-necessity to alter guess function unlike in the assumed mode technique is the main reason many researchers apply this method to their linear and nonlinear engineering problems. Jang, Bert, and Striz (1989), Malik and Bert (1998) and Striz, Jang, and Bert (1988) applied this method to the structural equation. Investigation of the tapered beam behavior on a two-parameter foundation in static and dynamic forms using DQM was conducted by Hassan and Nassar (2015). Kang and Kim (2002) studied the extensional vibration analysis of curved beams using the DQM.

Although there is a lot of research on flutter analysis in an aero-elastic system in a supersonic regime by DQM method, there are no studies in a subsonic regime, especially for an infinite wing, which uses different existing aerodynamics theory, along with DQM approaches. So in this paper, the instability condition of the typical airfoil and Goland wings in subsonic flow using differential quadrature method are investigated. In this method, an appropriate Wagner function is utilized as the aerodynamic theory. The presented model will be compared with those of other investigations in the frequency domain.

2. Governing equations

In this section, the two-dimensional equations of the wing are discretized by DQM method. The equations are presented in the time domain. The derivative of the functions at the sample point is approximated as a weighted linear summation in the computational domain. The number of the sample point is considered to the highest order of equations minus the number of boundary conditions.

2.1. Structural modeling

An altenative solution to the complex structural modeling of the wings can be obtained through considering simplification of the complex design components. For small deflection, the linear structural equation of wings can be modeled by a Euler-Bernoulli coupled beam as follows (Hodges & Pierce, 2011):

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + me\frac{\partial^2 \theta}{\partial t^2} = L$$
$$GJ\frac{\partial^2 \theta}{\partial x^2} + me\frac{\partial^2 w}{\partial t^2} + I_{ea}\frac{\partial^2 \theta}{\partial t^2} = M_{ea} \quad (1)$$

Where w is the transvers displacement, θ is the torsional displacement, EI is the bending stiffness, m is the mass of the system, e is the elastic axis, GJ is the torsional stiffness, I_{ea} is the mass moment of inertial, and L and M are lift and moment of the system, respectively. So by consideration of the wings as the cantilever beam, the boundary condition can be presented as:

$$x = L, \left(\frac{\partial^2 w}{\partial x^2} = 0,\ \frac{\partial^3 w}{\partial x^3} = 0\right), \left(\frac{\partial \theta}{\partial x} = 0,\ \frac{\partial^3 \theta}{\partial x^3} = 0\right)$$
$$x = 0, \left(w = 0,\ \frac{\partial w}{\partial x} = 0\right), \left(\theta = 0,\ \frac{\partial^2 \theta}{\partial x^2} = 0\right) \quad (2)$$

2.2. Aerodynamic modeling

The aerodynamic forces in quasi-steady and non-steady subsonic flows modeled by Wagner function in Duhamel integral form. In this model, lift and moment distribution around the elastic axis can be represented as below:

$$\begin{aligned} L &= \pi\rho_\infty b^2(\ddot{w} + U_\infty\dot{\theta} - ba\ddot{\theta}) + 2\pi\rho_\infty U_\infty b \\ &\times \left[\left(\dot{w} + U_\infty\theta + b\left(\frac{1}{2} - a\right)\dot{\theta}\right)\varphi(t) \right. \\ &\left. + \int_0^t \varphi(t-\lambda)\left[\ddot{w} + U_\infty\dot{\theta} + b\left(\frac{1}{2} - a\right)\ddot{\theta}\right]d\lambda\right] \end{aligned}$$

$$\begin{aligned} M_{ea} &= \pi\rho_\infty b^3\left(a\ddot{w} - U_\infty\left(\frac{1}{2} - a\right)\dot{\theta}\right. \\ &\left. + b\left(\frac{1}{8} + a^2\right)\ddot{\theta}\right) + 2\pi\rho_\infty U_\infty b^2\left(\frac{1}{2} + a\right) \\ &\times \left[\left(\dot{w} + U_\infty\theta + b\left(\frac{1}{2} - a\right)\dot{\theta}\right)\varphi(t) \right. \\ &\left. + \int_0^t \varphi(t-\lambda)\left[\ddot{w} + U_\infty\dot{\theta} + b\left(\frac{1}{2} - a\right)\ddot{\theta}\right]d\lambda\right] \end{aligned} \quad (3)$$

where a is the elastic axis location, b is the half cord of the wing, ρ_∞ is the air density, U_∞ is the air speed, and $\varphi(t)$ is the Wagner function. It is represented as:

$$\varphi(t) = 1 - \alpha_1 e^{\beta_1 t} - \alpha_2 e^{\beta_2 t} \quad (4)$$

The constants coefficients of the Wagner function have been approximated by Jones (1938) as follows:

$$\alpha_1 = 0.165,\ \alpha_2 = 0.335,\ \beta_1 = 0.0455(U_\infty/b),$$
$$\beta_2 = 0.3(U_\infty/b)$$

Using the Laplace transformation method with zero initial condition and definition of the Wagner function, Equation (3) can be written as:

$$L = \pi\rho_\infty b^2(S^2\bar{w} + U_\infty S\bar{\theta} - baS^2\bar{\theta})$$
$$+ 2\pi\rho_\infty U_\infty b[\bar{W}_a - \bar{C}_1(S) - \bar{C}_2(S)]$$

$$M_{ea} = \pi\rho_\infty b^3 \left(aS^2\bar{w} - U_\infty\left(\frac{1}{2}-a\right)S\bar{\theta} + b\left(\frac{1}{8}+a^2\right)S^2\bar{\theta}\right) + 2\pi\rho_\infty U_\infty b^2\left(\frac{1}{2}+a\right) \times [\bar{W}_a - \bar{C}_1(S) - \bar{C}_2(S)] \quad (5)$$

where S is the non-dimensional Laplace variable, $\bar{C}_1(S) = 0.165\, S\bar{W}_a/S + 0.0455$, $\bar{C}_2(S) = 0.335\, S\bar{W}_a/S + 0.35$, and $\bar{W}_a$ is the Laplace transformation of $W_a = \dot{w} + U_\infty\theta - ba\dot{\theta}$. Using Laplace inverse transformation, Equation (5), $\bar{C}_1(S)$ and $\bar{C}_2(S)$ respectively can be obtained as:

$$L = \pi\rho_\infty b^2(\ddot{w} + U_\infty\dot{\theta} - ba\ddot{\theta}) + 2\pi\rho_\infty U_\infty b[(\dot{w} + U_\infty\theta - ba\dot{\theta}) - C_1(t) - C_2(t)]$$

$$M_{ea} = \pi\rho_\infty b^3 \left(a\ddot{w} - U_\infty\left(\frac{1}{2}-a\right)\dot{\theta} + b\left(\frac{1}{8}+a^2\right)\ddot{\theta}\right) + 2\pi\rho_\infty U_\infty b^2\left(\frac{1}{2}+a\right) \times [(\dot{w} + U_\infty\theta - ba\dot{\theta}) - C_1(t) - C_2(t)] \quad (6)$$

and

$$\dot{C}_1(t) + 0.0455\,C_1(t) = 0.165\ W_a$$
$$\dot{C}_2(t) + 0.35\,C_2(t) = 0.335\ W_a \quad (7)$$

So lift and moment of the coupled system can be represented in the matrix form as follows:

$$\begin{bmatrix} L \\ M \end{bmatrix} = [\mathbf{M}_a]\begin{bmatrix} \ddot{w} \\ \ddot{\theta} \end{bmatrix} + [\mathbf{C}_a]\begin{bmatrix} \dot{w} \\ \dot{\theta} \end{bmatrix} + [\mathbf{K}_a]\begin{bmatrix} w \\ \theta \end{bmatrix} + [\mathbf{A}_a]\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (8)$$

where $\mathbf{M}_a$,$\mathbf{C}_a$, and $\mathbf{K}_a$ are aerodynamic mass, damping and stiffness matrix and introduced as:

$$\mathbf{M}_a = \pi\rho_\infty b^2 \begin{bmatrix} 1 & ba \\ ba & b^2\left(\frac{1}{8}+a^2\right) \end{bmatrix}$$

$$\mathbf{K}_a = 2\pi\rho_\infty {U_\infty}^2 b \begin{bmatrix} 0 & 1 \\ 0 & b\left(\frac{1}{2}+a\right) \end{bmatrix}$$

$$\mathbf{C}_a = 2\pi\rho_\infty bU_\infty \begin{bmatrix} 1 & -ba \\ b\left(\frac{1}{2}+a\right) & -b^2\left(\left(\frac{1}{4}+a^2\right)\right) \end{bmatrix}$$

$$\mathbf{A}_a = -2\pi\rho_\infty b\,U_\infty \begin{bmatrix} 1 & 1 \\ b\left(\frac{1}{2}+a\right) & b\left(\frac{1}{2}+a\right) \end{bmatrix} \quad (9)$$

And also the matrix form of Equation (7) as:

$$[\mathbf{I}]\begin{bmatrix} \dot{C}_1(t) \\ \dot{C}_2(t) \end{bmatrix} + [\mathbf{B}]\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = [\mathbf{D_1}]\begin{bmatrix} \dot{w} \\ \dot{\theta} \end{bmatrix} + [\mathbf{D_2}]\begin{bmatrix} w \\ \theta \end{bmatrix} \quad (10)$$

where **B**, $\mathbf{D_1}$, and $\mathbf{D_1}$ signified as:

$$[\mathbf{D}_2] = U_\infty \begin{bmatrix} 0 & 0.165 \\ 0 & 0.335 \end{bmatrix}$$

$$[\mathbf{D}_1] = \begin{bmatrix} 0.165 & -0.165ba \\ 0.335 & -0.335ba \end{bmatrix}$$

$$[\mathbf{B}] = \begin{bmatrix} 0.0455 & 0 \\ 0 & 0.335 \end{bmatrix}$$

2.3. DQM and aero-elastic equation

As mentioned earlier, aero-elastic formulation of the wing is estimated by the following equations:

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + me\frac{\partial^2\theta}{\partial t^2} = L$$
$$-GJ\frac{\partial^2\theta}{\partial x^2} + me\frac{\partial^2 w}{\partial t^2} + I_{ea}\frac{\partial^2\theta}{\partial t^2} = M_{ea} \quad (11)$$

By the implementing of a differential equation in the domain of the solution, Equation (12) is characterized as follows:

$$EI[I]\vec{w}_d'''' + m[I]\ddot{\vec{w}}_d + me\ [I]\ddot{\vec{\theta}}_d = \vec{L}$$
$$-GJ[I]\vec{\theta}_d'' + me[I]\ddot{\vec{w}}_d + I_{ea}[I]\ddot{\vec{\theta}}_d = \vec{M}_{ea} \quad (12)$$

Using the DQM technique Equation (13) is represented as:

$$EI[K_{B_d}]\,\vec{w}_d + m[I]\ddot{\vec{w}}_d + me\ [I]\ddot{\vec{\theta}}_d = \vec{L}$$
$$-GJ[K_{T_d}]\,\vec{\theta}_d + me[I]\ddot{\vec{w}}_d + I_{ea}\ [I]\ddot{\vec{\theta}}_d = \vec{M}_{ea} \quad (13)$$

So the stiffness matrix can be divided into the boundary and internal domain:

$$EI([K_{B_{dd}}] + [K_{B_{db}}])\,\vec{w}_d + m[I]\ddot{\vec{w}}_d + me\ [I]\ddot{\vec{\theta}}_d = \vec{L}$$
$$-GJ([K_{T_{dd}}] + [K_{T_{db}}]])\,\vec{\theta}_d + me[I]\ddot{\vec{w}}_d + I_{ea}[I]\ddot{\vec{\theta}}_d = \vec{M}_{ea} \quad (14)$$

By enforcing boundary condition on $(1, 2, \ldots, \mathrm{N}-1, \mathrm{N})$ points, the stiffness matrix on the boundary takes the following form:

$$[K_{B_b}]\,\vec{w} = 0$$
$$[K_{T_b}]\,\vec{w} = 0 \quad (15)$$

Using the stiffness matrix which related to the boundary points, the relation between the boundary and domain displacement is:

$$\begin{bmatrix}K_{B_{bb}} & K_{B_{bd}}\end{bmatrix}\begin{Bmatrix}\vec{w}_b\\ \vec{w}_d\end{Bmatrix} = 0 \Rightarrow \vec{w}_b = -([K_{B_{bb}}]^{-1}[K_{B_{bd}}])\vec{w}_d$$

$$\begin{bmatrix}K_{T_{bb}} & K_{T_{bd}}\end{bmatrix}\begin{Bmatrix}\vec{\theta}_b\\ \vec{\theta}_d\end{Bmatrix} = 0 \Rightarrow \vec{\theta}_b = -([K_{T_{bb}}]^{-1}[K_{T_{bb}}])\vec{\theta}_d \tag{16}$$

So the DQM form of structural equation is presented as:

$$\begin{aligned}&[K_B]\,\vec{w}_d + m[I]\ddot{\vec{w}}_d + me\,[I]\ddot{\vec{\theta}}_d = \vec{L}\\ &[K_T]\vec{\theta}_d + me[I]\ddot{\vec{w}}_d + I_{ea}\,[I]\ddot{\vec{\theta}}_d = \vec{M}_{ea}\end{aligned} \tag{17}$$

Where K_B and K_T are:

$$[K_B] = EI([K_{B_{dd}}] - [K_{B_{db}}][K_{B_{bb}}]^{-1}[K_{B_{bd}}])$$

$$[K_T] = -GJ([K_{T_{dd}}] - [K_{T_{db}}][K_{T_{bb}}]^{-1}[K_{T_{bd}}])$$

So the structural section of the aero-elastic equation can be obtained as:

$$\mathbf{M}_s\ddot{\mathbf{q}} + \mathbf{C}_s\dot{\mathbf{q}} + \mathbf{K}_s\mathbf{q} = \mathbf{f} \tag{18}$$

Where

$$\mathbf{q} = \begin{Bmatrix}\vec{w}\\ \vec{\theta}\end{Bmatrix} \quad \mathbf{M}_s = \begin{bmatrix}m[I] & me[I]\\ me[I] & I_{ea}[I]\end{bmatrix};$$

$$\mathbf{C}_s = \begin{bmatrix}[0] & [0]\\ [0] & [0]\end{bmatrix}; \mathbf{K}_s = \begin{bmatrix}[K_B] & [0]\\ [0] & [K_T]\end{bmatrix}$$

Also, the DQM form of aerodynamic model can be written as follows:

$$\begin{aligned}&[I]\dot{B}_1 + \left(\beta_1\frac{U_\infty}{b}\right)[I]B_1 = [I]\dot{w} - U_\infty[I]\theta + ba\,[I]\dot{\theta}\\ &\quad - \frac{b}{2}\left(\frac{C_{L_\alpha}}{\pi} - 1\right)[I]\dot{\theta}\\ &[I]\dot{B}_2 + \left(\beta_2\frac{U_\infty}{b}\right)[I]B_2 = [I]\dot{w} - U_\infty[I]\theta + ba[I]\,\dot{\theta}\\ &\quad - \frac{b}{2}\left(\frac{C_{L_\alpha}}{\pi} - 1\right)[I]\dot{\theta}\end{aligned} \tag{19}$$

The matrix form of aero-elastic equation can be shown as:

$$\mathbf{I}\,\dot{\mathbf{d}} + \mathbf{E}\,\mathbf{d} = \mathbf{D}_1\,\dot{\mathbf{q}} + \mathbf{D}_2\,\mathbf{q} \tag{20}$$

where

$$\mathbf{I} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix} \mathbf{E} = \frac{U_\infty}{b}\begin{bmatrix}\beta_1.\mathbf{I} & 0\\ 0 & \beta_2.\mathbf{I}\end{bmatrix}$$

$$\mathbf{D}_1 = \begin{bmatrix}\mathbf{I} & ba - \frac{b}{2}\left(\frac{C_{L_\alpha}}{\pi} - 1\right).\mathbf{I}\\ \mathbf{I} & ba - \frac{b}{2}\left(\frac{C_{L_\alpha}}{\pi} - 1\right).\mathbf{I}\end{bmatrix} \mathbf{D}_2 = -U_\infty\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}$$

3. Result and discussion

In this section, the verifications of the introduced coupled formulation are carried out for two test cases. The first test case is a clamped free typical airfoil and the second

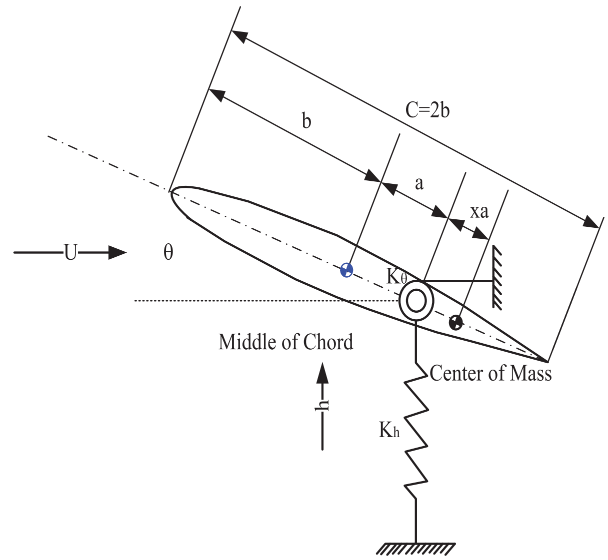


Figure 1. Two-dimensional typical section airfoil.

Table 1. Parameters of two-dimensional airfoils (Haddadpour & Firouz-Abadi, 2009).

PARAMETER	Description	Case 1
$\sigma = \omega_\theta/\omega_h$	Bending to torsion frequencies ratio	0.4
A	Elastic axis position on the airfoil	−0.2
x_α	Center of mass	.1
$\mu = m/\pi\rho b^2$	Mass ratio	20
$r_\alpha = I_\alpha/mb^2$	Radius of gyration	0.24

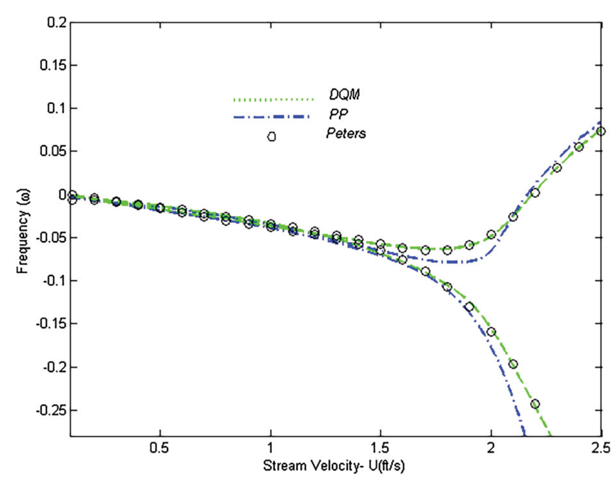


Figure 2. Real part variation at subsonic speed.

Table 2. The variation of flutter speed vs. number of nodes.

Number of nodes	8	20	25
Non-dimensional flutter speed	1.8	1.98	1.985

Table 3. Given wing model data.

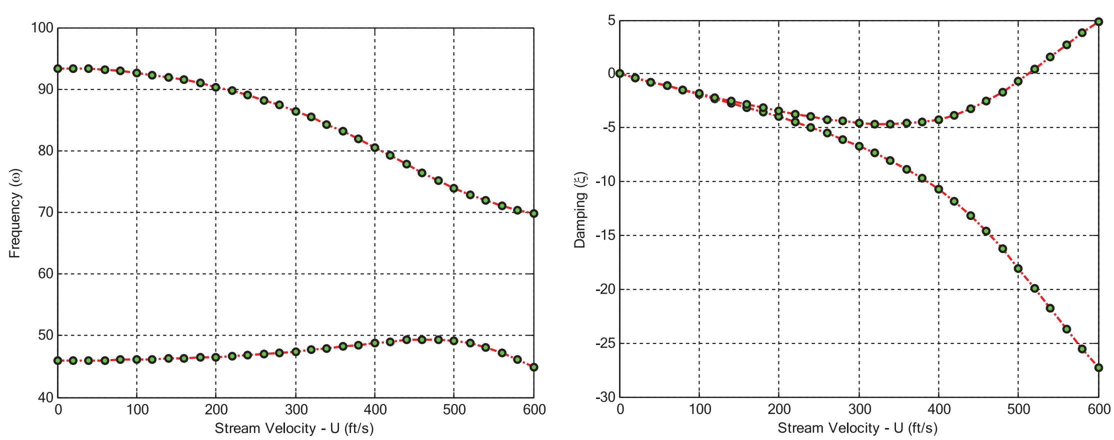
Parameters	Unit and discerption	Value
$\rho_{Sea\ Level}$	Density (sea level)$Slug/ft^3$	0.002378
$\rho_{20\ Kft}$	Density (20Kft)$Slug/ft^3$	0.001267
EI	Flexural stiffness ($lb.ft$)	23.6×10^6
GJ	Torsional stiffness ($lb.ft$)	2.39×10^6
I_{ea}	$Slug/ft^2/ft$	1.943
L	Wing span(ft)	20
$c = 2b$	Cord (ft)	6
a	Elastic axis location (ft)	$-1/3$
x_θ	Center of Mass	0.1997
m	Mass per length ($Slug/ft$)	0.746

one is a Goland shaped wing slender wing. Indeed, the airfoil is a finite wing with translational and torsional springs (Figure 1). Using unsteady Peters theory and also PP method (Haddadpour & Firouz-Abadi, 2009), the flutter speed estimation of typical section airfoil (Table 1) is evaluated (Figure 2).

The study of the DQM result which is extracted by different nodes for frequencies shows that the instability speed approximations are very close to the other models (Table 2).

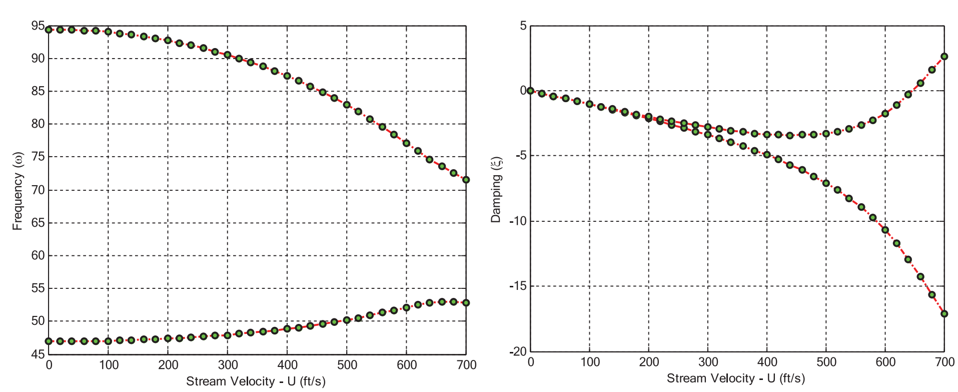
Investigations of the slender body are also performed for quasi-steady and non-steady aerodynamic models by the DQM method with the following specification (Table 3). The Results are extracted like previous examples: by P method at both sea level and 20,000 ft (Figures 3–6).

The flutter speeds of the given airfoil are estimated by 20 nodes for two aerodynamic models (Table 4). The results show minimum slight differences compared to the literature. In the following figures, in which are illustrated the results of flutter speed estimation, the U, ω, and ζ are air speed, frequency, and damping respectively.



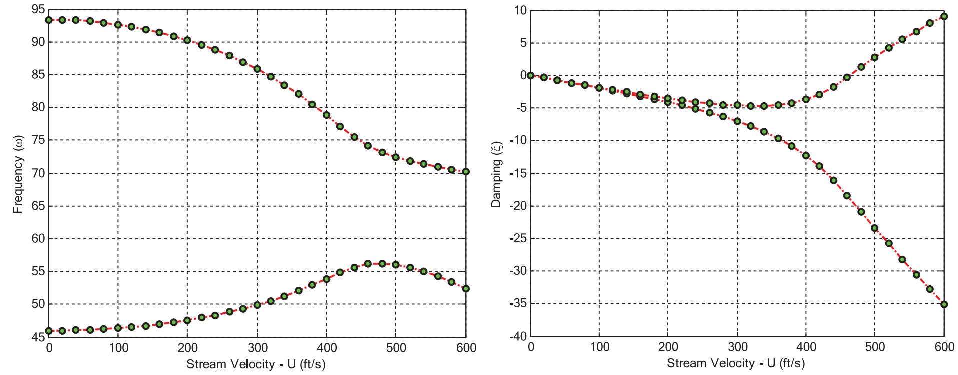
(a) **(b)**

Figure 3. (a) Frequency; and (b) damping part of the aero-elastic system for quasi-steady aerodynamic model at sea level.



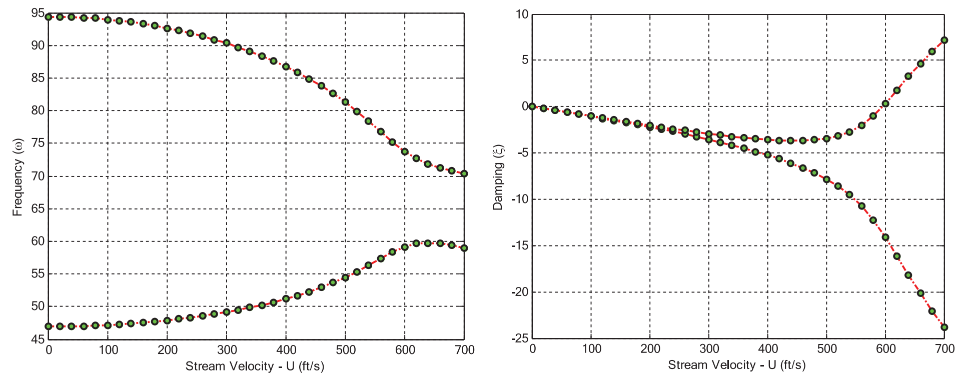
(a) **(b)**

Figure 4. (a) Frequency; and (b) damping part of the aero-elastic system for quasi-steady aerodynamic model at 20,000 ft.



(a) (b)

Figure 5. (a) Frequency; and (b) damping part of the aero-elastic system for non-steady aerodynamic model at sea level.



(a) (b)

Figure 6. (a) Frequency; and (b) damping part of the aero-elastic system for non-steady aerodynamic model at 20,000 ft.

Table 4. Comparison between presented models for aerodynamic model vs. literature.

	Flutter speed (ft/s)			
	At sea level		*At 20 Kft*	
	Unsteady	Quasi-steady	Unsteady	Quasi-steady
Present	446	446	573	573
Haddadpour & Firouz-Abadi (2009)	447	447	574	574

4. Conclusion

In this study, a DQM model for the flutter speed prediction of a simplified airfoil (typical section) was developed. Using unsteady and quasi-steady Wagner function the instability condition of airfoil was studied. Comparison between the presented model and PP, Theodorsen, and also Peter's aerodynamic methods for both a typical section airfoil and Goland wing for sea level and 20,000 ft above was conducted. The good agreements between the results, which are extracted by only 20 nodes and simplification of governing equations, show the tools powerful, with minimum 0.17% difference in aero-elastic instability estimation. Future studies, besides overcoming challenges which emerged from instability predicting differences especially in quasi-steady models, could be focused on the nonlinearity effect of aerodynamics beside the large deflection of structures in flutter speed approximation using a DQM approach.

Disclosure statement

No potential conflict of interest was reported by the authors.

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