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Multi-Power Reaching Law Based Discrete-Time Sliding-Mode Control

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ABSTRACT This paper proposes a multi-power reaching law-based sliding-mode control (SMC) for uncertain discrete-time systems. The proposed controller mainly consists of the multi-power function along with the perturbation estimation. Different from the existing similar works, the control gains of the controller are adaptively adjusted by the multi-power function, i.e., three power terms, according to different stages of the convergence process. Hence, the system trajectory of the controlled system can be forced toward the sliding surface with a faster convergence rate. The corresponding sliding-mode dynamics and the reaching steps to the sliding surface are theoretically analyzed. A practical example is given to examining the validity of the proposed method. The simulation results show that the proposed method reduces the reaching steps while guaranteeing better control accuracy than the single power method.

INDEX TERMS Multi-power function, discrete-time sliding-mode control (DSMC), reaching law.

I. INTRODUCTION

During the past few decades, sliding-mode control (SMC) has been widely employed to stabilize varieties of linear and nonlinear systems [1]–[3]. It has many attractive merits like easy realization, quick response and especially the invariability to parameter uncertainties and external disturbances [4]. Nowadays, more and more control methods are performed in the sampled-data system, the study on discrete-time SMC (DSMC) has attracted the attention of many researchers [5]. It is notable that, the properties valid for continuous-time SMC are incapable of extending directly to its discrete-time counterpart because of the finite sampling rate in the sampled-data system. Therefore, the redesign of DSMC becomes imperative and preferable. Among these design methods, the reaching law method, first presented in [6] and [7], has been certified to be a simple and effective one. It owns many advantages, like streamlining the design process of DSMC, and describing sliding-mode dynamics of DSMC systems [8]. Some improved reaching law methods have been proposed by other researchers, such as the non-switching reaching law [9]–[15], the observed based reaching law [16]–[20], the power reaching law [7], [9], [21], the generalized reaching law [22]–[24], and so on.

It is notable that the reaching law method for discrete-time systems is not flawless; indeed, in practical applications, the adoption of the discrete-time reaching law will result in chattering phenomenon in the vicinity of the sliding surface. This is unacceptable in some applications [25]–[27].

An interesting method in literature for chattering alleviation is the single power reaching law, which replaces the discontinuous gain k of the sign function by a power term of the switching function $k \cdot |s(k)|^{\beta}$ [7], [9], [21]. This method can mitigate chattering since its convergence rate varies in accordance with the distance variation. Nevertheless, when the state is far away from the sliding surface, the extremely small convergence rate results in long reaching time. A bi-power reaching law has been proposed in [28] for continuous-time systems. However, it cannot be directly employed to discrete-time systems. To the best of our knowledge, the multi-power reaching law based DSMC has not been properly investigated.

In this paper, a multi-power reaching law based DSMC is introduced, which contains the multi-power function and

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the perturbation estimation. In comparison with the single power method, the proposed control scheme enables a faster convergence rate and better control accuracy. The gains of the switching control terms are meaningfully and adaptively adjusted by the multi-power function according to different stages of the convergence process. Moreover, the sliding mode dynamics and the reaching steps of the developed method are discussed and analyzed theoretically. A practical example is given to illustrate the validity of the developed method.

This paper is organized as follows: in Section II, the system description along with the important property and assumption are given. The novel multi-power based DSMC is presented in Section III as well as the system dynamics analysis. Section IV presents simulation results. Conclusion is given in Section V.

II. SYSTEM DESCRIPTION

The following uncertain discrete-time system is considered:

$$x(k+1) = Ax(k) + Bu(k) + d(k), \tag{1}$$

where $x(k) \in R^n$ represents the state variable, $u(k) \in R^1$ stands for the control input variable. The disturbance $d(k) \in R^n$ owns a property as follows:

Property 1 [29]: $d(k) = O(T)$, $d(k) - d(k-1) = O(T^2)$, and $d(k) - 2d(k-1) + d(k-2) = O(T^3)$, where T is the sampling time interval.

With the state variable, a discrete-time switching function is constructed as follows:

$$s(k) = Cx(k), \tag{2}$$

where C is to be chosen such that CB is invertible.

Assumption 1: $\delta(k)$, which represents the change rate of the disturbance $d(k)$, is expected to be bounded as follows:

$$|\delta(k)| \leq \delta^*, \tag{3}$$

where $\delta(k)$ can be indicated as $\delta(k) = \delta_2(k) = C[d(k) - d(k-1)]$ [21] or $\delta(k) = \delta_3(k) = C[d(k) - 2d(k-1) + d(k-2)]$ [14]. Referring to *Property 1*, the upper bound δ^* is in the order of $O(T^2)$ or $O(T^3)$.

The following lemma is required in the demonstration of *Theorems 1* and *2*

Lemma 1 [30]: Let $f : I \subset \mathrm{R} \to \mathrm{R}$ be a convex function. If $x_i \in I$ $(i = 1, 2, \ldots, n)$, and $q_i \geq 0 (i = 1, 2, \ldots, n)$ with $\sum_{i=1}^{n} q_i = 1$, then

$$f\left(\sum_{i=1}^{n} q_i x_i\right) \leq \sum_{i=1}^{n} q_i f(x_i). \tag{4}$$

III. MULTI-POWER REACHING LAW BASED DSMC

The single power reaching law has been presented in previous works [9], [21]:

$$s(k+1) = (1 - qT_0)\, s(k) - k_0\, |s(k)|^{\tau} \operatorname{sgn}(s(k)) + \delta_2(k), \tag{5}$$

where $0 < 1 - qT_0 < 1$, $k_0 > 0$, $0 < \tau < 1$. Although the chattering can be reduced, the reaching time of this method is significantly increased when the state is far away from the sliding surface.

In this paper, a multi-power reaching law is presented to conquer the drawbacks of single power reaching law. The proposed reaching law is:

$$\begin{aligned} s(k+1) = {} & (1 - qT)\, s(k) \\ & - k_1 |s(k)|^{\alpha} \operatorname{sgn}(s(k)) - k_2 |s(k)|^{\beta} \operatorname{sgn}(s(k)) \\ & - k_3 |s(k)|^{\gamma} \operatorname{sgn}(s(k)) + \delta(k), \end{aligned} \tag{6}$$

where $0 < 1 - qT < 1$, $k_1, k_2, k_3 > 0$, $\alpha > 1$, $0 < \beta < 1$, $\delta(k) = \delta_2(k)$ or $\delta_3(k)$.

$$\gamma = \begin{cases} \max\{\alpha, |s(k)|\}, & \text{if } |s(k)| \geq 1 \\ \min\{\beta, |s(k)|\}, & \text{if } |s(k)| < 1. \end{cases} \tag{7}$$

Considering Eq. (7), it is deduced that γ will not equal to 1.

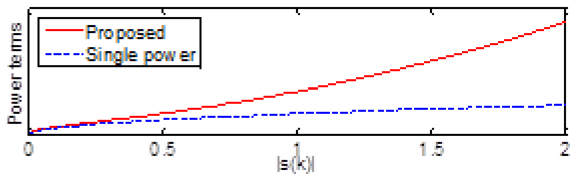

FIGURE 1. Power terms comparison.

Figure 1 depicts the power terms evolution employing the proposed method and the single power reaching law. The initial state $|s(0)| = 2$.

Remark 1: The reaching law (6) has three power terms, i.e., the multi-power function:

1) When the state variable is far away from the sliding surface ($|s(k)| \geq 1$), $k_1|s(k)|^{\alpha}\operatorname{sgn}(s(k)) + k_3|s(k)|^{\gamma} \operatorname{sgn}(s(k))$ plays a leading role and the effect of $k_2|s(k)|^{\beta}\operatorname{sgn}(s(k))$ can be ignored comparing with other power terms. Hence, the convergence rate toward the sliding surface is faster than the single power reaching law (5).
2) When the state variable approximates the sliding surface ($|s(k)| < 1$), $k_2|s(k)|^{\beta}\operatorname{sgn}(s(k)) + k_3|s(k)|^{\gamma}\operatorname{sgn}(s(k))$ plays a leading role and the effect of $k_1|s(k)|^{\alpha}\operatorname{sgn}(s(k))$ can be ignored. Hence, the proposed method ensures a slightly shorter convergence time than the single power reaching law. It is notable that the multi-power function approaches the single power function when $|s(k)|$ is quite small, as illustrated in Fig. 1.
3) $k_3|s(k)|^{\gamma}\operatorname{sgn}(s(k))$ further divides the convergence process into four stages: $s(0) \to s(k_l) = \alpha$, $s(k_l) = \alpha \to s(k_m) = 1$, $s(k_m) = 1 \to s(k_n) = \beta$, $s(k_n) = \beta \to s(k_o) = 0$. γ alters in the four stages, which ensures the system quickly and smoothly converge to the sliding surface from the initial state.

Because of the unavailable knowledge of $d(k)$, the disturbance term can be estimated by the perturbation estimation method [31]. The system states are measurable in most cases.

$$d(k-1) = x(k) - Ax(k-1) - Bu(k-1). \tag{8}$$

Inserting the system model (1) into (2) and in view of the reaching law (6) and (8) gives

$$u(k)\begin{cases} = -(CB)^{-1}\begin{bmatrix} CAx(k) - (1-qT)\,s(k) \\ +k_1\,|s(k)|^{\alpha}\,\mathrm{sgn}\,(s(k)) \\ +k_2\,|s(k)|^{\beta}\,\mathrm{sgn}\,(s(k)) \\ +k_3\,|s(k)|^{\gamma}\,\mathrm{sgn}\,(s(k)) \\ +2Cd(k-1) - Cd(k-2) \end{bmatrix}, \\ \qquad\qquad \text{if } \delta(k) = \delta_3(k) \\ = -(CB)^{-1}\begin{bmatrix} CAx(k) - (1-qT)\,s(k) \\ +k_1\,|s(k)|^{\alpha}\,\mathrm{sgn}\,(s(k)) \\ +k_2\,|s(k)|^{\beta}\,\mathrm{sgn}\,(s(k)) \\ +k_3\,|s(k)|^{\gamma}\,\mathrm{sgn}\,(s(k)) \\ +Cd(k-1) \end{bmatrix}, \\ \qquad\qquad \text{if } \delta(k) = \delta_2(k). \end{cases} \tag{9}$$

Next, the stability of the designed DSMC system will be discussed in the following aspects: the first two are the system dynamics in and out the vicinity of the sliding surface, and the last one is how many steps that the system trajectory needs to first cross the sliding surface.

Theorem 1: Noting the uncertain discrete-time system (1), suppose that the switching function (2) and the DSMC controller (9) are employed, then the system trajectory $s(k)$ can converge into the region Ψ defined as follows:

$$\begin{aligned} \Psi &= \chi(\gamma)\cdot\Phi \\ &= \chi(\gamma)\cdot\max\left\{\Phi_1 = \left(\frac{\delta^*}{k_f}\right)^{\frac{1}{k_{11}\alpha+k_{22}\beta+k_{33}\gamma}},\right. \\ &\qquad\qquad \left.\Phi_2 = \left(\frac{k_f}{1-qT}\right)^{\frac{1}{1-\gamma}}\right\}, \end{aligned} \tag{10}$$

with $k_{11}+k_{22}+k_{33} = 1$, $k_f\cdot k_{11} = k_1$, $k_f\cdot k_{22} = k_2$, $k_f\cdot k_{33} = k_3$, and

$$\chi(\gamma) = 1 + \gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}. \tag{11}$$

Proof: To demonstrate that $s(k)$ will be driven to Ψ, a Lyapunov function $V(k) = s^2(k)$ is selected. Inserting (6) into the Lyapunov function leads to

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= [s(k+1)-s(k)]\,[s(k+1)+s(k)]\,. \end{aligned} \tag{12}$$

Then, the following two cases, i.e., $s(k) > \Psi$ and $s(k) < \Psi$, will be considered.

Case 1: If $s(k) > \Psi$, considering (6), the difference between $s(k+1)$ and $s(k)$ can be computed as follows:

$$\begin{aligned} s(k+1)-s(k) = &-qTs(k) - k_1\,|s(k)|^{\alpha}\,\mathrm{sgn}\,(s(k)) \\ &- k_2\,|s(k)|^{\beta}\,\mathrm{sgn}\,(s(k)) \\ &- k_3\,|s(k)|^{\gamma}\,\mathrm{sgn}\,(s(k)) + \delta(k). \end{aligned} \tag{13}$$

Additionally, recalling *Lemma 1*, it can be derived from (13) that

$$\begin{aligned} &s(k+1)-s(k) \\ &\quad\le -qTs(k) - k_1s(k)^{\alpha} - k_2s(k)^{\beta} - k_3s(k)^{\gamma} + \delta^* \\ &\quad\le -k_1s(k)^{\alpha} - k_2s(k)^{\beta} - k_3s(k)^{\gamma} + \delta^* \\ &\quad\le -k_fs(k)^{k_{11}\alpha+k_{22}\beta+k_{33}\gamma} + \delta^*. \end{aligned} \tag{14}$$

Taking into account (14), it can be obtained that if $-k_fs(k)^{k_{11}\alpha+k_{22}\beta+k_{33}\gamma}s(k)^{k_{11}\alpha+k_{22}\beta+k_{33}\gamma} + \delta^* < 0$, then $s(k+1)-s(k) < 0$ holds. Solving inequality (14) yields

$$s(k) > \Phi_1 = \left(\frac{\delta^*}{k_f}\right)^{\frac{1}{k_{11}\alpha+k_{22}\beta+k_{33}\gamma}}. \tag{15}$$

Based on (6), we can get

$$\begin{aligned} s(k+1)+s(k) \ge (2-qT)\,s(k) - k_1s(k)^{\alpha} \\ - k_2s(k)^{\beta} - k_3s(k)^{\gamma} - \delta^*. \end{aligned} \tag{16}$$

Recalling (14), it can be derived that if

$$\begin{aligned} &(2-qT)\,s(k) - k_1s(k)^{\alpha} - k_2s(k)^{\beta} - k_3s(k)^{\gamma} - \delta^* \\ &\qquad \ge qTs(k) + k_1s(k)^{\alpha} + k_2s(k)^{\beta} + k_3s(k)^{\gamma} - \delta^*, \end{aligned} \tag{17}$$

then $s(k+1)+s(k) > 0$ holds. Inequality (17) is simplified into

$$(1-qT)\,s(k) - k_1s(k)^{\alpha} - k_2s(k)^{\beta} - k_3s(k)^{\gamma} \ge 0. \tag{18}$$

1) If $s(k) \ge 1$, then inequality (18) can be represented as

$$(1-qT)\,s(k) - k_fs(k)^{\gamma} \ge 0, \tag{19}$$

with $\gamma = \max\{s(k), \alpha\}$. It is derived that if condition (20) is met

$$s(0) \le \left(\frac{1-qT}{k_f}\right)^{\frac{1}{\gamma-1}}, \tag{20}$$

then inequality (19) holds.

2) If $s(k) < 1$, then inequality (18) can be represented as (19) with $\gamma = \min\{s(k), \beta\}$. It is deduced from (19) that

$$s(k) \ge \Phi_2 = \left(\frac{k_f}{1-qT}\right)^{\frac{1}{1-\gamma}}. \tag{21}$$

Case 2: If $s(k) < -\Psi$, the proof is similar to Case 1 and the relation $\Delta V(k) < 0$ still holds.

Considering *Assumption 1* and noting [21], δ^* is in the order of $O(T^2)$ or $O(T^3)$ and the system trajectory can converge into a small region whose width can approach δ^*. According to [32], it is deduced that $\chi(\gamma) \in\in (1, 2)$. Hence, if $s(k) > \Psi$, then $\Delta V(k) < 0$, and system trajectory can be driven onto the Ψ vicinity.

Next, we will demonstrate the fact that once the system trajectory gets into the Ψ region, it cannot escape from this region, i.e., $|s(k+1)| \in \Psi$, $\forall |s(k)| \in \Psi$. Before moving forward, some lemmas are given in the following. ■

Lemma 2: Function $\chi(x) = 1 + x^{\frac{x}{1-x}} - x^{\frac{1}{1-x}}$ is monotonically decreasing with $x \in (0, 1)$ and $(1, +\infty)$.

Proof: We should calculate the first order derivative of $\chi(x)$. For convenience, let $y = x^{\frac{x}{1-x}}$ and $z = x^{\frac{1}{1-x}}$, it yields

$$\text{In}y = \frac{x}{1-x}\text{In}x, \quad \text{In}z = \frac{1}{1-x}\text{In}x, \tag{22}$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{\text{In}x}{(1-x)^2} + \frac{1}{1-x}\right] x^{\frac{x}{1-x}},$$

$$\frac{dz}{dx} = \left[\frac{\text{In}x}{(1-x)^2} + \frac{1}{x(1-x)}\right] x^{\frac{1}{1-x}}. \tag{23}$$

Combining (22) and (23) gives

$$\frac{d\chi(x)}{dx} = \frac{\text{In}x}{1-x} x^{\frac{x}{1-x}}. \tag{24}$$

Moreover, it is deduced that $\lim_{x\to+\infty} y = 0$, $\lim_{x\to+\infty} z = 1$, $\lim_{x\to 1} y = 1/e$, $\lim_{x\to 1} z = 1/e$. If $x > 1$, then $d\chi(x)/dx < 0$, and $0 < \chi(x) < 1$; If $0 < x < 1$, then $d\chi(x)/dx < 0$, and $1 < \chi(x) < 2$. Hence, $\chi(x)$ is monotonically decreasing with respect to $x \in (0, +\infty)$. ■

Lemma 3: The following condition can be met in the Ψ region:

$$\delta^* \le k_f \Phi^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma} \le k_f \max\left\{\Phi^\alpha, \Phi^\beta, \Phi^\gamma\right\} \le (1-qT)\Phi. \tag{25}$$

Proof: As stated in *Theorem 1*, the Ψ region is quite small and $\gamma = \min\{s(k), \beta\}$ in this region. There are two situations to be considered:

Case 1: If $\Phi = \Phi_2 = \max\{\Phi_1, \Phi_2\}$, i.e.,

$$\Phi_1 = \left(\frac{\delta^*}{k_f}\right)^{\frac{1}{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}} \le \Phi = \Phi_2 = \left(\frac{k_f}{1-qT}\right)^{\frac{1}{1-\gamma}}, \tag{26}$$

then it can be derived from (26) that

$$\delta^* \le k_f \Phi^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}, \quad (1-qT)\Phi = k_f \Phi^\gamma = k_f \max\left\{\Phi^\alpha, \Phi^\beta, \Phi^\gamma\right\}. \tag{27}$$

In view of *Lemma 1*, the following relation is derived

$$k_f \max\left\{\Phi^\alpha, \Phi^\beta, \Phi^\gamma\right\} \ge k_1\Phi^\alpha + k_2\Phi^\beta + k_3\Phi^\gamma \ge k_f \Phi^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}. \tag{28}$$

Case 2: If $\Phi = \Phi_1 = \max\{\Phi_1, \Phi_2\}$, i.e.,

$$\Phi = \Phi_1 = \left(\frac{\delta^*}{k_f}\right)^{\frac{1}{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}} \ge \Phi_2 = \left(\frac{k_f}{1-qT}\right)^{\frac{1}{1-\gamma}}, \tag{29}$$

then it is deduced from (29) that

$$\delta^* = k_f \Phi^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}, \quad (1-qT)\Phi \ge k_f \Phi^\gamma = k_f \max\left\{\Phi^\alpha, \Phi^\beta, \Phi^\gamma\right\}. \tag{30}$$

Considering *Lemma 1*, inequality (28) still holds in this situation. ■

Theorem 2: Noting the discrete-time system represented by (1) and the switching function (2), the DSMC controller (9) and the condition (31), once the system trajectory gets into the Ψ region, the following condition $|s(k+1)| \le \Psi$, $\forall |s(k)| \le \Psi$ is met.

$$k_{11}\alpha + k_{22}\beta + k_{33}\gamma \le 1 \quad \text{or } 2(1-qT) \le \chi(\gamma). \tag{31}$$

Proof: By defining $s(k) = \mu \cdot \Psi = \mu\chi(\gamma)\Phi$ with $-1 \le \mu \le 1$ and $\text{sig}^x(s(k)) = |s(k)|^x \text{ sgn }(s(k))(x = \alpha, \beta, \gamma)$, Eq. (6) is represented as

$$\begin{aligned} s(k+1) &= (1-qT)\mu\chi(\gamma)\Phi - k_1\text{sig}^\alpha(\mu\chi(\gamma))\Phi^\alpha \\ &\quad - k_2\text{sig}^\beta(\mu\chi(\gamma))\Phi^\beta - k_3\text{sig}^\gamma(\mu\chi(\gamma))\Phi^\gamma + \delta(k) \\ &\le (1-qT)\mu\chi(\gamma)\Phi - k_1\text{sig}^\alpha(\mu\chi(\gamma))\Phi^\alpha \\ &\quad - k_2\text{sig}^\beta(\mu\chi(\gamma))\Phi^\beta - k_3\text{sig}^\gamma(\mu\chi(\gamma))\Phi^\gamma + \delta^*. \end{aligned} \tag{32}$$

Case 1: If $\mu \ge 0$, recalling *Lemma 1*, then it is deduced from (32) that

$$s(k+1) \le (1-qT)\mu\chi(\gamma)\Phi - k_f(\mu\chi(\gamma))^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}\Phi^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma} + \delta^*. \tag{33}$$

If $\mu\chi(\gamma) \ge 1$, in view of *Lemma 3* and (33) yields

$$\begin{aligned} s(k+1) &\le (1-qT)\mu\chi(\gamma)\Phi - (\mu\chi(\gamma))^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}\delta^* + \delta^* \\ &\le (1-qT)\mu\chi(\gamma)\Phi \\ &\le \chi(\gamma)\Phi = \Psi. \end{aligned} \tag{34}$$

If $0 \le \mu\chi(\gamma) < 1$, in view of *Lemma 3*, it can be derived from (33) that

$$\begin{aligned} &s(k+1) \\ &\le (1-qT)\mu\chi(\gamma)\Phi + \left[1 - (\mu\chi(\gamma))^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}\right](1-qT)\Phi \\ &= \left[1 + \mu\chi(\gamma) - (\mu\chi(\gamma))^{k_{11}\alpha + k_{22}\beta + k_{33}\gamma}\right](1-qT)\Phi. \end{aligned} \tag{35}$$

By noting condition (31), it is found that $s(k) \le \Psi$.

Case 2: If $\mu < 0$, in accordance with *Lemma 3* gives

$$\begin{aligned} &s(k+1) \\ &\le -(1-qT)|\mu\chi(\gamma)|\Phi + k_1|\mu\chi(\gamma)|^\alpha\Phi^\alpha \\ &\quad + k_2|\mu\chi(\gamma)|^\beta\Phi^\beta + k_3|\mu\chi(\gamma)|^\gamma\Phi^\gamma + \delta^* \\ &\le -(1-qT)|\mu\chi(\gamma)|\Phi \\ &\quad + \left[\begin{array}{l}\frac{k_1}{k_f}|\mu\chi(\gamma)|^\alpha + \frac{k_2}{k_f}|\mu\chi(\gamma)|^\beta \\ + \frac{k_3}{k_f}|\mu\chi(\gamma)|^\gamma\end{array}\right] k_f \max\left\{\begin{array}{c}\Phi^\alpha, \\ \Phi^\beta, \\ \Phi^\gamma\end{array}\right\} + \delta^* \\ &\le -(1-qT)|\mu\chi(\gamma)|\Phi \\ &\quad + \left[\begin{array}{l}\frac{k_1}{k_f}|\mu\chi(\gamma)|^\alpha + \frac{k_2}{k_f}|\mu\chi(\gamma)|^\beta \\ + \frac{k_3}{k_f}|\mu\chi(\gamma)|^\gamma\end{array}\right](1-qT)\Phi + \delta^*. \end{aligned} \tag{36}$$

If $\mu\chi(\gamma) \le -1$, the following deduction is obtained from (36) considering *Lemma 3*

$$\begin{aligned} &s(k+1) \\ &\le -(1-qT)|\mu\chi(\gamma)|\Phi \end{aligned}$$

$$+\frac{k_1+k_2+k_3}{k_f}\left|\mu\chi(\gamma)\right|^{\alpha}(1-qT)\Phi+(1-qT)\Phi$$
$$=-\left[\left|\mu\chi(\gamma)\right|-\left|\mu\chi(\gamma)\right|^{\alpha}-1\right](1-qT)\Phi. \quad (37)$$

Next, we will prove $[|\mu\chi(\gamma)|-|\mu\chi(\gamma)|^{\alpha}-1](1-qT)\geq -\chi(\gamma)$. Construct the following function with $0\leq x\leq 1$

$$g(x)=\left[x\chi(\gamma)-x^{\alpha}\chi(\gamma)^{\alpha}-1\right](1-qT)+\chi(\gamma). \quad (38)$$

In view of the expression of $f(x)$ leads to

$$g(0)=\chi(\gamma)-(1-qT)>0, \quad (39)$$
$$\frac{dg(x)}{dx}=\left[\chi(\gamma)-\alpha x^{\alpha-1}\chi(\gamma)^{\alpha}\right](1-qT)=0, \quad (40)$$
$$\Rightarrow x=\frac{\alpha^{\frac{1}{1-\alpha}}}{\chi(\gamma)}. \quad (41)$$

Substituting (41) into (38) gives

$$g(x)=\left(1+\gamma^{\frac{\gamma}{1-\gamma}}-\gamma^{\frac{1}{1-\gamma}}\right)-(1-qT)\left(1+\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}\right)$$
$$\geq\left(1+\gamma^{\frac{\gamma}{1-\gamma}}-\gamma^{\frac{1}{1-\gamma}}\right)-\left(1+\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}\right). \quad (42)$$

According to *Lemma 2*, $g(x)>0$ holds. Hence, $s(k)\leq\Psi$.

If $-1<\mu\chi(\gamma)\leq 0$, the expression of $s(k+1)$ can be represented as

$$s(k+1)\leq-(1-qT)\left|\mu\chi(\gamma)\right|\Phi+\left|\mu\chi(\gamma)\right|^{\gamma}(1-qT)\Phi+(1-qT)\Phi$$
$$=-\left[\left|\mu\chi(\gamma)\right|-\left|\mu\chi(\gamma)\right|^{\gamma}-1\right](1-qT)\Phi. \quad (43)$$

Considering [21, Lemma A.2], Eq. (43) is devised as follows

$$s(k+1)\leq(1-qT)\chi(\gamma)\Phi\leq\Psi. \quad (44)$$

Similarly, noting (6), the following deduction is generated

$$s(k+1)\geq(1-qT)\mu\chi(\gamma)\Phi-k_1\mathrm{sig}^{\alpha}(\mu\chi(\gamma))\Phi^{\alpha}-k_2\mathrm{sig}^{\beta}(\mu\chi(\gamma))\Phi^{\beta}-k_3\mathrm{sig}^{\gamma}(\mu\chi(\gamma))\Phi^{\gamma}-\delta^*. \quad (45)$$

Case 1: If $\mu\geq 0$ and $\mu\chi(\gamma)\geq 1$, combining *Lemma 1* and (45) yields

$$s(k+1)\geq(1-qT)\mu\chi(\gamma)\Phi-(\mu\chi(\gamma))^{\alpha}(1-qT)\Phi-(1-qT)\Phi$$
$$=\left[\mu\chi(\gamma)-(\mu\chi(\gamma))^{\alpha}-1\right](1-qT)\Phi$$
$$\geq-\Psi. \quad (46)$$

If $0\leq\mu\chi(\gamma)<1$, similar to (43), it is derived that

$$s(k+1)\geq\left[\mu\chi(\gamma)-(\mu\chi(\gamma))^{\gamma}-1\right](1-qT)\Phi\geq-\Psi. \quad (47)$$

Case 2: If $\mu<0$ and $\mu\chi(\gamma)\leq-1$, we can obtain that

$$s(k+1)\geq-(1-qT)\left|\mu\chi(\gamma)\right|\Phi+\left[\left|\mu\chi(\gamma)\right|^{k_{11}\alpha+k_{22}\beta+k_{33}\gamma}-1\right]\delta^*$$
$$\geq-(1-qT)\left|\mu\chi(\gamma)\right|\Phi\geq-\Psi. \quad (48)$$

If $0\leq\mu\chi(\gamma)<1$, similar to (35)

$$s(k+1)\geq\left[\left|\mu\chi(\gamma)\right|^{k_{11}\alpha+k_{22}\beta+k_{33}\gamma}-1-\left|\mu\chi(\gamma)\right|\right](1-qT)\Phi$$
$$\geq-\Psi. \quad (49)$$

Therefore, in the Ψ region, the following condition $|s(k+1)|\leq\Psi, \forall|s(k)|\leq\Psi$ is satisfied.

It is noted that the parameters in condition (31) are adjustable control gains. Hence, condition (31) can hold by selecting appropriate gains.

Theorem 3: For the system (1) with the DSMC controller (9), the system trajectory will take at most K^* steps (finite steps) to first cross the sliding surface, where

$$K^*=\lfloor m^*\rfloor+1 \text{ with}$$
$$m^*=\frac{s^2(0)-\alpha^2}{\mu_1^2}+\frac{\alpha^2-1}{\mu_2^2}+\frac{1-\beta^2}{\mu_3^2}+\frac{\beta^2}{\mu_4^2},$$
$$\mu_1=qT\alpha+(k_1+k_3)\alpha^{\alpha}-\delta^*,$$
$$\mu_2=qT+(k_1+k_3)-\delta^*,$$
$$\mu_3=qT\beta+(k_2+k_3)\beta^{\beta}-\delta^*,\quad \mu_4=-\delta^*. \quad (50)$$

Proof: Assume the initial state $|s(0)|>\alpha$, the convergence process can be divided into four stages: $|s(0)|\rightarrow|s(k_l)|=\alpha$, $|s(k_l)|=\alpha\rightarrow|s(k_m)|=1$, $|s(k_m)|=1\rightarrow|s(k_n)|=\beta$, $|s(k_n)|=\beta\rightarrow|s(k_o)|=0$.

Case 1 ($s(0)>\alpha$):

Stage 1: $s(0)\rightarrow s(k_l)=\alpha$. Owing to $s(k)>1$, the effect of $k_2|s(k)|^{\beta}$ sgn $(s(k))$ can be ignored comparing with other power terms [39]. Equation (6) can be written as

$$s(k+1)\approx(1-qT)s(k)-k_1\left|s(k)\right|^{\alpha}\mathrm{sgn}(s(k))-k_3\left|s(k)\right|^{\gamma}\mathrm{sgn}(s(k))+\delta(k). \quad (51)$$

Then, recalling *Theorem 1*, the following deduction is obtained, i.e.,

$$-[s(k+1)-s(k)]\geq qTs(k)+k_1s(k)^{\alpha}+k_3s(k)^{\gamma}-\delta^* \quad (52)$$
$$\geq qT\alpha+(k_1+k_3)\alpha^{\alpha}-\delta^*:=\mu_1,$$
$$s(k+1)+s(k)\geq-[s(k+1)-s(k)]\geq\mu_1. \quad (53)$$

It follows from (52) and (53) that

$$s^2(1)\leq s^2(0)-\mu_1^2, s^2(2)\leq s^2(0)-2\mu_1^2,\ldots,\alpha^2=s^2(0)-m_1\mu_1^2. \quad (54)$$

Solving (54) generates

$$m_1=\frac{s^2(0)-\alpha^2}{\mu_1^2}. \quad (55)$$

Stage 2: $s(k_l)=\alpha\rightarrow s(k_m)=1$. The effect of $k_2|s(k)|^{\beta}$ sgn $(s(k))$ is still ignored, and

$$s(k+1)\approx(1-qT)s(k)-(k_1+k_3)\left|s(k)\right|^{\alpha}\,sgn\,(s(k))+\delta(k) \quad (56)$$

$$\Rightarrow -[s(k+1)-s(k)] \geq qT + (k_1+k_3) - \delta^* := \mu_2, \quad (57)$$

$$\Rightarrow m_2 = \frac{\alpha^2-1}{\mu_2^2}. \quad (58)$$

Stage 3: $s(k_m) = 1 \rightarrow s(k_n) = \beta$. The effect of $k_1|s(k)|^\alpha$ sgn $(s(k))$ is still ignored, and we have

$$s(k+1) \approx (1-qT)\,s(k) - (k_2+k_3)\,|s(k)|^\beta \,\mathrm{sgn}\,(s(k)) + \delta(k). \quad (59)$$

$$\Rightarrow -[s(k+1)-s(k)] \geq qT\beta + (k_2+k_3)\,\beta^\beta - \delta^* := \mu_3, \quad (60)$$

$$\Rightarrow m_3 = \frac{1-\beta^2}{\mu_3^2}. \quad (61)$$

Stage 4: $s(k_n) = \beta \rightarrow s(k_o) = 0$. The effect of $k_1|s(k)|^\alpha$ sgn $(s(k))$ is still ignored, and we have

$$s(k+1) \approx (1-qT)\,s(k) - k_2\,|s(k)|^\beta\, sgn\,(s(k)) - k_3\,|s(k)|^\gamma \,\mathrm{sgn}\,(s(k)) + \delta(k). \quad (62)$$

$$\Rightarrow -[s(k+1)-s(k)] \geq -\delta^* := \mu_4, \quad (63)$$

$$\Rightarrow m_4 = \frac{\beta^2}{\mu_4^2}. \quad (64)$$

Case 1 $(s(0) < -\alpha)$*:*

A similar proof can be obtained, and will not be detailed here.

The above two cases reveal that the system trajectory is able to first cross the sliding surface within at most $K^* = \lfloor m^* \rfloor + 1 = \lfloor m_1+m_2+m_3+m_4 \rfloor + 1$ steps. ■

Remark 2: With same or smaller control gains, the developed method guarantees a faster convergence rate than the single power reaching law like [9] and [21].

For convenience, the convergence process of the single power reaching law is also divided into four stages: $|s(0)| \rightarrow |s(k_l)| = \alpha$, $|s(k_l)| = \alpha \rightarrow |s(k_m)| = 1$, $|s(k_m)| = 1 \rightarrow |s(k_n)| = \beta$, $|s(k_n)| = \beta \rightarrow |s(k_o)| = 0$. Similarly, considering (5), the reaching steps can be obtained

$$\begin{aligned} K_s^* &= \lfloor m_s^* \rfloor + 1 \quad \text{with } m_s^* \\ &= \frac{s^2(0)-\alpha^2}{\mu_{1s}^2} + \frac{\alpha^2-1}{\mu_{2s}^2} + \frac{1-\beta^2}{\mu_{3s}^2} + \frac{\beta^2}{\mu_{4s}^2}, \\ \mu_{1s} &= qT_0\alpha + k_0\alpha^\tau - \delta^*, \quad \mu_{2s} = qT_0 + k_0 - \delta^*, \\ \mu_{3s} &= qT_0\beta + k_0\beta^\tau - \delta^*, \quad \mu_{4s} = -\delta^*. \end{aligned} \quad (65)$$

If same control gains are selected, i.e., $qT = qT_0$, $\beta = \tau$, comparing (50) with (65), it can be deduced that $K^* < K_s^* K^* < K_s^*$ and the convergence rate of the proposed method is improved. Moreover, if smaller control gains are adopted for proposed method, $K^* < K_s^* K^* < K_s^*$ still holds by proper selection of α, k_1, k_2, k_3.

IV. EXAMPLES AND SIMULATIONS

In this section, the following piezomotor-driven linear stage [33] is employed to demonstrate the proposed DSMC method:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2 = -\frac{k_s}{m}x_1(t) - \frac{k_v}{m}x_2(t) + \frac{k_f}{m}u(t) - \frac{1}{m}d(x,t),\; y = x_1(t) \quad (66)$$

where $x_1(t)$ and $x_2(t)$ represent the linear displacement and the velocity, respectively. $u(t)$ stands for the voltage input. $m(=1$ kg$)$, $k_s(=0)$, $k_v(=144$ N$)$, and $k_f(=6$ kg$)$ represent the nominal mass, spring, damping, and force constants. $d(x, t)$ is the disturbance term.

The sampling time interval is selected as $T = 1$ms. Then, the discretized system (1) with the following parameters is obtained: $A = [1, 0.0009; 0, 0.8659]$, $B = [0; 0.0056]$, $C = [5, 1]$, $d(k) = [0; 0.0123\sin(0.5k\pi) + 0.0056]$. The single power DSMC (5) [21] is employed here for comparison.

Two cases are considered in the following simulation. In Case 1, same gains are selected for both methods. The change rate of the disturbance $\delta(k)$ is represented as $\delta(k) = \delta_2(k) = C[d(k) - d(k–1)]$ in the developed method (6). In order to further narrow down the width of the Ψ region, $\delta(k)$ is selected as $\delta_3(k) = C[d(k) - 2d(k–1) + d(k–2)]$ in the proposed method (6) in Case 2. For a fair comparison, same control gains, i.e., $qT = qT_0$, $\beta = \tau$, $k_2 = k_0$, are chosen for both methods. The control gains β, τ are selected as 0.5 to obtain small width of the Ψ region ([21, Remark 3.2]). Other control gains are elaborately adjusted via simulations to realize satisfactory reaching steps and relatively small width of the Ψ region. The gains are picked as: $qT = qT_0 = 0.6$, $k_2 = k_0 = 1 \times 10^{-3}$, $k_1 = 9.27 \times 10^{-2}$, $k_3 = 4.5 \times 10^{-6}$, $\alpha = 1.9$.

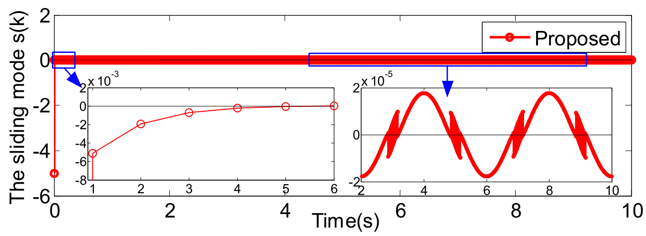

FIGURE 2. **Switching function of the proposed method: Case 1.**

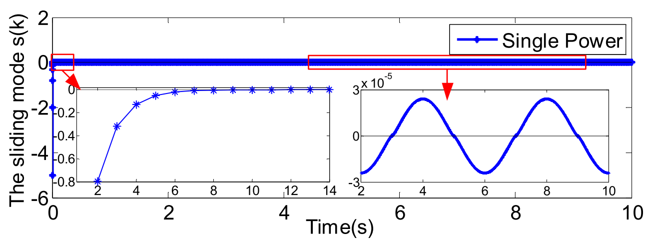

FIGURE 3. **Switching function of the single power DSMC: Case 1.**

Case 1: The switching functions of both methods are illustrated in Figs. 2 and 3. In comparison with the single power DSMC ($K_s^* = 13$), the proposed method improves the convergence rate ($K^* = 6$), which implies that the system trajectory converges faster to the sliding surface than that of the single power DSMC. Moreover, the proposed method generates a smaller width of the Ψ region (1.8×10^{-5}) than that of the single power DSMC (2.5×10^{-5}). Hence, the proposed method produces a better control accuracy while shortening the convergence time. The state variables of both methods are depicted in Figs. 4 and 5.

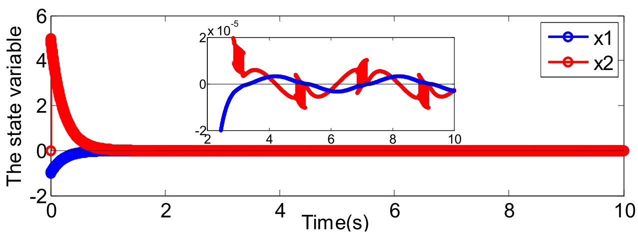

FIGURE 4. **State variable of the proposed method: Case 1.**

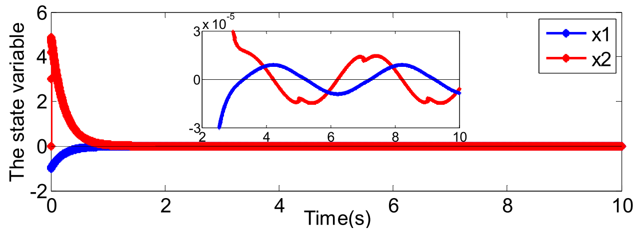

FIGURE 5. **State variable of the single power DSMC: Case 1.**

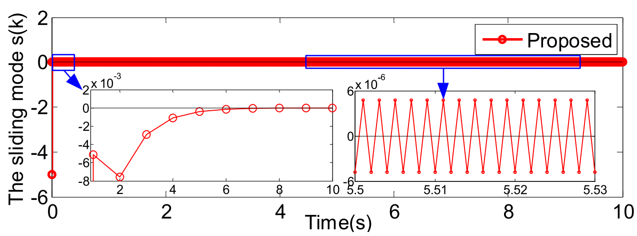

FIGURE 6. **Switching function of the proposed method: Case 2.**

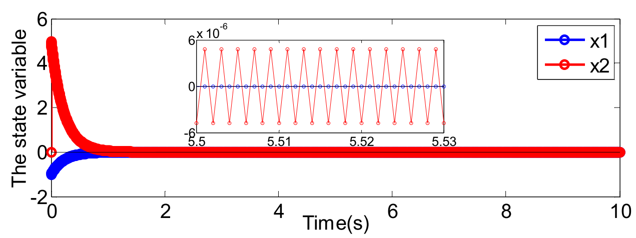

FIGURE 7. **State variable of the proposed method: Case 2.**

Case 2: For the purpose of further improving the control accuracy of the developed method, $\delta(k)$ in (6) is selected as $\delta_3(k) = C[d(k) - 2d(k{-}1) + d(k{-}2)]$ in this case. Hence, the impact of the disturbance is reduced. The control gains remain the same with that in Case 1. The switching function and the state variable of the proposed method are depicted in Figs. 6 and 7, respectively. It can be found that the proposed method produces a much smaller width of the Ψ region (5×10^{-6}) than that in Case 1. Moreover, as depicted in Fig. 6, the developed method still decreases the reaching steps with $K^* = 9$, as compared with that of [21] in Case 1, and system trajectory of developed method remains to cross the sliding surface in all subsequent steps.

V. CONCLUSION

This paper has given the design and verification of a multi-power reaching law based discrete-time sliding-mode control (DSMC). With the integration of the multi-power function and the perturbation estimation, the proposed method exhibits a superior performance over the single power DSMC. The proposed method has the ability to improve the convergence rate and guarantee better control accuracy in the same time. Theoretical analyses of the reaching steps and the convergence property under the impact of the disturbance have been conducted. Simulation results on a practical example verify that the proposed method is effective and feasible.

REFERENCES

[1] H. Thanh and S. Hong, "Quadcopter robust adaptive second order sliding mode control based on PID sliding surface," *IEEE Access*, vol. 6, pp. 66850–66860, 2018.
[2] L. Ma, Y. Zhang, X. Yang, S. Ding, and L. Dong, "Quasi-continuous second-order sliding mode control of buck converter," *IEEE Access*, vol. 6, pp. 17859–17867, 2018.
[3] Y. Wang, F. Yan, J. Chen, and B. Chen, "Continuous nonsingular fast terminal sliding mode control of cable-driven manipulators with super-twisting algorithm," *IEEE Access*, vol. 6, pp. 49626–49636, 2018.
[4] Q. Zhang, J. Zhang, and Y. Wang, "Robust sliding-mode control for fuzzy stochastic singular systems with different local input matrices," *IEEE Access*, vol. 6, pp. 29391–29406, 2018.
[5] A. Bartoszewicz and P. Latosiński, "Reaching law based discrete time sliding mode inventory management strategy," *IEEE Access*, vol. 4, pp. 10051–10058, 2016.
[6] W. Gao and J. C. Hung, "Variable structure control of nonlinear systems: A new approach," *IEEE Trans. Ind. Electron.*, vol. 40, no. 1, pp. 45–55, Feb. 1993.
[7] W. Gao, Y. Wang, and A. Homaifa, "Discrete-time variable structure control systems," *IEEE Trans. Ind. Electron.*, vol. 42, no. 2, pp. 117–122, Apr. 1995.
[8] C. Xiu and P. Guo, "Global terminal sliding mode control with the quick reaching law and its application," *IEEE Access*, vol. 6, pp. 49793–49800, 2018.
[9] P. Latosiński, "Sliding mode control based on the reaching law approach—A brief survey," in *Proc. Int. Conf. Methods Models Autom. Robot.(MMAR)*, Miedzyzdroje, Poland, 2017, pp. 519–524.
[10] A. Bartoszewicz, "Remarks on discrete-time variable structure control systems," *IEEE Trans. Ind. Electron.*, vol. 43, no. 1, pp. 235–238, Jan. 1996.
[11] A. Bartoszewicz, "Discrete-time quasi-sliding-mode control strategies," *IEEE Trans. Ind. Electron.*, vol. 45, no. 4, pp. 633–637, Apr. 1998.
[12] A. Bartoszewicz and P. Latosiński, "New switching and nonswitching type reaching laws for SMC of discrete time system," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 3, pp. 670–677, Mar. 2016.
[13] A. Bartoszewicz and P. Latosiński, "Discrete time sliding mode control with reduced switching—A new reaching law approach," *Int. J. Robust Nonlinear Control*, vol. 26, no. 1, pp. 47–68, Jan. 2016.
[14] H. Ma, J. Wu, and Z. Xiong, "Discrete-time sliding-mode control with improved quasi-sliding-mode domain," *IEEE Trans. Ind. Electron.*, vol. 63, no. 10, pp. 6292–6304, Oct. 2016.
[15] Y. Niu, D. W. C. Ho, and Z. Wang, "Improved sliding mode control for discrete-time systems via reaching law," *IET Control Theory Appl.*, vol. 4, no. 11, pp. 2245–2251, Nov. 2010.
[16] Y. Eun, J.-H. Kim, K. Kim, and D.-I. Cho, "Discrete-time variable structure controller with a decoupled disturbance compensator and its application to a CNC servomechanism," *IEEE Trans. Control Syst. Technol.*, vol. 7, no. 4, pp. 414–423, Jul. 1999.

[17] Y. Eun and D.-I. Cho, ''Robustness of multivariable discrete-time variable structure control,'' *Int. J. Control*, vol. 72, no. 12, pp. 1106–1115, Aug. 1999.

[18] S. Qu, X. Xia, and J. Zhang, ''Dynamics of discrete-time sliding-mode-control uncertain systems with a disturbance compensator,'' *IEEE Trans. Ind. Electron.*, vol. 61, no. 7, pp. 3502–3510, Jul. 2014.

[19] J. Zhang, P. Shi, Y. Xia, and H. Yang, ''Discrete-time sliding mode control with disturbance rejection,'' *IEEE Trans. Ind. Electron.*, to be published. doi: 10.1109/TIE.2018.2879309.

[20] H. Ma, J. Wu, and Z. Xiong, ''A novel exponential reaching law of discrete-time sliding-mode control,'' *IEEE Trans. Ind. Electron.*, vol. 64, no. 5, pp. 3840–3850, May 2017.

[21] H. Du, X. Yu, M. Z. Q. Chen, and S. Li, ''Chattering-free discrete-time sliding mode control,'' *Automatica*, vol. 68, no. 3, pp. 87–91, Jun. 2016.

[22] S. Chakrabarty and B. Bandyopadhyay, ''Minimum ultimate band design of discrete sliding mode control,'' *Asian J. Control*, vol. 17, no. 5, pp. 1889–1897, Sep. 2015.

[23] S. Chakrabarty and B. Bandyopadhyay, ''A generalized reaching law for discrete time sliding mode control,'' *Automatica*, vol. 52, pp. 83–86, Feb. 2015.

[24] S. Chakrabarty and B. Bandyopadhyay, ''A generalized reaching law with different convergence rates,'' *Automatica*, vol. 63, pp. 34–37, Jan. 2016.

[25] X. Yu, B. Wang, and X. Li, ''Computer-controlled variable structure systems: The state-of-the-art,'' *IEEE Trans. Ind. Informat.*, vol. 8, no. 2, pp. 197–205, May 2012.

[26] S. Janardhanan and B. Bandyopadhyay, ''Multirate output feedback based robust quasi-sliding mode control of discrete-time systems,'' *IEEE Trans. Autom. Control*, vol. 52, no. 3, pp. 499–503, Mar. 2007.

[27] S. Janardhanan and B. Bandyopadhyay, ''Output feedback sliding-mode control for uncertain systems using fast output sampling technique,'' *IEEE Trans. Ind. Electron.*, vol. 53, no. 5, pp. 1677–1682, Oct. 2006.

[28] L. Zheng, F. Jiang, J. Song, Y. Gao, and M. Tian, ''A discrete-time repetitive sliding mode control for voltage source inverters,'' *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 6, no. 3, pp. 1553–1566, Sep. 2018.

[29] K. Abidi, J.-X. Xu, and Y. Xinghuo, ''On the discrete-time integral sliding-mode control,'' *IEEE Trans. Autom. Control*, vol. 52, no. 4, pp. 709–715, Apr. 2007.

[30] J. Liu, X. Zhang, and B. Jiang, ''Some generalizations and improvements of discrete hardy's inequality,'' *Comput. Math. Appl.*, vol. 63, no. 3, pp. 601–607, Feb. 2012.

[31] W.-C. Su, S. V. Drakunov, and U. Ozguner, ''An O(T_2) boundary layer in sliding mode for sampled-data systems,'' *IEEE Trans. Autom. Control*, vol. 45, no. 3, pp. 482–485, Mar. 2000.

[32] S. Li, H. Du, and X. Yu, ''Discrete-time terminal sliding mode control systems based on Euler's discretization,'' *IEEE Trans. Autom. Control*, vol. 59, no. 2, pp. 546–552, Feb. 2014.

[33] K. Abidi, J. X. Xu, and J. H. She, ''A discrete-time terminal sliding-mode control approach applied to a motion control problem,'' *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3619–3627, Sep. 2009.

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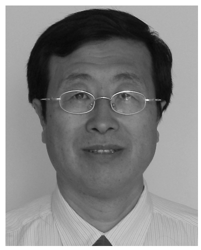

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