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Performance improvement of NOMA visible light communication system by adjusting superposition constellation: a convex optimization approach

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Abstract: Non-orthogonal multiple access (NOMA) can increase the overall spectral efficiency of visible light communication (VLC) system. In this paper, we propose a novel scheme to improve the bit error rate (BER) performance of a two-user NOMA VLC system by adjusting superposition constellation. The corresponding closed-form BER expressions are derived. Convex optimization is used to find the optimal parameters of the adjusted superposition constellation, where the overall BER is minimized. The BER performances are evaluated by theoretical analysis, Monte-Carlo (MC) simulation and experiment. The results illustrate that 8.9-dB and 8.1-dB signal-to-noise ratios (SNRs) are reduced to achieve BER of 10^{-3} for the two users, whose power allocation coefficients are 0.6 and 0.4, respectively.

1. Introduction

In the last decade, visible light communication (VLC) has aroused many researchers' interests [1–3]. VLC uses random data signal to drive light-emitting diode (LED). Hence, communication and illumination are accomplished at the same time. Due to the characteristics of LED, VLC system has the following advantages, including high data rate, high security, eye safety, and immunity to electromagnetic interference. So VLC is a great approach to fifth generation networks and beyond [4].

The performance of VLC system can be improved by using non-orthogonal multiple access (NOMA) [5–7]. NOMA is an attractive access means to let multiple terminals use the same time and frequency resources. It provides high spectral efficiency, high flexibility, and improved fairness [8, 9]. A lot of works on NOMA VLC system have been carried out. Yin *et al* investigated the capacity regions for NOMA and orthogonal frequency division multiple access (OFDMA), and pointed out that NOMA outperforms OFDMA [10]. In [11], Shi *et al* investigated the performance of offset quadrature amplitude modulation (OQAM) orthogonal frequency division multiplexing (OFDM) based multiple input multiple output (MIMO) NOMA VLC system, and studied its advantages over conventional MIMO system. Power allocation methods have been investigated to improve the NOMA VLC system performances, such as gain ratio power allocation method, normalized gain difference power allocation method, and enhanced

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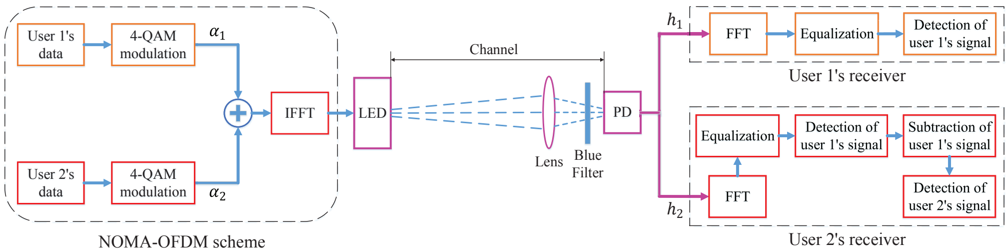

Fig. 1. Two-user NOMA VLC system.

power allocation method [5, 6, 12]. In [13], Lin *et al* demonstrated a bidirectional NOMA-OFDMA VLC system, where the optimum power allocation ratios were studied. Optimization of superposition constellation is another approach to improve the performance of NOMA VLC system. Constellation rotation was carried out before superposition to decrease the symbol error rate [14]. In [15, 16], Guan *et al* used phase pre-distortion method to improve the performance of uplink NOMA VLC system.

In the work, we propose a novel superposition constellation adjustment scheme to improve the bit error rate (BER) performance of a two-user NOMA VLC system. The modulation formats for the two users are both 4-quadrature amplitude modulation (QAM) OFDM. The closed-form BER expressions are derived for the two users. In order to obtain the minimized overall BER performance, the parameters in adjusting the constellation are obtained according to convex optimization. In such a case, the two users' BER performances are both improved. We verify such BER improvement by using theoretical analysis, Monte-Carlo (MC) simulation and experiment. The results show that the signal-to-noise ratios (SNRs) to achieve the BER of 10^{-3} decrease 8.9 dB and 8.1 dB for the two users, when their corresponding power allocation coefficients are 0.6 and 0.4, respectively.

The rest of this paper is organized as follows. Section 2 presents the principle of two-user NOMA VLC system using OFDM. The approach to improve the BER performance by adjusting the superposition constellation is also described. Section 3 investigates the theoretical analysis, MC simulation, and experiment results. Finally, the concluding remarks are summarized in Section 4.

2. Principles

The two-user NOMA VLC system is shown in Fig. 1. The random data of two users are mapped to 4-QAM. The power allocation coefficients for the two users are α_1 and α_2, where $\alpha_1 + \alpha_2 = 1$ [8]. The two 4-QAM signals are then converted to OFDM by taking inverse fast Fourier transform (IFFT). Hermitian symmetry is used to guarantee the real output of OFDM signal. The OFDM signal drives LED, whose output light propagates through indoor VLC channel. A lens is used to concentrate the modulated LED light, which is converted to electrical signal by photo-detector (PD). The channel gains for the two users are denoted by h_1 and h_2, respectively. In power-domain NOMA system, more power is allocated to the signal with lower channel gain, *i.e.*, $\alpha_1 \geqslant \alpha_2$, when $h_1 \leqslant h_2$ [8]. In this work, we assume that the received signal power of user 1 is higher than that of user 2. For the sake of convenience, we consider the case that $h_1 = h_2$ and $\alpha_1 > \alpha_2$. So at user 1, after taking fast Fourier transform (FFT) and equalization, user 1's receiver detects its signal directly, where user 2's signal is treated as the interference. At user 2, after taking FFT and equalization, user 1's signal is detected first, since it has higher power. And then, the detected signal is remodulated and subtracted from the composite signal [9]. Finally, the signal of user 2 is detected.

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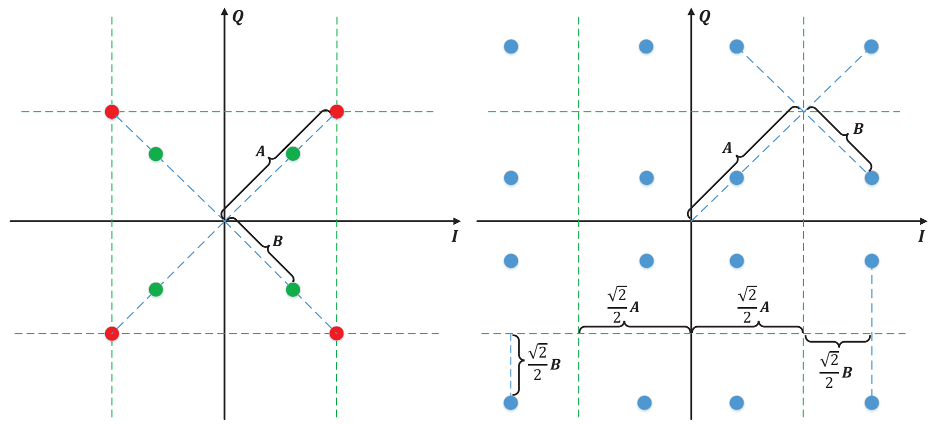

Fig. 2. (a) The two users' original constellations before superposition. Red: user 1; green: user 2. (b) The superposition constellation before adjustment for two-user NOMA VLC system.

As depicted in Fig. 2(a), we use A and B to denote the distances between origin and any point in the constellation before superposition for user 1 and user 2, respectively. The corresponding average power for the two users are $P_1 = A^2$ and $P_2 = B^2$. The constellation after superposition is shown in Fig. 2(b). The average power P of the superposition signal is the same as the average power of each quadrant, which is given by

$$P = \frac{1}{4}\left[4\left(\frac{A}{\sqrt{2}} - \frac{B}{\sqrt{2}}\right)^2 + 4\left(\frac{A}{\sqrt{2}} + \frac{B}{\sqrt{2}}\right)^2\right] = A^2 + B^2 = P_1 + P_2. \tag{1}$$

So the power of the superposition signal is the sum of the two signals' power. The power allocation coefficients of the two signals are denoted as $\alpha_1 = \frac{P_1}{P}$ and $\alpha_2 = \frac{P_2}{P}$. The Euclidean distances between any two adjacent points in the superposition constellation may not be identical due to the two signals' power allocation coefficients. As depicted in Fig. 2(b), there are 16 points in the superposition constellation, where 4 points are found in each quadrant. The Euclidean distance between two adjacent points those are in two adjacent quadrants is shorter than that between two adjacent points those are within one quadrant. Hence, the BER performance is deteriorated due to the superposition signal in NOMA system.

In order to improve the BER performance, we propose a scheme to adjust the superposition constellation, without affecting the total power of two signals. In this scheme, the differences among Euclidean distances between adjacent constellation points are made as small as possible, and the symmetry of superposition constellation is not destroyed. As shown in Fig. 3, user 1's constellation is not changed; while user 2's constellation is shrunk, where four parameters m, n, b, and c represent the shrink in different directions. After adjusting the superposition constellation, the two signals' overall power is given by

$$\begin{aligned} P &= \frac{1}{4}\left\{2\left[\left(\frac{A}{\sqrt{2}} - \frac{mB}{\sqrt{2}}\right)^2 + \left(\frac{A}{\sqrt{2}} + \frac{nB}{\sqrt{2}}\right)^2\right] + (A - bB)^2 + (A + cB)^2\right\} \\ &= \alpha_1 P + \frac{1}{4}\left(m^2 + n^2 + b^2 + c^2\right)\alpha_2 P + (n + c - m - b)\frac{\sqrt{\alpha_1\alpha_2}}{2}P. \end{aligned} \tag{2}$$

If all the four parameters m, n, b, and c are unity, the adjusted superposition constellation is nothing but the original superposition constellation. As shown in Eq. (2), after adjusting the

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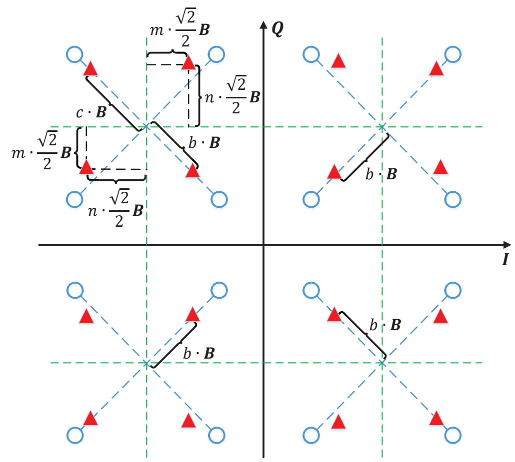

Fig. 3. The proposed superposition constellation after adjustment for two-user NOMA VLC system.

constellation, the first item of the superposition signal is the power of user 1, which is not affected by the constellation adjustment. The second item is the power of user 2, and the third item is the intermediate item. The summation of the second and third items should be the same as the user 2's power without adjusting the constellation, *i.e.*, $\frac{1}{4}\left(m^2+n^2+b^2+c^2\right)\alpha_2 P + (n+c-m-b)\frac{\sqrt{\alpha_1\alpha_2}}{2}P = \alpha_2 P$. In later descriptions, we will see that for the optimal adjusted constellation, the second item is lower than user 2's power before adjusting the constellation, and the third item is larger than zero.

After adjusting the superposition constellation, the BER of user 1 is described as follows

$$\begin{aligned}\mathrm{BER}_1 =& \frac{1}{4}\Bigg[Q\left(\sqrt{\mathrm{SNR}_1}+c\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right)+Q\left(\sqrt{\mathrm{SNR}_1}+n\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right)\\ &+Q\left(\sqrt{\mathrm{SNR}_1}-b\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right)+Q\left(\sqrt{\mathrm{SNR}_1}-m\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right)\Bigg],\end{aligned} \tag{3}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right)dt$ is the Q-function [17]. SNR_1 is the SNR of user 1, which is given by

$$\mathrm{SNR}_1 = \frac{P_1}{\sigma_\mathrm{N}^2} = \frac{\alpha_1 P}{\sigma_\mathrm{N}^2}. \tag{4}$$

σ_N^2 is the power of additive white Gaussian noise (AWGN). The BER of user 2 is described as follows

$$\begin{aligned}\mathrm{BER}_2 = \frac{1}{4}\Bigg[& Q\left(b\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) + Q\left(c\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) + Q\left(m\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) + Q\left(n\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) \\ & + Q\left(\sqrt{\mathrm{SNR}_1} - b\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) - Q\left(\sqrt{\mathrm{SNR}_1} + c\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) \\ & + Q\left(\sqrt{\mathrm{SNR}_1} - m\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) - Q\left(\sqrt{\mathrm{SNR}_1} + n\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) \\ & - Q\left(2\sqrt{\mathrm{SNR}_1} - b\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) + Q\left(2\sqrt{\mathrm{SNR}_1} + c\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) \\ & - Q\left(2\sqrt{\mathrm{SNR}_1} - m\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right) + Q\left(2\sqrt{\mathrm{SNR}_1} + n\sqrt{\frac{\alpha_2\mathrm{SNR}_1}{\alpha_1}}\right)\Bigg].\end{aligned} \tag{5}$$

The BERs of user 1 and user 2 in Eqs. (3) and (5) are those under the case without adjusting the superposition constellation, when the four parameters m, n, b, and c are all unity. In this case, Eqs. (3) and (5) are equivalent to Eqs. (3) and (6) in [18].

We investigate the selection of parameters m, n, b, and c to minimize the overall BER performance. As mentioned above, when the constellation is adjusted, the overall power of the two signals should not be changed, *i.e.*, the last two items in Eq. (2) should be the original power of user 2. The optimization problem is then given by

$$\begin{aligned} \min_{m,n,b,c} \quad & f(m,n,b,c) = \mathrm{BER}_1 + \mathrm{BER}_2 \\ s.t. \quad & \frac{1}{4}\left(m^2+n^2+b^2+c^2\right)\alpha_2 + (n+c-m-b)\frac{\sqrt{\alpha_1\alpha_2}}{2} - \alpha_2 = 0 \\ & m>0,\ \ n>0,\ \ b>0,\ \ c>0. \end{aligned} \tag{6}$$

The objective function $f(m,n,b,c)$ in Eq. (6) is convex, when its Hessian $\nabla^2 f$ is positive

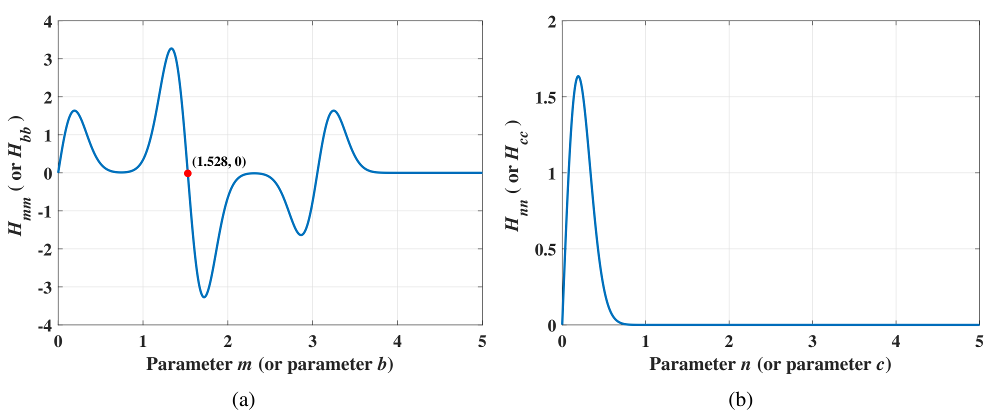

Fig. 4. The curves of (a) H_{mm} (or H_{bb}), and (b) H_{nn} (or H_{cc}) when $\alpha_1 = 0.7$ and $\mathrm{SNR}_1 = 18$ dB.

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semidefinite. The Hessian of $f(m,n,b,c)$ is a diagonal matrix, which is given by

$$\nabla^2 f = \begin{bmatrix} H_{mm} & 0 & 0 & 0 \\ 0 & H_{nn} & 0 & 0 \\ 0 & 0 & H_{bb} & 0 \\ 0 & 0 & 0 & H_{cc} \end{bmatrix}. \tag{7}$$

In Eq. (7), H_{mm}, H_{nn}, H_{bb}, and H_{cc} denote the second order partial derivatives of objective function $f(m,n,b,c)$ with respect to parameters m, n, b, and c, respectively. H_{mm}, H_{nn}, H_{bb}, and H_{cc} are denoted as

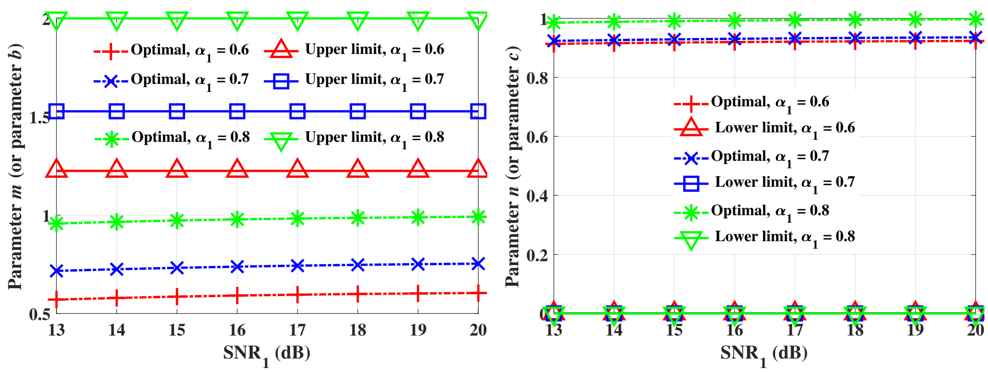
$$\begin{aligned} H_{mm} &= \frac{1}{4\sqrt{2\pi}}\left[mu^3 e^{-\frac{u^2m^2}{2}} - u^2(2v-um)e^{-\frac{(2v-um)^2}{2}} + 2u^2(v-um)e^{-\frac{(v-um)^2}{2}}\right], \\ H_{nn} &= \frac{1}{4\sqrt{2\pi}}\left[nu^3 e^{-\frac{u^2n^2}{2}} + u^2(2v+un)e^{-\frac{(2v+un)^2}{2}}\right], \\ H_{bb} &= \frac{1}{4\sqrt{2\pi}}\left[bu^3 e^{-\frac{u^2b^2}{2}} - u^2(2v-ub)e^{-\frac{(2v-ub)^2}{2}} + 2u^2(v-ub)e^{-\frac{(v-ub)^2}{2}}\right], \\ H_{cc} &= \frac{1}{4\sqrt{2\pi}}\left[cu^3 e^{-\frac{u^2c^2}{2}} + u^2(2v+uc)e^{-\frac{(2v+uc)^2}{2}}\right], \end{aligned} \tag{8}$$

where $u = \sqrt{\frac{\alpha_2}{\alpha_1}\mathrm{SNR}_1}$ and $v = \sqrt{\mathrm{SNR}_1}$. The positive semidefinite Hessian requires that $H_{mm} \geqslant 0$, $H_{nn} \geqslant 0$, $H_{bb} \geqslant 0$, and $H_{cc} \geqslant 0$. As shown in Eq. (8), H_{mm} and H_{bb} have the same expressions. The same thing happens to H_{nn} and H_{cc}. We can find the range of the four parameters m, n, b, and c to guarantee the positive semidefinite Hessian by investigating the curves of H_{mm}, H_{nn}, H_{bb}, and H_{cc}. For example, the curves of H_{mm} (or H_{bb}) and H_{nn} (or H_{cc}) are depicted in Fig. 4, when $\alpha_1 = 0.7$ and $\mathrm{SNR}_1 = 18$ dB. It can be found that H_{mm} (or H_{bb}) and H_{nn} (or H_{cc}) are both nonnegative, when parameter m (or parameter b) falls into the range $(0, 1.528]$ and parameter n (or parameter c) falls into the range $(0, \infty)$. In this case, 1.528 is called the upper limit of parameter m (or parameter b) , and 0 is called the lower limit of parameter n (or parameter c). So, the Hessian $\nabla^2 f$ is positive semidefinite, and consequently the objective function $f(m,n,b,c)$ in Eq. (6) is convex.

We use the method of Lagrange multipliers to find the optimal parameters m, n, b, and c. The Lagrange function is shown as follows

$$\begin{aligned} L(m,n,b,c,\lambda) = &\mathrm{BER}_1 + \mathrm{BER}_2 + \lambda\left[\frac{1}{4}\left(m^2+n^2+b^2+c^2\right)\alpha_2\right. \\ &\left. + (n+c-m-b)\frac{\sqrt{\alpha_1\alpha_2}}{2} - \alpha_2\right], \end{aligned} \tag{9}$$

where λ is the Lagrange multiplier. The optimal parameters $\tilde{m}$, $\tilde{n}$, $\tilde{b}$, and $\tilde{c}$ for Eq. (9) must satisfy



(a) (b)

Fig. 5. (a) Optimal values of parameter m (or parameter b) and their upper limits; (b) optimal values of parameter n (or parameter c) and their lower limits.

the following equations, which are the first order partial derivatives of $L(m, n, b, c, \lambda)$

$$\begin{aligned}
\frac{\partial L}{\partial m} &= \frac{1}{4\sqrt{2\pi}}\left[-ue^{-\frac{u^2\tilde{m}^2}{2}} - ue^{-\frac{(2v-u\tilde{m})^2}{2}} + 2ue^{-\frac{(v-u\tilde{m})^2}{2}}\right] + \frac{1}{2}\alpha_2\tilde{m}\lambda - \frac{\sqrt{\alpha_1\alpha_2}}{2}\lambda = 0,\\
\frac{\partial L}{\partial n} &= \frac{1}{4\sqrt{2\pi}}\left[-ue^{-\frac{u^2\tilde{n}^2}{2}} - ue^{-\frac{(2v+u\tilde{n})^2}{2}}\right] + \frac{1}{2}\alpha_2\tilde{n}\lambda + \frac{\sqrt{\alpha_1\alpha_2}}{2}\lambda = 0,\\
\frac{\partial L}{\partial b} &= \frac{1}{4\sqrt{2\pi}}\left[-ue^{-\frac{u^2\tilde{b}^2}{2}} - ue^{-\frac{(2v-u\tilde{b})^2}{2}} + 2ue^{-\frac{(v-u\tilde{b})^2}{2}}\right] + \frac{1}{2}\alpha_2\tilde{b}\lambda - \frac{\sqrt{\alpha_1\alpha_2}}{2}\lambda = 0,\\
\frac{\partial L}{\partial c} &= \frac{1}{4\sqrt{2\pi}}\left[-ue^{-\frac{u^2\tilde{c}^2}{2}} - ue^{-\frac{(2v+u\tilde{c})^2}{2}}\right] + \frac{1}{2}\alpha_2\tilde{c}\lambda + \frac{\sqrt{\alpha_1\alpha_2}}{2}\lambda = 0,\\
\frac{\partial L}{\partial \lambda} &= \frac{1}{4}\left(\tilde{m}^2 + \tilde{n}^2 + \tilde{b}^2 + \tilde{c}^2\right)\alpha_2 + \left(\tilde{n} + \tilde{c} - \tilde{m} - \tilde{b}\right)\frac{\sqrt{\alpha_1\alpha_2}}{2} - \alpha_2 = 0.
\end{aligned} \tag{10}$$

By solving Eq. (10), we can obtain the optimal parameters $\tilde{m}$, $\tilde{n}$, $\tilde{b}$, and $\tilde{c}$. It can be seen from Eq. (10) that $\tilde{m} = \tilde{b}$, and $\tilde{n} = \tilde{c}$. The optimal parameters $\tilde{m}$, $\tilde{n}$, $\tilde{b}$, and $\tilde{c}$ under different SNR_1 and power allocation coefficient α_1 are shown in Fig. 5. The parameters' corresponding upper or lower limits to guarantee the convexity of $f(m, n, b, c)$ in Eq. (6) are also shown. The optimal values of parameters always fall into the range that satisfies the convexity requirements, *i.e.*, the optimal values of parameters m and b are smaller than their upper limits, and the optimal values of parameters n and c are larger than their lower limits. Since the objective function $f(m, n, b, c)$ in Eq. (6) is convex, the solution of Eq. (9) is the global minimum of the overall BER $f(m, n, b, c)$.

3. Results and discussions

In this section, we investigate the BER performance of two-user NOMA VLC system with various power allocation coefficients. Theoretical analysis, MC simulation and experiment are carried out to evaluate the BER performance improvement by adjusting the superposition constellation. In both the MC simulation and experiment, the bandwidth of 4-QAM OFDM signal is 10 MHz, and the number of subcarriers is 64.

3.1. Theoretical analysis and MC simulation

In this subsection, we use both theoretical analysis and MC simulations to analyze the BER performances of two-user NOMA VLC system. Figure 6 shows the BER performance under the case that the two power allocation coefficients α_1 and α_2 are 0.7 and 0.3, respectively. Without adjusting the superposition constellation, the difference between the two users' BERs becomes smaller, and the two BER curves even overlap when SNR_1 is larger than 14 dB. The SNR_1s for the two users to achieve BER of 10^{-3} are both 18.4 dB. After adjusting the superposition constellation, both the BERs of user 1 and user 2 are decreased, and the difference between the two BERs becomes smaller as the SNR_1 increases. The SNR_1s for user 1 and user 2 to achieve BER of 10^{-3} are reduced to 15 dB and 15.8 dB, respectively. Consequently, the SNR reductions to achieve BER of 10^{-3} for the two users are 3.4 dB and 2.6 dB, respectively. The constellations inside Fig. 6 are those without and with constellation adjustment at SNR_1 of 17.5 dB. We can see that the overlapped constellation points are separated from each other after the constellation adjustment, which improves the BER performance.

Figure 7 shows the BER performance under the case that the two power allocation coefficients α_1 and α_2 are 0.6 and 0.4, respectively. Before adjusting the constellation, the BER curves for user 1 and user 2 overlap when the SNR_1 is larger than 12 dB, which is smaller than that for the case of $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$ as shown in Fig. 6. And 23.9-dB SNR_1 is required to achieve BER of 10^{-3} for both the two users. After adjusting the constellation, 15-dB and 15.8-dB SNR_1 are required to achieve BER of 10^{-3} for user 1 and user 2, respectively. Hence, the corresponding SNR reductions are 8.9 dB and 8.1 dB, respectively. The constellations inside Fig. 7 are those without and with constellation adjustment at SNR_1 of 17.5 dB. It can be seen that the superposition constellation is clearer after the adjustment. Therefore, the BER performance is improved. In Figs. 6 and 7, the results of theoretical analysis and MC simulation match very well, which demonstrates the validity of Eqs. (3) and (5).

3.2. *Experiment*

The improvement of BER performance for two-user NOMA VLC system is also verified by experiment. In the experiment, an arbitrary waveform generator (RIGOL DG1062Z) generates OFDM signal, which drives LED (Cree® XLamp® XM-L2). The modulated light is collected by PD (HAMAMATSU S10784), which converts it into electrical signal. The converted signal is captured by an oscilloscope (RIGOL MSO2302A), and is processed offline. As depicted in Fig. 5, the optimal values of parameters *m*, *n*, *b*, and *c* are almost constant. So in the experiment under a certain pair of α_1 and α_2, we use fixed parameters *m*, *n*, *b*, and *c*, which are obtained when SNR_1 is 18 dB.

First, we measure the frequency response of the whole transmission system, which is shown in Fig. 8(a). The fluctuation of frequency response is due to the characteristics of circuits only,

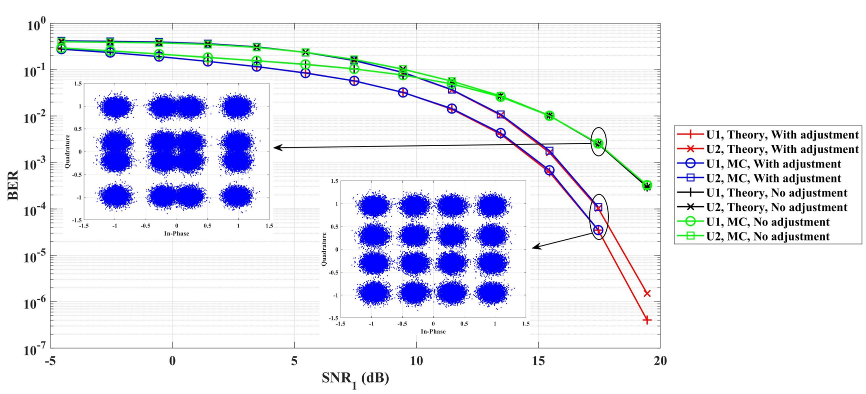


Fig. 6. BER performance of two-user NOMA VLC system when $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$. U1: user 1; U2: user 2.

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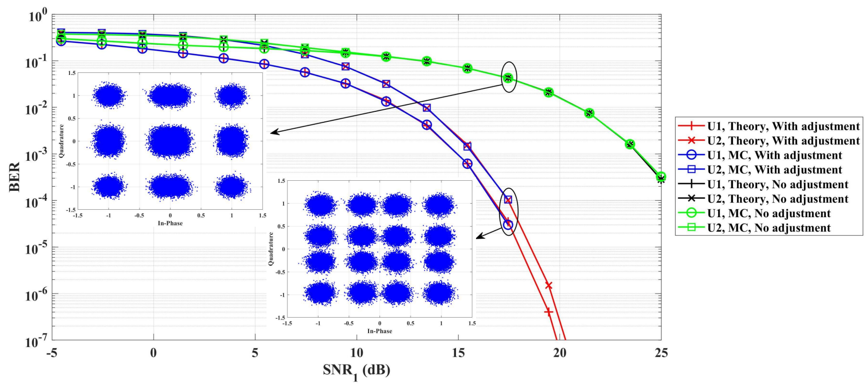
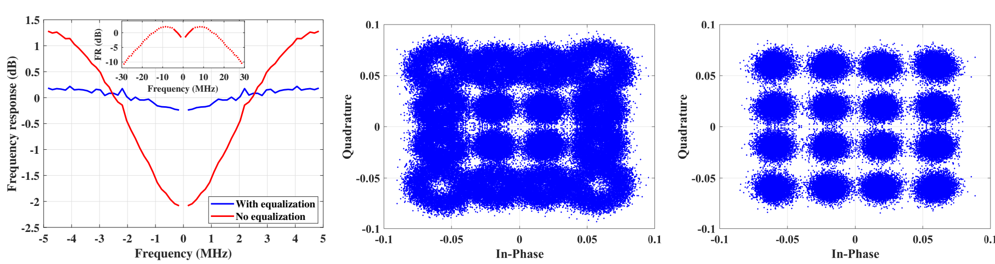


Fig. 7. BER performance of two-user NOMA VLC system when $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$. U1: user 1; U2: user 2.

since the indoor VLC channel gains h_1 and h_2 are flat over all the frequencies. Note that in the experiment we only use part of the whole bandwidth ([-30 MHz, 30 MHz]), as shown in the inside frequency response of Fig. 8(a). And then, we take the zero-forcing equalization (ZFE) to mitigate the fluctuation of frequency response, which makes the constellation clearer. The constellations before and after ZFE are shown in Figs. 8(b) and 8(c), respectively. The corresponding power allocation coefficients α_1 and α_2 are 0.7 and 0.3, respectively, and the transmission distance is 3.4 m. We use N_u to denote the number of used subcarriers in evaluating the BER performance. Since the SNR in terms of frequency response after ZFE is not flat in the whole spectrum, we should use average BER of all the used subcarriers as a function of SNR to evaluate the performance, *i.e.*,

$$\mathrm{BER}_{\mathrm{avg}} = \frac{1}{N_u}\sum_{i=1}^{N_u}\mathrm{BER}_i\left(\mathrm{SNR}_{1,i}\right). \tag{11}$$

The theoretical and experimental BER performances are depicted in Figs. 9 and 10, where the power allocation coefficients pairs (α_1, α_2) are (0.7, 0.3) and (0.6, 0.4), respectively. We evaluate the BER performances based on their fitting curves. Note that the SNR_1 in Figs. 9 and 10 is the overall SNR_1 that is obtained by analyzing the received data in all the used subcarriers. In the experiment, we change the transmission distances to obtain various measured SNR_1s. For the case that $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$, the transmission distance varies from 2.6 m to 3.8 m. The



(a) (b) (c)

Fig. 8. (a) The frequency responses before (red) and after (blue) ZFE, and the frequency response with larger frequency range inside. FR: frequency response; (b) signal constellation before ZFE; (c) signal constellation after ZFE. $\alpha_1 = 0.7$, $\alpha_2 = 0.3$, and the transmission distance is 3.4 m.

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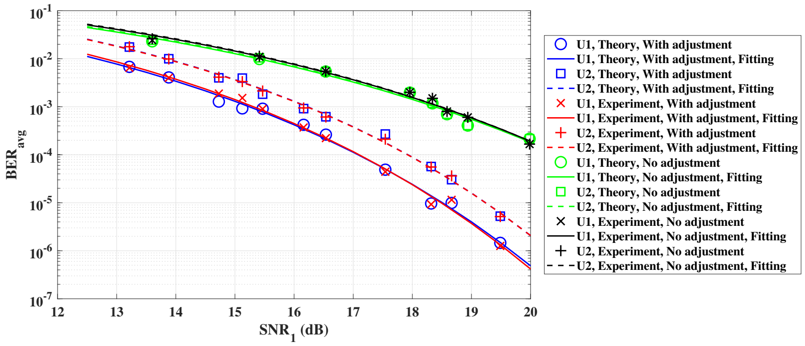

Fig. 9. Theoretical and experimental BER performance of two-user NOMA VLC system when $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$. U1: user 1; U2: user 2.

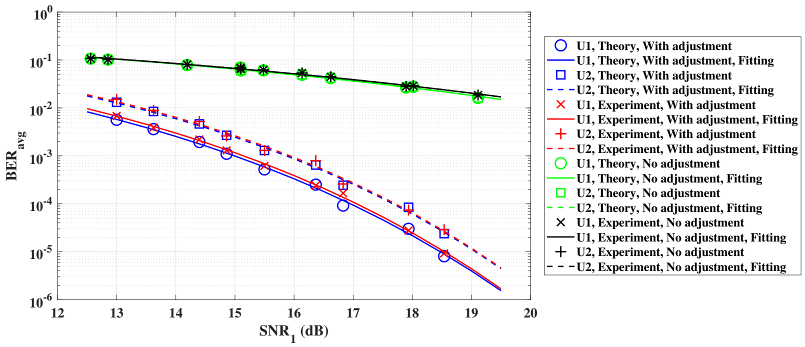

Fig. 10. Theoretical and experimental BER performance of two-user NOMA VLC system when $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$. U1: user 1; U2: user 2.

corresponding four parameters are $m = b = 0.747$, and $n = c = 0.934$, which are also shown in Table 1. In this case, the third item in Eq. (2) is larger than zero, *i.e.*, the power of user 2 is decreased after the adjustment of superposition constellation, and the BER performance is still improved. For the case that $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$, the transmission distance varies from 1.2 m to 2.2 m. The corresponding four parameters are $m = b = 0.599$, and $n = c = 0.922$. Consequently, we analyze the BER performance under the SNR_1 that varies from 12 dB to 20 dB due to the limitation of circuits. As shown in Figs. 9 and 10, the theoretical analysis and experiment results match very well, which demonstrates the validity of the theoretical BER performances by adjusting the superposition constellation as shown in Eqs. (3) and (5).

As shown in Table 1, the SNR reductions for the two users are increased as the difference

Table 1. SNR reductions to achieve BER of 10^{-3} by adjusting the superposition constellation under various power allocation coefficients. U1: user 1; U2: user 2.

Power allocation coefficients	m or b	n or c	SNR reduction for U1	SNR reduction for U2
$\alpha_1 = 0.6$, $\alpha_2 = 0.4$	0.599	0.922	8.9 dB	8.1 dB
$\alpha_1 = 0.7$, $\alpha_2 = 0.3$	0.747	0.934	3.4 dB	2.6 dB
$\alpha_1 = 0.8$, $\alpha_2 = 0.2$	0.986	0.995	0.2 dB	0 dB

between power allocation coefficients α_1 and α_2 is reduced. When α_1 and α_2 are 0.8 and 0.2, respectively, the SNR reductions to achieve BER of 10^{-3} are 0.2 dB and 0 dB for user 1 and user 2, respectively. When α_1 and α_2 are changed to 0.6 and 0.4, respectively, the SNR reductions to achieve BER of 10^{-3} are increased to 8.9 dB and 8.1 dB for the two users, respectively. So the effect of adjusting the superposition constellation for two-user NOMA VLC system are more significant, when the two users' power allocation coefficients are close to each other.

4. Conclusion

In this paper, we have proposed a novel scheme to improve the overall BER performance of two-user NOMA VLC system by adjusting the superposition constellation. The two users' closed-form BER expressions are derived consequently. The corresponding parameters of the superposition constellation are obtained by using convex optimization. The well-matched theoretical analysis, MC simulation and experiment results have demonstrated the validity of this proposed scheme. The scheme can be used to two-user NOMA VLC system under various power allocation coefficients. When the power allocation coefficients are 0.6 and 0.4 for the two users, respectively, the SNR reductions to achieve BER of 10^{-3} are 8.9 dB and 8.1 dB, respectively.

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