A Robust Optimisation Approach to the Aircraft Sequencing and Scheduling Problem with Runway Configuration Planning

K.K.H. NG, C.K.M. LEE, F.T.S. CHAN
Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hong Kong, China
kkh.ng@connect.polyu.hk , ckm.lee@polyu.edu.hk , f.chan@polyu.edu.hk

Abstract – **Unanticipated delays cause significant reduction of airport capacity management, decrease in customer satisfaction, and poor on-time performance. Safety in runway configuration planning is a top priority in aviation management, and air traffic control adopts the risk analysis in handing flight schedules under uncertainty. The degree of conservatism in handling airborne delays and airport traffic should be increased, as any accident due to improper runway usage causes dramatic loss, delay propagation and disruption to the airport management and subsequence activities. In this paper, the robust aircraft sequencing and scheduling problem with runway configuration planning using the min-max regret approach is proposed. The adoption of the mid-point scenario heuristic as an initial solution is able to reduce the computational burden in the computational experiment compared to the solution by using lower bound scenario by solving instances of moderate size (10 – 30 flights) in a two-runways system.**

Keywords – **Robust optimisation, aircraft sequencing and scheduling problem, runway configuration planning, min-max regret approach**

I. INTRODUCTION

Aircraft sequencing and scheduling problem (ASSP) considers the assignment of runway operation and formulates the sequence of the landings and take-offs schedule, which is usually assuming that the input parameters are deterministic in nature. Introducing uncertain parameter in ASSP model provides practical usage, as the aircraft landing and take-off times are often subject to unexpected deviations from the execution of planned runway schedule. Even for flights enter the Terminal Manoeuvring Area (TMA), the actual time of runway operation is still affected by the weather situation, air traffic of the congested airport and general airport delays. Inaccurate information for the approaching and take-off time leads to the infeasibility of the planned schedule, delay propagation, a re-scheduling effort by Air Traffic Control (ATC) and the flexibility of runway configuration planning.

The runway configuration includes: solely take-off, solely landing, mixed-mode operation, semi-mixed-mode operation and configuration switching. The selection of the runway for take-off or landing is determined by the orientation and length of the runway, surface winds and direction, the availability of approaching aids and the ATC preference. In practical situations, the runway configuration can be switched between landings and take-offs to tackle the problem of imbalanced arrival and departure rates. The aircraft Landing Problem (ALP) [1-4] and Aircraft Take-off Problem (ATP) [5] are the segregate runway operations that optimise the landing/take-off sequences for a set of flights in operation. Mixed-mode runway operation defined as the configuration of the runway can perform take-offs and landings [6]. As for semi-mixed mode operation, one runway is used exclusively for landing or take-offs, while other runways are configured as mixed-mode operation. Semi-mixed mode operation is usually applied when certain runways are segregated for approaches or departures due to terrain and runway specification constraints. Jacquillat, et al. [7] proposed a runway configuration switch, which can resolve the problem of limited arrival and departure runway capacities and enhance the resilience level of runway schedules by introducing runway clearance. Although the efficiency and effectiveness of a runway schedule can be further enhanced using mixed-mode runway operations, the applicability of the operation relies on clear monitoring of aircraft activities by satellite navigation in the near TMA. Such operation increases the ATC workload. Therefore, introducing temporary runway clearance in runway configuration switching in ASSP increases the resilience level in a runway schedule.

The ASSP, in general, requires formulation of a planned schedule from a given a set of deterministic arrival and departure times of the flights. However, it is crucial to consider the unexpected delay in ASSP with runway configuration planning. The disrupted flights may affect the scheduled runway clearance operation, subsequence landing and take-off operations in the schedule. To leverage the effect of unexpected delays, this research considers the uncertain arrival and departure times as an interval case. Stochastic and robust modelling are the general methods for handling uncertainty. The min-max regret approach is a typical criterion in robust optimisation. The regret value is defined as the makespan deviation from the optimal schedule under the worst-case scenario.

In this paper, we propose a robust optimisation to the ASSP model and runway configuration planning using the min-max regret criterion considering the uncertainty of arrival and departure delays. Due to the computational complexity, we adopt the mid-point scenario heuristic as an initial solution, which can reduce the computational burden and solution gaps in order to obtain better robust ASSP solution.

II. PROBLEM FORMULATION

A. Description of Deterministic Aircraft Sequencing and Scheduling Problem with the control of runway configuration

A robust ASSP with runway configuration planning is proposed. The formulation of the proposed model is the extension of parallel machine scheduling [8]. The descriptions of the parameters and decision variables are shown in Table 1. We consider a set of approaching and departing flights n $(i = 1,2,\dots,n)$ that are ready to perform landings/take-offs on a runway r $(r = ,1,2,\dots m)$. The set of approaching flights in the schedule must be reached in the near TMA, while the set of departing flights in the schedule is ready to take-offs soon. The maximum number of flights in a schedule is n and the maximum number of runway is m. Each flight must land/take-off on only one runway, and each runway is only reserved for one flight to the runway operation.

TABLE I
NOTATION AND DECISION VARIABLES

Parameters	Explanation
i	Aircraft ID i, (i= $1,2,\dots,n$)
n	The maximum number of aircraft
r	Runway ID r, $(r = 1,2,\dots,m), m \geq 2$
m	The maximum number of runway
b	Family of runway operation b, (b =*1,…,q)*
q	The maximum number of runway operation
β_{ib}	1, if the aircraft i is to perform runway operation b; 0, otherwise
z_{jib}	1, if aircraft i and j belongs to the same family of runway operation b, 0, otherwise
S_{ij}	The runway operation based separation time between aircraft i and j scheduled on the same runway, $S_{ij} \geq 0$
STO_{ir}	The scheduled landing/take-off time of aircraft i on runway r
$\underline{STO_{ir}}$	The lower bound value of scheduled landing/take-off time of aircraft i on runway r
$\overline{STO_{ir}}$	The upper bound value of scheduled landing/take-off time of aircraft i on runway r
STO_{ir}^s	The uniform distributions of landing/take-off time intervals, $STO_{ir}^s \in \left[\underline{STO_{ir}}, \overline{STO_{ir}}\right]$
k	The family-dependent switching time
ε	The unexpected arrival/departure delay or the deviation of predetermined operation time with or without interruption
s	The possible realised operation time in a scenario, $s = (STO_{1r}^s, STO_{2r}^s, \dots, STO_{(n-1)r}^s, STO_{nr}^s), s \in \delta$
ω	The set of all feasible schedules
M	Large number associated with the artificial variable
Decision variables	**Explanation**
X	A schedule X is constructed by x_{ir}, y_{jir} and $T_{ir}^s(x)$.
x_{ir}	$\begin{cases}1 & \text{If aircraft } i \text{ is assigned to runway } r\\ 0 & \text{otherwise}\end{cases}$
y_{jir}	$\begin{cases}1 & \text{If aircraft } j \text{ is before aircraft } i \text{ on the same runway } r \text{ (not necessarily immediately)}\\ 0 & \text{otherwise}\end{cases}$
$T_{ir}^s(X)$	The assigned operation time for aircraft i on the runway r in schedule X under scenario s, $T_{ir} \geq 0$
$C_r^s(X)$	The makespan of schedule X under scenario s, $C_r \geq 0$

The scheduled time of operation of flight i on runway r is STO_{ir}. Separation time S_{ji} separates the scheduled time of operation between STO_{jr} and STO_{ir} for flights j and i with regard to their runway operation. The scheduled time of operation is defined as an interval case $[\underline{STO_{ir}}, \overline{STO_{ir}}]$ in a uniform distribution. The unexpected delays ε represents the possible delay time in runway operation. The regulated separation requirement is shown in Table 2.

The runway operation of each flight i is formulated as a product family in general job-shop scheduling problem. Each flight belongs to a family of runway operations b, either landings or take-offs using the parameter β_{ib}. Therefore, the maximum number of runway operations is equal to 2. The parameter z_{jib} that determines the family relationship between flights j and i. z_{jib} is equal to one when flights j and i belong to the same family. The parameter k is represented by the switching time of changing the runway configuration between arrival and departure setting. The following is the complete formulation in the deterministic ASSP model with runway configuration planning.

$$\min C_r^s \tag{1}$$

$s.t.$

$$T_{ir}^s(X) - T_{jr}^s(X) \geq S_{ji} + kz_{jib} - M\left(1 - y_{jir}\right), \forall i,j,i \neq j,r,s,b \tag{2}$$

$$T_{ir}^s(X) \geq STO_{ir}^s - M(1 - x_{ir}), \forall i,r,s \tag{3}$$

$$x_{ir} + x_{jr} \leq 1 + y_{ijr} + y_{jir}, \forall i,j,i \neq j,r \tag{4}$$

$$y_{jir} + y_{ijr} \leq 1, \forall i,j,i \neq j,r \tag{5}$$

$$\sum_{r=1}^{m} x_{ir} = 1, \forall i \tag{6}$$

$$x_{ir} \in \{0,1\}, \forall i,r \tag{7}$$

$$y_{jir} \in \{0,1\}, \forall i,j,r \tag{8}$$

$$z_{jib} \in \{0,1\}, \forall i,j,b \tag{9}$$

Under scenario s, the objective function (1) in the deterministic case is to minimise the makespan value, which defined as the latest completion time among all runways. Constraint (2) computes the assigned operation time for flight i from flight j regarding the separation requirement S_{ji} and runway switching time kz_{jib}. Constraint (3) ensures that the assigned time of operation T_{ir}^s must be larger than the scheduled time of operation STO_{ir}^s under scenario s. Constraints (4) and (5) express the relationship between x_{ir} and y_{jir}. If flights i and j are assigned to the same runway, the value $(y_{ijr} + y_{jir})$ takes either 0 or 1 to illustrate the sequential relationship. Constraint (6) defined that each flight can only assign to one and only one runway to perform the operation. Each flight i must perform approaches or departures on only one runway r. Constraints (7), (8) and (9) denote the decision variables x_{ir} and y_{jir}, and parameter z_{jib} as binary.

TABLE II
SEPARATION TIME (IN SECONDS) BETWEEN TWO CONSECUTIVE FLIGHTS WITH SAFE PARALLEL OPERATION

			Trailing aircraft					
			Arr.			Dep.		
			SSF	MSF	LSF	SSF	MSF	LSF
Leading aircraft	Arr.	SSF	82	69	60	75	75	75
		MSF	131	69	60	75	75	75
		LSF	196	157	96	75	75	75
	Dep.	SSF	60	60	60	60	60	60
		MSF	60	60	60	60	60	60
		LSF	60	60	60	120	120	90

Arr. = Arrivals; Dep. = Departures, SSF = Small size flight; MSF = Medium size flight; LSF = Large size flight

B. Description of Robust Aircraft Sequencing and Scheduling Problem with the control of runway configuration

The scheduled time of operation is an interval case to represent the uncertain operation time and operate lack of complete information. The robust schedule provides guarantees about the performance of the schedule with less effort on rescheduling, a high resilience level to the configuration switching and the possibility of delay propagation in the runway schedule.

$$Regret(X,s) = F(X,s) - F_s^* \quad (10)$$

$$F(X,s) = \max_{r\in R}(C_r^s(X)) \quad (11)$$

$$Regret_{max}(X) = \max_{s\in S} Regret(X,s) \quad (12)$$

$$\min_{X\in\omega} Regret_{max}(X) = \min_{X\in\omega}\max_{s\in S}(F(X,s) - F_s^*) \quad (13)$$

In the min-max regret approach, the objective function is to minimise the maximum deviation of the makespan across all scenarios in robust ASSP with runway configuration switching. Equation (10) calculates the deviation of the makespan under scenario s from the optimal condition. The makespan under scenario s is computed by equation (11).

Definition 1. As for each scenario $s \in \delta$, the regret value is maximised under following scenario s_k as follows:

$$STO_i^{s^k} = \begin{cases} \overline{STO_i}, if\ x_{ik} = 1 \\ \underline{STO_i}, if\ x_{ik} = 0 \end{cases}, i = 1,2,\dots,n \quad (14)$$

Definition 2. Given a makespan-like objective, the maximum completion time of a schedule $X \in \omega$ under scenario $s \in \delta$ requires to declare the critical runway $\zeta \in m$.

$$C_\zeta^s(X) = \max_{r\in m}\{C_r^s(X)\} = F(X,s) \quad (15)$$

Proposition 1. The maximum completion time under scenario s must exist when runway ζ is critical under scenario s^ζ, and scenario s^ζ is the worst-case scenario. The proof is omitted due to limited space.

By definitions 1 and 2 of the extreme point scenario, the maximum regret value is calculated by equation (12). Therefore, minimising the maximal regret value can be obtained by equation (13).

III. SOLUTION PROCEDURES

A. Exact Algorithm in the Iterative Relaxation Procedure

The objective function in the min-max regret approach in ASSP model with runway configuration includes two operators, which is in a non-linear form, and therefore, the optimal condition cannot be obtained directly by the exact method. An iterative relaxation procedure is introduced by adding the regret cuts iteratively.

Proposition 2. The maximum regret value under each scenario can be obtained by identifying the critical runway.

$$Regret_{max}(X) = \max_{r\in R}(C_r^{s^\zeta} - F_{s^\zeta r}^*) \quad (15)$$

According to proposition 2, the worst-case scenario s^ζ can be identified, in which r^* is critical, by using the equation of $r^* = arg\max_{r\in R}(C_r^{s^\zeta} - F_{s^\zeta r}^*)$. The regret cuts of robust optimsation are limited by a set of scenarios $\Omega = (s_1, s_2, \dots, s_\omega)$. Given a limited set of scenarios, the regret cut of each scenario is added into the robust optimisation model using $C_r^{s^\zeta} - F_{s^\zeta r}^* \leq RV$. The minimized maximum regret value can be calculated using the iterative relaxation procedure using the reformulation of the relaxed model as follows:

$$min\ RV \quad (16)$$

$$s.t.$$

$$C_r^{s^\zeta} - F_{s^\zeta r}^* \leq RV, s \in S, r = 1,2,\dots,m \quad (17)$$

And

(2) – (9)

The pseudo code of the iterative relaxation model is shown as follows:

Step 1: Set the lower bound regret value $\underline{RV}$ as 0 and the upper bound regret value $\overline{RV}$ as ∞.
Step 2: Obtain an initial solution in deterministic model by defining the STO_{ir} as $\underline{STO_{ir}}$.
Step 3: Define the upper bound regret value under extreme-point scenario $\overline{RV} = Regret_{max}(X)$.
Step 4: Identify the worst-case scenario s of the solution X
Step 5: Add regret cuts to the relaxed robust model and calculate the lower bound regret value $\underline{RV}$.
Step 6: Repeat step 3 until the $\underline{RV} \geq \overline{RV}$ in the iterative relaxation procedure is met.

B. Midpoint-scenario heuristic in the Iterative Relaxation Procedure

The branching in the min-max regret approach starts from the lower bound of scheduled time of operation $\underline{STO_{ir}}, i = 1,2, \ldots, n$. The computational burden is complex when the branch-and-bound algorithm is initialised with a set of lower bound scheduled times of operation $\underline{STO_{ir}}$. The midpoint-scenario heuristic is proposed from the literature with mathematical proof, which simplifies the computational process in the iterative relaxation procedure [9, 10]. The mid-point scenario of the scheduled time of operation is equal to $\widetilde{STO_{ir}} = (\overline{STO_{ir}} + \underline{STO_{ir}})/2$.

IV. COMPUTATIONAL EXPERIMENTS

A. Description of test instances

The instance in this study is randomly generated in accordance with the description shown in Table 3. The size of the test instance is $n = (10,20,30)$ in a dual-runways system, with runway configuration switching. The lower bound of scheduled time of operation $\underline{STO_{ir}}$ falls into a uniform interval of $[0, {}^{60m}/_{n}]$ to indicate an average one minute per flight (either approaches or departures) per runway. The unexpected delay of each flight ε_i is randomly generated in an interval of $[0, \alpha]$, where $\alpha = (30,60)$. Therefore, the upper bound of the scheduled time of operation $\overline{STO_{ir}} = \underline{STO_{ir}} + \varepsilon_i$. The switching time between landings and take-offs k is 300 seconds. The computational unit is configured as Intel Core i7 3.60 GHz CPU and 16GB random-access memory under the Windows 7 Enterprise 64-bit operating system. The exact algorithm is calculated by *IBM ILOG CPLEX Optimisation Studio 12.6.3* with a 3600 seconds computational limit.

$$\overline{STO_{ir}} = \underline{STO_{ir}} + \varepsilon_i$$

TABLE III
DESCRIPTION OF TEST INSTANCES

Robust ASSP Instance	ID	%LSF=20%, % MSF=70%, %SSF=10%	
		n	α
Two runways – runway configuration switching	1	10	30
	2	10	60
	3	20	30
	4	20	60
	5	30	30
	6	30	60

SSF = Small size flight; MSF = Medium size flight; LSF = Large size flight

The lower-bound scenario and mid-point scenario heuristic are evaluated by the following measurement. Equation (18) calculates the deviation of the upper bound and lower bound regret values $Regret\ Gap$ %. If the $Regret\ Gap$ % is equal to zero, this indicates an optimal condition of the robust solution.

$$Regret\ Gap\ \% = \frac{RegretUB - RegretLB}{RegretUB} \quad (18)$$

B. Computational results

In this section, we evaluate the performance of lower bound scenario and mid-point scenario heuristic in the Robust ASSP model with runway configuration planning using the min-max regret approach. The number of iterations in the iterative relaxation procedure depends on the number of scenarios, which is unable to be known beforehand. The optimal condition in the iterative relaxation procedure is to measure the deviation of the upper bound and lower bound regret values. Once the lower bound regret value is larger than or equal to the upper bound regret value, the optimal condition is reached. The calculated regret deviation with the lower-bound scenario and mid-point scenario heuristic are shown in Tables 4 and 5 correspondingly.

TABLE IV
COMPUTATIONAL PERFORMANCE OF THE LOWER BOUND SCENARIO BY EXACT ALGORITHM

ID	$\overline{RV}$	$\underline{RV}$	$Regret\ Gap$ %	CPU (sec)
1	0	0	0%	1.7
2	20	26	0%	2.12
3	30	4	86.67%	--
4	36	3	91.67%	--
5	319	18	94.36%	--
6	220	45	79.55%	--

--: computational time over 3600 seconds

The results indicates that the mid-point scenario heuristic significantly reduces the computational complexity to obtain an optimal solution. Furthermore, the min-max regret approach using the mid-point scenario heuristic as an initial solution can reach an optimal condition in our experiments.

TABLE V
COMPUTATIONAL PERFORMANCE OF THE MID-POINT SCENARIO HEURISTIC BY EXACT ALGORITHM

ID	$\overline{RV}$	$\underline{RV}$	$Regret\ Gap$ %	CPU (sec)
1	0	0	0%	1.7
2	20	26	0%	3.68
3	11	11	0%	--
4	28	28	0%	--
5	230	990	0%	--
6	206	207	0%	--

--: computational time over 3600 seconds

V. CONCLUDING REMARKS

In this paper, we propose a formulation for robust optimisation in aircraft sequencing and scheduling problem with runway configuration planning. Delay management is an important issue that affects the robustness of a schedule. The aggregate delays also affect the subsequent activities, and therefore, airport management and agents are required to re-schedule activities using the latest landings and take-offs schedule. Runway configuration can be changed along with the demand patterns of approaches and departures. Although runway configuration planning helps ATC to maintain a smooth schedule to reduce terminal and airport traffic, any delays will affect the schedule of runway clearance. Therefore, the robust aircraft schedule insists on accommodating unexpected delays in a resilience-driven schedule approach. The min-max regret approach is considered to resolve the robust ASSP model. The iterative relaxation process, indeed, can resolve the nonlinear function in a min-max regret approach. However, the computational time is significantly lengthened. The mid-point scenario heuristic can moderate the computational burden in the iterative process. The computational experiments measure the performance of the construction of the initial solution with lower bound and the mid-point scenario heuristic. The results suggest that mid-point scenario heuristic obtains a better robust solution in terms of the solution quality.

Future research is recommended in the following aspects. (1) The computational time will increase along with the complexity of the model, heuristics and meta-heuristics and so are the possible research direction to extend the robust optimisation in practical usage. (2) Other Terminal Manoeuvring Area resources can be integrated in a model, as the models of near terminal/terminal traffic and airport traffic are interdependent. Therefore, an integrated model can reduce the uncertain level to obtain accuracy results.

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