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On the Empirical Likelihood Option Pricing[1]

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Abstract

The Black-Scholes model is the golden standard for pricing derivatives and options in the modern financial industry; however, this method imposes some parametric assumptions of the stochastic process and the performance of it becomes doubtful when these assumptions are violated. This paper investigates the application of a nonparametric method, namely the empirical likelihood method, in the study of option pricing. A blockwise empirical likelihood procedure is proposed to deal with the dependence in the data. Simulation and real data studies show that the new method performs reasonably well. More importantly, it outperforms classical models developed to account for jumps and stochastic volatility, thanks to the fact that nonparametric methods capture information about higher order moments.

1 Introduction

Since the seminal works by Black and Scholes (1973) and Merton (1973), option valuation methodologies have been extensively developed. The Black-Scholes model has become one of the most well-known discoveries in finance literature, and relates the cross-sectional properties of option prices with the underlying assets' returns distribution. However, Rubinstein (1985)and

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Melino and Turnbull (1990) point out several limitations in the Black-Scholes model due to strong assumptions, such as non-normality of returns, stochastic volatility (implied volatility smile), jumps and others. Both parametric and nonparametric approaches have been proposed to deal with these issues.

Scott (1987), Hull and White (1987), and Wiggins (1987) extend the Black and Scholes model and allow the volatility to be stochastic. Heston (1993) develops a closed-form solution for option pricing with the underlying asset's volatility being stochastic. Duan (1995) proposes a GARCH option pricing model in an attempt to explain some systematic biases associated with the Black-Scholes model. Later, Heston and Nandi (2000) provide a closed-form solution for option pricing with the underlying asset's volatility following the GARCH(p,q) process. Bates (1996), Bakshi and Cao and Chen (1997) derive an option pricing model with stochastic volatility and jumps. Kou (2002) provides a solution to pricing the option with the double exponential jumps diffusion process. Carr and Madan (1999) introduce the fast Fourier transform approach to option pricing given a specified characteristic function of the return, which provides an efficient computational algorithm to calculate the option prices. For further reference, see Duffie et al. (2000), Bakshi and Madan (2000), and Carr and Madan (2009), among others. All these methods assume a parametric form of either the distribution of the underlying assets returns or the characteristic function of the underlying assets returns.

Nonparametric approaches have also been proposed to capture the underlying asset and option price data to reconstruct the structure of the diffusion process. For example, Hutchinson, Lo, and Poggio (1994) apply the neural network techniques to price the derivatives. Ait-Sahalia and Lo (1998) use the kernel regression to fit the state-price density implicitly in option pricing. Ait-Sahalia (1996) proposes a nonparametric pricing estimation procedure for interest rate derivative securities under the assumption that the unknown volatility is independent of time. Stutzer (1996) adopts the canonical valuation method, which incorporates the non-arbitrary principle embodied in the formula for calculating the expectation of the discounted value of assets under the risk-neutral probability distribution.

One of the most important nonparametric methodologies is empirical likelihood (EL), which conducts likelihood-based statistical inference by profiling a nonparametric likelihood. See Owen (1988, 1990, 2001), DiCiccio and Romano (1989), and Hall and La Scala (1990) for examples. For the application of EL method to time series, see Mykland (1995), Chuang and Chan (2002), and Ling and Chan (2006) among others. Kitamura (1997) introduced a blockwise empirical likelihood method for a weakly dependent time series. Nordman, Sibbertsen, and Lahiri (2007) modifiy the blockwise methods to cope with various dependence structures and achieve better finite sample performance. Yau (2012) studies the application of EL to long-memory time series.

In this paper, we implement the EL method to price the derivative or options under risk neutral measures. We firstly construct an empirical probability constraint using the historical holding period return time series observations, without assuming the distribution family of the returns. Further, we view the derivative / option price as the parameter of interest directly in the empirical likelihood optimization procedure. An empirical likelihood-based estimate of the parameter (e.g. call option price) is obtained and the asymptotic properties of the EL ratio are studied. We further introduce a blockwise empirical likelihood procedure for the weakly dependent processes. Monte Carlo simulation and empirical results for S&P 500 index option are discussed.

The remainder of the paper is organized as follows. Section 2 provides a detailed empirical likelihood procedure in option pricing. Asymptotic properties are discussed and a robust confidence interval is constructed. Section 3 provides some empirical performance of the empirical likelihood option pricing including both Monte Carlo simulation and S&P 500 Index options. Section 4 concludes the paper with discussions.

2 Empirical Likelihood in Option Pricing

Let $P(t)$ be the underlying asset price at time t, $D(t)$ be the future dividend at time t, $r(s,t)$ be the gross risk-free interest rate during time s and t with $r(t,t) = 1$, $\mathcal{P}$ be the physical probability measure, and $\mathcal{Q}$ be the risk-neutral probability measure (See Huang and Litzenberger (1988)), under which the price process plus the accumulated dividends are martingales after normalization if no arbitrage exists in the pricing systems. To be specific, the latter leads to the following pricing formula:

$$\begin{aligned} P(t) &= E^{\mathcal{Q}}\left[\frac{P(T)+\sum_{s=t}^{T}D(s)r(s,T)}{r(t,T)}\right] \\ &= E^{\mathcal{P}}\left[\frac{P(T)+\sum_{s=t}^{T}D(s)r(s,T)}{r(t,T)}\frac{\mathrm{d}\mathcal{Q}}{\mathrm{d}\mathcal{P}}\right]. \end{aligned} \tag{1}$$

Here $\frac{\mathrm{d}\mathcal{Q}}{\mathrm{d}\mathcal{P}}$ is the Radon-Nykodym density of the marginal measure. One can price an option or a derivative security by evaluating the expected discounted value under $\mathcal{Q}$. For example, the call option price with strike price K and expiring date T is given by

$$C(t,T) = \frac{E^{\mathcal{Q}}\max[P_T - K, 0]}{r(t,T)}. \tag{2}$$

The following subsection illustrates the idea of estimating $C(t,T)$ through EL coupled with the change-of-measure constraint.

2.1 The Estimating Procedure

Suppose historical data are available in the format of $\{(P(t), D(t)), t = -1, -2, ..., -H\}$. A nonparametric way of estimating the option price could be built on approximating $\mathcal{Q}$ by a discrete distribution supported on the observed value of option price; namely, $\mathrm{HPR}(-i-T,-i)/r(-i-T,-i)$, $1 \le i \le H-T$ with the corresponding probability denoted by π_i. Here $\mathrm{HPR}(s,t)$ is the holding period return between times s and t. If there is no dividend, $\mathrm{HPR}(-i-T,i) = P(-i)/P(-i-T)$. Then (1) can be approximated by

$$1 = \sum_{i=1}^{H-T}\frac{\mathrm{HPR}(-i-T,-i)}{r(-i-T,-i)}\pi_i. \tag{3}$$

Correspondingly, we can estimate the option price by approximating (2) by

$$\hat{C}(t,T) = \sum_i \frac{max[P_i(T)-K,0]}{r(t,T)}\pi_i. \tag{4}$$

Note that the choice of π_i subjecting to (3) is not unique. Stutzer (1996) uses the idea of maximum entropy, namely maximizing $\sum_{i=1}^{H-T}\pi_i\log\pi_i$, subjecting to (3). Here we adopt the empirical likelihood method (Owen, 1988) by changing the objective function from entropy to empirical likelihood, namely maximizing $\sum_{i=1}^{H-T}\log\pi_i$. This objective function can be easily interpreted as a nonparametric log likelihood function and, hence, the whole optimization procedure in our method can be interpreted as a maximum likelihood method, which is considered more efficient than a maximum entropy method. Moreover, Baggerly (1998) proposes a general class of empirical likelihood type methods, which contains both $\sum \log \pi_i$ and $\sum \pi_i \log \pi_i$ as special cases. Additionally, Baggerly (1998) proves the empirical likelihood used in this paper is the only method in the general class which has a higher order correction of the large sample properties. We refer to Kitamura (1997) for the detailed form of the higher order correction. Meanwhile, by noticing that the sequence $\mathrm{HPR}(-i-T,-i)/r(-i-T,-i)$, $1 \le i \le H-T$ possess a reasonable amount of dependence, we suggest the adoption of the blockwise version of the algorithm as follows: group the data into Q blocks where length M is the length of the moving block. Set L to be the step size of the moving block. We obtain block weight π_i^* by maximizing $\sum_{i=1}^{H-T} \log \pi_i^*$ subjecting to

$$1 = \sum_{i=1}^{Q} \pi_i^* \left[\frac{1}{M} \sum_{j=1}^{M} \frac{\mathrm{HPR}(-i*L-j-T,-i*L-j)}{r(-i*L-j-T,-i*L-j)} \right]. \tag{5}$$

Then estimate the option price by

$$C = \sum_{i=1}^{Q} \left[\frac{1}{M} \sum_{j=1}^{M} \frac{\max[P_{i*L-j}(T)-K,0]}{r(-i*L-j-T,-i*L-j)} \right] \pi_i^*. \tag{6}$$

This blocking idea is studied by Kitamura (1997), who argues that using blockwise methods has a much better empirical performance for weakly dependent processes by moving average noise terms. The estimation procedure in the spirit of Kitamura (1997) is slightly different:

$$\max_{C,\pi_i^*} \sum_{i=1}^{Q} \log \pi_i^*, \tag{7}$$

subject to constraints (5), (6), and

$$\begin{cases} \sum_{i=1}^{Q} \pi_i^* = 1, \\ \pi_i^* > 0, \end{cases}$$

and the maximizing C is our estimator. The estimated risk neutral measure weights pi_i^* have the following form:

$$\pi_i^* = \left\{ Q \left(1 + \gamma \left[\frac{1}{M} \sum_{j=1}^{M} \frac{\mathrm{HPR}(-i*L-j-T,-i*L-j)}{r(-i*L-j-T,-i*L-j)} - 1 \right] \right) \right\}^{-1},$$

where γ is a Lagrange multiplier. These weights are similar to the Gibbs canonical probability in Stutzer (1996) because they put small weights when rate of returns of underlyings are far from risk-free returns. Additionally, Peng (2015) shows that these two approaches yield the same asymptotic property. In our simulation below, we adopt the second method since it is well known and there is an existing package for implementation. Particularly, Qin and Lawless (1994) provide a Lagrangian with multipliers approach to solve the above-mentioned optimization problem. We can either apply the numerical optimization process or derive the solution, similar to Qin and Lawless (1994). For more details about the Lagrangian optimization or the basic properties of the empirical likelihood procedure, see Owen (1990) and Qin and Lawless (1994).

2.2 Asymptotic Properties

In this subsection, we discuss some basic asymptotic properties of the option price respect to the empirical likelihood process (Equation (6) / (7)), which helps us to understand the asymptotic distribution of our estimate and conduct further inference.

Theorem 1 *Consider that*

$$f(\mathrm{HPR}_t, C) = \left(\frac{\max[P_i(T) - K, 0]}{r(t,T)} - C, \frac{\mathrm{HPR}(-t-T,t)}{r(t-T,t)} - 1 \right)^T$$

and further assume that:

(i) the derivative price (C) is in a compact set Θ*;*

(ii) C_0 *is a unique solution of* $E(f(\mathrm{HPR}_t, C)) = 0$*;*

(iii) For sufficient small $\delta > 0$ *and* $\eta > 0$*,*

$$E[\sup_{C^* \in O(C,\delta)} ||f(\mathrm{HPR}, C^*)|| < \infty$$

for all $C \in \Theta$*;*

(iv) If a sequence of C_j*,* $j = 1, 2, ...$ *converges to some* C *as* $j \to \infty$*,* $f(\mathrm{HPR}_t, C_j)$ *converges to* $f(\mathrm{HPR}_t, C)$ *for all* HPR_t *except on a null set, which may vary with* C*;*

(v) C_0 *is an interior point of* Θ*;*

(vi) $\mathrm{Var}(H^{-\frac{1}{2}} \sum_{i=1}^{H} f(\mathrm{HPR}_i, C_0)) \to S > 0$*;*

(vii) For blockwise empirical likelihood approach, we further assume the weak dependent condition, $\sum_{k=1}^{\infty} \alpha_X(k)^{1-1/d} < \infty$ *for some constant* $d > 1$*. And we require additional assumptions,*

$$E||f(\mathrm{HPR}_t, C_0)||^{2d} < \infty, \ \textit{for } d > 1$$

$$E \sup_{C^* \in O(C_0,\delta)} ||f(\mathrm{HPR}_t, C^*)||^{2+\epsilon} < K, \ \textit{for some } \epsilon > 0.$$

Then,

$$\mathrm{LR}_0 = 2\sum_{i=1}^{Q} \log(1 + \gamma(\hat{C})^T f(\mathrm{HPR}_i, \hat{C})) \rightarrow_{\mathrm{dist.}} \chi_1^2$$

where K is a finite number, $\gamma(\hat{C})$ is the Lagrange multiplier vector and Q is the total number of states. Particularly for non-blockwise empirical likelihood case (i.e. Equation (6)), $Q = H - T$.

Theorem 1 provides an asymptotic distribution of the likelihood ratio LR_0, which can be further applied to inference of the estimate. We omit the detailed proof here.[2] For independent observations of HPR_i, we only require the assumptions (i)-(vi) to have the asymptotic property of the likelihood ratio; and, for weak-dependent observations of HPR_i, assumption (vii) is additionally required. Given the simple fact that the chi-square distribution is the square of a normal distribution, the distribution of the errors measured by the likelihood ratio will be close to the white noise when sample size goes to infinity. This means that our estimator will eventually capture almost all the information in the data.

3 Empirical Results

In this section, we first compare our method with several popular option pricing models through Monte Carlo simulation and then conduct an empirical analysis on the option pricing for the S&P 500 index call options.

3.1 Monte Carlo Simulation

3.1.1 Black-Scholes Model

Following Hutchinson et al. (1994), Ait-Sahalia and Lo (1995), and Stutzer (1996), we generate a geometric Brownian motion process with a 10 percent drift and 20 percent annualized

[2]Our proof is a direct consequence of Theorems 1 & 2 in Kitamura (1997).

volatility. Firstly, we simulate 2 years of historical daily stock returns with $253 \times 2 = 506$ observations. We repeat 200 samples, and, for each sample, three different prices are calculated: 1. the estimated price by the empirical likelihood option pricing procedure; 2. the estimated price by the Black-Scholes model with historical volatility; 3. the actual price by the Black-Scholes model with actual volatility.

Table 1: Monte Carlo Simulation in a Black-Scholes Market

This table reports the mean absolute percentage error (MAPE) of the empirical likelihood (EL) option price to the ideal Black-Scholes price (Panel A), the historical volatility based Black Scholes price to the ideal Black-Scholes price (Panel B) for different combination of the relative exercise prices (P/K) and time to expiration date. The price dynamics follow the Geometric Brownian Motion with $\mu = 0.1$ and $\sigma = 0.2$. The relative exercise prices (P/K) are chosen as Rubinstein (1985), Stutzer (1996). The time to expiration date are $1/13, 1/4, 1/2$ years, respectively.

Panel A:

	Hist Var vs Ideal BS	Years to Maturity		
		1/13	1/4	1/2
	9/10	0.2124	0.0987	0.0687
Moneyness (P/K)	1	0.0327	0.0305	0.0284
	9/8	6.375×10^{-4}	0.0047	0.0080

Panel B:

	EL vs Ideal BS	Years to Maturity		
		1/13	1/4	1/2
	9/10	0.724	0.514	0.537
Moneyness (P/K)	1	0.088	0.149	0.230
	9/8	0.003	0.025	0.058

The performance of the first two prices are compared based on the mean absolute percentage error (MAPE) with respect to the third price, which is considered to be the true price. The comparison is made at different price-to-strike price ratios (i.e. $P/X = \frac{9}{10}, 1, \frac{9}{8}$) and different expiration dates (i.e. $T = \frac{1}{13}, \frac{1}{4}, \frac{1}{2}$).

Table 1 provides the simulation performance: Panel A reports the MAPE of the empirical likelihood (EL) option price, and Panel B reports the MAPE of the historical volatility based Black Scholes price. In the perfect Black-Scholes world, the Black-Scholes formula using the historical volatility outperforms the empirical likelihood option pricing methodology. This is because the Black-Scholes formula only requires the second moment information and 506 observations can provide a very good estimate of the second moment; however the empirical likelihood methods automatically capture the higher order moment information, which does not benefit in pricing the options in the perfect Black-Scholes world.

We are also interested in the accuracy of the empirical likelihood option pricing for different moneyness and days-to-maturity. The empirical likelihood option pricing method provides very good performance in pricing the in-the-money options with small MAPE; however, the MAPE are very significant for out-of-the-money options. At-the-money option pricing error is in between. On the other hand, the pricing errors have different patterns for in-the-money, at-the-money, and out-of-the-money options. For in-the-money and at-the-money options, the fewer days to maturity, the smaller the pricing errors are. For out-of-the-money options, the fewest days to maturity case has the largest pricing error, with a possible reason being that the price magnitude of the out-of-the-money options with very few days to maturity is already very small.

3.1.2 Stochastic Volatility Jump Model

Bates (1996) adds a compound Poisson process to Heston stochastic volatility model to account for the rare sudden drift of some financial assets. The stochastic processes are defined as follows:

$$\begin{aligned}
&\mathrm{d}S/S = \mu \mathrm{d}t + \sqrt{V}\mathrm{d}Z + k\mathrm{d}q, \\
&\mathrm{d}V = (\alpha - \beta V) + \sigma_v \sqrt{V}\mathrm{d}Z_v, \\
&\mathrm{cov}\left(\mathrm{d}Z, \mathrm{d}Z_v\right) = \rho \mathrm{d}t, \\
&P\left(\mathrm{d}q = 1\right) = \lambda \mathrm{d}t, \\
&\ln\left(1 + k\right) \sim N\left(\log\left(1 + \kappa\right), \delta^2\right).
\end{aligned}$$

Here we use the paramters estimates in Bates (1996) to produce simulated stock prices and European option prices. We compare our nonparametric option pricing method with Stutzer (1996), historical Black-Scholes, and Heston models. We summarize the results in Table 2. The κ and δ are the mean and standard deviations of the sizes of jumps. From the table, we can find that when jump sizes are small, as is the case in the second row, our method beats the historical Black-Scholes, but loses to the Heston model. That is because when jump sizes are sufficiently small, the Bates model is extremely close to the Heston model, and, hence, calibration of the Heston model is more or less the same as using a parametric method with the true likelihood function. We know that the parametric likelihood method always achieves the lowest error bound when we use the right likelihood functions. When the jump sizes are large, however, as is the case in the third row, our nonparametric method not only outperforms the other methods, but also performs consistently well no matter whether the jump sizes are large or small.

3.2 S&P 500 Index Options

We also implement the empirical likelihood option pricing method in pricing the S&P 500 index options. The daily return data is from the Center for Research in Security Prices (CRSP) and the option data is from OptionMetrics. The daily return data is from Jan 2011 to Dec 2012. We use the year 2011 daily return data as the formation period, and then test its performance in the year 2012 daily index options pricing, comparing with the historical volatility-based

Black-Scholes model and the true values. We only keep the options which have the Moneyness closest to 1 and days to maturity between 15 to 50.

Table 2: MAPE of the Four Methods under Bates Model

Jump Parameters	Empirical Likelihood	Stutzer	Historical Black-Scholes	Heston Model
$\kappa = -0.001$, $\delta = 0.019$.	0.3680	0.3804	0.5734	2.428×10^{-7}
$\kappa = 0.1$, $\delta = 0.5$.	0.3759	0.4448	0.6062	0.4765

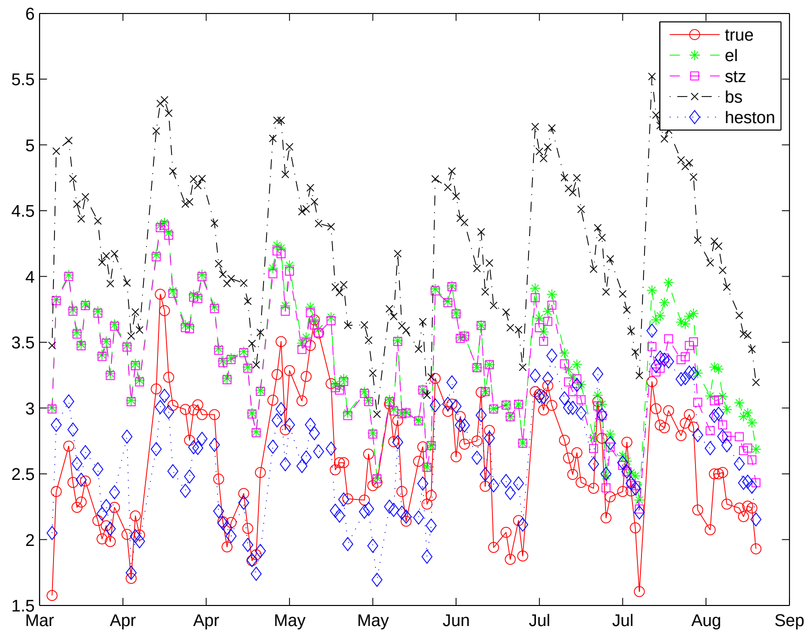
Figure 1 shows the time series of the option prices. The red line is the true value of the market daily close price, the green line is the empirical likelihood option price, the black line is the Black-Scholes option price using the historical volatility, the blue line is the Heston stochastics volatility option pricing using least square calibration, and the purple line is the method from Stutzer (1996). Due to the stock price movement, the true option prices vary from 1.5 to 3.7; however, the historical volatility-based Black-Scholes option prices are consistently overpriced for the at-the-money call options, as is documented in Hull and White (1987). In contrast, our EL option prices are closer to the true option market prices. This is because our methodology also captures the high order moment information in the empirical likelihood procedure, while the historical volatility-based Black-Scholes option model only captures the second moment information.

4 Conclusion

In this paper, we introduce an empirical likelihood method to price derivatives under a risk-neutral measure. Based on Monte Carlo simulations and on S&P 500 index option data, we show that our method outperforms classical alternative models (Black-Scholes, Heston, and Bates), thanks to our advantage in capturing higher order moment information.

Figure 1: Comparison of the S&P 500 index option prices and EL option prices

This figure shows the time series of three S&P 500 index option prices. We only keep the options have the Moneyness closest to 1 and days to maturity between 15 to 50. The red line is true value of the market daily close price, the green line is the empirical likelihood option price, the black line is the Black Scholes option price using the historical volatility, the blue line is Heston stochastics volatility option pricing using least square calibration and the purple line is method from Stutzer (1996).



References

Ait-Sahalia, Y. (1996). Nonparametric Pricing of Interest Rate Derivative Securities. *Econometrica*, 64(3), 527-560. DOI: 10.2307/2171860

Ait-Sahalia, Y., & Lo, A. W. (1998). Nonparametric estimation of state-price densities implicit in financial asset prices. *The Journal of Finance*, 53(2), 499-547. DOI: 10.1111/0022-1082.215228

Baggerly. K.A. (1997). Empirical likelihood as a goodness-of-fit measure. *Biometrika*, 85(3), 535-547. DOI: 10.1093/biomet/85.3.535

Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical performance of alternative option pricing models. *The Journal of Finance*, 52(5), 2003-2049. DOI: 10.1111/j.1540-6261.1997.tb02749.x

Bakshi, G., & Madan, D. (2000). Spanning and derivative-security valuation. *Journal of Financial Economics*, 55(2), 205-238. DOI:10.1016/S0304-405X(99)00050-1

Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *Review of Financial Studies*, 9(1), 69-107. DOI: 10.1093/rfs/9.1.69

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy*, 637-654. DOI: 10.1086/260062

Carr, P., & Madan, D. (1999). Option valuation using the fast Fourier transform. *Journal of Computational Finance*, 2(4), 61-73.

Carr, P., & Madan, D. (2009). Saddlepoint methods for option pricing. *Journal of Computational Finance*, 13(1), 49.

Chan, N. H., & Ling, S. (2006). Empirical likelihood for GARCH models. *Econometric Theory*, 22(03), 403-428. DOI: http://dx.doi.org/10.1017/S0266466606060208

Chuang, C. S., & Chan, N. H. (2002). Empirical likelihood for autoregressive models, with applications to unstable time series. *Statistica Sinica*, 12(2), 387-408.

Diciccio, T. J., & Romano, J. P. (1989). On adjustments based on the signed root of the empirical likelihood ratio statistic. *Biometrika*, 76(3), 447-456. DOI: 10.2307/2336111

Duan, J. C. (1995). The GARCH option pricing model. *Mathematical Finance*, 5(1), 13-32. DOI: 10.1111/j.1467-9965.1995.tb00099.x

Duffie, D., Pan, J., & Singleton, K. (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 68(6), 1343-1376. DOI: 10.1111/1468-0262.00164

Hall, P., & La Scala, B. (1990). Methodology and algorithms of empirical likelihood. *International Statistical Review/Revue Internationale de Statistique*, 109-127. DOI: 10.2307/1403462

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2), 327-343. DOI: 10.1093/rfs/6.2.327

Heston, S. L., & Nandi, S. (2000). A closed-form GARCH option valuation model. *Review of Financial Studies*, 13(3), 585-625. DOI: 10.1093/rfs/13.3.585

Huang, C., & Litzenberger, R. H. (1988). Foundations for financial economics. *Princeton University.*

Hull, J., & White, A. (1987). The pricing of options on assets with stochastic volatilities. *The Journal of Finance*, 42(2), 281-300. DOI: 10.1111/j.1540-6261.1987.tb02568.x

Hutchinson, J. M., Lo, A. W., & Poggio, T. (1994). A nonparametric approach to pricing and hedging derivative securities via learning networks. *The Journal of Finance*, 49(3), 851-889. DOI: 10.1111/j.1540-6261.1994.tb00081.x

Kou, S. G. (2002). A jump-diffusion model for option pricing. *Management science*, 48(8), 1086-1101. DOI: 10.1287/mnsc.48.8.1086.166

Kitamura, Y. (1997). Empirical likelihood methods with weakly dependent processes. *The Annals of Statistics*, 25(5), 2084-2102. DOI:10.1214/aos/1069362388

Melino, A., & Turnbull, S. M. (1990). Pricing foreign currency options with stochastic volatility. *Journal of Econometrics*, 45(1), 239-265. DOI:10.1016/0304-4076(90)90100-8

Merton, R. C. (1980). On estimating the expected return on the market: An exploratory investigation. *Journal of Financial Economics*, 8(4), 323-361. DOI:10.1016/0304-405X(80)90007-0

Mykland, P. A. (1995). Dual likelihood. *The Annals of Statistics*, 396-421. DOI:10.1214/aos/1176324527

Nordman, D. J., Sibbertsen, P., & Lahiri, S. N. (2007). Empirical likelihood confidence intervals for the mean of a long-range dependent process. *Journal of Time Series Analysis*, 28(4), 576-599. DOI: 10.1111/j.1467-9892.2006.00526.x

Owen, A. B. (1988). Empirical likelihood ratio confidence intervals for a single functional. *Biometrika*, 75(2), 237-249. DOI: 10.1093/biomet/75.2.237

Owen, A. (1990). Empirical likelihood ratio confidence regions. *The Annals of Statistics*, 90-120. DOI:10.1214/aos/1176347494

Owen, A. B. (2001). Empirical likelihood. *CRC press.*

Peng, H. (2015). On a class of maximum empirical likelihood estimators. Manuscript

Qin, J., & Lawless, J. (1994). Empirical likelihood and general estimating equations. *The Annals of Statistics*, 300-325. DOI:10.1214/aos/1176325370

Rubinstein, M. (1985). Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, 1978. *The Journal of Finance*, 40(2), 455-480. DOI: 10.1111/j.1540-6261.1985.tb04967.x

Scott, L. O. (1987). Option pricing when the variance changes randomly: Theory, estimation, and an application. *Journal of Financial and Quantitative Analysis*, 22(04), 419-438. DOI: 10.2307/2330793

Stutzer, M. (1996). A simple nonparametric approach to derivative security valuation. *The Journal of Finance*, 51(5), 1633-1652. DOI: 10.2307/2329532

Wiggins, J. B. (1987). Option values under stochastic volatility: Theory and empirical estimates. *Journal of Financial Economics*, 19(2), 351-372. DOI:10.1016/0304-405X(87)90009-2

Yau, C. Y. (2012). Empirical likelihood in long-memory time series models. *Journal of Time Series Analysis*, 33(2), 269-275. DOI: 10.1111/j.1467-9892.2011.00756.x