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Contact Acoustic Nonlinearity (CAN)-based Continuous Monitoring of Bolt Loosening: Hybrid Use of High-order Harmonics and Spectral Sidebands

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Abstract

The significance of evaluating bolt tightness in engineering structures, preferably in a continuous manner, cannot be overemphasized. With hybrid use of high-order harmonics (*HOH*) and spectral sidebands, a contact acoustic nonlinearity (*CAN*)-based monitoring framework is developed for detecting bolt loosening and subsequently evaluating the residual torque on a loose bolt. Low-frequency pumping vibration is introduced into the bolted joint to produce a "breathing" effect at the joining interface that modulates the propagation characteristics of a high-frequency probing wave when it traverses the bolt, leading to the generation of *HOH* and vibro-acoustic nonlinear distortions (manifested as sidebands in the signal spectrum). To gain insight into the mechanism of *CAN* generation and to correlate the acquired nonlinear responses of a loose joint with the residual torque remaining on the bolt, an analytical model based on microcontact theory is established. Two types of nonlinear index, respectively exploiting the induced *HOH* and spectral sidebands, are defined without dependence on excitation intensity and are experimentally demonstrated to be effective in continuously monitoring bolt loosening in both aluminum-aluminum and composite-composite bolted joints. Taking a step further, variation of the index pair is quantitatively associated with the residual torque on a loose bolt. The approach developed provides a reliable method of continuous evaluation of bolt tightness in both composite and metallic joints, regardless of their working conditions, from early awareness of bolt loosening at an embryonic stage to quantitative estimation of residual torque.

Keywords**:** bolt loosening; nonlinear distortion; high-order harmonic; vibro-acoustic modulation; structural health monitoring; bolted joint

Nomenclature

AL-AL	Aluminum-aluminum
AS	Average of left and right sidebands
CAN	Contact acoustic nonlinearity
C-C	Composite-composite
HOH	High-order harmonics
HF	High-frequency probing wave
LF	Low-frequency pumping vibration
LS	Left sideband
RS	Right sideband
SEM	Scanning electron microscope
SOH	Second-order harmonic
TOF	Time of flight
TOH	Third-order harmonic
VAM	Vibro-acoustic modulation

1. Introduction

With safety a paramount priority, reliability, integrity and durability criteria must be strictly met for bolted joints, and this entails early awareness of bolt loosening and continuous monitoring of bolt tightness. Primary but pervasive, detection methods involving visual inspection, tapping during off-line operation, and a high degree of human interaction, performed regularly by sophisticated personnel after halting the normal service of an inspected joint, still prevail to serve the purpose of such monitoring. To circumvent the deficiencies encountered, continuing efforts have been made to enhance existing approaches. Examples of some well-demonstrated enhanced monitoring methods [1-3] include linear acoustic methods using a torque wrench [4], strain gages[5], embedded sensors [6, 7] and piezoelectric wafers [8, 9]. In particular, various linear signal features, such as delay in time-of-flight (*TOF*) [10, 11], mode conversion [12-14] and wave attenuation [15], are used for estimating the tightening state of a bolt. However, these linear signal features may not be remarkable when a bolt is slightly loose. The difficulty in extracting unremarkable signal changes from acquired signals has created a bottleneck for detection methodologies using linear acoustic features [16].

In contrast to the use of linear signal features, the exploitation of nonlinear acoustic properties [17, 18] has attracted increased preference, represented by methods using *HOH* (particularly the second-order harmonic (*SOH*) and the third-order harmonic (*TOH*)) or vibro-acoustic modulation (*VAM*)). With proven effectiveness in detecting small-scale damage such as fatigue cracks [19, 20], these approaches are based on the premise that when acoustic waves traverse the contact interface of a fatigue crack, the "breathing" motion pattern of the crack – manifested as the two contact surfaces moving to close the gap during wave compression and to open the gap during wave tension – induces asymmetry in the contact restoration forces. Consequently, those forces cause parametric modulation of stiffness and introduce localized nonlinearity to the propagating waves guided by the medium, known as *contact acoustic nonlinearity* (*CAN*) [21]. With this principle, in an *HOH*-based approach, the magnitude of the "breathing" crack-induced (*CAN*-induced) *SOH* and *TOH* can be correlated with the occurrence and further with the severity of the crack. Currently, *SOH*-based method, which requires only one excitation and is capable of locating damage[21, 22], has been widely used to quantitively evaluate micro cracks in both metallic and composite structures while few articles regarding *TOH*-based method have been reported; in a *VAM*-based method, two distinct excitations, namely a low-frequency pumping vibration and a high-frequency probing wave, are simultaneously introduced into the inspected structure. With the presence of a "breathing" crack that produces *CAN*, additional sidebands around the probing signal components are expected to be present in the signal spectrum – called *left sideband* (if lower than the frequency of the probing signal, denoted by *LS*) or *right sideband* (if higher than the frequency of the probing signal, denoted by *RS*). The magnitude of the sidebands can be linked to the crack severity with much less influence of nonlinearity contributed by measurement apparatus compared to *SOH* and *HOH* [23, 24]. In a bolted joint, the joining interface presents nonlinear responses when a bolt is loose, that is somewhat similar to the *CAN* engendered by a “breathing” crack. Along the same line of thinking, it is feasible to detect and evaluate, either qualitatively or quantitatively, the tightness condition of a bolt, in virtue of the variation in *CAN* produced at the jointing interface [23-28] – a task difficult to accomplish using conventional techniques based on linear acoustic features. Representatively, Yan et al. [28] presented a nonlinear ultrasonic approach using the *SOH* to evaluate kissing bonds in an adhesive aluminum joint, in conjunction with the use of a natural filtering method to minimize non-damage-related nonlinearities (*e.g.,* instrument-induced nonlinearity) and improve evaluation accuracy. Zhang et al. [26] compared a *VAM*-based nonlinear method with a wave energy dissipation-based linear approach to detect bolt loosening in three types of bolted aluminum joints, on which basis the conclusion was drawn that ‘detectability’ in the nonlinear approach was not restricted by the joint type, contrasted with the high dependence of the linear approach on the joint type. Taking a further step, Zhang et al. [23] extended the approach to composite structures, demonstrating that the effectiveness of nonlinear acoustic methods was independent of material properties. Amerini and Meo [29] configured two individual experimental setups to evaluate the health condition of a metallic bolted structure, using *SOH*-based and *VAM*-based approaches, both producing good results. It would therefore be cost-effective to integrate *HOH*-based and *VAM*-based detection philosophies through a single measurement. Furthermore, a bolt-loosening indicator for the *VAM*-based method adopted in the above works [23, 26, 29]was defined, ignoring the effect of pumping vibration. It is noteworthy that, in most reported studies, these two nonlinear methods have been dedicated to detecting loose bolts in a metallic structure. Rather fewer studies have been devoted to identifying bolt loosening in a bolted composite structure using nonlinear acoustic methods [23].

The present paper is dedicated to the development of a monitoring framework for bolted joints, based on the *CAN* engendered at the interface of a loose bolt, from early awareness of bolt loosening to continuous and quantitative evaluation of residual torque on a loose bolt. This framework rests on the hybrid use of *HOH* and *VAM*. An analytical model based on micro-contact theory regarding rough surface contact in joints is first established to facilitate understanding of the mechanism of *CAN* generation by a loose bolt and further to link the nonlinear responses of a loose bolt to its residual torque. *HOH*- and *VAM*-based detection philosophies are integrated and mutually supplemented in the framework via a single measurement. Two types of nonlinear indices independent of excitation parameters (*e.g.,* frequency or magnitude), respectively based on *HOH* and *VAM*, are defined, for qualitative detection of loosening of a bolt and quantitative estimate of its residual torque. For experimental validation, two representative types of bolted joints, an aluminum-aluminum (*AL-AL*) and a composite-composite (*C-C*) joint, are prepared, and the proposed monitoring framework is verified by continuously evaluating bolt loosening in these two joints, from fully tightened to fully loose. Metallic and composite materials are used comparatively in order to examine the dependence of the framework on joint material properties. Based on the detection results, the accuracy of *HOH*- and *VAM*-based methods is compared quantitatively.

2. Theoretical Modeling of *CAN* in Joint with Loose Bolt

The preload F and pressure p produced by a bolt at the contact interface between two joining components in a bolted structure, as a result of the torque T applied by the bolt, depend upon the bolt diameter d and the friction coefficient τ between nut and bolt, which can be described by [30]

$$F = T / \tau d. \qquad \textbf{(1)}$$

Variation in T leads to consequent changes in F and p, considering $p \propto F$. In reality, the surfaces of two adjoining components are rough, with irregular contact [31] as illustrated schematically in **Figures 1** and **2** that present a typical bolted composite joint comprised of two adjoining components with partial contact on the interface. The degree of the rough contact (see the composite surface profile measured by scanning electron microscope (*SEM*), **inset in Figure 2**) can be calibrated in terms of the average distance between the two contact surfaces, denoted by X in the **inset in Figure 1**.

When a pumping vibration is introduced into the joint, the vibration drives the rough contact surfaces to open and close periodically via the tensional and compressive forces applied by the vibration, with a consequent change in X. As a result, a periodic perturbation is imposed on the contact pressure p. Together, this leads to a collective contact stiffness at the interface, including linear (K_1) and nonlinear (K_2) components, depicted in [32] by considering the roughness of contact interface in the micro-scale.

$$K_1 = Cp^m \propto T^m, \tag{2a}$$

$$K_2 = 0.5mC^2 p^{2m-1} \propto T^{2m-1}, \tag{2b}$$

where C and m are associated with the surface properties of the joint material. For most engineering cases, $m < 0.5$ and $C > 0$ [32]. It is apparent that K_1 increases whereas K_2 decreases as the contact pressure p increases. Eqs. (2a) and (2b) reveal the direct dependence of K_1 and K_2 on the torque T applied on a bolt.

To put the above into perspective, consider a single-lap bolted joint subject to pumping vibration with an equivalent force $F_1 \cos(\omega_1 t)$. The joint can be simplified using a single-*DOF* system, as shown in **Figure 2**. The equation of motion of the bolted joint can be expressed as

$$M\ddot{x} + K_1 x - \varepsilon K_2 x^2 = F_1 \cos(\omega_1 t)\,, \tag{3}$$

where ω_1 signifies the excitation frequency of the pumping vibration. M denotes the mass and t is the time. The term with K_2 represents a second-order perturbation in which ε is a small quantity to scale the perturbation to be minute. According to perturbation theory, to obtain the nonlinear responses of the second-order and third-order harmonics, the solution to Eq. (3) takes the form

$$x = x_1 + \varepsilon x_2 + \varepsilon^2 x_3\,, \tag{4}$$

where x_1 represents the linear response of the bolted joint, x_2 the second-order harmonic response, and x_3 contains the third-order harmonic response of the joint. Substituting Eq. (4) into Eq. (3) and forcing the coefficients of ε-related terms to be identical on the left and right sides of the equation, we have

$$M\ddot{x}_1 + K_1 x_1 = F_1 \cos(\omega_1 t)\,, \tag{5a}$$

$$M\ddot{x}_2 + K_1 x_2 = K_2 x_1^2\,, \tag{5b}$$

$$M\ddot{x}_3 + K_1 x_3 = 2K_2 x_1 x_2\,. \tag{5c}$$

Further, neglecting the transient component, the linear and nonlinear responses can be obtained as

$$x_1 = \frac{F_1}{K_1 - M\omega_1^{\,2}} \cos\omega_1 t = A_1 \cos(\omega_1 t)\,, \tag{6a}$$

$$x_2 = \frac{0.5K_2}{K_1 - 4M\omega_1^{\,2}} A_1^2 \cos(2\omega_1 t)\,, \tag{6b}$$

$$x_3 = \frac{K_2^2}{2(K_1 - 9M\omega_1^{\,2})(K_1 - 4M\omega_1^{\,2})} A_1^3 \cos(3\omega_1 t) + \frac{K_2^2}{2(K_1 - 4M\omega_1^{\,2})(K_1 - M\omega_1^{\,2})} A_1^3 \cos(\omega_1 t)\,. \tag{6c}$$

In Eqs. (6a) and (6c), the terms involving ω_1 concern the fundamental mode linear response of the joint and the terms $2\omega_1$ and $3\omega_1$ in Eqs. (6b) and (6c) regulate the characteristics of second-order harmonic and third-order harmonic in the signal spectrum, respectively. Considering that the nonlinear response (*i.e.,* third-order harmonic) is much weaker than the linear one, the magnitude of the linear response (A_{LF}, at the frequency of ω_1), second-order harmonic (A_{SOH}, at the frequency of $2\omega_1$), and third-order harmonic (A_{TOH}, at the frequency of $3\omega_1$) can be written as

$$A_{LF} \approx \frac{F_1}{K_1 - M\omega_1^2}, \tag{7a}$$

$$A_{SOH} = \frac{0.5K_2}{K_1 - 4M\omega_1^2} A_{LF}^2, \tag{7b}$$

$$A_{TOH} = \frac{0.5K_2^2}{(K_1 - 9M\omega_1^2)(K_1 - 4M\omega_1^2)} A_{LF}^3. \tag{7c}$$

In Eqs. (7b) and (7c), it is apparent that the magnitudes of second-order harmonic and third-order harmonic in the signal spectrum are decided by the nonlinear contact stiffness K_2, which is dependent on (i) the contact properties at the rough contact interface of the joint under the pumping vibration, and (ii) the residual torque T on the bolt, according to Eq. 2(b). Consequently, based on Eqs. (7b) and (7c), two nonlinear indices, β_{SOH}^{Theory} and β_{TOH}^{Theory}, are defined, embracing the magnitude of second-order harmonic (*i.e.,* A_{SOH}) and third-order harmonic (*i.e.,* A_{TOH}) of the pumping vibration (to be ascertained in the signal spectrum), as

$$\beta_{SOH}^{Theory} = \frac{A_{SOH}}{A_{LF}^2} = \frac{K_2}{2} \frac{1}{K_1 - M(2\omega_1)^2}, \tag{8a}$$

$$\beta_{TOH}^{Theory} = \frac{A_{TOH}}{A_{LF}^3} = \frac{K_2}{2} \frac{K_2}{(K_1 - M(2\omega_1)^2)} \frac{1}{(K_1 - M(3\omega_1)^2)}, \tag{8b}$$

where "*Theory*" in the superscript reflects that this index is derived from theoretical modeling by using linear (A_{LF}) and nonlinear (A_{SOH} or A_{TOH}) responses, distinguished from the indices obtained by using linear (LF) and nonlinear (SOH or TOH) acceleration responses in the unit of dB via actual experimental measurement in this study. Provided that only 1st order classical nonlinearity[33] considered, the response of *TOH* can be much smaller than that of *SOH*. While for materials in which the hysteretic nonlinearity and the 2nd order classical nonlinearity (denoted by K_3) dominate [33], *TOH*-based method would be preferable because the materials may generate only *TOH* [33, 34]. Allowing for this, Eq. (3) is re-written as $M\ddot{x}+K_1x+\varepsilon K_3x^3=F_1\cos(\omega_1 t)$. Using the perturbation method, one can obtain the solutions with only the linear and third-order harmonic responses. Given that nonlinearity source in a joint may be complex and uncertain [35], it is wise to develop both *SOH*-based and *TOH*-based nonlinear indices for bolt loosening evaluation.

Provided the joint is subject to a mixed excitation (*i.e.,* the pumping vibration at the frequency of ω_1 and probing wave at the frequency of ω_2), the other nonlinear indices, β_{LS}^{Theory} , β_{RS}^{Theory} and β_{VAM}^{Theory} , can be established in accordance with the authors' previous work [26], embracing magnitudes of the left sideband (*i.e.,* A_{LS} , at the frequency of $\omega_2-\omega_1$) and right sideband (*i.e.,* A_{RS} , at the frequency of $\omega_1+\omega_2$), as well as the magnitudes of the low-frequency pumping vibration (*i.e.,* A_{LF} , at the frequency of ω_1) and high-frequency probing wave (*i.e.,* A_{HF} , at the frequency of ω_2) in the signal spectrum, as

$$\beta_{LS}^{Theory}=\frac{A_{LS}}{A_{LF}A_{HF}}=\frac{K_2}{2}\frac{1}{K_1-M(\omega_2-\omega_1)^2}, \qquad \textbf{(9a)}$$

$$\beta_{RS}^{Theory}=\frac{A_{RS}}{A_{LF}A_{HF}}=\frac{K_2}{2}\frac{1}{K_1-M(\omega_1+\omega_2)^2}, \qquad \textbf{(9b)}$$

$$\beta_{VAM}^{Theory}=\frac{1}{2}(\beta_{LS}^{Theory}+\beta_{RS}^{Theory})=\frac{K_2}{2}\left(\frac{1}{K_1-M(\omega_2-\omega_1)^2}+\frac{1}{K_1-M(\omega_1+\omega_2)^2}\right). \qquad \textbf{(9c)}$$

Forming an index pair, these three nonlinear indices (β_{SOH}^{Theory} , β_{TOH}^{Theory} and β_{VAM}^{Theory}), respectively defined in terms of high-order harmonic-based (second-order harmonic-based and third-order harmonic-based) and *VAM*-based nonlinearity of the responses of a bolted joint, quantitatively reflect the changes in structural dynamic characteristics (*i.e.,* K_1 and K_2) induced by a loose bolt in the joint. Both indices exhibit the same reliance on the nonlinear contact stiffness (K_2) associated with the *CAN* produced by a loose bolt (*i.e.,* $\beta_{SOH}^{Theory} \propto K_2 \propto T^{2m-1}$, $\beta_{TOH}^{Theory} \propto K_2^2 \propto T^{4m-2}$ and $\beta_{VAM}^{Theory} \propto K_2 \propto T^{2m-1}$). Theoretically, both types of index decrease with an increase in the residual torque T on the bolt (*i.e.,* the lower the indices, the tighter is the bolt, and *vice versa*), providing a quantitative indication of the tightness of a bolt. Furthermore, for bolted structures, the three indices are determined by the tightness of the bolts only, even when the structures are under varying intensities of excitation, as validated in the Discussion section. This independence of the excitation parameters (*e.g.*, frequency or magnitude) endows the defined nonlinear indices with additional flexibility to accommodate various working conditions. Therefore, the proposed detection framework using nonlinear indices has potential to be adopted for monitoring bolted structures in real practice with good implementability.

Considering that, in the experiment that follows, the linear and nonlinear responses of the loose joint are presented in dB units, the nonlinear indices defined theoretically in Eqs. (8a), (8b), and (9a)- (9c) are here redefined as

$$\begin{aligned}\beta_{SOH}^{Experiment} &= 20\log A_{SOH} - 2*20logA_{LF} \\ &= SOH\text{ - }2*LF,\end{aligned} \quad \textbf{(10a)}$$

$$\begin{aligned}\beta_{TOH}^{Experiment} &= 20\log A_{TOH} - 3*20logA_{LF} \\ &= TOH\text{ - }3*LF,\end{aligned} \quad \textbf{(10b)}$$

and

$$\begin{aligned}\beta_{LS}^{Experiment} &= 20logA_{LS} - (20logA_{HF} + 20logA_{LF}) \\ &= LS - (HF + LF),\end{aligned} \quad \textbf{(11a)}$$

$$\begin{aligned}\beta_{RS}^{Experiment} &= 20logA_{RS} - (20logA_{HF} + 20logA_{LF}) \\ &= RS - (HF + LF),\end{aligned} \quad \textbf{(11b)}$$

$$\beta_{VAM}^{Experiment} = \frac{1}{2}(\beta_{LS}^{Experiment} + \beta_{RS}^{Experiment}) = (LS + RS) / 2 - HF - LF. \quad \textbf{(11c)}$$

where $SOH = 20\log A_{SOH}$, and so on. Here, "*Experiment*" in the superscript implies that this index is obtained via actual experiment measurement using the linear (*LF* and *HF*) and nonlinear (*SOH*, *TOH*, *LS* and *RS*) accelerometer responses in the unit of dB. For the purpose of comparison, another index, β_{VAM}, reported elsewhere [26, 29], neglecting the effect of pumping vibration, is recalled, defined as

$$\beta_{VAM} = \left(LS + RS\right) / 2 - HF . \quad \textbf{(11d)}$$

3. Proof-of-Concept Validation

Integrating the two types of index, a *CAN*-based (*i.e.,* the *HOH*-based and *VAM*-based) monitoring framework is developed for detecting bolt loosening and continuously and quantitatively evaluating the residual torque remaining on a loose bolt, to be implemented via a single measurement in experiment. For proof-of-concept validation and also for examining the dependence of the proposed framework on joint material properties (metallic or composite structures), two sets of single-lap joints, made of metallic and composite materials, respectively, were prepared, and the two types of nonlinear index were comparatively acquired via experiment.

3.1. Experimental Setups

In each joint set, two plate-like interconnecting components, respectively made of aluminum

(AL7071) and carbon-fiber-reinforced epoxy composite (T700/7901), were assembled with an M6 bolt and then clamped as a cantilever system. To indicate material properties, the aluminum and composite joints are here denoted *AL-AL* and *C-C* joints, respectively. A schematic illustration of the experimental setup is displayed in **Figure 1** and photographed in **Figure 3**. For the *AL-AL* joint, two aluminum components of the same geometric dimension of 245×30×2.8 mm^3 were assembled with a lap length of 20 mm (the bolt was positioned in the middle of the lap area) and a clamped length of 10 mm (see **Figure 3**). For the *C-C* joint, two composite components, both measuring 245×30×2.0 mm^3, were assembled with the same lap and clamped lengths as those used in the *AL-AL* joint. The composite specimen was a zero-degree unidirectional laminate fabricated in accordance with a standard hot-press process. For both joints, a shaker (B&K®, Model type: 4809) was used to introduce a point-force-like pumping vibration, 39 mm from the free end; a piezo stack actuator (PI®, P-885.11) was surface-mounted 39 mm from the bolt to generate a probing wave. The excitation conditions of the *AL-AL* joint are the same as those adopted in the authors' previous work that is relevant to this study [26].Two sinusoidal signals of a lower frequency (992 Hz for the *AL-AL* joint and 758 Hz for the *C-C* joint; selection criteria to be detailed later) and a higher frequency (14.24 kHz for the *AL-AL* joint and 14.99 kHz for the *C-C* joint) were generated with a waveform generator (HIOKI®, Model Type: 7075), to supply the shaker and the stack actuator, respectively. The low-frequency pumping vibration was magnified by a linear power amplifier (B&K®, Model Type: 2706) before being applied to the joint. The input voltages of pumping vibration and probing wave were both 10 V for the *AL-AL* joint and 8 V and 16 V respectively for the *C-C* joint. The response signals of the joints under the mixed excitation were captured by an accelerometer (B&K®, Model Type: 4393) 36 mm from the bolt and registered by an oscilloscope (Agilent® DSO9064A) at a sampling frequency of 200 kHz.

It is noteworthy that with the above setup, the *SOH*-based, *TOH*-based, and *VAM*-based nonlinear indices, defined by Eqs. (10) and (11), respectively, were ascertained simultaneously via a single measurement. In addition, in order to strengthen the effect of the loose-bolt-induced *CAN* at the joining interface under the mixed excitation, the frequencies of the pumping vibration and probing wave were selected prudently. The excitation frequency of the pumping vibration was selected from low-order natural frequencies. For the probing wave, the strongest response frequency was ascertained in the signal spectrum when the joint was subjected to white noise and subsequently selected as the excitation frequency of the probing wave. Details of the selection principle can be found in [26].

For both joints, a torque of 13 N·m guaranteed full tightness of the bolt. For the state of full tightness, a series of 13 scenarios was considered, with the residual torque remaining on the bolt varying from 1 (fully loosened) to 13 N·m (fully tightened) at an increment of 1 N·m. The bolt was loosened and retightened five times. Under each scenario, signal acquisition was repeated and averaged to minimize operational error and measurement uncertainty.

3.2. Results

As representative results, the spectra obtained for three typical scenarios: (i) fully tightened (with a torque of 13 N·m applied on the bolt), (ii) intermediately fastened (5 N·m), and (iii) fully loosened (1 N·m), are compared in **Figures 4** and **5**, respectively, for the *AL-AL* and *C-C* joints. All measurements in the experiment are in dB. It can be clearly observed from the spectra that the nonlinear components, sidebands (denoted by *LS* and *RS* in the figure) in particular, exhibit a strong correlation with the residual torque on the bolt when the joint is subject to the mixed excitation. To be more specific, the magnitudes of these nonlinear components increase with a decrease in the residual torque – the looser a bolt, the greater the expected magnitudes of *SOH*, *TOH*, *LS*, and *RS*. In most scenarios, for example in **Figures 5(a)-(c)**, magnitudes of *TOH* are much higher than those of *SOH*. In addition, strong inter-harmonic responses to the pumping vibration are also observed in **Figure 4**, generated due to the breathing effects of the contact surfaces [24].

This observation revealed that the tightness status of a bolt could be quantitatively calibrated in terms of the intensity of the acquired nonlinear components in spectra. To achieve such quantitative calibration, the linear and nonlinear responses of the two sets of joints under different degrees of bolt tightness were obtained, compared in **Figure 6**. For convenience of discussion, the average of *LS* and *RS*, denoted by *AS* (in the unit of dB), is used, which is defined as

$$AS = (LS + RS)/2. \tag{12}$$

For the *AL-AL* joint, as can be observed in **Figure 6(a)**, (i) the linear response of the joint when subject to the low-frequency pumping vibration (denoted by *LF*) remains largely unchanged, regardless of variation in the torque remaining on the bolt; (ii) the linear response of the joint when subject to the high-frequency probing wave (denoted by *HF*) remains the same in the early stage of bolt loosening (when the residual torque is greater than 6.5 N·m – 50% of the torque in the fully tightened scenario (*i.e.,* 13 N·m, hereafter called *full torque*)) and tends to increase slightly when the loosening progresses (when the residual torque reduces to less than 6.5 N·m); (iii) the nonlinear responses (*i.e., SOH*, *TOH*, and *AS*) show similar trends and comparable sensitivity to the change in the torque on the bolt when the residual torque is greater than 5 N·m (*i.e.,* approximately 40% of the full torque). As the bolt loosening progresses (with the residual torque less than 40% of the full torque), *AS* persists its monotonic increase whereas *SOH* and *TOH* fluctuate. Similar phenomena can be observed in **Figure 6(b)** for the *C-C* joint. To conclude, the linear responses (*i.e.*, *LF* and

HF) of the two joint types of joint are insensitive to variation in the residual torque on a loose bolt. In contrast to the linear responses, the nonlinear responses of the joint (*i.e., SOH, TOH*, and *AS*) manifest much higher sensitivity to bolt loosening than their linear counterparts.

In a further step towards quantitatively linking the nonlinear responses of the joint to the residual torque, the four nonlinear indices ($\beta_{SOH}^{Experiment}, \beta_{TOH}^{Experiment}, \beta_{VAM}$ and $\beta_{VAM}^{Experiment}$) in dB, defined by Eqs. (10a) and (10b) for *HOH*-based and Eqs. (11c) and (11d) for *VAM*-based, were calculated using the linear and nonlinear responses in the unit of dB acquired from the experiment. For the *AL-AL* joint, $\beta_{VAM}^{Experiment}$, subject to the residual torque, obtained from the five tests are presented in **Figure 7(a)**. Slight discrepancies are observed among the five tests. A similar phenomenon is found in **Figure 7(c)** for the *C-C* joint. Therefore, the average of the nonlinear indices from the five tests is used for evaluating bolt loosening in what follows. For the *AL-AL* joint, **Figure 7(b)**, $\beta_{SOH}^{Experiment}, \beta_{TOH}^{Experiment}$, and $\beta_{VAM}^{Experiment}$ show similar sensitivity to the bolt loosening at an early stage of loosening (when the residual torque is greater than 40% of the full torque), a finding that is consistent with the qualitative theoretical prediction obtained using Eqs. (8) -(11). A similar phenomenon is observed for the *C-C* bolted joint, **Figure 7(d)**. Observations from **Figure 7** confirm that the occurrence of *SOH*, *TOH*, and spectral sidebands is due to the *CAN* effect arising from a loose bolt. Moreover, the *VAM*-based indices ($\beta_{VAM}^{Experiment}$ and β_{VAM}) show comparable sensitivity but enhanced stability when compared with the *HOH*-based indices ($\beta_{SOH}^{Experiment}$ and $\beta_{TOH}^{Experiment}$).

4. Discussion

The effect of the input voltages of *LF* and *HF* excitation on the accuracy of the proposed nonlinear indices is examined, with the experimental setup and configurations the same as those described in Section 3.

For the *AL-AL* joint under a torque of 5 N·m, the input voltage of pumping vibration varied from 4 to 13 V with a step of 1 V, while the voltage of the probing wave remained at 10 V. The linear (*i.e., LF* and *HF*) and nonlinear (*i.e., SOH*, *TOH*, *LS*, and *RS*) responses of the bolt accordingly obtained are displayed in **Figure 8(a)**, illustrating that (i) *LF* exhibits a linear dependence on the input voltage of pumping vibration; and (ii) the nonlinear responses (*i.e., SOH*, *TOH*, *LS*, and *RS*) also increase linearly with an increase in the input voltage of pumping vibration. When the voltage of the pumping vibration was kept at 10 V and the voltage of the probing wave increased from 7 to 16 V, both the linear and nonlinear responses showed similar dependence on the input voltage of the probing wave (see **Figure 8(b)**). Similar phenomena are also observed for the *C-C* bolted joint, in **Figure 8(c)** (with a constant voltage of 16 V for the probing wave) and **Figure 8(d)** (with a constant voltage of 8 V for the pumping vibration).

It can be argued, therefore, that for both the *AL-AL* and the *C-C* bolted joints, the nonlinear responses (including *SOH*, *TOH*, and sidebands) manifested similar reliance on the input voltages of the pumping vibration and probing wave. For instance, *SOH* increased with an augment in the voltage of pumping vibration, because a higher voltage intensified the "breathing" effect of a loose bolt. Considering the magnitude of pumping vibration is linearly dependent on the input voltage, this observation agrees with the theoretical prediction regarding qualitative dependence of harmonic responses on the magnitude of pumping vibration by Eq. (7b) ($A_{SOH} \propto A_{LF}^2$, if presented in the unit of dB: $SOH \propto LF$). On the other hand, sidebands increased with an increase in the magnitudes of the pumping vibration and the probing wave, also consistent with the qualitative theoretical prediction proposed in [26] ($A_{LS} / A_{RS} \propto A_{LF} \cdot A_{HF}$, if presented in the unit of dB: $LS / RS \propto LF + HF$). The obtained reliance between nonlinear responses and magnitude of the excitation is also consistent with experimental results of other researchers obtained from cracks [33].

To further examine the rationality of the developed nonlinear indices, the dependence of $\beta_{SOH}^{Experiment}$, $\beta_{TOH}^{Experiment}$ and $\beta_{VAM}^{Experiment}$, along with β_{VAM} (for comparison), on the intensity of *LF* and *HF* was obtained for each joint with a residual torque of 5 N·m remaining on the bolt, displayed in **Figure 9**, where the dots are experimental data and the curves are linearly fitting results. It can be found that $\beta_{SOH}^{Experiment}$ and $\beta_{VAM}^{Experiment}$ are independent of the magnitudes of *LF* and *HF*, whereas β_{VAM} phenomenally shows a linear correlation with the magnitude of *LF* (**Figures 9(a)** and **(c)**). In our previous paper [26], β_{VAM} was adopted for the evaluation of bolt loosening in the *AL-AL* joint due to magnitudes of *LF* remaining almost the same for the joint under different torques. Meanwhile, the nonlinear index (*i.e.*, $\beta_{VAM}^{Experiment}$) proposed in this study can extend the application of the *VAM* method through relaxing the rigorous requirement of excitation conditions. The dependence of $\beta_{TOH}^{Experiment}$ on the magnitude of *LF* can be divided into two stages. Before the response magnitude of *LF* reaches a certain value (approximately -22 dB for the *AL-AL* joint, **Figure 9(a)**, and -17.5 dB for the *C-C* joint, **Figure 9 (c)**), $\beta_{TOH}^{Experiment}$ decreases with an increase in *LF*, after which it remains almost unchanged regardless of an increase in *LF*. These observations further demonstrate that our experimental findings agree well with the qualitative theoretical prediction, and $\beta_{SOH}^{Experiment}$ $\beta_{VAM}^{Experiment}$ can be used to evaluate the severity of bolt loosening, even when the magnitudes of *LF* and *HF* vary during measurement. Meanwhile, the use of $\beta_{TOH}^{Experiment}$ should be based on the premise that the magnitude of *LF* has reached a certain intensity. However, the use of β_{VAM} should ideally be limited to cases in which the magnitude of *LF* remains the same during the entire measurement. These merits of the two indices ($\beta_{SOH}^{Experiment}$ and $\beta_{VAM}^{Experiment}$) endow the framework with enhanced flexibility and tolerance to varying measurement conditions during practical implementation.

5. Concluding Remarks

A contact acoustic nonlinearity (*CAN*)-based monitoring framework for detecting bolt loosening in a bolted joint, for both *C-C* and *AL-AL* bolted joints, was developed theoretically and experimentally, with hybrid use of high-order harmonics and spectral sidebands. Theoretical analysis facilitated understanding of the generation mechanism of second-order and third-order harmonics along with sidebands when a loose joint interacts with a mixed excitation (*i.e.,* a low-frequency pumping vibration and a high-frequency probing wave), and quantitatively linked the nonlinear responses of the loose joint to the residual torque on a loose bolt. Two types of nonlinear index, respectively exploiting high-order harmonics and spectral sidebands, were proposed and experimentally demonstrated effective continuous monitoring of bolt loosening in joints made of metallic or composite materials. With use of the *SOH*-based and *TOH*-based method, the nonlinear indices $\beta_{SOH}^{Experiment}$ and $\beta_{TOH}^{Experiment}$ showed efficiency in detecting bolt loosening at its early stage; with use of the improved *VAM*-based method, the nonlinear index $\beta_{VAM}^{Experiment}$ remains detectable throughout the entire progress of bolt loosening. With three nonlinear indices ($\beta_{SOH}^{Experiment}$, $\beta_{TOH}^{Experiment}$, and $\beta_{VAM}^{Experiment}$) sensitive to early bolt loosening, this developed framework provides a reliable and cost-effectively solution to integrating two *CAN*-based methods (*i.e., HOH*-based and *VAM*-based) in a single testing, from early awareness of bolt loosening at an embryonic stage to quantitative estimation of residual torque.

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Appendix

A perturbation method is used to solve Eq. (3). Substituting Eq. (4) into Eq. (3), one can obtain

$$\begin{aligned}&(M\ddot{x}_1+K_1x_1-F_1\cos\omega_1 t)+\varepsilon(M\ddot{x}_2+K_1x_2-K_2x_1^2)+\varepsilon^2(M\ddot{x}_3+K_1x_3-2K_2x_1x_2)\\&-\varepsilon^3K_2(x_2^2+2x_1x_3+2\varepsilon x_2x_3+\varepsilon^2x_3^2)=0.\end{aligned} \quad \textbf{(13)}$$

Forcing the coefficients of ε-related terms to be zero, Eqs. (5a)- (5c) can be derived. Eq. (5b) can now be re-written as follows by substituting Eq. (6a) for Eq. (5b):

$$M\ddot{x}_2+K_1x_2=0.5K_2A_1^2[\cos(2\omega_1)+1]. \quad \textbf{(14)}$$

Then the second-order harmonic response x_2, as expressed in Eq. (6b), can be obtained by solving the following equation and neglecting the transient component:

$$M\ddot{x}_2+K_1x_2=0.5K_2A_1^2\cos(2\omega_1 t). \quad \textbf{(15)}$$

Substituting Eqs. (6a) and (6b) into Eq. (5c), one can obtain Eq. (16) and solve x_3 as expressed in Eq. (6c).

$$M\ddot{x}_3+K_1x_3=\frac{K_2^2}{2(K_1-4M\omega_1^2)}A_1^3[\cos(3\omega_1 t)+\cos(\omega_1 t)]. \quad \textbf{(16)}$$

Note that $x_3 << x_1$ and then Eqs. (7a)- (7c) can be derived.

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