

## Review

# Advances in structural vibration control application of magneto-rheological visco-elastomer

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## HIGHLIGHTS

- New advances in structural vibration control using magnetorheological viscoelastomer.
- Dynamic optimization of periodic MR viscoelastomer sandwich structures.
- Optimal vibration control of parameter-excited MR viscoelastomer sandwich structures.

## ARTICLE INFO

## Article history:

Received 8 October 2016

Received in revised form 3 December 2016

Accepted 27 December 2016

Available online 24 January 2017

\*This article belongs to the Solid Mechanics

## Keywords:

Magneto-rheological visco-elastomer

Controllable sandwich structures

Nonlinear stochastic vibration

Periodic parameter distribution

Optimal control

## ABSTRACT

Magneto-rheological visco-elastomer (MRVE) as a new smart material developed in recent years has several significant advantages over magneto-rheological liquid. The adjustability of structural dynamics to random environmental excitations is required in vibration control. MRVE can supply considerably adjustable damping and stiffness for structures, and the adjustment of dynamic properties is achieved only by applied magnetic fields with changeless structure design. Increasing researches on MRVE dynamic properties, modeling, and vibration control application are presented. Recent advances in MRVE dynamic properties and structural vibration control application including composite structural vibration mitigation under uniform magnetic fields, vibration response characteristics improvement through harmonic parameter distribution, and optimal bounded parametric control design based on the dynamical programming principle are reviewed. Relevant main methods and results introduced are beneficial to understanding and researches on MRVE application and development.

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## 1. Introduction

The vibration control of both engineering structures subjected to strong excitations and vibration-sensitive apparatuses subjected to micro disturbances is a significant research subject. The purpose of structural vibration control is to improve vibration

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response characteristics which can be performed by dynamic optimization and dynamical programming. Supplementing structural damping and stiffness is adopted generally for reducing vibration response. The adjustability of structural dynamics is required due to the randomness of environmental excitations. Smart materials such as magneto-rheological liquid can supply adjustable damping and stiffness for structures, and the adjustment of dynamic properties is achieved only by applied magnetic fields with changeless structure design. Extensive researches on structural vibration suppression using magneto-rheological liquid dampers and magneto-rheological liquid composite structures have been presented [1–8]. In recent years, magneto-rheological visco-elastomer (MRVE) has been developed, because it has several significant advantages over magneto-rheological liquid, for example, the improvement of magnetic particle settlement in magneto-rheological liquid and suitability for composite structure cores. Researches on MRVE dynamic properties, modeling and vibration control application have been presented [9–45].

MRVE is a smart composite material, which consists of magnetically polarizable particles and non-magnetic polymers, for example, iron particles, silicone oil, and rubber [10,24]. It combines the advantageous dynamic properties of magneto-rheological fluid and visco-elastic substrate materials, including stiffness and damping changed reversibly under external magnetic fields applied in milliseconds. Many researches have been presented on MRVE fabrication, improvement, and test for magnetic-mechanical properties and dynamic behaviors [11–20]. A static modeling for MRVE shear modulus has been given based on magnetic dipole interaction and polymeric nonlinear elasticity [14,21,22]. A complex modulus based on polymer dynamics in frequency domain has been used frequently for describing MRVE dynamic properties [19,23]. However, the complex modulus describes only MRVE linear dynamic properties in small deformation. A nonlinear hysteretic model for MRVE dynamics has been proposed further for certain large deformation [24]. MRVE based tunable vibration isolators, absorbers and dampers, and magneto-rheological fluid-elastomer composite dampers have been designed and tested for vibration control application [25–29].

In particular, MRVE composite structures with controllable dynamic properties have been studied, including periodic vibration, frequency response characteristics, dynamic stability, stochastic micro-vibration response, and dynamics under localized uniform magnetic fields [30–42]. The MRVE composite structures with area energy dissipation effectively suppress deterministic and stochastic vibration. However, the structural vibration control effectiveness is improvable and two new progresses have been made recently in the vibration control of MRVE composite structures. The non-uniform spatial distribution of structural dynamic responses has been considered, and MRVE sandwich structures with continuous harmonic distribution parameters under stochastic excitations have been studied [43]. The harmonic parameter distribution can improve greatly structural vibration response characteristics and reduce stochastic responses. On the other hand, MRVE properties such as stiffness and damping are adjustable by applied magnetic fields. The optimal adjustment of MRVE composite structures based on the dynamical programming principle can exert complete MRVE properties and achieve further vibration control effectiveness. Note that MRVE stiffness and damping are represented as composite structure parameters and bounded due to magnetic-mechanical saturation. The optimal bounded parametric control for MRVE composite structures under stochastic and shock excitations has been proposed and further remarkable vibration suppression effectiveness has been obtained [45]. This review paper focuses on the MRVE dynamic properties and modeling, composite structural vibration mitigation under uniform magnetic fields, vibration response characteristics improvement through

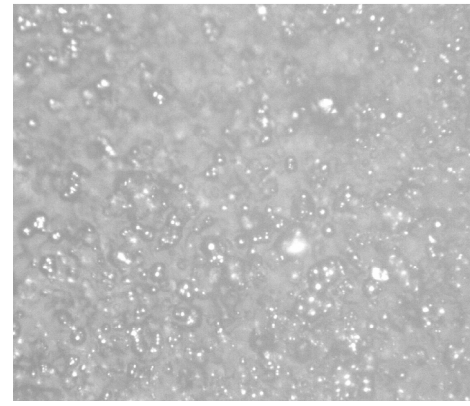


Fig. 1. MRVE section micrograph (500 optical magnification). Source: Reproduced from Ref. [24].

harmonic parameter distribution, and optimal bounded parametric control design based on the dynamical programming principle. Main methods and results relevant to MRVE dynamic modeling and vibration control application are introduced respectively in the following four sections.

## 2. Dynamic models of MRVE

MRVE specimens are fabricated generally by using silicone rubber, silicone oil, and carbonyl iron particles. Iron particles are used as magnetic fillers, silicone oil is used for performance regulator and silicone rubber is used as matrix material. According to certain ingredient percentage, iron particles are dispersed thoroughly in silicone oil, and the blended liquid is mixed with silicone rubber. The homogeneous mixture is poured into a mold and cured under certain temperature conditions. The micrograph of inner planar section of an MRVE specimen is shown in Fig. 1 [24]. In curing, magnetic fields can be applied to produce aligned MRVE specimens which are transversely isotropic and have better magnetic-mechanical properties. Tension, compression, and shear tests of MRVE specimens under applied magnetic fields were performed firstly for static magnetic-mechanical properties. The magnetic softening rigidity in tension, hardening rigidity in compression, and nonlinear stress-strain relationship for larger deformation of MRVE were observed [24]. A static modeling for MRVE shear modulus has been proposed based on magnetic dipole interaction and polymeric nonlinear elasticity [14,21,22]. However, dynamic characteristics have not been incorporated in the model. Dynamic behaviors of MRVE specimens under applied magnetic fields are tested by tension-compression and shear circulations with different frequencies. The MRVE dynamic stiffness increasing with magnetic field intensity was observed. A complex modulus based on polymer dynamics in frequency domain has been proposed for describing MRVE dynamic properties [19,23]. MRVE dynamic force generally consisting of elastic force and damping force can be separated equivalently. The complex modulus can be separated correspondingly into real part and imaginary part. For example, the complex shear modulus is expressed as

$$G(\omega, B_m) = G_R(\omega, B_m)[1 + j\Delta(\omega, B_m)], \quad (1)$$

where real part  $G_R$  is called storage modulus representing viscoelastic stiffness, imaginary part  $G_R\Delta$  is loss modulus,  $\Delta$  is called loss factor representing viscoelastic damping,  $\omega$  is vibration frequency,  $B_m$  is magnetic field intensity, and  $j = \sqrt{-1}$ . The real modulus and loss factor dependent on vibration frequency can be expressed approximately as

$$G_R(\omega, B_m) = \sum_{i=0}^{N_m} \alpha_i(B_m)\omega^i, \quad \Delta(\omega, B_m) = \sum_{i=0}^{N_l} \beta_i(B_m)\omega^i, \quad (2)$$

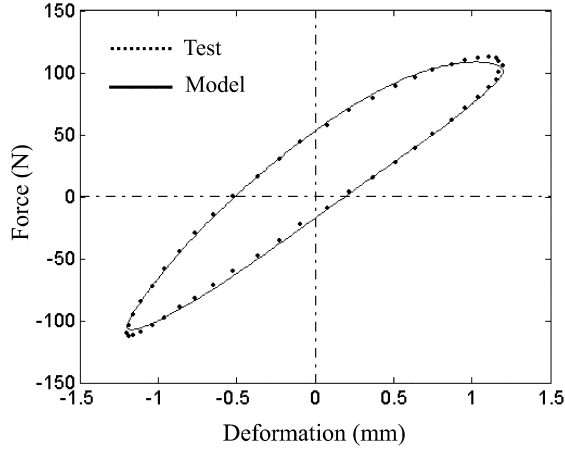


Fig. 2. Dynamic force versus deformation for 8 Hz and 0.14 T. Source: Reproduced from Ref. [24].

where coefficients  $\alpha_i$  and  $\beta_j$  depend only on magnetic field intensity,  $N_m$  and  $N_l$  are integers. The dependence of real modulus on magnetic field intensity and vibration frequency is more remarkable than that of loss factor.

For large deformation, the dynamic behavior including nonlinear elasticity (curve slope) and damping (loop area) of an MRVE specimen is shown in Fig. 2 [24]. The non-elliptical curve without rigidity indicates nonlinear damping and hysteresis characteristics as exhibited by magneto-rheological fluid under applied magnetic fields. A phenomenological model for MRVE dynamics in time domain has been constructed, which includes nonlinear elasticity for the backbone, viscous damping and nonlinear hysteresis for the loop and coupling effect. The nonlinear hysteretic model or dynamic force is given by

$$f(u) = f_e(u) + f_v(\dot{u}) + \lambda f_h(u),$$

$$f_e(u) = \sum_{i=0}^{N_e} a_i u^i, \quad f_v(\dot{u}) = c \dot{u},$$

$$\dot{f}_h = \alpha \dot{u} - \beta \dot{u} |f_h|^n - \gamma |\dot{f}_h| f_h |f_h|^{n-1}, \quad (3)$$

where  $u$  denotes dynamic deformation,  $N_e$  is integer,  $\lambda$ ,  $a_i$ ,  $c$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $n$  are coefficients dependent on magnetic field intensity. The MRVE model (3) in time domain can describe well dynamic behaviors under different magnetic fields, but it is complicated for vibration control application. However, for small nonlinearity, the MRVE model (3) can be converted equivalently into another model in frequency domain as expressed by the complex modulus (1) or Kelvin–Voigt model in time domain expressed by differential operators.

### 3. Structural vibration mitigation using MRVE

MRVE is firstly used to vibration control devices including tunable vibration absorber, shear-type damper, magneto-rheological fluid-elastomer composite dampers and isolators [25–29]. The control devices have been applied to helicopter stability augmentation and vehicle seat suspension vibration suppression [26,27]. However, more direct application is constructing MRVE composite structures with controllable dynamic properties. The adjustable stiffness of MRVE sandwich beams under periodic excitations has been studied and 40% change of vibration frequency was obtained [30,31]. The frequency response characteristics and vibration damping of thick MRVE sandwich beams and plates have been analyzed [32,33]. Also the harmonic acoustic wave transmission characteristics of MRVE sandwich plate have been presented [34]. The dynamic stability of MRVE sandwich beams under

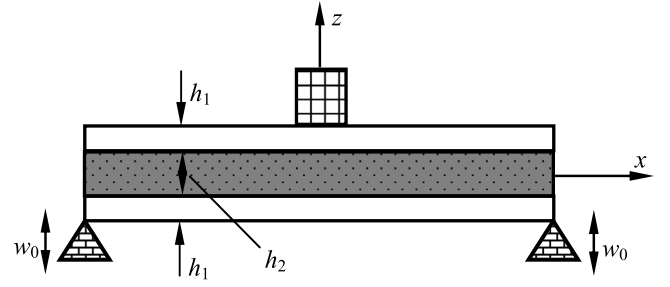


Fig. 3. MRVE sandwich beam with supported mass. Source: Reproduced from Ref. [38].

periodic axial excitations has been studied and improvable stability was observed by unstable regions [35,36].

Vibration-sensitive precise apparatuses require extremely stable operation environments. However, there exist inevitable environmental disturbances in random with wide frequency bands and long time periods. Thus stochastic micro-vibration control different from conventional strong vibration control is very important to vibration-sensitive apparatuses. The vibration-sensitive apparatus and its support structure can be modeled as a beam or plate with concentrated mass. The stochastic micro-vibration response reduction of MRVE sandwich beams and plates with supported mass under support motion excitations has been studied and great micro-vibration suppression using MRVE cores was obtained [37,38,40–42].

For example, the horizontal MRVE sandwich beam with supported mass is shown in Fig. 3 [38]. The non-dimensional partial differential equations for horizontal and vertical coupling motions of the MRVE sandwich beam are [38]

$$\frac{E_1 h_1^3}{6} \frac{\partial^4 \bar{w}}{\partial y^4} - G_2 h_a L^2 \frac{\partial^2 \bar{w}}{\partial y^2} - E_1 h_1^2 L \frac{\partial^3 \bar{u}}{\partial y^3} - 2G_2 L^3 \frac{\partial \bar{u}}{\partial y} + L^4 [\rho h_t + m \delta(y - y_0)] (\ddot{\bar{w}} + \ddot{w}_0) = 0,$$

$$E_1 h_1 h_2 \frac{\partial^2 \bar{u}}{\partial y^2} - 2G_2 L^2 \bar{u} - G_2 h_a L \frac{\partial \bar{w}}{\partial y} = 0, \quad (4)$$

where  $\bar{w}$  and  $\bar{u}$  are respectively dimensionless vertical and horizontal displacements,  $y = x/L$ ,  $L$  is beam length,  $h_a = h_1 + h_2$ ,  $\rho h_t = 2\rho_1 h_1 + \rho_2 h_2$ ,  $E_1$ ,  $\rho_1$ , and  $h_1$  are respectively elastic modulus, mass density, and thickness of facial layers,  $G_2$ ,  $\rho_2$ , and  $h_2$  are respectively shear modulus, mass density, and thickness of core layer,  $m \times bL$  and  $y_0$  are respectively supported mass and coordinate,  $b$  is beam width, and  $w_0$  is vertical support displacement excitation. By using vibration modes, Eq. (4) can be converted into ordinary differential equations and expressed in the matrix form as

$$\mathbf{M} \ddot{\mathbf{Q}} + \mathbf{K} \mathbf{Q} = \mathbf{F}(t), \quad (5)$$

where  $\mathbf{Q}$  is modal displacement vector,  $\mathbf{F}$  is modal excitation vector,  $\mathbf{M}$  is non-diagonal modal mass matrix, and  $\mathbf{K}$  is modal stiffness matrix dependent on vibration frequency and controllable by applied magnetic fields. The frequency response function and response spectral density matrices of system (5) are

$$\mathbf{H}(\omega) = (\mathbf{K} - \omega^2 \mathbf{M})^{-1}, \quad \mathbf{S}_Q(\omega) = \mathbf{H}(\omega) \mathbf{S}_F(\omega) \mathbf{H}^T(\omega), \quad (6)$$

where  $\mathbf{S}_F$  is the power spectral density matrix of excitation  $\mathbf{F}$ . The displacement response spectral density function of the MRVE sandwich beam is

$$S_{\bar{w}}(\omega, y) = \Phi^T(y) \mathbf{S}_Q(\omega) \Phi(y), \quad (7)$$

where  $\Phi$  is modal function vector. The frequency response characteristics and response statistics can be evaluated by using

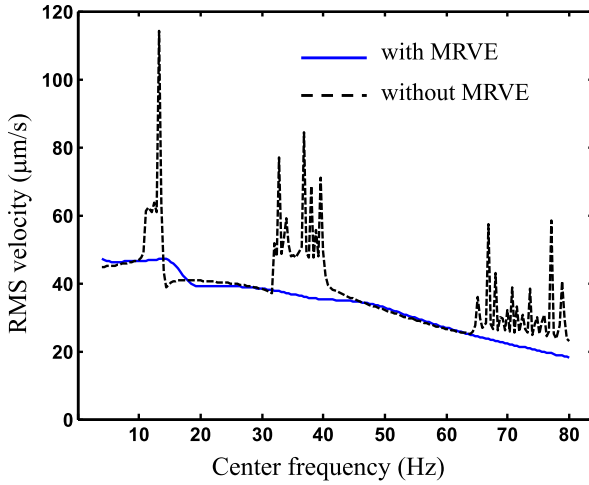


Fig. 4. Logarithmic root-mean-square velocity response spectra. Source: Reproduced from Ref. [38].

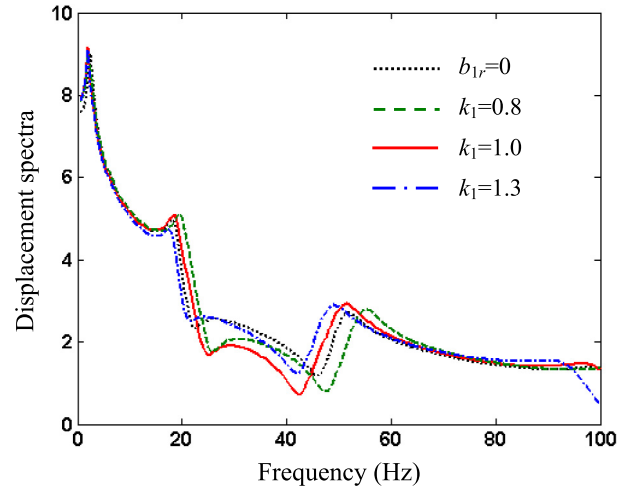


Fig. 5. Logarithmic non-dimensional displacement response spectra for different thickness wave numbers  $k_1$  ( $b_{1r} = 0.3c_{1m}$ ,  $b_{3r} = 0$ ). Source: Reproduced from Ref. [43].

Eqs. (6) and (7). In terms of the one-third octave frequency spectra for micro-vibration, logarithmic root-mean-square velocity response spectra of the MRVE sandwich beam and corresponding beam without MRVE under wide-band stochastic excitation are shown in Fig. 4 [38]. MRVE can greatly reduce the micro-vibration responses, especially for larger responses.

Note that a uniform magnetic field covering completely large structure is not so realistic. Analytical and experimental researches on MRVE sandwich beams and plates under localized uniform magnetic fields have been presented [39–42]. The localized magnetic fields with covering region larger than certain area have slight effect on the micro-vibration responses.

#### 4. Dynamic characteristics optimization of period-parametric MRVE sandwich structures

Researches on periodic structures with periodic distribution parameters showed special dynamic characteristics due to spatial and temporal periodicity coupling. The dynamic characteristics of periodic structures can be used for improving structural vibration response characteristics. In most researches, spaced supports, spaced stiffeners and alternate distribution of uniform physical and geometrical parameters were considered. The Bloch wave expansion method was used and dimensionless wave numbers were confined to  $2\pi$  in the analysis. Only one linear sub-structure was analyzed to determine dynamic characteristics under periodicity conditions so that structural vibration (or wave) has the period equal to parametric period. Recently, the vibration response characteristics of finite-size MRVE composite structures with the continuous harmonic spatial distribution of geometrical parameters and adjustable physical parameters have been studied [43]. For example, the finite harmonic MRVE sandwich beam with supported mass is as shown in Fig. 3. By the harmonic distribution of applied magnetic field intensity, the complex shear modulus varies harmonically with coordinate  $x$  and is expressed as

$$G_2(\omega, x) = e_{2m}(\omega) + b_{3r}(\omega) \cos \frac{2k_3 \pi x}{L}, \quad (8)$$

where  $e_{2m}$  is non-harmonic shear modulus component,  $b_{3r}$  is the amplitude of harmonic shear modulus component, and  $k_3$  is the wave number of harmonic shear modulus. Also the facial layer thickness is designed to be harmonic distribution as

$$h_1(x) = c_{1m} + b_{1r} \cos \frac{2k_1 \pi x}{L}, \quad (9)$$

where  $c_{1m}$  is non-harmonic thickness component,  $b_{1r}$  is the amplitude of harmonic thickness component, and  $k_1$  is the wave number of harmonic thickness. The non-dimensional partial differential equations for horizontal and vertical coupling motions of the harmonic MRVE sandwich beam are [43]

$$\frac{\partial}{\partial y} \left( \frac{E_1 h_1^3}{6} \frac{\partial^3 \bar{w}}{\partial y^3} - G_2 h_a L^2 \frac{\partial \bar{w}}{\partial y} - E_1 h_1^2 L \frac{\partial^2 \bar{u}}{\partial y^2} - 2G_2 L^3 \bar{u} \right) + L^4 [\rho h_t + m \delta(y - y_0)] (\ddot{\bar{w}} + \ddot{\bar{w}}_0) = 0,$$

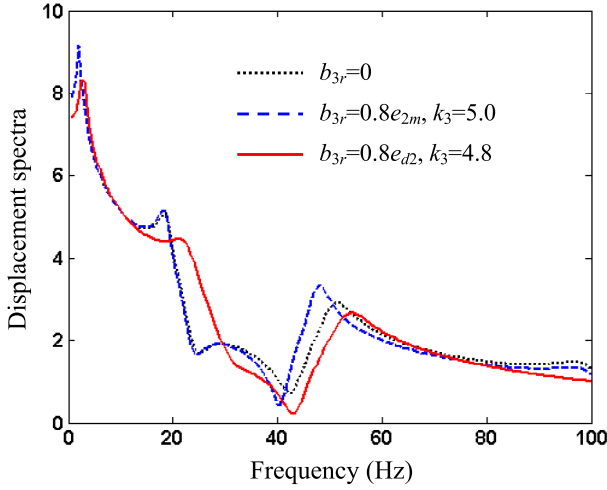
$$E_1 h_1 h_2 \frac{\partial^2 \bar{u}}{\partial y^2} - 2G_2 L^2 \bar{u} - G_2 h_a L \frac{\partial \bar{w}}{\partial y} = 0, \quad (10)$$

where variables and parameters are as given before. By using vibration modes of non-harmonic beam, Eq. (10) can be converted into ordinary differential equations and expressed in the matrix form as Eq. (5). However, the complex modal stiffness  $\mathbf{K}$ , modal mass  $\mathbf{M}$ , and modal excitation  $\mathbf{F}$  depend on harmonic distribution parameters  $k_1$ ,  $k_3$ ,  $b_{1r}$ , and  $b_{3r}$  due to  $G_2$  and  $h_1$ . The expressions of frequency response function and response spectral density are as given by Eqs. (6) and (7). The frequency response characteristics and response statistics depend on harmonic distribution parameters through shear modulus and facial layer thickness. Logarithmic non-dimensional displacement response spectra of the harmonic MRVE sandwich beam under wide-band stochastic excitation are shown in Fig. 5 for different thickness wave numbers  $k_1$  of only harmonic thickness and Fig. 6 for different harmonic core layer moduli of both harmonic thickness and modulus [43]. The harmonic distribution of geometrical and physical parameters can greatly improve the vibration response characteristics including resonant and anti-resonant response amplitudes and frequencies.

#### 5. Optimal vibration control of parameter-excited MRVE sandwich structures

MRVE properties such as stiffness and damping can be adjustable by applied magnetic fields. The passive control effectiveness of structural vibrations as given before depends fully on MRVE dynamic characteristics and then is confined, because MRVE properties such as adjustable stiffness and damping are restricted within certain limits. The optimal adjustment of MRVE composite structures based on the dynamical programming principle can





**Fig. 6.** Logarithmic non-dimensional displacement response spectra for different harmonic core layer moduli ( $k_1 = 1.0$ ,  $b_{1r} = 0.3c_{1m}$ ,  $e_{d2} = 2e_{2m}$ ). Source: Reproduced from Ref. [43].

exert complete MRVE properties and achieve further vibration control effectiveness. Note that MRVE stiffness and damping are represented as composite structure parameters and bounded due to magnetic-mechanical saturation. The optimal bounded parametric control for MRVE composite structures under stochastic and shock excitations has been proposed for further vibration suppression [44,45]. For example, the MRVE sandwich beam with supported mass is as shown in Fig. 3, which subjected to stochastic and deterministic support motion excitations. Based on the Kelvin–Voigt model, MRVE dynamic shear modulus is expressed by differential operator in time domain as

$$G_2 = G \left( 1 + \beta \frac{\partial}{\partial t} \right), \quad (11)$$

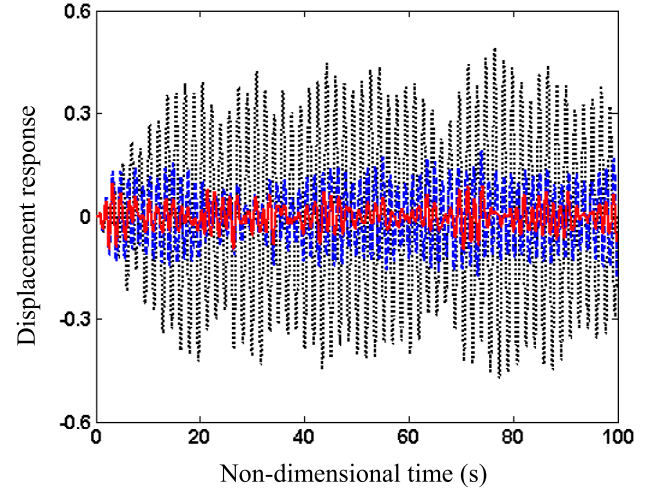
where  $G$  is equivalent real shear modulus and  $\beta$  is equivalent damping ratio. The shear modulus  $G$  is controllable by applied magnetic fields and considered as parametric control with certain limits. The non-dimensional partial differential equations for horizontal and vertical coupling motions of the MRVE sandwich beam are [45]

$$\begin{aligned} & \frac{E_1 h_1^3}{6} \frac{\partial^4 \bar{w}}{\partial y^4} - E_1 h_1^2 L \frac{\partial^3 \bar{u}}{\partial y^3} - GL^3 \left( \frac{h_a}{L} \frac{\partial^2 \bar{w}}{\partial y^2} + 2 \frac{\partial \bar{u}}{\partial y} \right) \\ & - \beta GL \sqrt{\frac{E_1 h_1^3}{\rho h_t}} \left( \frac{h_a}{L} \frac{\partial^2 \dot{\bar{w}}}{\partial y^2} + 2 \frac{\partial \dot{\bar{u}}}{\partial y} \right) \\ & + E_1 h_1^3 \left[ 1 + \frac{m}{\rho h_t} \delta(y - y_0) \right] (\ddot{\bar{w}} + \ddot{\bar{w}}_0) = 0, \\ & E_1 h_1 h_2 \frac{\partial^2 \bar{u}}{\partial y^2} - GL^2 \left( 2\bar{u} + \frac{h_a}{L} \frac{\partial \bar{w}}{\partial y} \right) - \beta G \sqrt{\frac{E_1 h_1^3}{\rho h_t}} \left( 2\dot{\bar{u}} + \frac{h_a}{L} \frac{\partial \dot{\bar{w}}}{\partial y} \right) \\ & = 0, \end{aligned} \quad (12)$$

where variables and parameters are as given before. By using vibration modes, Eq. (12) can be converted into ordinary differential equations and expressed as state equation

$$\dot{\mathbf{Z}} = \mathbf{A}\mathbf{Z} - \mathbf{B}(G)\mathbf{Z} + \mathbf{W}(\tau), \quad (13)$$

where  $\mathbf{Z}$  is state vector,  $\mathbf{A}$  is coefficient matrix,  $\mathbf{B}$  is control matrix, and  $\mathbf{W}$  is excitation vector. Bounded control constraint is  $G_l \leq G \leq G_h$ , where  $G_l$  and  $G_h$  are positive constants. The optimal bounded



**Fig. 7.** Optimally controlled and passively controlled non-dimensional displacement responses of MRVE sandwich beam and displacement response of beam without MRVE under stochastic excitation (dotted line: without MRVE; dashed line: passively controlled; solid line: optimally controlled). Source: Reproduced from Ref. [45].

control of system (13) has performance index

$$J(G) = E \left[ \int_0^{\tau_f} \mathbf{Z}^T \mathbf{S}_c \mathbf{Z} d\tau + \Psi(\mathbf{Z}(\tau_f)) \right], \quad (14)$$

where  $E[\cdot]$  is expectation operator,  $\mathbf{S}_c$  is semi-positive definite symmetric weight matrix,  $\Psi(\tau_f)$  is terminal cost, and  $\tau_f$  is terminal time. Based on the stochastic and deterministic dynamical programming principle, dynamical programming equations can be derived and optimal bounded parametric control law is determined as

$$G^* = G|_{\max\{\mathbf{B}(G)\mathbf{Z}^T \partial V / \partial \mathbf{Z}\}, G \in [G_l, G_h]}, \quad (15)$$

where  $V$  is value function. By using optimal control (15), dynamic responses of controlled system (13) and MRVE sandwich beam (12) can be obtained, and response statistics such as standard deviation can be estimated. Displacement response samples of the MRVE sandwich beam under stochastic excitation by using the optimal control and passive control are shown in Fig. 7 [45]. The optimal bounded parametric control can reduce the MRVE sandwich beam vibration responses further compared with corresponding passive control so that the adjustable MRVE properties are used completely.

## 6. Summary

The vibration control of engineering structures subjected to strong excitations and micro disturbances is a significant research subject. The adaptability or adjustability of structural dynamics is required due to the randomness of environmental excitations. MRVE as a new smart material has been developed in recent years, which has several significant advantages over magneto-rheological liquid. MRVE can supply considerably adjustable damping and stiffness for structures, and the adjustment of dynamic properties is achieved only by applied magnetic fields with changeless structure design. Increasing researches on MRVE dynamic properties, modeling and vibration control application have been presented. The advances in MRVE dynamic properties and application to structural vibration control have been reviewed, in particular, including composite structural vibration mitigation under uniform magnetic fields, vibration response characteristics improvement through harmonic parameter distribution, and optimal

bounded parametric control design based on the dynamical programming principle. Main methods and results relevant to MRVE dynamic modeling and vibration control application have been introduced, which are helpful for understanding and researches on MRVE application and development.

## Acknowledgments

This work was supported by the National Natural Science Foundation of China (11572279, 11432012, and U1234210), the Zhejiang Provincial Natural Science Foundation of China (LY15A020001), and the Hong Kong Polytechnic University Fund (1-BBY5).

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