

A General Noncoherent Chaos-Shift-Keying Communication System and its Performance Analysis

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Abstract— A general noncoherent chaos-shift-keying (CSK) communication system with adjustable weights is proposed in this paper. The performance of the system under additive white Gaussian noise (AWGN) and multipath channel conditions are evaluated. Analytical expressions of the bit error rates are derived. The performance of the system is compared with that of the DCSK system and existing noncoherent CSK systems. The results show that the general CSK system can achieve the same performance as that of the DCSK system. A detailed analysis is presented to show that the DCSK system is the optimal form of such noncoherent systems.

I. INTRODUCTION

The discovery by Pecora and Carroll [1], [2] that two chaotic systems can synchronize has triggered some interest in the development of chaos-based communication systems in the past decade [3]–[5]. The early few proposed chaos-based communication schemes were all coherent systems which were based upon the idealized assumption that two or more chaotic systems can be synchronized. In digital implementations, the chaos shift keying (CSK) has been most widely studied and analyzed [6], [7]. However, as the assumption of the synchronizability of chaotic systems has been found unrealistic or invalid under practical channel conditions, research direction has shifted towards noncoherent schemes. The differential chaos shift keying (DCSK) has been the most widely studied noncoherent scheme which does not require the synchronization of chaotic systems [8]. Furthermore, with frequency modulation (FM), the FM-DCSK scheme attains constant bit energy and hence represents a practical form of digital chaos-based communication scheme [9], [10].

In the past few years, research interest in the development of alternative noncoherent detection schemes for digital chaos-based communication systems has grown rapidly. Despite being inferior to coherent schemes in terms of bit-error performance, noncoherent schemes still inherit the advantages of carrying information using chaotic signals and being simple. A number of modulation schemes have been proposed, e.g., the maximum likelihood scheme [11], the tracker-based scheme [12], [13], the extended Kalman filter (EKF) scheme with channel equalization [14], the return-map-based scheme [15], the generalized maximum likelihood scheme for the detection

of frequency modulated differential chaos shift keying (FM-DCSK) signals [10], optimal [16], [17] and suboptimal [18] noncoherent detection method for CSK, and so on. In this paper, we consider a general weighted noncoherent CSK communication system and evaluate its performance in additive white Gaussian noise (AWGN) and multipath channel environments.

II. GENERAL WEIGHTED CSK SYSTEM

We begin with a formal description of the system to be studied in this paper. Motivated by the fact that the DCSK system is a well-studied noncoherent chaos communication system and weighting is a commonly used technique in communication system design, we design the general weighted noncoherent CSK system following the concept of DCSK. Fig. 1 shows the block diagram of the proposed noncoherent CSK communication system. The chaotic transmitter may simply employ a one-dimensional chaotic map, such as the logistic map:

$$x_{n+1} = 1 - \lambda x_n^2, \quad -1 < x_n < 1, \quad (1)$$

where $0 < \lambda \leq 2$. When $1.4 \leq \lambda \leq 2$, the map is chaotic.

Suppose the spreading factor of the system is $2N$. The chaotic signal \mathbf{x} is generated by choosing $\lambda = 2$ and an appropriate initial condition, and then iterating over N steps. In our proposed scheme, \mathbf{x} can be used to generate carriers $\mathbf{y}^{(1)}$ and $\mathbf{y}^{(2)}$, which are given by

$$y_i^{(1)} = \begin{cases} w_{11}x_i, & i = 1, \dots, N \\ w_{21}x_{i-N}, & i = N+1, \dots, 2N, \end{cases} \quad (2)$$

and

$$y_i^{(2)} = \begin{cases} w_{12}x_i, & i = 1, \dots, N \\ w_{22}x_{i-N}, & i = N+1, \dots, 2N, \end{cases} \quad (3)$$

where $w_{11}, w_{21}, w_{12}, w_{22}$ are non-zero adjustable weights (In order to obtain low BER, the weights should satisfy: $w_{11}w_{21} > 0, w_{12}w_{22} < 0$, see (10), (12), (15)). When $w_{11} = 1, w_{21} = 1, w_{12} = 1, w_{22} = -1$, the proposed system reduces to the DCSK system in [8], [19]. Considering the l th symbol. Suppose the transmitted symbol is denoted by b^l , which is

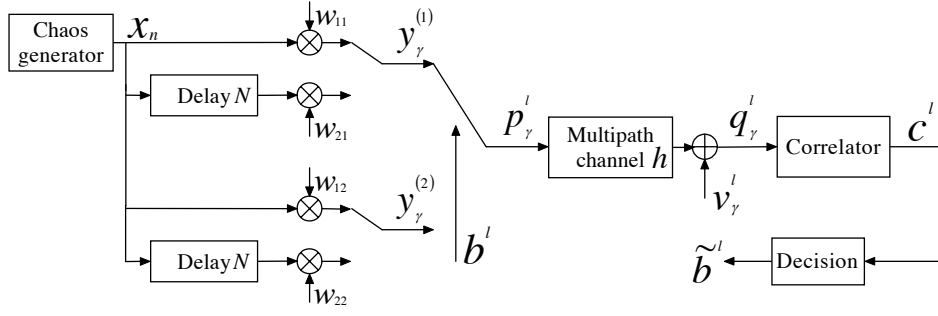


Fig. 1. Block diagram of the noncoherent CSK communication system.

either 1 or 0 which occurs with equal probabilities. During the l th symbol duration, the transmitted signal is

$$p_\gamma^l = b^l y_\gamma^{(1)} + (1 - b^l) y_\gamma^{(2)}, \quad \gamma = 1, 2, \dots, 2N. \quad (4)$$

In the receiver, a correlation-based detection is used, as shown in Fig. 1. The signal received by the receiver is given by

$$q_\gamma^l = h(p_\gamma^l) + v_\gamma^l, \quad l = 1, 2, \dots, M, \quad (5)$$

where v is AWGN with mean equal to zero and variance $N_0/2$, and $h(\cdot)$ is the multipath channel impulse response which can be represented by

$$h(p_\gamma^l) = \sum_{j=1}^L \alpha_j p_{\gamma-j+1}^l, \quad \gamma = 1, 2, \dots, 2N, \quad (6)$$

where $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_L]$ is the normalized path gain vector, L is the number of paths in the channel [20]. Here, we consider the case where the path gains are time-invariant. The decision variable is the output of the correlator, which is given by

$$c^l = \sum_{\beta=1}^N q_\beta^l q_{\beta+N}^l. \quad (7)$$

Then, the l th decoded symbol is determined according to the following rule:

$$\tilde{b}^l = \begin{cases} 1, & \text{if } c^l \geq 0 \\ 0, & \text{if } c^l < 0. \end{cases} \quad (8)$$

In the next section, we will derive the approximate bit error rates (BERs) of the proposed noncoherent CSK system over multipath channels.

III. PERFORMANCE ANALYSIS

In the following analysis, we assume that $0 < L \ll N$. According to (2) through (7), we obtain

$$\begin{aligned} c^l &= \sum_{\beta=1}^N \sum_{k=1}^L \{ \alpha_k^2 [(b^l)^2 w_{11} w_{21} + (1 - b^l)^2 w_{12} w_{22}] \\ &\quad + \alpha_k b^l (1 - b^l) (w_{11} w_{22} + w_{12} w_{21}) \} x_{\beta-k+1}^2 \\ &\quad + \sum_{\beta=1}^N \sum_{k=1}^L \alpha_k \{ v_\beta^l [b^l w_{21} + (1 - b^l) w_{22}] \end{aligned}$$

$$\begin{aligned} &\quad + v_{\beta+N}^l [b^l w_{11} + (1 - b^l) w_{12}] \} x_{\beta-k+1} \\ &\quad + \sum_{\beta=1}^N v_\beta^l v_{\beta+N}^l \\ &= A + B + C, \end{aligned} \quad (9)$$

where

$$\begin{aligned} A &= \sum_{\beta=1}^N \sum_{k=1}^L \{ \alpha_k^2 [(b^l)^2 w_{11} w_{21} + (1 - b^l)^2 w_{12} w_{22}] \\ &\quad + \alpha_k b^l (1 - b^l) (w_{11} w_{22} + w_{12} w_{21}) \} x_{\beta-k+1}^2, \\ B &= \sum_{\beta=1}^N \sum_{k=1}^L \alpha_k \{ v_\beta^l [b^l w_{21} + (1 - b^l) w_{22}] \\ &\quad + v_{\beta+N}^l [b^l w_{11} + (1 - b^l) w_{12}] \} x_{\beta-k+1}, \\ C &= \sum_{\beta=1}^N v_\beta^l v_{\beta+N}^l. \end{aligned}$$

Assuming that “1” is transmitted (i.e., $b^l = 1$), for large N , the following statistics are easily obtained:

$$\begin{aligned} E\{A|b^l = 1\} &= w_{11} w_{21} (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2) \\ &\quad \cdot NE\{x_n^2\}, \\ E\{B|b^l = 1\} &= E\{C|b^l = 1\} = 0, \\ \text{var}\{A|b^l = 1\} &= w_{11}^2 w_{21}^2 (\alpha_1^4 + \alpha_2^4 + \dots + \alpha_L^4) \\ &\quad \cdot N \text{var}\{x_n^2\}, \\ \text{var}\{B|b^l = 1\} &= (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2) \\ &\quad \cdot (w_{21}^2 + w_{11}^2) N N_0 E\{x_n^2\} / 2, \\ \text{var}\{C|b^l = 1\} &= N N_0^2 / 4, \\ \text{cov}\{A, B|b^l = 1\} &= \text{cov}\{B, C|b^l = 1\} \\ &= \text{cov}\{A, C|b^l = 1\} = 0 \end{aligned}$$

where $E[\cdot]$ and $\text{var}[\cdot]$ are the expectation and variance operators, respectively, and $\text{cov}[X, Y]$ denotes the covariance of X and Y . Then, we have

$$E\{c^l|b^l = 1\} = w_{11} w_{21} (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2) \cdot NE\{x_n^2\}, \quad (10)$$

$$\begin{aligned}
\text{var}\{c^l|b^l=1\} &= w_{11}^2 w_{21}^2 (\alpha_1^4 + \alpha_2^4 + \dots + \alpha_L^4) \\
&\quad \cdot N \text{var}\{x_n^2\} \\
&\quad + (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2) \\
&\quad \cdot (w_{21}^2 + w_{11}^2) N N_0 E\{x_n^2\}/2 \\
&\quad + N N_0^2/4. \tag{11}
\end{aligned}$$

The case of sending a symbol “0” may be computed in a likewise fashion, i.e.,

$$\begin{aligned}
E\{c^l|b^l=0\} &= w_{12} w_{22} (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2) \\
&\quad \cdot N E\{x_n^2\}, \tag{12}
\end{aligned}$$

$$\begin{aligned}
\text{var}\{c^l|b^l=0\} &= w_{12}^2 w_{22}^2 (\alpha_1^4 + \alpha_2^4 + \dots + \alpha_L^4) \\
&\quad \cdot N \text{var}\{x_n^2\} \\
&\quad + (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2) \\
&\quad \cdot (w_{22}^2 + w_{12}^2) N N_0 E\{x_n^2\}/2 \\
&\quad + N N_0^2/4. \tag{13}
\end{aligned}$$

For the logistic map, we have

$$\text{var}\{x_n^2\} = 1/8 \quad \text{and} \quad E\{x_n^2\} = 1/2. \tag{14}$$

If $\mathbf{y}^{(1)}$ is transmitted, the bit energy is $E_1 = (w_{11}^2 + w_{21}^2) N E\{x_n^2\}$; else if $\mathbf{y}^{(2)}$ is transmitted, the bit energy is $E_0 = (w_{12}^2 + w_{22}^2) N E\{x_n^2\}$. We define $S_1 = \alpha_1^4 + \alpha_2^4 + \dots + \alpha_L^4$ and $S_2 = \alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2$. According to (10) through (14), and assuming that c^l follows a normal distribution under the given conditions, the conditional BER may be computed as

$$\begin{aligned}
\text{BER}_c &= \frac{1}{2} \text{Prob}(c^l < 0|b^l=1) \\
&\quad + \frac{1}{2} \text{Prob}(c^l \geq 0|b^l=0) \\
&= \frac{1}{4} \left[\text{erfc} \left(\frac{E\{c^l|(b^l=1)\}}{\sqrt{2 \text{var}\{c^l|(b^l=1)\}}} \right) \right. \\
&\quad \left. + \text{erfc} \left(\frac{-E\{c^l|(b^l=0)\}}{\sqrt{2 \text{var}\{c^l|(b^l=0)\}}} \right) \right] \\
&= \frac{1}{4} \left\{ \text{erfc} \left(\left[\frac{S_1}{S_2^2 N} + \frac{(w_{11}^2 + w_{21}^2)^2 N_0}{w_{11}^2 w_{21}^2 S_2 E_1} \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{N(w_{11}^2 + w_{21}^2)^2 N_0^2}{2 S_2^2 w_{11}^2 w_{21}^2 E_1^2} \right]^{-1/2} \right) \right. \\
&\quad \left. + \text{erfc} \left(\left[\frac{S_1}{S_2^2 N} + \frac{(w_{12}^2 + w_{22}^2)^2 N_0}{w_{12}^2 w_{22}^2 S_2 E_0} \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{N(w_{12}^2 + w_{22}^2)^2 N_0^2}{2 S_2^2 w_{12}^2 w_{22}^2 E_0^2} \right]^{-1/2} \right) \right\} \\
&= \frac{1}{4} \left\{ \text{erfc} \left(\left[\frac{S_1}{S_2^2 N} + \frac{W(w_{11}^2 + w_{21}^2) N_0}{2 w_{11}^2 w_{21}^2 S_2 E_b} \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{W^2 N N_0^2}{8 S_2^2 w_{11}^2 w_{21}^2 E_b^2} \right]^{-1/2} \right) \right. \\
&\quad \left. + \text{erfc} \left(\left[\frac{S_1}{S_2^2 N} + \frac{W(w_{12}^2 + w_{22}^2) N_0}{2 w_{12}^2 w_{22}^2 S_2 E_b} \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{W^2 N N_0^2}{8 S_2^2 w_{12}^2 w_{22}^2 E_b^2} \right]^{-1/2} \right) \right\}, \tag{15}
\end{aligned}$$

where $W = w_{11}^2 + w_{21}^2 + w_{12}^2 + w_{22}^2$, E_b is the average bit energy and is represented by

$$E_b = (E_1 + E_0)/2 = W N E\{x_n^2\}/2. \tag{16}$$

A. Comparison with DCSK System

In [19], BER for the DCSK system under multipath channel conditions is given by

$$\text{BER}_d = \frac{1}{2} \text{erfc} \left(\left[\frac{S_1}{S_2^2 N} + \frac{4 N_0}{S_2 E_b} + \frac{2 N N_0^2}{S_2^2 E_b^2} \right]^{-1/2} \right). \tag{17}$$

when $w_{11} = 1, w_{21} = 1, w_{12} = 1, w_{22} = -1$, the noncoherent CSK system is equal to the DCSK system, so they have identical multipath performance. For other weights, from (15) we can further derive that if $w_{11}^2 = w_{21}^2 = w_{12}^2 = w_{22}^2 = 1$ and $w_{11} w_{21} > 0, w_{12} w_{22} < 0$, the two systems have identical multipath performance. However, If the weights do not take these values, performance of the noncoherent CSK system degrades.

B. Comparison with Existing Noncoherent CSK systems

The AWGN performance of other noncoherent CSK systems proposed in [11], [13], [15], which do not possess any adaptive algorithm like those in [21], are all inferior to that of the DCSK system. Our proposed system can achieve the same AWGN performance as that of the DCSK system, so it has better AWGN performance than those noncoherent CSK systems, such as those in [11], [13], [15].

IV. SIMULATION RESULTS

We consider two different multipath channel models [20]: (I) $\alpha = [1, 0.45, -0.22]$, (II) $\alpha = [1, 0.7, -0.3, 0.5, -0.1]$. Numerical calculation and computer simulation of the BERs of the proposed system are performed for two cases.

Case 1: The weights are $w_{11} = 1, w_{21} = 1, w_{12} = 1, w_{22} = -1$. The calculated and simulated BERs are plotted for $N = 50$ in Fig. 2.

Case 2: The weights are $w_{11} = 1, w_{21} = 1, w_{12} = -1, w_{22} = 1$. The calculated and simulated BERs are plotted for $N = 50$ in Fig. 3.

Numerical calculation and computer simulation of the BERs of the DCSK system is also performed. The calculated and simulated BERs are plotted for $N = 50$ in Fig. 4. It can be seen from Fig. 2–Fig. 4 that the simulated results are consistent with numerical results and the results are almost the same.

V. CONCLUSION

The performance of the proposed noncoherent CSK system is studied and compared with that of the DCSK system and existing noncoherent CSK systems. The results show that the proposed system can achieve the same multipath performance as that of the DCSK system, and is of better AWGN performance than some existing noncoherent CSK systems. This formally proves that the performance of such noncoherent CSK schemes is bounded under that of the DCSK system. Possible performance enhancement may be obtained through the use of adaptive algorithms [21].

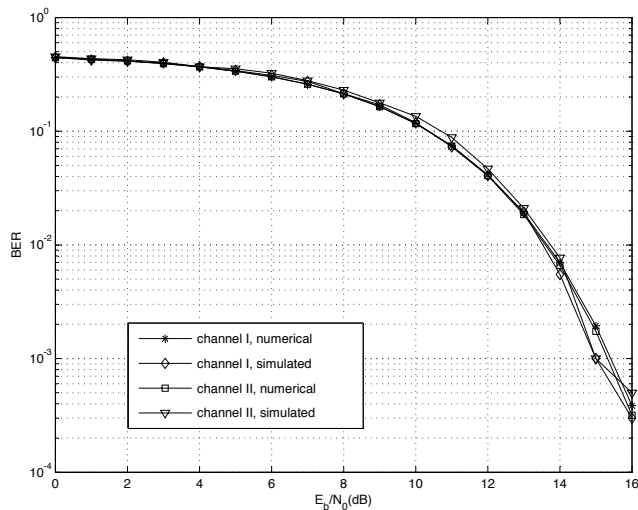


Fig. 2. BER performance of the proposed system (Case 1).

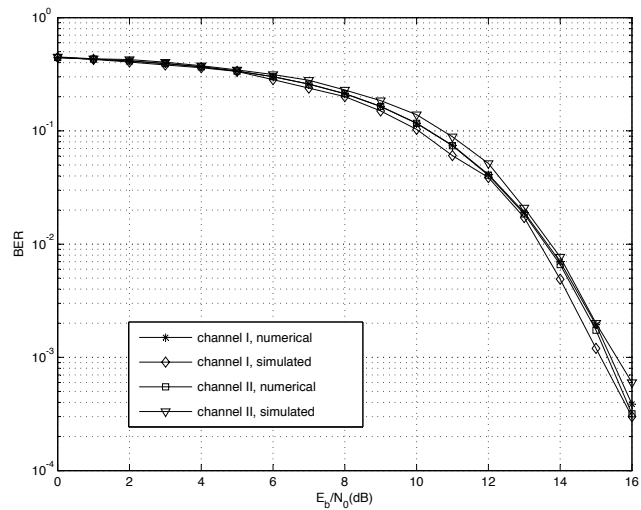


Fig. 4. BER performance of the DCSK system.

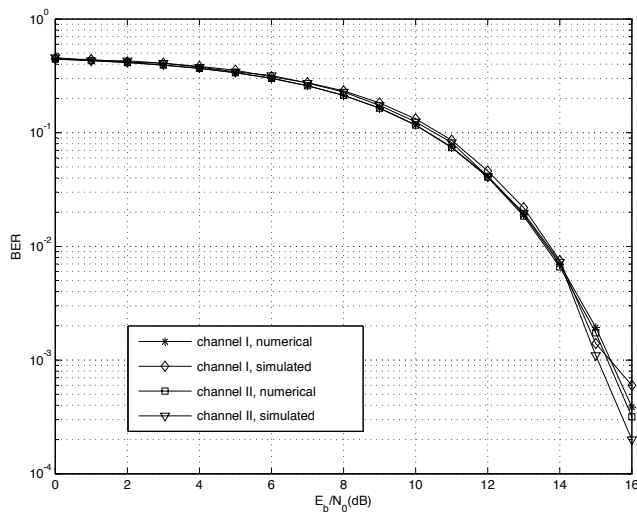


Fig. 3. BER performance of the proposed system (Case 2).

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REFERENCES

- [1] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, Vol. 64, pp. 821–824, 1990.
- [2] T. L. Carroll and L. M. Pecora, "Synchronizing chaotic circuits," *IEEE Trans. Circ. Syst. I*, Vol. 38, pp. 453–456, 1991.
- [3] A. Abel and W. Schwarz, "Chaos communications—principles, schemes, and system design," *Proc. IEEE*, vol. 90, pages 691–710, 2002.
- [4] M. Hasler and T. Schimming, "Optimal and suboptimal chaos receivers," *Proc. IEEE*, Vol. 90, pp. 733–746, 2002.
- [5] G. Kolumbán, M. P. Kennedy, Z. Jákó and G. Kis, "Chaotic communications with correlator receivers: Theory and performance limits," *Proc. IEEE*, Vol. 90, pp. 711–732, 2002.
- [6] H. Dedieu, M. P. Kennedy and M. Hasler, "Chaos shift keying: Modulation and demodulation of a chaotic carrier using self-synchronizing Chua's circuits," *IEEE Trans. Circ. Syst. II*, Vol. 40, pp. 634–642, 1993.
- [7] F. C. M. Lau and C. K. Tse, *Chaos-based Digital Communication Systems*, Heidelberg, Germany, Springer Verlag, 2003.
- [8] G. Kolumbán, G. K. Vizvari, W. Schwarz and A. Abel, "Differential chaos shift keying: A robust coding for chaos communication," *Proc. Int. Workshop on Nonl. Dynamics of Electron. Syst.*, pp. 87–92, 1996.
- [9] G. Kolumbán, G. Kis, Z. Jákó and M. P. Kennedy, "FM-DCSK: A robust modulation scheme for chaotic communications," *IEICE Trans. Fundamentals of Electronics, Communications & Computer Sciences*, Vol. E81-A, pp. 1798–1802, 1998.
- [10] G. Kolumbán, G. Kis, F. C. M. Lau and C. K. Tse, "Optimum noncoherent FM-DCSK detector: Application of chaotic GML decision rule," *IEEE Int. Symp. Circ. Syst.*, pp. 597–600, 2004.
- [11] A. Kisel, H. Dedieu and T. Schimming, "Maximum likelihood approaches for noncoherent communications with chaotic carriers," *IEEE Trans. Circ. Syst. I*, Vol. 48, pp. 533–542, 2001.
- [12] J. Feng and C. K. Tse, "On-line adaptive chaotic demodulator based on radial-basis-function neural networks," *Phys. Rev. E*, Vol. 63, pp. 1–10, 2001.
- [13] J. Feng, C. K. Tse and F. C. M. Lau, "A chaos tracker applied to non-coherent detection in chaos-based digital communication systems," *Proc. IEEE Int. Symp. Circ. Syst.*, pp. 795–798, 2001.
- [14] Z. Zhu and H. Leung, "Combined demodulation with adaptive blind-channel equalization for chaotic modulation communication systems," *IEEE Trans. Circ. Syst. I*, Vol. 49, pp. 1811–1820, 2002.
- [15] C. K. Tse, F. C. M. Lau, K. Y. Cheong and S. F. Hau, "Return-map-based approaches for noncoherent detection in chaotic digital communications," *IEEE Trans. Circ. Syst. I*, Vol. 49, pp. 1495–9, 2002.
- [16] M. Hasler and T. Schimming, "Chaos communication over noisy channels," *Int. J. Bifurcation and Chaos*, Vol. 10, pp. 719–735, 2000.
- [17] F. C. M. Lau and C. K. Tse, "On optimal detection of noncoherent chaos-shift-keying signals in a noisy environment," *Int. J. Bifurcation and Chaos*, Vol. 13, pp. 1587–1597, 2003.
- [18] S. Arai and Y. Nishio, "Variable sequence length transmitter for noncoherent chaos shift keying," *Proc. Int. Workshop on Nonlinear Circuits and Signal Processing*, pp. 301–304, 2006.
- [19] Y. Xia, C. K. Tse and F. C. M. Lau, "Performance of differential chaos-shift-keying digital communication systems over a multipath fading channel with delay spread," *IEEE Trans. Circ. Syst. II*, Vol. 51, pp. 680–684, 2004.
- [20] Z. Zhu and H. Leung, "Adaptive blind equalization for chaotic communication systems using extended-Kalman filter," *IEEE Trans. Circ. Syst. I*, Vol. 48, pp. 979–989, 2001.
- [21] W. M. Tam, F. C. M. Lau and C. K. Tse, "Generalized correlation-delay-shift-keying scheme for noncoherent chaos-based communication systems," *IEEE Trans. Circ. Syst. I*, Vol. 53, pp. 712–721, 2006.