

Model Independent Vibration Control

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1. Introduction

Active vibration control (AVC) and active noise control (ANC) are closely related research areas sharing many common approaches. Feedforward control is one of the common approaches if the primary signal is available as the reference signal (Nelson & Elliott, 1992, Hansen & Snyder, 1997). Parameters of feedforward controllers can be adjusted according to path transfer functions from actuators to error sensors, to generate destructive interference at sensed locations. The filtered-x least mean squares (FxLMS) algorithm is a popular tool for such systems.

In order for the FxLMS to be stable, phase errors in the secondary path model must be less than 90° in the entire frequency range (Snyder & Hansen, 1990, Bjarnason, 1995, Vandrey et al., 2003). Since path models are sensitive to variations of loading and boundary conditions, many researchers integrate online path modeling with AVC/ANC systems to keep path models as accurate as possible. The results are model independent feedforward controllers (MIFCs), most of which inject invasive probing signals into vibration systems for accurate online path modeling (Goodwin & Sin, 1984, Eriksson & Allie, 1989, Qiu & Hansen, 2001). Some researchers try to avoid the probing signals by controller perturbations (Nowlin et al., 2000, Kim & Swanson, 2005). Unfortunately controller perturbations also cause invasive effects to noise/vibration fields.

In this chapter, a noninvasive MIFC (NMIFC) is recommended for broadband vibration control. It has two important features. F1: the NMIFC is stable without introducing any invasive effects, such as probing signals or controller perturbations, into the vibration system; and F2: the NMIFC is applicable to vibration systems with either finite impulse response (FIR) or infinite impulse response (IIR) path models.

Feature F1 of the NMIFC is intended to solve the problem of invasive effects in most existing MIFC systems. This is made possible by replacing the FxLMS with online controller optimization. The NMIFC operates two online tasks. One is online path modeling and the other is online controller optimization. Theoretical analysis is presented to discuss the convergence of the two online tasks. The coupling effects of the two online tasks make it possible for the NMIFC to be stable without introducing any invasive effects to the vibration system.

Feature F2 is targeted at the fact that some vibration systems are well-damped whose path transfer functions may be approximated by FIR filters with negligible errors; others are lightly-damped whose path transfer functions must be approximated by IIR filters. For broadband control applications in lightly-damped vibration systems, it is more difficult to implement MIFC systems, since most available MIFC systems are based on FIR path models.

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Mismatches between FIR models and real IIR path transfer functions degrade control performance or even threaten closed-loop stability.

In the related ANC research, some ANC systems adopt IIR path models or IIR controllers (Eriksson, 1991, Lu et al., 2003). Adaptation of an IIR filter is a nonlinear process with multiple stationary points. There is no guarantee that an adaptive IIR filter will converge to its global optimal (Regalia, 1995). Adaptive AVCs, with FIR or IIR path models, have the same problem of depending on invasive effects as persistent excitations in vibration systems to ensure accurate parameter estimation and stable operation.

The NMIFC system is proposed to solve the above existing problems. Mathematical analysis is presented to explain the two features of the NMIFC; and how it is possible to achieve the control objectives. Experimental results are presented to verify the performance of NMIFC when it is applied to well- or lightly-damped vibration systems with FIR or IIR path models.

2. Background information

Mathematically, propagation of vibration waves in flexible structures can be described with transfer functions. When an external disturbance $f(z)$ is applied to a flexible structure at point A, its effect will propagate to other points of the structure. If one measures the vibration effect at point B and denotes the signal by $w(z)$, then there must exist a path transfer function $T(z)$, such that $w(z)=T(z)f(z)$. Propagation of vibration signals from disturbance $f(z)$ to measured signal $w(z)$ is mathematically modeled with a path model or a path transfer function $T(z)$.

2.1 Control objective

Let $P(z)$ and $S(z)$ denote, respectively, the primary and secondary path transfer functions. These are transfer functions from the primary/secondary sources to the error sensor respectively. If the primary signal, which is actually the disturbance signal, is available as the reference $r(z)$, the actuation signal $u(z)=C(z)r(z)$ can be synthesized by a controller with transfer function $C(z)$ in order to generate destructive interference at sensed locations. Mathematically, this is described by

$$e(z)=P(z)r(z)+S(z)u(z)=[P(z)+S(z)C(z)]r(z). \quad (1)$$

In a lightly-damped vibration system, path transfer functions are IIR rational functions, such as $P(z)=N_p(z)/D(z)$ and $S(z)=N_s(z)/D(z)$. These transfer functions have a common denominator polynomial $D(z)$ whose roots are eigenvalues of the vibration system (Vipperman & Burdisso, 1995). In lightly-damped vibration systems, eigenvalues represent resonant frequencies. In well-damped vibration systems, resonant effects are not significant due to strong damping effects, which implies reasonably accurate approximation of $D(z)=1$ such that path transfer functions can be approximated by FIR filters with negligible errors. It is desired that a vibration controller is able to work in either well- or lightly-damped vibration system by simply switching between a specific $D(z)=1$ and an arbitrary $D(z)$ with a conservatively estimated degree. This is feature F2 of the NMIFC system.

For broadband control applications, power spectral density (PSD) of reference $r(z)$ is almost constant in the entire frequency range. The objective of feedforward control is to minimize the norm of $P(z)+S(z)C(z)$, which requires accurate knowledge of $P(z)$ and $S(z)$. If either $P(z)$ or $S(z)$ is not available accurately, $C(z)$ may not be properly designed and the intended

destructive interference may become constructive interference to enhance vibration instead of suppressing it as the actuators keep on pumping energy into the vibration system. Unfortunately, both $P(z)$ and $S(z)$ are not available exactly in most practical applications. An important problem in feedforward controller design is to integrate online path modeling and obtain accurate estimate of at least $S(z)$, if the FxLMS algorithm is adopted for $C(z)$ tuning.

2.2 Approaches

The existing problem with FxLMS is the requirement that phase errors in the estimate of $S(z)$ must be less than 90° in the entire frequency range (Snyder & Hansen, 1990, Bjarnason, 1995, Vandrey et al., 2003). How to obtain accurate estimate of $S(z)$ with phase errors less than 90° in the entire frequency range is still not solved completely in available online path modeling algorithms. Some sort of invasive effects, either probing signals (Goodwin & Sin, 1984, Eriksson & Allie, 1989, Qiu & Hansen, 2001) or controller perturbations (Nowlin et al., 2000, Kim & Swanson, 2005), must be introduced in a MIFC system as persistent excitations to ensure accurate online estimation of path transfer functions in existing system identification techniques. The NMIFC is intended to avoid any invasive effects. It estimates both $P(z)$ and $S(z)$ simultaneously and replaces the FxLMS with an online controller optimization task. In adaptive controllers with online system identification (path modeling) tasks, parameters of transfer functions are expressed in regressions in order to be identified with the least squares (LS) methods. The NMIFC system also adopts this approach. Equation (1) is expressed in the time domain as a discrete-time regression

$$e_t = -\sum_{k=1}^{m_r} d_k e_{t-k} + \sum_{k=0}^m p_k r_{t-k} + \sum_{k=0}^m s_k u_{t-k} \tag{2}$$

where $\{d_k\}$, $\{p_k\}$ and $\{s_k\}$ are coefficients of polynomials $D(z)$, $N_p(z)$ and $N_s(z)$ to be identified with the LS methods. Using coefficient vector $\theta^T = [-d^T, p^T, s^T]$ and regression vector $\phi_t^T = [e_{t-1}, e_{t-2}, \dots, e_{t-m_r}, r_t, r_{t-1}, \dots, r_{t-m}, u_t, u_{t-1}, \dots, u_{t-m}]^T$, one may re-write Eq. (2) as an inner product

$$e_t = \theta^T \phi_t = -d^T x_t + p^T v_t + s^T \phi_t \tag{3}$$

where $x_t = [e_{t-1}, e_{t-2}, \dots, e_{t-m_r}]^T$, $v_t = [r_t, r_{t-1}, \dots, r_{t-m}]^T$, and $\phi_t = [u_t, u_{t-1}, \dots, u_{t-m}]^T$. If the NMIFC system is applied to well-damped vibration systems with FIR path models, there is no need to estimate $D(z)=1$ and Eq. (3) becomes

$$e_t = \theta^T \phi_t = p^T v_t + s^T \phi_t \tag{4}$$

with a reduced data vector $\phi_t = [r_t, r_{t-1}, \dots, r_{t-m}, u_t, u_{t-1}, \dots, u_{t-m}]^T$. The NMIFC system depends on two online tasks to ensure stability without introducing any invasive effects. Task 1 is a recursive least squares (RLS) path modeling process. Task 2 is a controller optimization process.

2.3 Online task 1

In task 1, $D(z)$, $N_p(z)$ and $N_s(z)$ are estimated as $\hat{D}(z)$, $\hat{N}_p(z)$ and $\hat{N}_s(z)$ respectively. The parameter vector is estimated as $\hat{\theta}^T = [-\hat{d}^T, \hat{p}^T, \hat{s}^T]$, where components of \hat{d} , \hat{p} and \hat{s} are coefficients of $\hat{D}(z)$, $\hat{N}_p(z)$ and $\hat{N}_s(z)$ respectively. The estimation residue is

$$\varepsilon_t = e_t - \hat{\theta}^T \phi_t = \Delta \theta^T \phi_t = -\Delta d^T x_t + \Delta p^T v_t + \Delta s^T \phi_t \tag{5}$$

where $\Delta\theta = \theta - \hat{\theta}$, $\Delta d = d - \hat{d}$, $\Delta p = p - \hat{p}$ and $\Delta s = s - \hat{s}$. If the NMIFC is applied to well-damped vibration systems with FIR path models and $D(z)=1$, the parameter vector is estimated as $\hat{\theta}^T = [\hat{p}^T, \hat{s}^T]$ because there is no need to estimate $D(z)$. In that case, Eq. (5) becomes

$$\varepsilon_t = e_t - \hat{\theta}^T \varphi_t = \Delta\theta^T \varphi_t = \Delta p^T v_t + \Delta s^T \phi_t, \quad (6)$$

In either case, the objective of task 1 is to drive $\varepsilon_t \rightarrow 0$, which is achievable with available system identification algorithms. A simple RLS algorithm will be adopted with convergence of $\varepsilon_t \rightarrow 0$ explained in Section 3.

2.4 Online task 2

In task 2, partial online estimates, \hat{p} and \hat{s} , are used to optimize controller transfer function $C(z)$ by minimizing $\|\hat{N}_p(z) + \hat{N}_s(z)C(z)\|_2$. In broadband noise control, power spectral density of $r(z)$ is almost constant in the entire frequency range. Minimizing $\|\hat{N}_p(z) + \hat{N}_s(z)C(z)\|_2$ is equivalent to minimizing the power spectrum of

$$\mu_t = \sum_{k=0}^m \hat{p}_k r_{t-k} + \sum_{k=0}^m \hat{s}_k u_{t-k} = \hat{p}^T v_t + \hat{s}^T \phi_t. \quad (7)$$

Several methods will be presented in Section 4 to minimize $\|\hat{N}_p(z) + \hat{N}_s(z)C(z)\|_2$. This is an important step in the NMIFC system to ensure stable operation without introducing any invasive effects into the system.

2.5 Expected results

One may use $\mu_t = \hat{p}^T v_t + \hat{s}^T \phi_t$, Eqs. (3) and (5) to derive

$$\left| e_t + \hat{d}^T x_t \right| = \left| \varepsilon_t + \mu_t \right| \leq \left| \varepsilon_t \right| + \left| \mu_t \right|. \quad (8)$$

If the NMIFC is applied to well-damped vibration systems with FIR path models and $D(z)=1$, Eq. (8) becomes

$$\left| e_t \right| = \left| \varepsilon_t + \mu_t \right| \leq \left| \varepsilon_t \right| + \left| \mu_t \right|. \quad (9)$$

Task 2 makes the NMIFC system significantly different from those MIFC systems that depend on FxLMS for controller adaptation. If the FxLMS algorithm is adopted to update controller transfer function $C(z)$, then phase errors in online estimate of $S(z)$ must be less than 90° in the entire frequency range. Invasive effects are inevitable to ensure accurate online estimation of $S(z)$ (Goodwin & Sin, 1984, Eriksson & Allie, 1989, Qiu & Hansen, 2001, Nowlin et al., 2000, Kim & Swanson, 2005). In Section 5, it will be explained why replacing the FxLMS with task 2 is able to ensure stability without introducing any forms of invasive effects.

3. Parameter adaptation

Driving the convergence of $\varepsilon_t = \Delta\theta^T \varphi_t \rightarrow 0$ is the objective of online task 1, which is achievable with available system identification algorithms. In this chapter, a simple RLS algorithm is used to explain how to estimate the parameter vector

$$\hat{\theta}_{t+1} = \hat{\theta}_t + \frac{\varepsilon_t \varphi_t}{\gamma + \varphi_t^T \varphi_t}, \tag{10}$$

where $0 < \gamma$ is a positive constant. The parameter is introduced to avoid the possibility that $\varphi_t^T \varphi_t$ may become too small at some instants to effect stable adaptation.

To see the convergence of $\varepsilon_t = \Delta \theta^T \varphi_t \rightarrow 0$, one may use positive definite function $V(t) = \Delta \theta_t^T \Delta \theta_t$. Similar to identity $a^2 - b^2 = (a-b)(a+b)$, one can see that

$$V(t+1) - V(t) = (\Delta \theta_{t+1} - \Delta \theta_t)^T (\Delta \theta_{t+1} + \Delta \theta_t). \tag{11}$$

Substituting Eq. (10) and $\Delta \theta = \theta - \hat{\theta}$, one obtains

$$\Delta \theta_{t+1} - \Delta \theta_t = \hat{\theta}_t - \hat{\theta}_{t+1} = \frac{-\varepsilon_t \varphi_t}{\gamma + \varphi_t^T \varphi_t}, \tag{12}$$

and

$$\Delta \theta_{t+1} + \Delta \theta_t = 2\theta - \hat{\theta}_t - \hat{\theta}_{t+1} = 2\Delta \theta_t - \frac{\varepsilon_t \varphi_t}{\gamma + \varphi_t^T \varphi_t}. \tag{13}$$

One may further substitute Eqs. (12) and (13) into Eq. (11) and derive

$$V(t+1) - V(t) = \frac{-\varepsilon_t \varphi_t^T}{\gamma + \varphi_t^T \varphi_t} \left(2\Delta \theta_t - \frac{\varepsilon_t \varphi_t}{\gamma + \varphi_t^T \varphi_t} \right). \tag{14}$$

Since $\varepsilon_t \varphi_t^T \Delta \theta_t = \varepsilon_t^2$, Eq. (14) is equivalent to

$$V(t+1) - V(t) = -\frac{\varepsilon_t^2 (2\gamma + \varphi_t^T \varphi_t)}{(\gamma + \varphi_t^T \varphi_t)^2} \leq -\frac{\varepsilon_t^2}{\gamma + \varphi_t^T \varphi_t} \leq 0. \tag{15}$$

It means monotonous decrease of $V(t) = \Delta \theta_t^T \Delta \theta_t$ until $\varepsilon_t \rightarrow 0$. The objective of online task 1 is therefore achievable with available RLS algorithms.

Mathematically, $\varepsilon_t = \Delta \theta^T \varphi_t \rightarrow 0$ does not mean $\Delta \theta \rightarrow 0$ unless φ_t is persistently excited (Goodwin & Sin, 1984). In those MIFC systems which depend on FxLMS to update controllers, invasive effects are introduced into the system as persistent excitations to ensure $\Delta \theta \rightarrow 0$ and accurate estimation of $S(z)$. In the NMIFC system, online task 2 is introduced to replace the FxLMS. It is an important measure to relax the phase error requirement on $S(z)$.

4. Controller optimization

In task 2, controller optimization would be a simple task if $\hat{N}_s(z)$ is minimum phase (MP) whose roots have magnitudes all less than 1. In that case, an IIR filter $C(z) = -\hat{N}_s^{-1}(z)\hat{N}_p(z)$ will achieve $\mu_t = 0$. In vibration systems, continuous-time secondary path transfer functions are sure to be MP only if the sensors collocate with the actuators. Even in case of sensor-actuator collocation, the discrete-time version of secondary transfer function and its online estimate may still be non-minimum phase (NMP). In many vibration control applications, sensors do not necessarily collocate with actuators. The secondary path transfer functions

are very likely to be NMP. Magnitudes of some roots of a NMP polynomial are equal to or larger than 1. Task 2 becomes complicated when $\hat{N}_s(z)$ is NMP with an unstable $\hat{N}_s^{-1}(z)$. A stable feedforward controller $C(z)$ must be sought to cancel the effects of $\hat{N}_s^{-1}(z)$ as much as possible. The NMIFC system is prepared for the worst case when $N_s(z)$ is NMP.

4.1 Optimal IIR controller

Applying available mathematical tools, it is possible to minimize

$$\left\| \hat{N}_p(z) + \hat{N}_s(z)C(z) \right\|_2 = \left\| \hat{N}(z) \right\|_2, \quad (16)$$

to the best extent by an optimal IIR controller with even if $N_s(z)$ is NMP. The NMIFC system will check roots of $\hat{N}_s(z)$ to split $\hat{N}_s(z) = N_m(z)N_n(z)$. In this case, $N_m(z)$ is a polynomial that includes those roots of $\hat{N}_s(z)$ with magnitudes less than 1 while $N_n(z)$ is another polynomial that includes the rest roots of $\hat{N}_s(z)$. Mathematically, $N_m(z)$ is the MP part of $\hat{N}_s(z)$ such that $N_m^{-1}(z)$ is stable; $N_n(z)$ is the NMP part of $\hat{N}_s(z)$ such that $N_n^{-1}(z)$ is unstable. The strategy is to cancel the MP part and avoid the NMP part of $\hat{N}_s(z)$.

To implement this strategy, the NMIFC system obtains a MP polynomial $M(z)$ by using coefficients of $N_n(z)$ in the reversed order. It can be shown that $A(z) = N_n(z)/M(z)$ is a stable all-pass filter (Regalia, 1995) and $\|A(z)\|_2 = \text{constant}$. With the help of $\hat{N}_s(z) = N_m(z)N_n(z)$, one may express $\hat{N}_s(z)$ as a product of $A(z)$ with a MP polynomial

$$\hat{N}_s(z) = N_n(z)N_m(z) = \frac{N_n(z)}{M(z)}M(z)N_m(z) = A(z)B(z) \quad (17)$$

where $B(z) = M(z)N_m(z)$ is MP. Equation (17) is used for substitution into Eq. (16). The result reads

$$\left\| \hat{N}_p(z) + \hat{N}_s(z)C(z) \right\|_2 = \left\| \hat{N}_p(z) + A(z)B(z)C(z) \right\|_2 \quad (18)$$

The above equation may be rewritten to

$$\begin{aligned} \left\| \hat{N}_p(z) + \hat{N}_s(z)C(z) \right\|_2 &= \|A(z)\|_2 \left\| A^{-1}(z)\hat{N}_p(z) + B(z)C(z) \right\|_2 \\ &\propto \left\| A^{-1}(z)\hat{N}_p(z) + B(z)C(z) \right\|_2 \end{aligned} \quad (19)$$

where $\|A(z)\|_2 = \text{constant}$ has been substituted. The next step is to apply the long-division to Eq. (19) and obtain

$$A^{-1}(z)\hat{N}_p(z) = \frac{M(z)\hat{N}_p(z)}{N_n(z)} = \frac{N_r(z)}{N_n(z)} + N_q(z) \quad (20)$$

where $N_q(z)$ and $N_r(z)$ are the quotient and remainder polynomials. Mathematically, $N_r(z)/N_n(z)$ and $N_q(z)$, on the right hand side of Eq. (20), are orthogonal to each other in the H_2 norm sense such that Eq. (19) may be expressed as

$$\begin{aligned} \left\| \hat{N}_p(z) + \hat{N}_s(z)C(z) \right\|_2 &\propto \left\| A^{-1}(z)\hat{N}_p(z) + B(z)C(z) \right\|_2 \\ &= \left\| \frac{N_r(z)}{N_n(z)} \right\|_2 + \left\| N_q(z) + B(z)C(z) \right\|_2. \end{aligned} \tag{21}$$

Since $N_n(z)$ is the NMP part of $\hat{N}_s(z)$, $N_r(z)/N_n(z)$ is unstable and cannot be cancelled by any stable feedforward controller. The best achievable result is to cancel the stable part $\left\| N_q(z) + B(z)C(z) \right\|_2 = 0$ with an optimal IIR controller

$$C_{opt}(z) = -\frac{N_q(z)}{B(z)} = \frac{-N_q(z)}{M(z)N_m(z)}. \tag{22}$$

Any other feedforward controllers only increase the H_2 norm in Eq. (21), which is equivalent to the H_2 norm in Eq. (16), if the controller transfer functions are not given by Eq. (22).

4.2 Sub-optimal FIR controller

The optimal IIR $C(z)$, with transfer function given by Eq. (22), is very computational expensive. In each step of RLS, it involves online root-solving of a potentially high-degree polynomial $\hat{N}_s(z) = N_m(z)N_n(z)$ and many other computations in the polynomial long division of Eq. (20). To avoid heavy online computations, a sub-optimal FIR controller is recommended to minimize Eq. (16).

For simplicity, one may set the degree of $C(z)$ to m . The impulse response of Eq. (16) can be written as

$$\hat{n} = \begin{bmatrix} \hat{n}_0 \\ \hat{n}_1 \\ \vdots \\ \vdots \\ \hat{n}_{2m} \end{bmatrix} = \begin{bmatrix} \hat{p}_0 \\ \hat{p}_1 \\ \vdots \\ \hat{p}_m \end{bmatrix} + \begin{bmatrix} \hat{s}_0 & & & & \\ \hat{s}_1 & \hat{s}_0 & & & \\ \vdots & \hat{s}_1 & \ddots & & \\ \hat{s}_m & \vdots & \ddots & \hat{s}_0 & \\ & \hat{s}_m & \vdots & \hat{s}_1 & \vdots \\ & & & \ddots & \hat{s}_m \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_m \end{bmatrix} = \hat{b}_p + \hat{N}_s c \tag{23}$$

where $\hat{n} = [\hat{n}_0, \dots, \hat{n}_{2m}]^T$ and $c = [c_0, \dots, c_m]^T$ are coefficient vectors of $\hat{N}(z)$ and $C(z)$. Coefficients of $\hat{N}_s(z)$ are used to construct matrix

$$\hat{N}_s = \begin{bmatrix} \hat{s}_0 & & & & \\ \hat{s}_1 & \hat{s}_0 & & & \\ \vdots & \hat{s}_1 & \ddots & & \\ \hat{s}_m & \vdots & \ddots & \hat{s}_0 & \\ & \hat{s}_m & \vdots & \hat{s}_1 & \vdots \\ & & & \ddots & \hat{s}_m \end{bmatrix}. \tag{24}$$

According to the Parseval's theorem, minimizing $\|\hat{n}\|_2^2$ is equivalent to minimizing the H_2 norm in Eq. (16). Using Eq. (23), one obtains

$$\hat{n}^T \hat{n} = (\hat{p} + \hat{N}_s c)^T (\hat{p} + \hat{N}_s c) = \hat{p}^T \hat{p} + \hat{p}^T \hat{N}_s c + c^T \hat{N}_s^T \hat{p} + c^T \Pi_s c, \quad (25)$$

where $\Pi_s = \hat{N}_s^T \hat{N}_s$ is the autocorrelation matrix of \hat{N}_s . Let $\psi = \hat{N}_s^T \hat{p}$, one can substitute $c^T \hat{N}_s^T \hat{p} = c^T \Pi_s \Pi_s^{-1} \psi$ into Eq. (25) and derive

$$\hat{n}^T \hat{n} = \hat{p}^T \hat{p} - \rho^T \Pi_s^{-1} \rho + (\Pi_s c + \rho)^T \Pi_s^{-1} (\Pi_s c + \rho). \quad (26)$$

In Eq. (26), vector c only affects $(\Pi_s c + \psi)^T \Pi_s^{-1} (\Pi_s c + \psi)$. Therefore it is possible to force $(\Pi_s c + \psi)^T \Pi_s^{-1} (\Pi_s c + \psi) = 0$ and minimize $\|\hat{n}\|_2^2$ by

$$c = -\Pi_s^{-1} \psi = -\Pi_s^{-1} \hat{N}_s^T \hat{p}. \quad (27)$$

This equation may be used to minimize the FIR controller $C(z)$ in task 2. It is less optimal than the IIR controller given by Eq. (22), but it is the best solution among all possible FIR controllers.

4.3 Economic FIR controller

In many vibration control systems, $N_p(z)$ and $N_s(z)$ are polynomials with high degrees. The dimension of Π_s is $(m \times m)$ if the degree of $C(z)$ is m . Calculation of Π_s still requires significant online computations after each step of RLS. Online solution of Eq. (27) is still expensive, though it is significantly less expensive than Eq. (22).

Alternatively, one may consider an economic FIR controller. Its coefficients are calculated by a recursive algorithm. One may use a positive definite function $O = 0.5 \hat{n}^T \hat{n}$ to study the convergence of the recursive algorithm that updates c and minimizes O . The time derivative of O is given by

$$\dot{O} = -\hat{n}^T \hat{N}_s \dot{c}, \quad (28)$$

where c is the coefficient vector of the FIR controller and Eq. (23) is used to link $d\hat{n}/dt$ to \dot{c} . The above equation suggests a very simple way to modify coefficient vector c . It is given by

$$\dot{c} = -\mu \hat{N}_s^T \hat{n}, \quad \text{or} \quad c_{t+1} = c_t - \mu \hat{N}_s^T \hat{n} \delta t, \quad (29)$$

where μ is a small positive constant and δt is the sampling interval. Combining Eqs. (28) and (29), one obtains $\dot{O} = -\mu \hat{n}^T \Pi_s \hat{n} \leq 0$. Therefore $O = 0.5 \hat{n}^T \hat{n}$ will be minimized by the recursive use of Eq. (29).

The advantage of this solution is to avoid online inverse of Π_s . The method is not able to minimize $O = 0.5 \hat{n}^T \hat{n}$ in every step of identification, though it reduces the computational cost. Mathematically, this solution is equivalent to the sub-optimal solution upon the convergence of Eq. (29).

5. Controller stability

It is time to answer the question why the NMIFC system is able to ensure stability without introducing any invasive effects into the system. Since the NMIFC is applicable to both well- and lightly-damped vibration systems, the answer is given to the respective cases when the NMIFC is applied to the two types of vibration systems.

5.1 Stability in well-damped vibration systems

If the NMIFC system is applied to well-damped vibration systems, path transfer functions are FIR filters with $D(z)=1$. There is no need to estimate $\hat{d}_i = 0$ and the control objective is described by Eq. (9).

On the right hand side of Eq. (9), $\varepsilon_t = \Delta\theta^T \varphi_t$ and $\mu_t = \hat{p}^T v_t + \hat{s}^T \phi_t$ represent two inner products. Online task 1 drives $\varepsilon_t \rightarrow 0$ and online task 2 minimizes $|\mu_t| \approx 0$ respectively. The joined effects of the two online tasks bound $|e_t| \approx 0$ from above, as suggested by Eq. (9). In the analysis of Section 3, no probing signals are required in online task 1. All available system identification algorithms are applicable to driving $\varepsilon_t \rightarrow 0$, which does not imply $\Delta\theta \rightarrow 0$ and the NMIFC system does not require $\Delta\theta \rightarrow 0$ as well. This is an important difference between the NMIFC and those feedforward controllers which are based on the FxLMS algorithm.

Figure 1 presents an inner product illustration of why the NMIFC is stable when its online path modeling process does not apply any measures to ensure $\Delta\theta \rightarrow 0$. It is assumed that online tasks in the NMIFC system have converged to a steady state where $\varepsilon_t \rightarrow 0$ and $|\mu_t| \approx 0$, but $\Delta\theta \neq 0$.

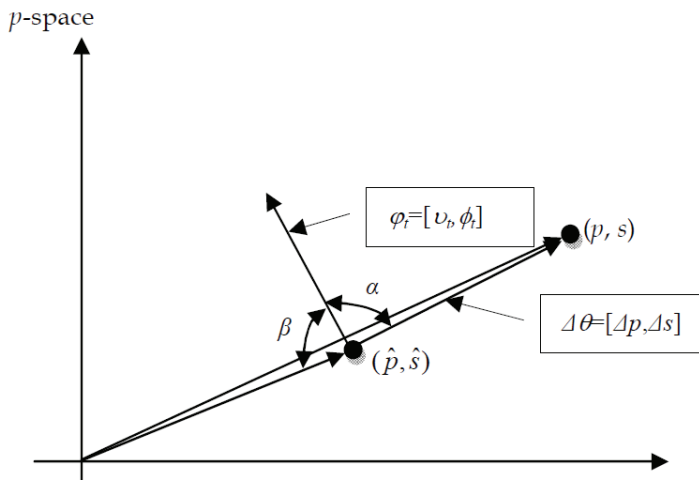


Fig. 1. Illustration of stable NMIFC without introducing invasive effects

In a FxLMS based MIFC system, $\Delta\theta \neq 0$ may cause phase errors in the estimate of secondary path transfer function to exceed 90° in some frequencies such that the closed-loop becomes unstable (Snyder & Hansen, 1990, Bjarnason, 1995, Vandrey et al., 2003). Avoiding $\Delta\theta \neq 0$ is the major issue in all FxLMS based AVC/ANC systems. Either probing signals are injected or controller perturbations are introduced as persistent excitations. Both measures introduce invasive effects to sound/vibration systems.

The NMIFC system, however, does not have such a problem at all. Online task 1 updates $\hat{\theta}^T = [\hat{p}^T, \hat{s}^T]$ to drive $\varepsilon_t = \Delta\theta^T \varphi_t \rightarrow 0$. This is enough to force φ_t and $\Delta\theta$ to be orthogonal with respect to each other such that α in Fig. 1 converges to 90° . Online task 2 optimizes controller $C(z)$. It is equivalent to adjusting $v_t = [r_t, r_{t-1}, \dots, r_{t-m}]^T$ and $\phi_t = [u_t, u_{t-1}, \dots, u_{t-m}]^T$ such that $|\mu_t| = |\hat{p}^T v_t + \hat{s}^T \phi_t| \approx 0$. This is enough to make φ_t and $\hat{\theta}^T = [\hat{p}^T, \hat{s}^T]$ almost orthogonal with respect to each other such that β in Fig. 1 becomes almost 90° .

As a result of the coupling effects of the two online tasks, φ_t and $\theta^T = [p^T, s^T]$ are almost orthogonal with respect to each other even if $\Delta\theta \neq 0$. This is not possible with FxLMS based MIFC, yet is possible with the NMIFC system. It is the ultimate objective of feedforward controllers to minimize the H_2 norm in Eq. (1), whose time domain expression becomes $e_t = \theta^T \varphi_t = p^T v_t + s^T \phi_t$ when $D(z) = 1$. Although direct minimization of $|e_t| = |\theta^T \varphi_t|$ is not possible when $\theta^T = [p^T, s^T]$ is not available accurately, indirect minimization of $|e_t| = |\theta^T \varphi_t|$ becomes possible by the NMIFC system using an imperfect $\hat{\theta}^T = [\hat{p}^T, \hat{s}^T]$ without introducing any invasive effects.

5.2 Stability in lightly-damped vibration systems

If the NMIFC system is applied to lightly-damped vibration systems with IIR path models, $\varepsilon_t \rightarrow 0$ and $|\mu_t| \approx 0$ implies $e_t + \hat{d}^T x_t \approx 0$ as suggested by Eq. (8). This means $\hat{D}(z)e(z) \approx 0$ since $\hat{D}(z) = 1 + \sum_{k=1}^{m_r} \hat{d}_k z^{-k}$ and $x_t = [e_{t-1}, e_{t-2}, \dots, e_{t-m_r}]^T$. One must prove $\hat{D}(z) \rightarrow D(z)$ to show stable minimization of $|e_t|$. Let $N(z) = N_p(z) + N_s(z)C(z)$, one can obtain an equivalent expression of Eq. (5) as

$$e_t = -\sum_{k=1}^{m_r} d_k e_{t-k} + \sum_{k=0}^{2m} n_k r_{t-k}, \tag{30}$$

where $\{n_k\}$ are coefficients of $N(z) = N_p(z) + N_s(z)C(z)$. Estimate of $N(z)$ is denoted by $\hat{N}(z)$ and defined in Eq. (16). Similar to the derivation of Eq. (3), one can obtain

$$\varepsilon_t = -\sum_{k=1}^{m_r} \Delta d_k e_{t-k} + \sum_{k=0}^{2m} \Delta n_k r_{t-k}, \tag{31}$$

where $\{\Delta d_k = d_k - \hat{d}_k\}$ and $\{\Delta n_k = n_k - \hat{n}_k\}$ are estimation errors of $D(z)$ and $N(z)$ respectively. Goodwin and Sin (1984) showed that convergence of estimation residue $\varepsilon_t = \phi_t^T \Delta\theta \rightarrow 0$ implies

$$\begin{bmatrix} e_{t-l} & e_{t-l-1} & \dots & e_{t-m_r-l+1} & r_{t-l-1} & r_{t-l-2} & \dots & r_{t-2m-l} \\ e_{t-l+1} & e_{t-l} & \dots & e_{t-m_r-l+2} & r_{t-l} & r_{t-l-1} & \dots & r_{t-2m-l+1} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ e_{t-2} & e_{t-3} & \dots & e_{t-m_r-1} & r_{t-1} & r_{t-2} & \dots & r_{t-2m-1} \\ e_{t-1} & e_{t-2} & \dots & e_{t-m_r} & r_t & r_{t-1} & \dots & r_{t-2m} \end{bmatrix} \begin{bmatrix} \Delta d_1 \\ \Delta d_2 \\ \vdots \\ \Delta d_{m_r} \\ -\Delta n_0 \\ -\Delta n_1 \\ \vdots \\ -\Delta n_{2m} \end{bmatrix} \rightarrow 0 \tag{32}$$

where $t > l \geq m_r + 2m + 1$. If r_t contains more than $0.5(m_r + 2m + 1)$ frequency components, then the above data matrix is full rank (Goodwin & Sin, 1984). The convergence of $\varepsilon_t \rightarrow 0$ implies $\{\Delta d_k \rightarrow 0\}$ and $\{\Delta n_k \rightarrow 0\}$. As a result, $\hat{D}(z)e(z) \approx 0$ converges to $D(z)e(z) \approx 0$ in broadband control operations.

The z -transform domain version of Eq. (21) is $D(z)e(z) = N(z)r(z)$ where $D(z)$ and $N(z)$ are, respectively, the auto-regressive and moving-average parts. Analytically, one may write

$$|N(z)r| \leq |\Delta N(z)r| + \left\| \left[\hat{N}_p(z) + \hat{N}_s(z)C(z) \right] r \right\| \leq |\Delta N(z)r| + \left\| \hat{N}_p(z) + \hat{N}_s(z)C(z) \right\|_2 \|r(z)\|_2 \quad (33)$$

Since $\varepsilon_t \rightarrow 0$ implies $|\Delta N(z)r| \rightarrow 0$ and $\mu_t \approx 0$ is the result of minimizing $\left\| \hat{N}_p(z) + \hat{N}_s(z)C(z) \right\|_2$, the NMIFC system minimizes $|N(z)r|$ by the joint effects of two online tasks. The objective is also achieved without probing signals or controller perturbations.

Based on the convergence of $\{\Delta d_k \rightarrow 0\}$ and $\hat{D}(z) \rightarrow D(z)$, the closed-loop system is stable if $|N(z)r|$ is stable and minimized. In view of Eq. (30), the time domain expression of $|N(z)r|$ is also an inner product $p^T v_t + s^T \phi$. Minimization of $|N(z)r|$ can also be illustrated by Fig. 1 to explain why the NMIFC is stable without any invasive effects, when it is applied to lightly-damped vibration systems.

6. Experiments

Experiments were conducted to demonstrate advantages of the NMIFC system, with experiment setup shown in Fig. 2. Cross-sectional area of the steel beam was $50 \times 1.5 \text{ mm}^2$. The error sensor was a B&K 4382 accelerometer installed near the end of the beam. The sampling rate was 500 Hz. All signals were low-pass filtered with cutoff frequency 200 Hz. The controllers were implemented in a dSPACE 1103 board and the primary source $r(z)$ was a broadband pseudo-random signal.

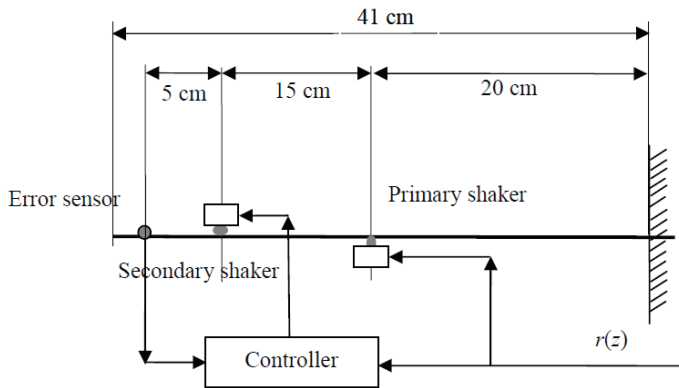


Fig. 2. Experimental setup

6.1 A well-damped beam

The first experiment was conducted to test the NMIFC in a well-damped beam shown in Fig. 2. Both actuators were shakers with shafts connected to the beam and solenoids attached to fixed bases. Damping coefficients of the shakers were different since the shakers

were made by different manufacturers. The primary shaker was one with weak damping and the secondary shaker was one with strong damping.

In the first experiment, the controller was turned off first to collect the error signal $e(z)$. Power spectral densities (PSD's) of $e(z)$ and $r(z)$ were computed with the MATLAB command "pmtm()" and denoted as $P_e = \text{pmtm}(e)$ and $P_r = \text{pmtm}(r)$ respectively. Here e and r are vectors containing data of $e(z)$ and $r(z)$. The normalized PSD of $e(z)$ is calculated as $P_{ne} = 10 \log(P_e/P_r)$ and plotted as the gray curve in Fig. 3. It is the reference for comparison. The PSD indicates a well-damped vibration system with fat resonant peaks.

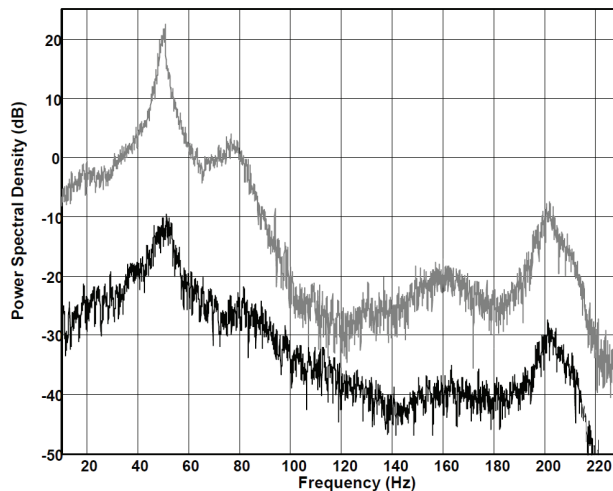


Fig. 3. Active control of a well-damped beam

Afterwards, the NMIFC was tested with $D(z)$ set to 1. The degrees of $C(z)$, $\hat{N}_p(z)$ and $\hat{N}_s(z)$ were $m=300$. The black curve in Fig. 3 represents the normalized PSD of the error signal collected after the convergence of the NMIFC system. The NMIFC was able to suppress vibration with good control performance.

6.2 A lightly-damped beam

In the second experiment, the setup was the same as in the first case. Only the secondary shaker was replaced by an inertial actuator with very weak damping. The inertial actuator was attached to the beam without other support. Its mass caused significant loading effect and reduced the fundamental frequency of the vibration system. The system sampling rate was still 500 samples/sec and all signals were low-pass filtered with cutoff frequency 200 Hz.

Again, the controllers were turned off first to collect the error signal $e(z)$. The normalized PSD of $e(z)$ is shown as the gray curve in Fig. 4. With very weak damping in both primary and secondary actuators, the system became lightly-damped. In particular, the low-frequency resonant peaks of the PSD are very narrow, which implies IIR path transfer functions.

The NMIFC system was tested in both FIR and IIR modes. When it was tested in FIR mode with $D(z)=1$, the degrees of $C(z)$, $\hat{N}_p(z)$ and $\hat{N}_s(z)$ were $m=300$. The normalized PSD of the error signal is plotted as the dashed-back curve in Fig. 4. Nose control effect is not satisfactory due to significant mismatches between FIR path models and real IIR path transfer functions. The NMIFC was also tested in IIR mode. The degrees of $C(z)$, $\hat{N}_p(z)$ and

$\hat{N}_s(z)$ was not changed. The only difference was the additional $\hat{D}(z)$ with degree $m_r=300$. The solid-black curve in Fig. 4 represents the normalized PSD of the error signal collected after the convergence of the NMIFC system. Comparing all curves in Fig. 4, one can see that the NMIFC system suppressed vibration much more significantly when it was set in IIR mode.

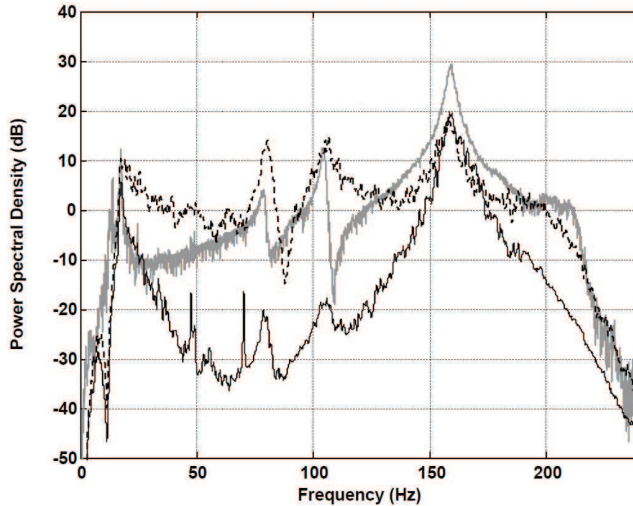


Fig. 4. Active control of a lightly-damped beam

6.3 FIR or IIR path models

In experiment 2, the beam was so lightly-damped that its vibration, if caused by an impulsive impact, would last for at least 6 seconds. The long-lasting impulse response must be modeled by a FIR filter with at least $m=3000$ taps when the sampling rate was 500 Hz. The author did not have the fast hardware to test the NMIFC with so many adaptive coefficients. Increasing FIR filter taps has other negative effects, because convergence rate and estimation accuracy of system identification algorithms are related to rank and condition number of the data matrix in Eq. (32) (Goodwin & Sin, 1984). Here the matrix had $l=m_r+2m+1=901$ columns for the NMIFC when it works in IIR mode, but at least $l=2m+1=6001$ columns for the NMIFC if the FIR path model had been increased to $m=3000$ taps. More than 6 times of columns in the data matrix of Eq. (32) would have required more than 6 times longer t to meet the necessary full rank condition. It means a much slower convergence rate. Besides, condition number of a large matrix is sensitive to quantization errors, finite word-length effects, *etc.* With a large size, the data matrix of Eq. (32) may have been analytically full rank, but numerically ill-conditioned to degrade estimate accuracy. The IIR mode is preferred with lower implementation cost, faster convergence rate and better estimation accuracy, due to a much smaller data matrix in Eq. (32). From this point of view, it is recommended to use IIR path models when the NMIFC is applied to a lightly-damped vibration system.

7. Conclusion

A NMIFC system is proposed for broadband vibration control. It has two important features. Feature F1 is that the NMIFC is stable without introducing any invasive effects, such as probing signals or controller perturbations, into the vibration system; feature F2 is

that the NMIFC is applicable to well- or lightly-damped vibration systems with either FIR or IIR path models. Mathematical analysis is presented to explain the two important features of the NMIFC; and how it is possible to achieve the intended objectives.

A significant difference between the NMIFC system and most existing MIFC systems is to replace the FxLMS algorithm with online controller optimization. The NMIFC system operates two online tasks. One is online path modeling and the other is online controller optimization. The coupling effects of the two online tasks make it possible for the NMIFC to be stable without introducing any invasive effects to the vibration system.

The NMIFC system is applicable to well- or lightly-damped vibration systems. Switching between the two kinds of systems is done by setting the degree of common denominator $D(z)$. When $D(z)=1$, the NMIFC is applicable to well-damped vibration systems with FIR path models. Otherwise, the NMIFC is applicable to lightly-damped vibration systems with IIR path models.

Experimental results are presented to demonstrate the performance of the NMIFC system when it is applied to both well- and lightly-damped vibration systems.

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Vibrations are a part of our environment and daily life. Many of them are useful and are needed for many purposes, one of the best example being the hearing system. Nevertheless, vibrations are often undesirable and have to be suppressed or reduced, as they may be harmful to structures by generating damages or compromise the comfort of users through noise generation of mechanical wave transmission to the body. The purpose of this book is to present basic and advanced methods for efficiently controlling the vibrations and limiting their effects. Open-access publishing is an extraordinary opportunity for a wide dissemination of high quality research. This book is not an exception to this, and I am proud to introduce the works performed by experts from all over the world.

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