

# AN IMPROVED MULTIPLE ACCESS SCHEME FOR CHAOS-BASED DIGITAL COMMUNICATIONS USING ADAPTIVE RECEIVERS

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## ABSTRACT

In this paper, a multiple access technique is proposed for chaos-based communication systems in which chaotic reference signals are transmitted together with the information-bearing signals. Chaotic reference signals modulated by a binary training sequence are sent periodically. The same chaotic signals are then modulated by binary data and transmitted. To achieve multiple access, different chaotic signals and training sequences are assigned to different users. At each receiver, an adaptive filter is employed to perform the demodulation based on the user's pre-assigned training sequence. The bit error rates of the proposed scheme are simulated and compared.

## 1. INTRODUCTION

Digital communications based on chaotic circuits were first proposed more than a decade ago [1]. Since then, various modulation and demodulation schemes have been suggested and studied [2]–[5]. Recently, researchers have begun looking into the multiple access capability of chaos-based communication systems which are inherently broadband. At the code level, spreading codes based on chaos have been applied to conventional direct-sequence code-division-multiple-access systems [6, 7]. At the signal level, coherent and non-coherent multiple access systems have been proposed and analysed [8]–[11]. Due to the lack of robust chaos synchronization techniques for the practical noise levels concerned, the study of coherent detection schemes remains only of academic interest and the performances of coherent systems are used as mainly benchmark indicators. Non-coherent systems are more practical and improvements are still being made [10, 11].

In this paper, a non-coherent multiple access scheme is proposed for chaos-based digital communication systems utilizing transmitted references. In this scheme, each user transmits reference chaotic signals modulated by a training sequence to the receiver periodically. Also, users are differentiated by their chaotic signals and distinct training sequences. Based on the incoming reference signals, each receiver adjusts the parameters of an adaptive filter according to the user's pre-assigned training sequence. The adaptive filter will then be used to demodulate the information-bearing chaotic signals. Improved performance of the proposed scheme will be demonstrated by computer simulations.

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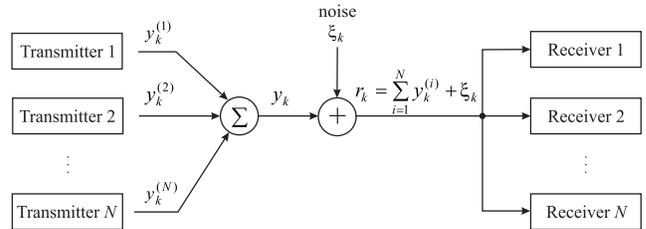


Fig. 1. Multiple access chaos-based communication system under an additive white Gaussian noise environment.

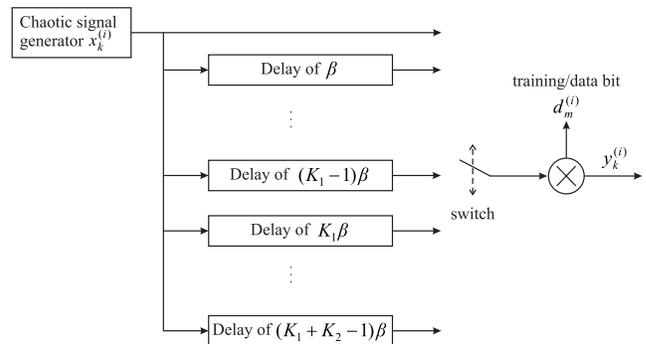


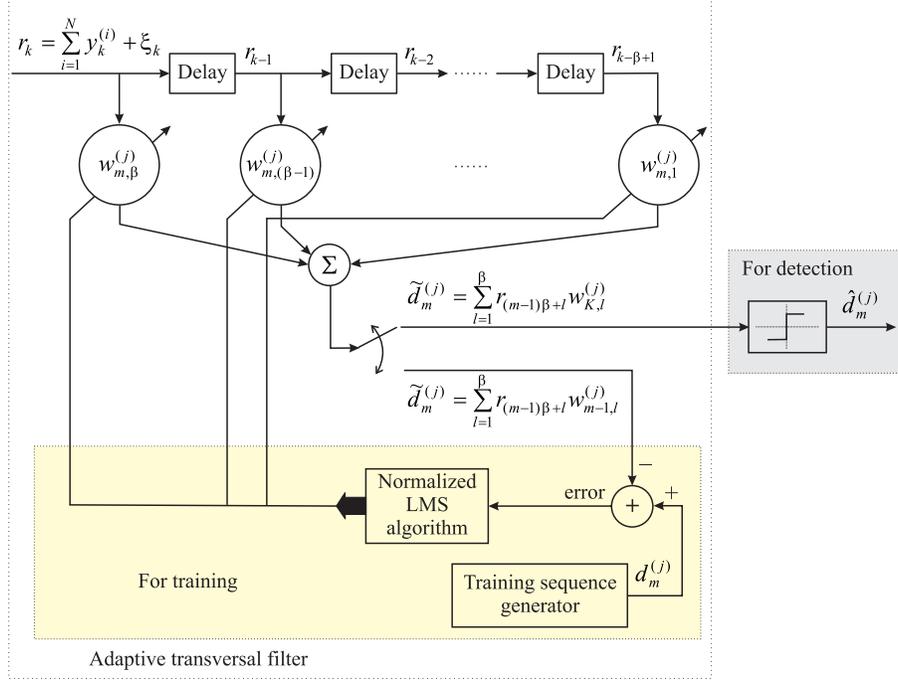
Fig. 2. Transmitter structure of the adaptive-receiver multiple-access system.

## 2. ADAPTIVE-RECEIVER MULTIPLE-ACCESS (ARMA) SCHEME FOR NONCOHERENT CHAOS-BASED COMMUNICATIONS

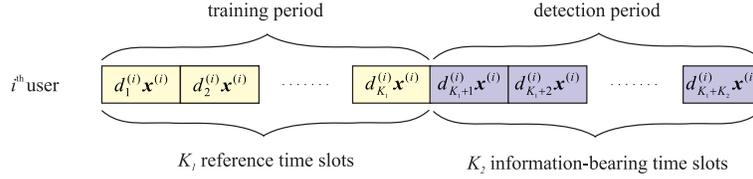
Shown in Fig. 1 is a multiple access chaos-based communication system under a noisy environment. Here, we make use of discrete-time baseband equivalent models of the transmitter and receiver to describe the proposed multiple access scheme [3, 5, 12]. Figures 2 and 3 show the transmitter and receiver, respectively, of the  $i$ th user.

### 2.1. Transmitter Structure

Each transmitter consists of a chaos generator and a number of delay blocks. The transmitted signal is organized into frames which are further sub-divided into a number of time slots, as shown in Fig. 4. The first  $K_1$  slots are used to send the reference chaotic



**Fig. 3.** Receiver structure of the adaptive-receiver multiple-access system.



**Fig. 4.** Transmitted frame structure of the  $i$ th user.

samples for training, while the information-bearing chaotic samples are sent in the remaining  $K_2$  slots. Without loss of generality, we consider the transmitted signal for a single frame period. Suppose  $\beta$  chaotic samples are sent within one slot. For each frame, the same  $\beta$  chaotic samples will be used in each slot. We denote the chaotic samples used by the  $i$ th user within this frame by  $\{x_k^{(i)} : k = 1, 2, \dots, \beta\}$ . For algebraic brevity, we also define a chaotic-sample vector  $\mathbf{x}^{(i)}$  as follows:

$$\mathbf{x}^{(i)} = [x_1^{(i)} \ x_2^{(i)} \ \dots \ x_\beta^{(i)}]^T. \quad (1)$$

Moreover, the chaotic-sample vector will be modulated by the training bits or the data bits before transmission. For the  $m$ th time slot, the sample vector sent by the  $i$ th user, denoted by  $\mathbf{y}_m^{(i)}$ , is given by

$$\begin{aligned} \mathbf{y}_m^{(i)} &= [y_{(m-1)\beta+1}^{(i)} \ y_{(m-1)\beta+2}^{(i)} \ \dots \ y_{m\beta}^{(i)}]^T \\ &= d_m^{(i)} \mathbf{x}^{(i)} \end{aligned} \quad (2)$$

where  $d_m^{(i)}$  is either “+1” or “−1”. When  $1 \leq m \leq K_1$ ,  $d_m^{(i)}$  represents the training bit, and when  $K_1 + 1 \leq m \leq K_1 + K_2$ ,  $d_m^{(i)}$  denotes the information bit. In other words, if the training/data bit

is “+1”, the transmitted samples are the same as the chaotic samples. If the training/data bit equals “−1”, the sign of the chaotic samples will be inverted and then transmitted. A typical frame structure for the  $i$ th user is shown in Fig. 4.

Consider the  $N$ -user system shown in Fig. 1. We assume that the frame sizes of all users are identical and the frames transmitted for all users are synchronized. Also, the number of slots per frame and the number of chaotic samples transmitted per slot are identical for all users. We further assume that  $K_1 = K_2 = K$  such that on the average, two slots are required to send one data bit. Thus the average number of chaotic samples transmitted per bit (spreading factor) equals  $2\beta$ . The overall sample vector passing into the channel, denoted by  $\mathbf{y}_m$ , is obtained by summing the outputs from all users, i.e.,

$$\begin{aligned} \mathbf{y}_m &= [y_{(m-1)\beta+1} \ y_{(m-1)\beta+2} \ \dots \ y_{m\beta}]^T \\ &= \sum_{i=1}^N \mathbf{y}_m^{(i)}. \end{aligned} \quad (3)$$

## 2.2. Receiver Structure

We make the usual assumption that the channel is additive white Gaussian. Thus, during the  $m$ th time slot, the received sample

vector,  $\mathbf{r}_m$ , is

$$\begin{aligned} \mathbf{r}_m &= [r_{(m-1)\beta+1} \quad r_{(m-1)\beta+2} \quad \cdots \quad r_{m\beta}]^T \\ &= \mathbf{y}_m + \Phi_m \end{aligned} \quad (4)$$

where

$$\Phi_m = [\xi_{(m-1)\beta+1} \quad \xi_{(m-1)\beta+2} \quad \cdots \quad \xi_{m\beta}]^T \quad (5)$$

and  $\xi_k$  represents the  $k$ th noise sample, the mean and variance (power spectral density) of which are zero and  $N_0/2$ , respectively.

The first  $K$  received sample vectors, i.e.,  $\{\mathbf{r}_m : m = 1, 2, \dots, K\}$ , are the references. They will be used by an adaptive transversal filter at the receiving end for training. Figure 3 shows the structure of an adaptive transversal filter. For the  $j$ th user, the estimated training data corresponding to the  $m$ th slot, denoted by  $\tilde{d}_m^{(j)}$ , is equal to

$$\tilde{d}_m^{(j)} = \mathbf{r}_m^T \mathbf{w}_{m-1}^{(j)} \quad (6)$$

where

$$\mathbf{w}_m^{(j)} = [w_{m,1}^{(j)} \quad w_{m,2}^{(j)} \quad \cdots \quad w_{m,\beta}^{(j)}]^T \quad (7)$$

is a vector containing the tap weights of the adaptive filter after the  $m$ th iteration (time slot). During the training period, at the end of each time slot, i.e.,  $m = 1, 2, \dots, K$ , the tap weights of the adaptive filter are updated using the normalized least-mean-square (LMS) algorithm, which is summarized in the following steps [13]:

$$\mathbf{w}_0 = 0 \quad (8)$$

$$e_m^{(j)} = d_m^{(j)} - \tilde{d}_m^{(j)} = d_m^{(j)} - \mathbf{r}_m^T \mathbf{w}_{m-1}^{(j)} \quad (9)$$

$$\begin{aligned} \mathbf{w}_m^{(j)} &= \mathbf{w}_{m-1}^{(j)} + \frac{\mu}{a + \|\mathbf{r}_m\|^2} e_m^{(j)} \mathbf{r}_m, \\ 0 < \mu < 2, \quad a &\geq 0 \end{aligned} \quad (10)$$

where  $e_m^{(j)}$  represents the error between the desired data ( $d_m^{(j)}$ ) and the estimated data ( $\tilde{d}_m^{(j)}$ ) during the  $m$ th time slot, and  $\|\mathbf{r}_m\|$  is the Euclidean norm of the input vector  $\mathbf{r}_m$  defined as

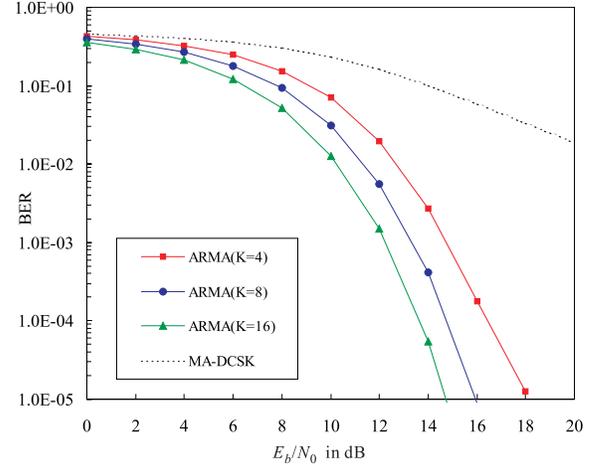
$$\|\mathbf{r}_m\| = \sqrt{\sum_{l=1}^{\beta} [r_{(m-1)\beta+l}]^2}. \quad (11)$$

At the end of the training period, i.e., after  $K$  iterations, we will obtain the tap-weight vector  $\mathbf{w}_K$  which can then be used to estimate the data symbols embedded in the remaining time slots of the frame. The decoded data symbol, i.e.,  $\hat{d}_m^{(j)}$ ,  $m = K + 1, K + 2, \dots, 2K$ , is then determined according to the following rule:

$$\hat{d}_m^{(j)} = \begin{cases} +1 & \text{if } \hat{d}_m^{(j)} = \mathbf{r}_m^T \mathbf{w}_K^{(j)} > 0 \\ -1 & \text{if } \hat{d}_m^{(j)} = \mathbf{r}_m^T \mathbf{w}_K^{(j)} \leq 0. \end{cases} \quad (12)$$

### 2.3. Choice of Training Sequences

In the proposed ARMA scheme, the chaotic samples used to carry the data vary once every frame. At the beginning of each frame, the tap weights of the adaptive filter need to be reset and updated so as to track the next set of chaotic samples. Hence the training period should not be too long. To shorten the convergence time and to avoid getting the same tap weights for more than one users at the



**Fig. 5.** BER versus  $E_b/N_0$  for a 4-user adaptive-receiver multiple-access (ARMA) system. Spreading factor is 200.

end of the training period, sets of orthogonal codes are assigned to the users as the training sequences.

Suppose  $K = 2^n$  where  $n$  is a positive integer. We can construct  $K$  orthogonal Walsh functions based on the Hadamard matrix  $\mathbf{H}_K$  [12]. The choice of  $K$  depends on the number of users in the system. For example,  $\mathbf{H}_4$  gives four orthogonal Walsh codes, and hence up to four users can be supported.

### 3. RESULTS AND DISCUSSIONS

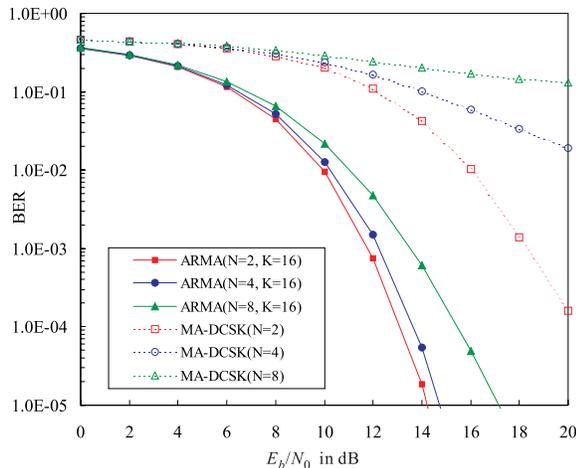
In our simulations, all users use the same map, each with a different initial condition, to generate the chaotic samples. The form of the map is given by  $x_{k+1} = 4x_k^3 - 3x_k$ . With this map, the average transmission power of each user, denoted by  $P_s$ , is readily shown equal to  $P_s = E[x_k^2] = 0.5$  [10]. At the receivers, the parameters  $\mu$  and  $a$  in the LMS algorithm are set to 0.5 and  $10^{-20}$ , respectively. The bit error rates of the proposed system are then simulated to reveal the effects of varying the following parameters:

- bit-energy-to-noise-power-spectral-density ratio ( $E_b/N_0$ ) which is given by  $2\beta P_s/N_0$ ;
- number of slots per frame; and
- spreading factor.

For comparison, we will show also the BERs of the non-coherent multiple access differential chaos-shift-keying (MA-DCSK) system.

Figure 5 plots the BERs versus  $E_b/N_0$  for a 4-user ARMA system. The number of slots  $K$  used for training is equal to 4, 8 and 16. A spreading factor ( $2\beta$ ) of 200 is employed. The BER curve for the MA-DCSK system [10] is also shown for comparison. Here, we see clearly that the ARMA system significantly outperforms the MA-DCSK scheme in all cases, especially for large values of  $K$ .

Next, we study the ARMA system when the number of users  $N$  varies. Figure 6 plots the BER curves for 2-, 4- and 8-user systems with  $K = 16$ . The spreading factor used is 200. The corresponding curves for the MA-DCSK system are also shown in the



**Fig. 6.** BER versus  $E_b/N_0$  for the adaptive-receiver multiple-access (ARMA) system with 2, 4 and 8 users.  $K = 16$  and spreading factor is 200.

same figure. It can be observed that the BER increases (degrades) as the number of users increases due to the increased inter-user interference. Compared to the MA-DCSK scheme, the ARMA technique has the capability of reducing the effects of inter-user interference effectively. Thus, the ARMA scheme can achieve a better BER compared with the MA-DCSK scheme. Note that the ARMA scheme with 8 users outperforms the MA-DCSK system with only 2 users.

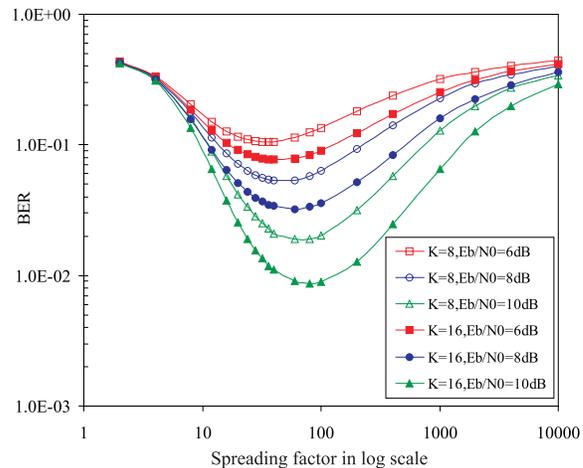
Finally, we study the variation of BER against the spreading factor in a 4-user system with  $K = 8$  and 16. As shown in Fig. 7, as the spreading factor increases, the BER improves initially. This is because a larger spreading factor will generally ease the auto- and cross-correlation estimation problems [11]. But as the spreading factor further increases, the improvement in the estimation problems is not so significant while the increase in noise level simultaneously worsens the bit error performance.

#### 4. CONCLUSIONS

In this paper, we have proposed an adaptive-receiver multiple-access (ARMA) scheme for noncoherent chaos-based communications. The transmission scheme is simple and easy to implement. Essentially, training signals are sent to train an adaptive filter at the receiving side. By using the normalized least-mean-square algorithm to update the tap weights of the filter, the mean-square-error between the incoming training symbols and the expected symbols is reduced, which in turn mitigates the interference between users when performing the actual data demodulation. Results show that the ARMA method outperforms a previously proposed multiple access technique.

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**Fig. 7.** BER versus spreading factor for the adaptive-receiver multiple-access (ARMA) system. The number of users is  $N=4$ .

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