# An Approach to Calculating the Bit-Error Rate of a Coherent Chaos-Shift-Keying Digital Communication System Under a Noisy Multiuser Environment

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***Abstract*—Assuming ideal synchronization at the receivers, an approach to calculating the approximate theoretical bit-error rate (BER) of a coherent chaos-shift-keying (CSK) digital communication system under an additive white Gaussian noise environment is presented. The operation of a single-user coherent CSK system is reviewed and the BER is derived. Using a simple cubic map as the chaos generator, it is demonstrated that the calculated BERs are consistent with those found from simulations. A multiuser coherent CSK system is then defined and the BER is derived in terms of the noise intensity and the number of users. Finally, the computed BERs under a multiuser environment are compared with the simulation results.**

***Index Terms*—Bit-error rate, chaos communication, chaos-shift-keying, multiple access.**

## I. Introduction

SEVERAL chaotic digital modulation schemes [1]–[9] have been proposed over the past few years. The basic approach is to map binary symbols to nonperiodic chaotic basis functions. For example, chaos-shift-keying (CSK) maps different symbols to different chaotic attractors, which are produced either from a dynamical system with different values of a bifurcation parameter or from a set of different dynamical systems [4], [6]. A coherent correlation CSK receiver is then required at the receiving end to decode the signals. Noncoherent detection is also possible provided the signals generated by the different attractors have different attributes, such as mean of the absolute value, variance and standard deviation. The optimal decision level of the threshold detector, however, will depend on the signal-to-noise ratio of the received signal.

To overcome the threshold-level shift problem, differential CSK (DCSK) has been proposed [4], [6], [8]. In DCSK, every transmitted symbol is represented by two chaotic signal samples. The first one serves as the reference (reference sample) while the second one carries the data (data sample). If a "$+1$" is to be transmitted, the data sample will be identical to the reference sample and if a "$-1$" is to be transmitted, an inverted version of the reference sample will be used as the data sample. The advantage of DCSK over CSK is that the threshold level is always set at zero and is independent of the noise effect. It has shown that by increasing the length of the chaotic sample or the bandwidth of the carrier, the variance of the estimation can be reduced, resulting in a lower bit-error rate (BER) [10]. No analytical solution for the BER has been derived, however, in terms of the chaotic sample length or the signal-to-noise power ratio.

Because of the nonperiodic nature of chaos, the bit energy of the transmitted symbol using CSK and DCSK varies from one bit to another. To produce a wideband chaotic signal with constant power, a chaotic signal generated by an appropriately designed analog phase-locked-loop is fed to a frequency modulator. The output, having a bandlimited spectrum with uniform power spectral density, can then be further combined with other modulation techniques such as DCSK to give frequency-modulated DCSK (FM-DCSK) [2], [4].

In almost all communication systems, the allocated frequency spectrum is shared by a number of users. Conventional multiple-access techniques such as frequency-division multiple access, time-division multiple access and code-division multiple access (CDMA) are commonly used. Some basic work has been reported on multiple access in chaos-based communication such as multiplexing chaotic signals [11] and exploiting chaotic functions for generating spreading codes under the conventional CDMA scheme [12]. The effect of the number of users on the system BER, however, has not been studied thoroughly.

Our purpose in this paper, is to present a simple and systematic way of deriving the approximate theoretical BER of a coherent CSK digital modulation system under the influence of additive white Gaussian noise (AWGN), assuming ideal synchronization at the receivers. A simple one-dimensional cubic map is used to generate the chaotic signals with different initial conditions assigned to different users. The calculation of the BER for a single-user system and a multiuser system will be presented. Simulations are also performed to verify the accuracy of the calculated values.

## II. Single-User CSK Communication System

In this section, the performance of a single-user coherent CSK communication system under a noisy condition is examined. The baseband equivalent model of a CSK digital communication system is depicted in Fig. 1 [7], [13]. Assume that a chaotic signal is generated by the map $x_{k+1} = g(x_k)$. With two different initial conditions, we can generate two sets of chaotic sequences which can be used to represent two binary symbols. Let

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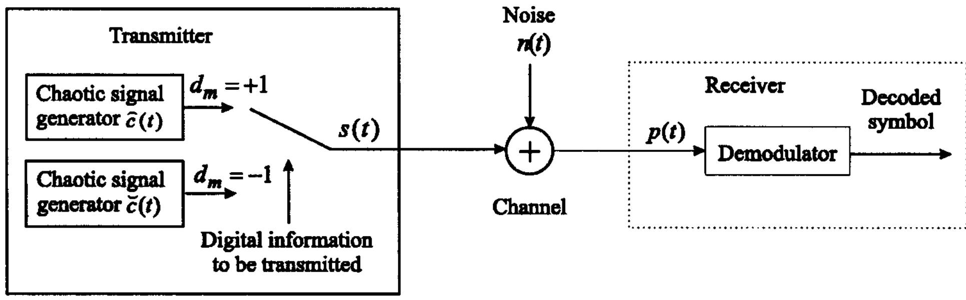


Fig. 1. Baseband equivalent model of a chaos-shift-keying digital communication system.

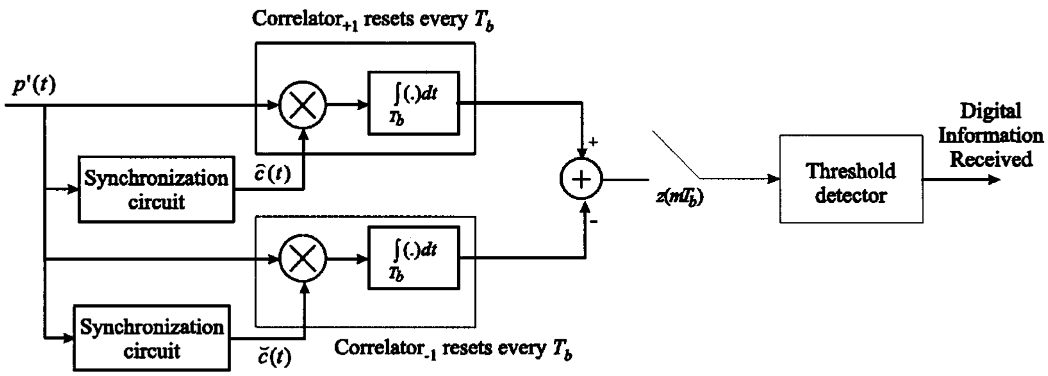


Fig. 2. Coherent CSK demodulator.

$\{\breve{x}_k\}$ and $\{\hat{x}_k\}$ be the two chaotic sequences representing "$-1$" and "$+1$", respectively. The outputs of the chaotic signal generators, denoted by $\breve{c}(t)$ and $\hat{c}(t)$, are given by

$$\breve{c}(t) = \sum_{k=0}^{\infty} \breve{x}_k r(t - kT_c) \tag{1}$$

and

$$\hat{c}(t) = \sum_{k=0}^{\infty} \hat{x}_k r(t - kT_c) \tag{2}$$

where $r(t)$ is a rectangular pulse of unit amplitude and width $T_c$, i.e.,

$$r(t) = \begin{cases} 1, & 0 \le t < T_c \\ 0, & \text{elsewhere.} \end{cases} \tag{3}$$

Assume that the system starts at $t = 0$ and the binary data to be transmitted has a period $T_b$. Denote the transmitted data by $\{d_1, d_2, d_3, \ldots\}$, where $d_m \in \{-1, +1\}$. In CSK, if a binary "$-1$" is to be transmitted during the interval $[(m-1)T_b, mT_b)$, $\breve{c}(t)$ will be sent. Likewise, if "$+1$" is to be transmitted, $\hat{c}(t)$ will be sent. Let $\beta$ be the spreading factor, defined as $T_b/T_c$, which is an integer. Thus, denoted by $v^{(m)}(t)$, the transmitted waveform for the $m$th bit is given in (4) shown at the bottom of the page where

$$y_{k+(m-1)\beta}^{(m)} = \begin{cases} \breve{x}_{k+(m-1)\beta}, & \text{if } d_m = -1 \\ \hat{x}_{k+(m-1)\beta}, & \text{if } d_m = +1 \end{cases}. \tag{5}$$

The overall transmitted waveform, $s(t)$, is

$$s(t) = \sum_{m=1}^{\infty} v^{(m)}(t). \tag{6}$$

Assume that the only channel distortion is due to the noise source $n(t)$, which is an additive white Gaussian noise (AWGN) with a two-sided power spectral density given by

$$S_n(f) = \frac{N_0}{2}, \qquad \text{for all} f. \tag{7}$$

In the following analysis, $n(t)$ is replaced by an equivalent noise source $n'(t)$ given by

$$n'(t) = \sum_{k=0}^{\infty} \xi_k r(t - kT_c) \tag{8}$$

$$\begin{aligned} v^{(m)}(t) &= \begin{cases} \sum_{k=0}^{\beta-1} \breve{x}_{k+(m-1)\beta} r\left[t - (kT_c + (m-1)T_b)\right], & \text{if } d_m = -1 \\ \sum_{k=0}^{\beta-1} \hat{x}_{k+(m-1)\beta} r\left[t - (kT_c + (m-1)T_b)\right], & \text{if } d_m = +1 \end{cases} \\ &= \sum_{k=0}^{\beta-1} y_{k+(m-1)\beta}^{(m)} r\left[t - (kT_c + (m-1)T_b)\right] \end{aligned} \tag{4}$$

where the coefficients $\{\xi_k\}$ are independent Gaussian random variables with zero mean and variance

$$\sigma_{n'}^2 = \frac{N_0}{2T_c}. \tag{9}$$

A proof of the equivalence of $n'(t)$ and $n(t)$ is given in Appendix A. As a consequence, the input signal at the receiver can be re-written as $p'(t) = s(t) + n'(t)$. Fig. 2 shows a coherent CSK demodulator where it is assumed that the synchronization circuits can reproduce the chaotic signals perfectly and the synchronization time is assumed to be negligibly small compared with the bit period. The BERs are now derived.

### A. Derivation of Bit Error Rate

Referring to Fig. 2, the input to the demodulator is given by

$$\begin{aligned} p'(t) =& s(t) + n'(t) \\ =& \sum_{m=1}^{\infty} v^{(m)}(t) + \sum_{k=0}^{\infty} \xi_k r(t - kT_c) \\ =& \sum_{m=1}^{\infty} \sum_{k=0}^{\beta-1} y_{k+(m-1)\beta}^{(m)} r[t - (kT_c + (m-1)T_b)] \\ &+ \sum_{k=0}^{\infty} \xi_k r(t - kT_c). \end{aligned} \tag{10}$$

For the $m$th received symbol, the output of correlator$_{-1}$ at the end of the bit duration equals

$$\begin{aligned} \text{Corr}_{-1}(mT_b) =& \int_{(m-1)T_b}^{mT_b} p'(t)\check{c}(t)dt \\ =& T_c \sum_{k=(m-1)\beta}^{m\beta-1} \left( y_k^{(m)} \check{x}_k + \xi_k \check{x}_k \right). \end{aligned} \tag{11}$$

Similarly, the output of correlator$_{+1}$ can be shown equal to

$$\text{Corr}_{+1}(mT_b) = T_c \sum_{k=(m-1)\beta}^{m\beta-1} \left( y_k^{(m)} \hat{x}_k + \xi_k \hat{x}_k \right). \tag{12}$$

The output of the adder, i.e., the input to the threshold detector, at this time instant is as shown in (13) at the botttom of the page. If a "+1" has been transmitted for the $m$th symbol, i.e., $d_m = +1$ and $y_k^{(m)} = \hat{x}_k$ for $(m-1)\beta \le k \le m\beta - 1$, the input of the detector will be given by

$$\begin{aligned} z_{+1}(mT_b) =& T_c \sum_{k=(m-1)\beta}^{m\beta-1} \left( \hat{x}_k^2 + \xi_k \hat{x}_k - \hat{x}_k \check{x}_k - \xi_k \check{x}_k \right) \\ =& \beta T_c \{ \gamma[\hat{x}, \hat{x}, (m-1)\beta, (m-1)\beta, \beta] \\ &+ \gamma[\xi, \hat{x}, (m-1)\beta, (m-1)\beta, \beta] \\ &- \gamma[\hat{x}, \check{x}, (m-1)\beta, (m-1)\beta, \beta] \\ &- \gamma[\xi, \check{x}, (m-1)\beta, (m-1)\beta, \beta] \} \end{aligned} \tag{14}$$

where $\gamma[a, b, k, l, \beta]$ denotes the partial discrete correlation between the chaotic sequences $\{a_k, a_{k+1}, \ldots, a_{k+\beta-1}\}$ and $\{b_l, b_{l+1}, \ \ldots, b_{l+\beta-1}\}$. Formal definitions of the various $\gamma$ functions are given in Appendix B. Using the central limit theorem [13], each of the four terms in (14) can be approximated by a normal random variable. Assume further that the random variables are independent of each other. Then, $z_{+1}(mT_b)$ will be normally distributed [14] with mean equal to (15) shown at the bottom of the page where overlining denotes the mean value over all $m$. Also, the variance of $z_{+1}(mT_b)$ is given by (16) shown at the bottom of the next page where var$[\xi]$ represents the variance of $\xi$.

If $z_{+1}(mT_b)$ is larger than zero, a "+1" is decoded for the $m$th symbol. Otherwise, a "$-1$" is detected. The conditional error probability given a "+1" has been transmitted is given by[13]

$$\begin{aligned} &\text{Prob}\left(z_{+1}(mT_b) < 0 \mid \text{“} +1 \text{” is transmitted}\right) \\ &\qquad = Q\left( \frac{\overline{z_{+1}(mT_b)}}{\sqrt{\text{var}[z_{+1}(mT_b)]}} \right) \end{aligned} \tag{17}$$

where $Q(\cdot)$ is defined as

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left( \frac{-t^2}{2} \right) dt. \tag{18}$$

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$$\begin{aligned} z(mT_b) =& \text{Corr}_{+1}(mT_b) - \text{Corr}_{-1}(mT_b) \\ =& T_c \sum_{k=(m-1)\beta}^{m\beta-1} \left( y_k^{(m)} \hat{x}_k + \xi_k \hat{x}_k - y_k^{(m)} \check{x}_k - \xi_k \check{x}_k \right) \end{aligned} \tag{13}$$

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$$\begin{aligned} \overline{z_{+1}(mT_b)} =& \beta T_c \Big\{ \overline{\gamma[\hat{x}, \hat{x}, (m-1)\beta, (m-1)\beta, \beta]} \\ &+ \overline{\gamma[\xi, \hat{x}, (m-1)\beta, (m-1)\beta, \beta]} \\ &- \overline{\gamma[\hat{x}, \check{x}, (m-1)\beta, (m-1)\beta, \beta]} \\ &- \overline{\gamma[\xi, \check{x}, (m-1)\beta, (m-1)\beta, \beta]} \Big\} \end{aligned} \tag{15}$$

TABLE I
MEANS AND VARIANCES OF THE PARTIAL DISCRETE CORRELATION FUNCTIONS OF CHAOTIC SERIES

| Type of partial discrete correlation function | Correlation function | Mean | Variance |
|---|---|---|---|
| Auto-correlation with $k=l$ | $\gamma(\breve{x},\breve{x},k,k,L) = \frac{1}{L}\sum_{n=0}^{L-1}\breve{x}_{k+n}^2$ | $m_{\breve{x},k=l} = \bar{\gamma}(\breve{x},\beta) = \begin{cases}0.5 & \text{if } L=100\\ 0.5 & \text{if } L=1000\end{cases}$ | $\sigma^2_{\breve{x},k=l} = \begin{cases}1.24\times10^{-3} & \text{if } L=100\\ 1.24\times10^{-4} & \text{if } L=1000\end{cases}$ |
| Auto-correlation with $k \neq l$ | $\gamma(\breve{x},\breve{x},k,l,L) = \frac{1}{L}\sum_{n=0}^{L-1}\breve{x}_{k+n}\breve{x}_{l+n}$ | $m_{\breve{x},k\neq l} = \begin{cases}0 & \text{if } L=100\\ 0 & \text{if } L=1000\end{cases}$ | $\sigma^2_{\breve{x},k\neq l} = \begin{cases}2.50\times10^{-3} & \text{if } L=100\\ 2.50\times10^{-4} & \text{if } L=1000\end{cases}$ |
| Cross-correlation | $\gamma(\breve{x},\hat{x},k,l,L) = \frac{1}{L}\sum_{n=0}^{L-1}\breve{x}_{k+n}\hat{x}_{l+n}$ | $m_{\breve{x}\hat{x}} = \begin{cases}0 & \text{if } L=100\\ 0 & \text{if } L=1000\end{cases}$ | $\sigma^2_{\breve{x}\hat{x}} = \begin{cases}2.50\times10^{-3} & \text{if } L=100\\ 2.50\times10^{-4} & \text{if } L=1000\end{cases}$ |
| Cross-correlation with normal random sequences with zero mean and unit variance | $\gamma(\breve{x},\phi,k,l,L) = \frac{1}{L}\sum_{n=0}^{L-1}\breve{x}_{k+n}\phi_{l+n}$ | $m_{\breve{x}\phi} = \begin{cases}0 & \text{if } L=100\\ 0 & \text{if } L=1000\end{cases}$ | $\sigma^2_{\breve{x}\phi} = \begin{cases}5.0\times10^{-3} & \text{if } L=100\\ 5.0\times10^{-4} & \text{if } L=1000\end{cases}$ |
| Cross-correlation with normal random sequences with zero mean and variance $\sigma^2_\lambda$ | $\gamma(\breve{x},\lambda,k,l,L) = \frac{1}{L}\sum_{n=0}^{L-1}\breve{x}_{k+n}\lambda_{l+n}$ | $m_{\breve{x}\lambda} = \begin{cases}0 & \text{if } L=100\\ 0 & \text{if } L=1000\end{cases}$ | $\sigma^2_{\breve{x}\lambda} = \begin{cases}5.0\times10^{-3}\sigma^2_\lambda & \text{if } L=100\\ 5.0\times10^{-4}\sigma^2_\lambda & \text{if } L=1000\end{cases}$ |

Likewise, it can be shown that when a "$-1$" has been transmitted, the conditional error probability is given by

$$\begin{aligned}\text{Prob}\left(z_{-1}(mT_b) > 0 \mid \text{“}-1\text{” is transmitted}\right) \\ = Q\left(\frac{-\overline{z_{-1}(mT_b)}}{\sqrt{\text{var}\left[z_{-1}(mT_b)\right]}}\right).\end{aligned} \quad (19)$$

In the sequel, we assume that the map

$$x_{k+1} = g(x_k) = 4x_k^3 - 3x_k \quad (20)$$

is used to generate the chaotic signals. It can be readily shown that with this choice of $g(\cdot)$, the partial discrete correlation functions are normally distributed [15]. The means and variances of the functions for spreading factors 100 and 1000 are tabulated in Table I.

Using the values in Table I, (15) and (16), we easily get (21) shown at the botttom of the page. Therefore, the overall BER of a single-user coherent CSK system under AWGN is as shown in (22) at the botttom of the next page where $P^+$ is the probability of a "$+1$" being transmitted $P^-$ is the probability of a "$-1$" being transmitted. Note that $P^+ + P^- = 1$. The average bit energy of the system is given by

$$\overline{E_b} = T_b \lim_{T\to\infty}\frac{1}{T}\int_0^T s^2(t)dt = T_c[\beta\bar{\gamma}(\hat{x},\beta)P^+ + \beta\bar{\gamma}(\breve{x},\beta)P^-] \quad (23)$$

where $\bar{\gamma}(\hat{x},\beta)$ and $\bar{\gamma}(\breve{x},\beta)$ denote the mean-squared values of chaotic sequence samples of length $\beta$ taken from the chaotic series $\{\hat{x}_k\}$ and $\{\breve{x}_k\}$ respectively (see Appendix C). Note that $\bar{\gamma}(\hat{x},\beta)$ and $\bar{\gamma}(\breve{x},\beta)$ equal the mean of $\gamma[\hat{x},\hat{x},(m-1)\beta,(m-1)\beta,\beta]$ and the mean of $\gamma[\breve{x},\breve{x},(m-1)\beta,(m-1)\beta,\beta]$ respectively over all positive integers $m$. Since both series $\{\hat{x}_k\}$ and $\{\breve{x}_k\}$ are generated from the same map with different initial conditions, we have $\bar{\gamma}(\hat{x},\beta) = \bar{\gamma}(\breve{x},\beta)$ and (23) can be simplified to

$$\overline{E_b} = T_c\beta\bar{\gamma}(\hat{x},\beta)\left[P^+ + P^-\right] = T_c\beta\bar{\gamma}(\hat{x},\beta) = 0.5T_b \quad (24)$$

$$\begin{aligned}\text{var}\left[z_{+1}(mT_b)\right] = {}&\beta^2T_c^2\left\{\text{var}\left[\gamma\left[\hat{x},\hat{x},(m-1)\beta,(m-1)\beta,\beta\right]\right] + \text{var}\left[\gamma\left[\xi,\hat{x},(m-1)\beta,(m-1)\beta,\beta\right]\right]\right.\\ &\left.+\text{var}\left[\gamma\left[\hat{x},\breve{x},(m-1)\beta,(m-1)\beta,\beta\right]\right] + \text{var}\left[\gamma\left[\xi,\breve{x},(m-1)\beta,(m-1)\beta,\beta\right]\right]\right\}\end{aligned} \quad (16)$$

$$\begin{aligned}\text{Prob}\left(z_{+1}(mT_b) < 0 \mid \text{“}+1\text{” is transmitted}\right) &= \text{Prob}\left(z_{-1}(mT_b) > 0 \mid \text{“}-1\text{” is transmitted}\right)\\ &= \begin{cases} Q\left(\frac{0.5}{\sqrt{3.74\times10^{-3}+1.0\times10^{-2}\frac{N_0}{2T_c}}}\right), & \text{if } \beta=100\\ Q\left(\frac{0.5}{\sqrt{3.74\times10^{-4}+1.0\times10^{-3}\frac{N_0}{2T_c}}}\right), & \text{if } \beta=1000\end{cases}\end{aligned} \quad (21)$$

TABLE II
CALCULATED BIT ERROR RATES OF A SINGLE-USER COHERENT CSK SYSTEM FOR VARIOUS $\overline{E_b}/N_0$

| $\overline{E_b}/N_0$ in dB | Spreading factor 100 | 1000 |
|---|---|---|
| 0 | 0.1605 | 0.1588 |
| 2 | 0.1067 | 0.1043 |
| 4 | 0.0599 | 0.0568 |
| 6 | 0.0263 | 0.0233 |
| 8 | 0.0082 | 0.0062 |
| 10 | 0.0016 | 0.0008 |
| 12 | 0.0002 | 0.0000 |
| 14 | 0.0000 | 0.0000 |
| ∞ | 0.0000 | 0.0000 |

TABLE III
SIMULATED BIT ERROR RATES OF A SINGLE-USER COHERENT CSK SYSTEM FOR VARIOUS $\overline{E_b}/N_0$

| $\overline{E_b}/N_0$ in dB | Spreading factor 100 | 1000 |
|---|---|---|
| 0 | 0.1604 | 0.1596 |
| 2 | 0.1050 | 0.1038 |
| 4 | 0.0577 | 0.0558 |
| 6 | 0.0259 | 0.0225 |
| 8 | 0.0072 | 0.0065 |
| 10 | 0.0011 | 0.0004 |
| 12 | 0.0000 | 0.0000 |
| 14 | 0.0000 | 0.0000 |
| ∞ | 0.0000 | 0.0000 |

for $\beta = 100$ or 1000. Using $T_b = \beta T_c$ and (24), (22) can also be expressed as

$$\mathrm{BER}_{1-\mathrm{user}} = \begin{cases} Q\left(\dfrac{\overline{E_b}}{\sqrt{3.74\times10^{-3}T_b^2+0.5T_bN_0}}\right), & \text{if } \beta = 100 \\ Q\left(\dfrac{\overline{E_b}}{\sqrt{3.74\times10^{-4}T_b^2+0.5T_bN_0}}\right), & \text{if } \beta = 1000. \end{cases} \tag{25}$$

Assume that $T_b = 10^{-4}$ s. For a range of $\overline{E_b}/N_0$ values, the BERs can be computed using (25), as tabulated in Table II. As expected, the BER decreases with increasing $\overline{E_b}/N_0$. Moreover, for the same $\overline{E_b}/N_0$, the BER for $\beta = 100$ is worse than that for $\beta = 1000$. This is because when $\beta$ is increased from 100 to 1000, the variances of the discrete partial correlation functions decrease (central limit theorem [13]), resulting in a lower variance of the signal at the input of the threshold detector (16). Hence, the chance of an incorrect detection is reduced.

### *B. Simulations*

Computer simulations have been performed based on the equivalent system model shown in Fig. 1 (with the noise source replaced by the equivalent noise source) and the demodulator shown in Fig. 2. The bit period $T_b$ is assumed to be $10^{-4}$ s and $\beta = 100$ and 1000. For each of the different $\overline{E_b}/N_0$ values simulated, 10 000 bits have been transmitted and the BERs are tabulated in Table III.

### *C. Comparisons*

The calculated and simulated BERs are plotted against $\overline{E_b}/N_0$ in Fig. 3. It can be observed that the results are in good agreement. The theoretical BER for a coherent antipodal CSK system [16] is also plotted in the same figure for comparison. It can be seen that for the same BER, coherent antipodal CSK modulation shows an advantage of about 3 dB, which agrees with the theoretical gain of an antipodal modulation system. In the next section, we extend our analysis to a multiuser system.

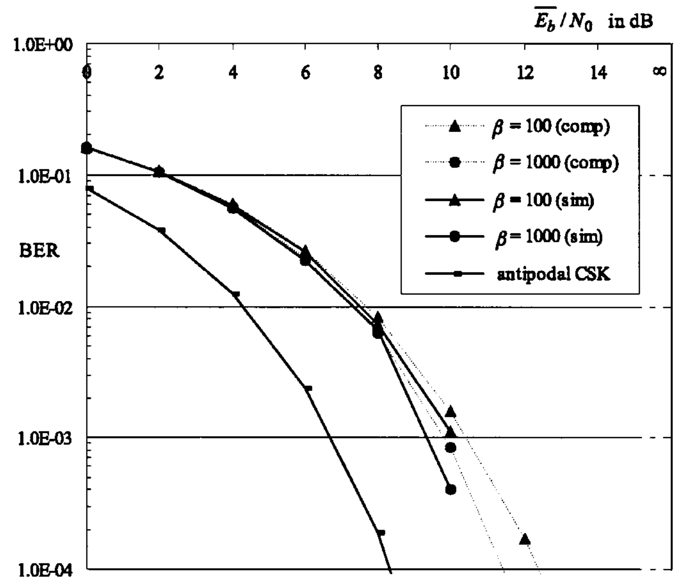


Fig. 3. Computed (comp) and simulated (sim) bit error rates against $\overline{E_b}/N_0$ in a single-user coherent CSK system.

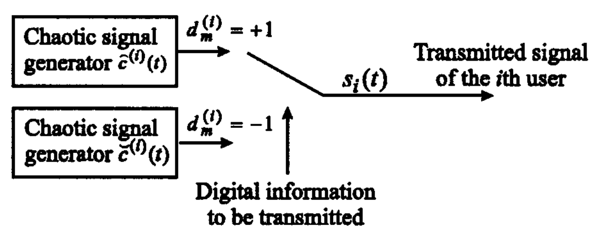


Fig. 4. Transmitter of the $i$th user in a multiuser coherent CSK digital communication system.

## III. MULTIUSER CSK COMMUNICATION SYSTEM

Assume that there are $N$ users within the system. The transmitter for the $i$th user is shown in Fig. 4. The pair of chaotic

$$\begin{aligned}\mathrm{BER}_{1-\mathrm{user}} &= \mathrm{Prob}\left(z_{+1}\left(mT_b\right) < 0 \mid \text{“}+1\text{” is transmitted}\right)P^{+} + \mathrm{Prob}\left(z_{-1}\left(mT_b\right) > 0 \mid \text{“}-1\text{” is transmitted}\right)P^{-} \\ &= \begin{cases} Q\left(\dfrac{0.5}{\sqrt{3.74\times10^{-3}+1.0\times10^{-2}\frac{N_0}{2T_c}}}\right), & \text{if } \beta = 100 \\ Q\left(\dfrac{0.5}{\sqrt{3.74\times10^{-4}+1.0\times10^{-3}\frac{N_0}{2T_c}}}\right), & \text{if } \beta = 1000 \end{cases}\end{aligned} \tag{22}$$

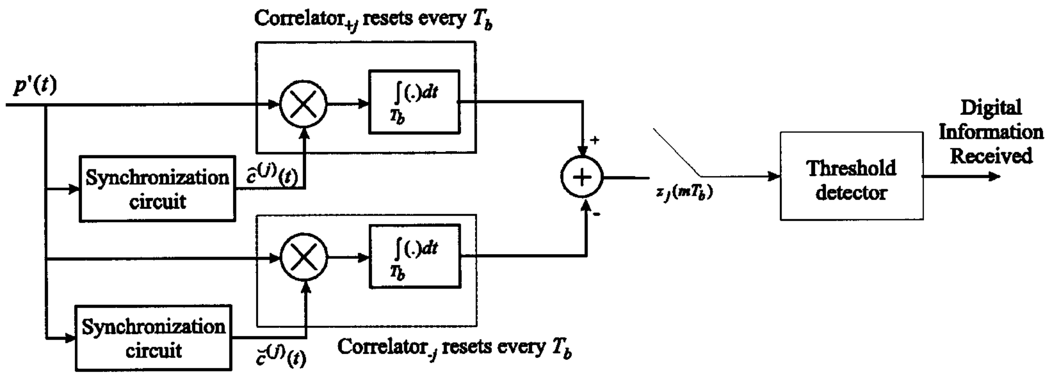


Fig. 5. Coherent demodulator for user $j$ in a multiuser coherent CSK communication system.

series for user $i$ are denoted by $\{\breve{x}_k^{(i)}\}$ and $\{\hat{x}_k^{(i)}\}$. It is also assumed that all series are generated by the same map $x_{k+1} = g(x_k)$ but with different initial conditions, where $g(x)$ is defined as in (20). Using similar notations as in the previous section, the outputs of the chaotic signal generators for user $i$ are given by

$$\breve{c}^{(i)}(t) = \sum_{k=0}^{\infty} \breve{x}_k^{(i)} r\left(t - kT_c\right) \tag{26}$$

and

$$\hat{c}^{(i)}(t) = \sum_{k=0}^{\infty} \hat{x}_k^{(i)} r\left(t - kT_c\right). \tag{27}$$

Denote the transmitted data for user $i$ by $\{d_1^{(i)}, d_2^{(i)}, d_3^{(i)}, \ldots\}$, where $d_m^{(i)} \in \{-1, +1\}$. The transmitted waveform for the $m$th bit of user $i$, $v^{(m,i)}(t)$, is given by (28) shown at the bottom of the page where

$$y_{k+(m-1)\beta}^{(m,i)} = \begin{cases} \breve{x}_{k+(m-1)\beta}^{(i)}, & \text{if } d_m^{(i)} = -1 \\ \hat{x}_{k+(m-1)\beta}^{(i)}, & \text{if } d_m^{(i)} = +1 \end{cases}$$
$$k = 0, 1, \ldots, \beta - 1. \tag{29}$$

The transmitted waveform for user $i$, $s^{(i)}(t)$, is therefore

$$s^{(i)}(t) = \sum_{m=1}^{\infty} v^{(m,i)}(t). \tag{30}$$

The overall signal component arriving at the receiver, $s(t)$, is derived by summing the signals of all individual users, i.e.,

$$s(t) = \sum_{i=1}^{N} s^{(i)}(t) = \sum_{i=1}^{N} \sum_{m=1}^{\infty} v^{(m,i)}(t). \tag{31}$$

### A. Derivation of Bit Error Rate

Corrupted by AWGN, the received signal arrives at the receiver. The demodulator for user $j$ is depicted in Fig. 5. The input to the demodulator is given by

$$\begin{aligned} p'(t) =& s(t) + n'(t) \\ =& \sum_{i=1}^{N} \sum_{m=1}^{\infty} v^{(m,i)}(t) + \sum_{k=0}^{\infty} \xi_k r\left(t - kT_c\right). \end{aligned} \tag{32}$$

$$\begin{aligned} v^{(m,i)}(t) &= \begin{cases} \sum_{k=0}^{\beta-1} \breve{x}_{k+(m-1)\beta}^{(i)} r\left[t - (kT_c + (m-1)T_b)\right], & \text{if } d_m^{(i)} = -1 \\ \sum_{k=0}^{\beta-1} \hat{x}_{k+(m-1)\beta}^{(i)} r\left[t - (kT_c + (m-1)T_b)\right], & \text{if } d_m^{(i)} = +1 \end{cases} \\ &= \sum_{k=0}^{\beta-1} y_{k+(m-1)\beta}^{(m,i)} r\left[t - (kT_c + (m-1)T_b)\right] \end{aligned} \tag{28}$$

$$\begin{aligned} \mathrm{Corr}_{-j}\left(mT_b\right) &= \int_{(m-1)T_b}^{mT_b} p'(t) \breve{c}^{(j)}(t) dt \\ &= \int_{(m-1)T_b}^{mT_b} \left[ \sum_{i=1}^{N} v^{(m,i)}(t) + \sum_{k=(m-1)\beta}^{m\beta-1} \xi_k r\left(t - kT_c\right) \right] \breve{c}^{(j)}(t) dt \\ &= T_c \left[ \sum_{i=1}^{N} \sum_{k=(m-1)\beta}^{m\beta-1} y_k^{(m,i)} \breve{x}_k + \sum_{k=(m-1)\beta}^{m\beta-1} \xi_k \breve{x}_k^{(j)} \right] \end{aligned} \tag{33}$$

Assume perfect synchronization at the receiver. For the $m$th received symbol, the output of correlator$_{-j}$ at the end of the symbol period is given by (33) shown at the bottom of the previous page. Similarly, the output of correlator$_{+j}$ can be shown equal to

$$\text{Corr}_{+j}(mT_b) = T_c \left[ \sum_{i=1}^{N} \sum_{k=(m-1)\beta}^{m\beta-1} y_k^{(m,i)} \hat{x}_k + \sum_{k=(m-1)\beta}^{m\beta-1} \xi_k \hat{x}_k^{(j)} \right]. \quad (34)$$

The input to the threshold detector at this instant is

$$\begin{aligned} z_j(mT_b) =& \text{Corr}_{+j}(mT_b) - \text{Corr}_{-j}(mT_b) \\ =& T_c \left[ \sum_{i=1}^{N} \sum_{k=(m-1)\beta}^{m\beta-1} y_k^{(m,i)} \hat{x}_k + \sum_{k=(m-1)\beta}^{m\beta-1} \xi_k \hat{x}_k^{(j)} \right. \\ & \left. - \sum_{i=1}^{N} \sum_{k=(m-1)\beta}^{m\beta-1} y_k^{(m,i)} \check{x}_k - \sum_{k=(m-1)\beta}^{m\beta-1} \xi_k \check{x}_k^{(j)} \right]. \end{aligned} \quad (35)$$

Suppose for user $j$, a "+1" has been transmitted for the $m$th symbol, i.e., $d_m^{(j)} = +1$ and $y_k^{(m,j)} = \hat{x}_k^{(j)}$ for $(m-1)\beta \le k \le m\beta - 1$, the input of the detector will be given by (36) shown at the bottom of the page. Applying the central limit theorem again, all the $\gamma$ functions in (36) can be approximated by normal random variables (see Appendix D). Assume that the random variables are independent of each other. $z_{j,+1}(mT_b)$ will also be normally distributed with mean

$$\overline{z_{j,+1}(mT_b)} = 0.5\beta Tc \quad \beta = 100 \text{ or } 1000 \quad (37)$$

and variance. [See (38) at the bottom of the page.] See Appendix E for a detailed derivation. The conditional error probability given a "+1" has been transmitted is given by (39) shown at the bottom of the next page. Likewise, it can be shown that if a "$-1$" is transmitted, the conditional error probability is the same as in (39). Thus, it can be shown easily that the overall BER of a multiuser coherent CSK system under AWGN is as shown in (40) at the bottom of the next page. Similar to the single-user case, the average bit energy for user $i$ is defined as

$$\overline{E_{b_i}} = T_b \lim_{T\to\infty} \frac{1}{T} \int_0^T \left( s^{(i)}(t) \right)^2 dt. \quad (41)$$

Using the same procedure as in Section II-A, it can be readily shown that

$$\overline{E_{b_i}} = 0.5 T_b \quad \beta = 100 \text{ or } 1000 \quad (42)$$

for all users $i = 1, 2, \ldots, N$. Denoting $\overline{E_{b_i}}$ by $\overline{E_b}$ and putting (42) into (40), we have (43) shown at the bottom of the next page. For $T_b = 10^{-4}$ s, the BERs are computed using (43) for a range of users and $\overline{E_b}/N_0$ values. The results are tabulated in Table IV. With an increasing number of users transmitting their chaotic signals at the same time, more interference will

$$\begin{aligned} z_{j,+1}(mT_b) =& T_c \sum_{k=(m-1)\beta}^{m\beta-1} \left( \hat{x}_k^{(j)} \right)^2 + T_c \sum_{\substack{i=1 \\ i\neq j}}^{N} \sum_{k=(m-1)\beta}^{m\beta-1} y_k^{(m,i)} \hat{x}_k^{(j)} + T_c \sum_{k=(m-1)\beta}^{m\beta-1} \xi_k \hat{x}_k^{(j)} - T_c \sum_{k=(m-1)\beta}^{m\beta-1} \hat{x}_k^{(j)} \check{x}_k^{(j)} \\ & - T_c \sum_{\substack{i=1 \\ i\neq j}}^{N} \sum_{k=(m-1)\beta}^{m\beta-1} y_k^{(m,i)} \check{x}_k^{(j)} - T_c \sum_{k=(m-1)\beta}^{m\beta-1} \xi_k \check{x}_k^{(j)} \\ =& \beta T_c \left( \gamma\left[ \hat{x}^{(j)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta \right] + \sum_{\substack{i=1 \\ i\neq j}}^{N} \gamma\left[ y^{(m,i)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta \right] \right. \\ & + \gamma\left[ \xi, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta \right] - \gamma\left[ \hat{x}^{(j)}, \check{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta \right] \\ & \left. - \sum_{\substack{i=1 \\ i\neq j}}^{N} \gamma\left[ y^{(m,i)}, \check{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta \right] - \gamma\left[ \xi, \check{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta \right] \right) \end{aligned} \quad (36)$$

$$\text{var}\left[ z_{j,+1}(mT_b) \right] = \begin{cases} \beta^2 T_c^2 \left( 1.24 \times 10^{-3} + 2.5 \times 10^{-3}(2N-1) + 1.0 \times 10^{-2} \frac{N_0}{2T_c} \right), & \text{if } \beta = 100 \\ \beta^2 T_c^2 \left( 1.24 \times 10^{-4} + 2.5 \times 10^{-4}(2N-1) + 1.0 \times 10^{-3} \frac{N_0}{2T_c} \right), & \text{if } \beta = 1000 \end{cases} \quad (38)$$

be introduced into the system and more errors occur. Hence, as shown in the table, the BER increases with an increasing number of users. Moreover, as in the single-user case, the BER decreases with increasing $\overline{E_b}/N_0$ whereas for the same $\overline{E_b}/N_0$, the BER for $\beta = 100$ is worse than that for $\beta = 1000$. Note that in a multiuser system, the performance of the system may be limited by the interference between the users and not by the noise power. For example, when there are 20 users in the system and the spreading factor equals 100, a BER of 0.1986 is obtained for $\overline{E_b}/N_0 = 0$ dB. When the powers of the signals increase, or equivalently the noise power diminishes, the effect of noise is reduced and the BER improves. For $\overline{E_b}/N_0$ tending to $\infty$, i.e., a noiseless environment, the BER still remains at a nonzero value of 0.0558. Under such circumstances, the errors are caused by the mutual interference between users. If further reduction of such interference-limited BER is required, a larger spreading factor can be used. In this particular example, if the spreading factor is changed to 1000, the BER goes to zero as $\overline{E_b}/N_0 \to \infty$. Notice that when the spreading factor is increased by 10 times, the data rate will drop by the same factor if the chip duration $T_c$ is kept constant, or the bandwidth of the transmitted signal will be increased by 10 times if the bit duration $T_b$ is fixed.

### B. Simulations

Computer simulations have been performed based on the demodulator shown in Fig. 5. The bit period is assumed to be $10^{-4}$s with $\beta = 100$ and 1000. For each of the different $\overline{E_b}/N_0$ values simulated, 10 000 bits have been transmitted for each user and the average BERs are tabulated in Table V.

### C. Comparisons

The computed and simulated BERs are plotted against $\overline{E_b}/N_0$ in Figs. 6 and 7 for $\beta = 100$ and 1000, respectively. In both cases, the numerical BERs agree well with the simulated results. Hence, we can conclude that the calculation described above provides an accurate means to estimate the BERs for a multiuser coherent CSK communication system.

## IV. Conclusions

In this paper, we have reviewed the operation of a single-user coherent CSK communication system. Using a cubic map as the chaos generating function, we present an approach to calculating the approximate bit error rates for various average-bit-energy-to-noise-power-spectral-density ratios ($\overline{E_b}/N_0$). Simulations are performed to verify the calculated BERs.

Assuming that perfect synchronization is achieved at all receivers, we then extend our analysis on the coherent CSK system to a multiuser environment. BERs are again derived and compared with the simulation results. It is found that the computed BERs provide a very good estimation of the actual performance. As expected, the BER decreases with increasing $\overline{E_b}/N_0$. Moreover, the BERs can be reduced for the same $\overline{E_b}/N_0$ by using a higher spreading factor. In a multiuser environment, more interference will be introduced into the system when more users are

$$\begin{aligned}
&\text{Prob}\left(z_{j,+1}\left(mT_b\right) < 0 \mid \text{“}+1\text{” is transmitted}\right)\\
&\quad = Q\left(\frac{\overline{z_{j,+1}\left(mT_b\right)}}{\sqrt{\text{var}\left[z_{j,+1}\left(mT_b\right)\right]}}\right)\\
&\quad = \begin{cases} Q\left(\frac{0.5}{\sqrt{1.24\times10^{-3}+2.5\times10^{-3}(2N-1)+1.0\times10^{-2}\frac{N_0}{2T_c}}}\right), & \text{if } \beta=100\\ Q\left(\frac{0.5}{\sqrt{1.24\times10^{-4}+2.5\times10^{-4}(2N-1)+1.0\times10^{-3}\frac{N_0}{2T_c}}}\right), & \text{if } \beta=1000 \end{cases}
\end{aligned} \tag{39}$$

$$\text{BER}_{\text{multiuser}} = \begin{cases} Q\left(\frac{0.5}{\sqrt{1.24\times10^{-3}+2.5\times10^{-3}(2N-1)+1.0\times10^{-2}\frac{N_0}{2T_c}}}\right), & \text{if } \beta=100\\ Q\left(\frac{0.5}{\sqrt{1.24\times10^{-4}+2.5\times10^{-4}(2N-1)+1.0\times10^{-3}\frac{N_0}{2T_c}}}\right), & \text{if } \beta=1000 \end{cases} \tag{40}$$

$$\text{BER}_{\text{multiuser}} = \begin{cases} Q\left(\frac{\overline{E_b}}{\sqrt{1.24\times10^{-3}T_b^2+2.5\times10^{-3}(2N-1)T_b^2+0.5N_0T_b}}\right), & \text{if } \beta=100\\ Q\left(\frac{\overline{E_b}}{\sqrt{1.24\times10^{-4}T_b^2+2.5\times10^{-4}(2N-1)T_b^2+0.5N_0T_b}}\right), & \text{if } \beta=1000 \end{cases} \tag{43}$$

TABLE IV
CALCULATED BERs OF A MULTI-USER COHERENT CSK SYSTEM FOR VARIOUS $\overline{E_b}/N_0$: (a) $\beta = 100$ AND (b) $\beta = 1000$

(a)

| $\overline{E_b}/N_0$ in dB | Number of users | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 0 | 0.1605 | 0.1696 | 0.1801 | 0.1898 | 0.1986 | 0.2067 | 0.2142 | 0.2212 | 0.2277 | 0.2338 | 0.2395 |
| 2 | 0.1067 | 0.1203 | 0.1356 | 0.1493 | 0.1618 | 0.1730 | 0.1832 | 0.1926 | 0.2012 | 0.2091 | 0.2164 |
| 4 | 0.0599 | 0.0772 | 0.0971 | 0.1148 | 0.1307 | 0.1450 | 0.1578 | 0.1694 | 0.1800 | 0.1896 | 0.1984 |
| 6 | 0.0263 | 0.0446 | 0.0672 | 0.0880 | 0.1067 | 0.1235 | 0.1385 | 0.1520 | 0.1641 | 0.1751 | 0.1852 |
| 8 | 0.0082 | 0.0235 | 0.0463 | 0.0688 | 0.0894 | 0.1080 | 0.1246 | 0.1395 | 0.1529 | 0.1650 | 0.1759 |
| 10 | 0.0016 | 0.0118 | 0.0328 | 0.0558 | 0.0776 | 0.0974 | 0.1152 | 0.1310 | 0.1452 | 0.1580 | 0.1696 |
| 12 | 0.0002 | 0.0059 | 0.0245 | 0.0473 | 0.0698 | 0.0903 | 0.1088 | 0.1254 | 0.1402 | 0.1535 | 0.1655 |
| 14 | 0.0000 | 0.0032 | 0.0195 | 0.0420 | 0.0647 | 0.0857 | 0.1047 | 0.1217 | 0.1369 | 0.1505 | 0.1628 |
| $\infty$ | 0.0000 | 0.0006 | 0.0118 | 0.0328 | 0.0558 | 0.0776 | 0.0974 | 0.1152 | 0.1310 | 0.1452 | 0.1580 |

(b)

| $\overline{E_b}/N_0$ in dB | Number of users | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 0 | 0.1588 | 0.1598 | 0.1610 | 0.1622 | 0.1633 | 0.1645 | 0.1656 | 0.1668 | 0.1679 | 0.1690 | 0.1701 |
| 2 | 0.1043 | 0.1057 | 0.1075 | 0.1093 | 0.1110 | 0.1127 | 0.1144 | 0.1161 | 0.1177 | 0.1194 | 0.1210 |
| 4 | 0.0568 | 0.0586 | 0.0609 | 0.0631 | 0.0653 | 0.0675 | 0.0697 | 0.0718 | 0.0739 | 0.0761 | 0.0782 |
| 6 | 0.0233 | 0.0251 | 0.0273 | 0.0295 | 0.0318 | 0.0341 | 0.0364 | 0.0387 | 0.0410 | 0.0433 | 0.0456 |
| 8 | 0.0062 | 0.0073 | 0.0089 | 0.0105 | 0.0123 | 0.0142 | 0.0161 | 0.0181 | 0.0202 | 0.0223 | 0.0245 |
| 10 | 0.0008 | 0.0013 | 0.0019 | 0.0027 | 0.0037 | 0.0049 | 0.0061 | 0.0076 | 0.0091 | 0.0108 | 0.0126 |
| 12 | 0.0000 | 0.0001 | 0.0003 | 0.0005 | 0.0009 | 0.0014 | 0.0021 | 0.0030 | 0.0040 | 0.0052 | 0.0066 |
| 14 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0004 | 0.0008 | 0.0012 | 0.0019 | 0.0027 | 0.0037 |
| $\infty$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0004 | 0.0008 |

TABLE V
SIMULATED BERs OF A MULTI-USER COHERENT CSK SYSTEM FOR VARIOUS $\overline{E_b}/N_0$: (a) $\beta = 100$ AND (b) $\beta = 1000$

(a)

| $\overline{E_b}/N_0$ in dB | Number of users | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 0 | 0.1604 | 0.1692 | 0.1824 | 0.1883 | 0.1970 | 0.2065 | 0.2128 | 0.2198 | 0.2267 | 0.2343 | 0.2405 |
| 2 | 0.1050 | 0.1196 | 0.1363 | 0.1471 | 0.1603 | 0.1722 | 0.1828 | 0.1916 | 0.2010 | 0.2102 | 0.2178 |
| 4 | 0.0577 | 0.0760 | 0.0961 | 0.1137 | 0.1294 | 0.1441 | 0.1568 | 0.1693 | 0.1798 | 0.1911 | 0.2004 |
| 6 | 0.0259 | 0.0441 | 0.0655 | 0.0872 | 0.1043 | 0.1225 | 0.1383 | 0.1517 | 0.1636 | 0.1776 | 0.1861 |
| 8 | 0.0072 | 0.0222 | 0.0436 | 0.0689 | 0.0879 | 0.1070 | 0.1248 | 0.1400 | 0.1523 | 0.1678 | 0.1765 |
| 10 | 0.0011 | 0.0106 | 0.0309 | 0.0563 | 0.0755 | 0.0961 | 0.1148 | 0.1302 | 0.1451 | 0.1594 | 0.1705 |
| 12 | 0.0000 | 0.0049 | 0.0226 | 0.0480 | 0.0671 | 0.0896 | 0.1087 | 0.1249 | 0.1401 | 0.1547 | 0.1662 |
| 14 | 0.0000 | 0.0025 | 0.0176 | 0.0426 | 0.0620 | 0.0844 | 0.1037 | 0.1205 | 0.1367 | 0.1512 | 0.1637 |
| $\infty$ | 0.0000 | 0.0003 | 0.0106 | 0.0313 | 0.0534 | 0.0759 | 0.0949 | 0.1151 | 0.1291 | 0.1444 | 0.1618 |

(b)

| $\overline{E_b}/N_0$ in dB | Number of users | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 0 | 0.1620 | 0.1597 | 0.1596 | 0.1631 | 0.1630 | 0.1637 | 0.1660 | 0.1661 | 0.1669 | 0.1700 | 0.1702 |
| 2 | 0.1060 | 0.1051 | 0.1086 | 0.1107 | 0.1125 | 0.1118 | 0.1141 | 0.1155 | 0.1166 | 0.1201 | 0.1209 |
| 4 | 0.0510 | 0.0584 | 0.0602 | 0.0631 | 0.0649 | 0.0665 | 0.0690 | 0.0716 | 0.0732 | 0.0761 | 0.0779 |
| 6 | 0.0208 | 0.0260 | 0.0265 | 0.0291 | 0.0314 | 0.0336 | 0.0360 | 0.0388 | 0.0403 | 0.0431 | 0.0451 |
| 8 | 0.0057 | 0.0077 | 0.0090 | 0.0103 | 0.0121 | 0.0136 | 0.0160 | 0.0179 | 0.0197 | 0.0224 | 0.0241 |
| 10 | 0.0007 | 0.0011 | 0.0018 | 0.0025 | 0.0035 | 0.0046 | 0.0060 | 0.0074 | 0.0090 | 0.0109 | 0.0123 |
| 12 | 0.0000 | 0.0001 | 0.0002 | 0.0005 | 0.0009 | 0.0014 | 0.0021 | 0.0029 | 0.0039 | 0.0051 | 0.0063 |
| 14 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0004 | 0.0007 | 0.0012 | 0.0018 | 0.0025 | 0.0035 |
| $\infty$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0003 | 0.0007 |

transmitting at the same time. Hence, the interference between users can sometimes pose a lower limit on the BER.

In our analysis, the distributions of the partial discrete correlation functions are obtained by simple computer simulations. An alternate approach to determine the distributions is based on the empirical probability density function (pdf) of the chaotic signal. The technique is under investigation and it is expected that the empirical pdf, found by simpler simulations, will provide us with all the data needed to calculate the BERs. In addition, in our derivations of the BERs, it is observed that the correlation properties of the chaotic sequences have a determining factor on the system performance. Chaotic sequences with high auto-correlation and low cross-correlation values would give better error performance. The search of such chaotic sequences would therefore be of great interest to researchers.

## APPENDIX A
## JUSTIFICATION OF USING AN EQUIVALENT NOISE SOURCE FOR ANALYSIS

In a coherent CSK system corrupted by additive white Gaussian noise (AWGN), the input to the demodulator is given by

$$\begin{aligned} p(t) &= s(t) + n(t) \\ &= \sum_{m=1}^{\infty} v^{(m)}(t) + n(t) \end{aligned} \tag{44}$$

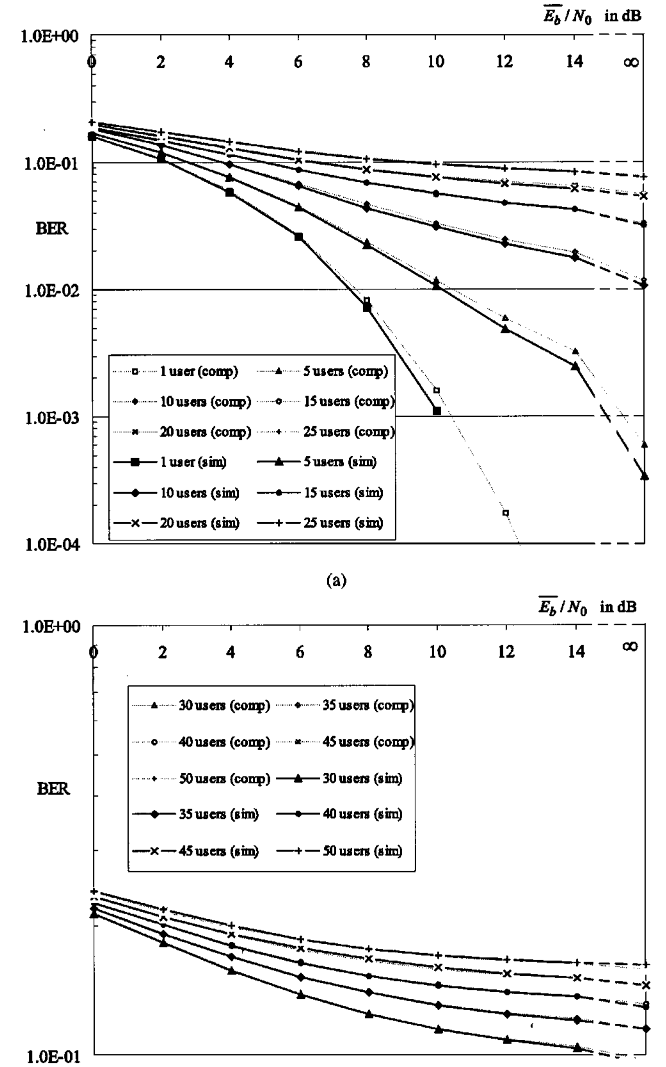


Fig. 6. Computed (comp) and simulated (sim) bit error rates against $\overline{E_b}/N_0$ in a multiuser coherent CSK system ($\beta = 100$): (a) 1–25 users and (b) 30–50 users.

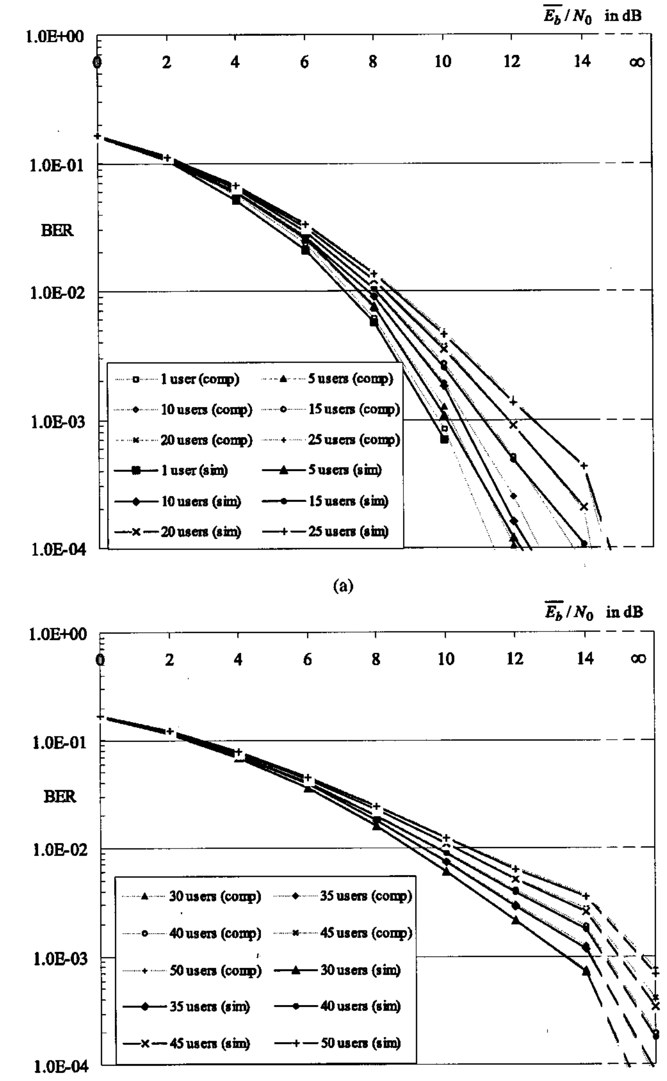


Fig. 7. Computed (comp) and simulated (sim) bit error rates against $\overline{E_b}/N_0$ in a multiuser coherent CSK system ($\beta = 1000$): (a) 1–25 users and (b) 30–50 users.

where $n(t)$ is an AWGN with a two-sided power spectral density given by

$$S_n(f) = \frac{N_0}{2} \quad \text{for all } f. \tag{45}$$

For the $m$th received symbol, the output of correlator$_{-1}$ shown in Fig. 2 at the end of the bit period is given by

$$\begin{aligned}
\text{Corr}_{-1}(mT_b) &= \int_{(m-1)T_b}^{mT_b} p(t)\check{c}(t)dt \\
&= \int_{(m-1)T_b}^{mT_b} \left[v^{(m)}(t) + n(t)\right] \check{c}(t)dt \\
&= \int_{(m-1)T_b}^{mT_b} v^{(m)}(t)\check{c}(t)dt \\
&\quad + \int_{(m-1)T_b}^{mT_b} n(t)\check{c}(t)dt.
\end{aligned} \tag{46}$$

Consider the output component due to noise

$$I_{\text{noise}} = \int_{(m-1)T_b}^{mT_b} n(t)\check{c}(t)dt. \tag{47}$$

It can be divided into several subcomponents shown below.

$$\begin{aligned}
I_{\text{noise}} &= \int_{(m-1)T_b}^{mT_b} n(t)\check{c}(t)dt \\
&= \int_{(m-1)T_b}^{mT_b} n(t) \sum_{k=(m-1)\beta}^{m\beta-1} \check{x}_k r(t - kT_c)\, dt \\
&= \sum_{k=(m-1)\beta}^{m\beta-1} \int_{kT_c}^{(k+1)T_c} n(t)\check{x}_k r(t - kT_c)\, dt \\
&= \sum_{k=(m-1)\beta}^{m\beta-1} \check{x}_k \int_{kT_c}^{(k+1)T_c} n(t)dt \\
&= \sum_{k=(m-1)\beta}^{m\beta-1} n_k
\end{aligned} \tag{48}$$

where

$$n_k = \breve{x}_k \int_{kT_c}^{(k+1)T_c} n(t)dt. \quad (49)$$

Given $\breve{x}_j$ and $\breve{x}_k$, the distribution of the noise sub-components $\{n_k\}$ is Gaussian [13]. Their mean values are

$$E[n_k] = \breve{x}_k \int_{kT_c}^{(k+1)T_c} E[n(t)]dt = 0 \quad (50)$$

for all $k$. Their covariances [14] are

$$\begin{aligned}\text{cov}[n_j, n_k] =& E[n_j n_k] - E[n_j]E[n_k] \\ =& E[n_j n_k] \\ =& \breve{x}_j \breve{x}_k \int_{jT_c}^{(j+1)T_c} \int_{kT_c}^{(k+1)T_c} E[n(t)n(\tau)]\, d\tau dt \\ =& \breve{x}_j \breve{x}_k \int_{jT_c}^{(j+1)T_c} \int_{kT_c}^{(k+1)T_c} \frac{N_0}{2}\delta(t-\tau)d\tau dt \\ =& \breve{x}_k^2 \frac{N_0 T_c}{2}\delta_{jk}\end{aligned} \quad (51)$$

where

$$\delta_{jk} = \begin{cases} 1 & j = k \\ 0 & \text{otherwise.}\end{cases} \quad (52)$$

Hence, the noise sub-components $\{n_k\}$ are zero mean uncorrelated Gaussian variables with variances $\sigma_n^2 = \breve{x}_k^2(N_0 T_c/2)$. Since these noise sub-components are uncorrelated Gaussian random variables, they are also statistically independent.

Consider another noise source given by

$$n'(t) = \sum_{k=0}^{\infty} \xi_k r(t - kT_c) \quad (53)$$

where the coefficients $\{\xi_k\}$ are independent Gaussian random variables with mean $\bar{\xi}$ and variance $\sigma_{n'}^2$. Replacing the AWGN noise source in (44) by $n'(t)$ and using the aforementioned procedures, the correlator output component due to the new noise source can be shown to be

$$\begin{aligned}I'_{\text{noise}} =& \sum_{k=(m-1)\beta}^{m\beta-1} \breve{x}_k \int_{kT_c}^{(k+1)T_c} n'(t)dt \\ =& \sum_{k=(m-1)\beta}^{m\beta-1} \breve{x}_k \xi_k T_c \\ =& \sum_{k=(m-1)\beta}^{m\beta-1} n'_k\end{aligned} \quad (54)$$

where $n'_k = \breve{x}_k \xi_k T_c$. Given $\breve{x}_j$ and $\breve{x}_k$, the distribution of the new noise sub-components $\{n'_k\}$ is also Gaussian. Their mean values are

$$E[n'_k] = \breve{x}_k T_c E[\xi_k] = \breve{x}_k T_c \bar{\xi} \quad (55)$$

and their covariances are

$$\begin{aligned}\text{cov}[n'_j, n'_k] =& E[n'_j n'_k] - E[n'_j]E[n'_k] \\ =& \breve{x}_j \breve{x}_k T_c^2 E[\xi_j \xi_k] - \breve{x}_j \breve{x}_k T_c^2 \bar{\xi}^2 \\ =& \breve{x}_k^2 T_c^2 \sigma_{n'}^2 \delta_{jk} - \breve{x}_j \breve{x}_k T_c^2 \bar{\xi}^2\end{aligned} \quad (56)$$

where the last equality is due to the independence of $\xi_k$'s. Therefore, if the noise source $n'(t)$ is to generate the same effect as the AWGN source at the coherent CSK receiver

$$E[n'_k] = E[n_k] \Rightarrow \breve{x}_k T_c \bar{\xi} = 0 \Rightarrow \bar{\xi} = 0 \quad (57)$$

and

$$\begin{aligned}\text{cov}[n'_j, n'_k] =& \text{cov}[n_j, n_k] \\ \Rightarrow \breve{x}_k^2 T_c^2 \sigma_{n'}^2 \delta_{jk} - \breve{x}_j \breve{x}_k T_c^2 (0)^2 =& \breve{x}_k^2 \frac{N_0 T_c}{2}\delta_{jk} \\ \Rightarrow \sigma_{n'}^2 =& \frac{N_0}{2T_c}.\end{aligned} \quad (58)$$

Since the mean and variance of $\{\xi_k\}$ are independent of the chaotic signals, it can be concluded that in the analyzes of a coherent CSK system with AWGN, the original AWGN noise source can be replaced by an equivalent noise $n'(t)$ given by

$$n'(t) = \sum_{k=0}^{\infty} \xi_k r(t - kT_c) \quad (59)$$

where the coefficients $\{\xi_k\}$ are independent Gaussian random variables with mean

$$\bar{\xi} = 0 \quad (60)$$

and variance

$$\sigma_{n'}^2 = \frac{N_0}{2T_c}. \quad (61)$$

## Appendix B
## Definitions of Partial Discrete Correlation Functions

In the following, $L$ denotes the length of the sequence sample and $k$, $l \geq 0$. The partial discrete auto-correlation function of the chaotic series $\{\breve{x}_k\}$ is defined as

$$\gamma[\breve{x}, \breve{x}, k, l, L] = \frac{1}{L}\sum_{n=0}^{L-1} \breve{x}_{k+n}\breve{x}_{l+n}. \quad (62)$$

The partial discrete cross-correlation function between the chaotic series $\{\breve{x}_k\}$ and $\{\hat{x}_k\}$ is defined as

$$\gamma[\breve{x}, \hat{x}, k, l, L] = \frac{1}{L}\sum_{n=0}^{L-1} \breve{x}_{k+n}\hat{x}_{l+n}. \quad (63)$$

Given a normal random sequence $\Lambda = \{\lambda_0, \lambda_1, \lambda_2, \ldots\}$, the elements of which are normal random variables with zero mean and variance $\sigma_\lambda^2$. The partial discrete cross-correlation function between the chaotic series $\{\breve{x}_k\}$ and the normal random sequence $\{\lambda_k\}$ is defined as

$$\gamma[\breve{x}, \lambda, k, l, L] = \frac{1}{L}\sum_{n=0}^{L-1} \breve{x}_{k+n}\lambda_{l+n}. \quad (64)$$

It has been studied in [15] that using the iterative map $x_{k+1} = g(x_k) = 4x_k^3 - 3x_k$ to generate the chaotic series, the partial discrete correlation functions defined above are normally distributed with fixed means and spreading-factor-dependent variances. The means and variances for $L = 100$ and 1000 are tabulated in Table I.

## APPENDIX C
## DERIVATION OF AVERAGE BIT ENERGY

Applying (4)–(6) to the first part of (23), the average bit energy can be derived to obtain (65) shown at the bottom of the page where $\bar{\gamma}(\hat{x},\beta)$ and $\bar{\gamma}(\breve{x},\beta)$ denote the mean-squared values of chaotic sequence samples of length $\beta$ taken from the chaotic series $\{\hat{x}_k\}$ and $\{\breve{x}_k\}$ respectively; $P^+$ and $P^-$ represent the probability of a "+1" and "−1" being transmitted respectively. Note that $\bar{\gamma}(\hat{x},\beta)$ and $\bar{\gamma}(\breve{x},\beta)$ equal the mean of $\gamma[\hat{x},\hat{x},(m-1)\beta,(m-1)\beta,\beta]$ and the mean of $\gamma[\breve{x},\breve{x},(m-1)\beta,(m-1)\beta,\beta]$, respectively, over all positive integers $m$.

## APPENDIX D
## ANALYSES OF THE DISCRETE CORRELATION FUNCTIONS AT THE INPUT TO THE DETECTOR

Given the input to the detector is

$$\begin{aligned} z_{j,+1}(mT_b) =& \beta T_c \Big(\gamma\left[\hat{x}^{(j)},\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &+\sum_{\substack{i=1\\ i\neq j}}^{N}\gamma\left[y^{(m,i)},\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &+\gamma\left[\xi,\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &-\gamma\left[\hat{x}^{(j)},\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &-\sum_{\substack{i=1\\ i\neq j}}^{N}\gamma\left[y^{(m,i)},\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &-\gamma\left[\xi,\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right]\Big). \end{aligned} \tag{66}$$

Inside the bracket, the first term $\gamma[\hat{x}^{(j)},\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta]$ represents the partial discrete auto-correlation function of the chaotic series $\{\hat{x}_k^{(j)}\}$. The third and sixth terms, $\gamma[\xi,\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta]$ and $\gamma[\xi,\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta]$, correspond to the partial discrete cross-correlation functions between the normal random sequence $\{\xi_k\}$ and the chaotic series $\{\hat{x}_k^{(j)}\}$ and $\{\breve{x}_k^{(j)}\}$ respectively. The fourth term $\gamma[\hat{x}^{(j)},\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta]$ is the partial discrete cross-correlation function between the chaotic series $\{\hat{x}_k^{(j)}\}$ and $\{\breve{x}_k^{(j)}\}$.

In the first summation, the terms $\gamma[y^{(m,i)},\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta]$ ($i=1,2,\ldots,N$ and $i\neq j$) refer to the partial discrete cross-correlation functions between the chaotic series pairs $\{\breve{x}_k^{(i)}\}$ and $\{\hat{x}_k^{(j)}\}$, or $\{\hat{x}_k^{(i)}\}$ and $\{\hat{x}_k^{(j)}\}$, depending on the $m$th transmitted symbol of the $i$th user is "−1" or "+1". Since $i\neq j$, the two series undergoing correlation will not be derived from the same initial condition. Similarly, the terms in the second summation, $\gamma[y^{(m,i)},\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta]$ ($i=1,2,\ldots,N$ and $i\neq j$), refer to the partial discrete cross-correlation functions between the chaotic series pairs $\{\breve{x}_k^{(i)}\}$ and $\{\breve{x}_k^{(j)}\}$, or $\{\hat{x}_k^{(i)}\}$ and $\{\breve{x}_k^{(j)}\}$. Also, the two series undergoing correlation will not be generated from the same initial condition.

Applying the central limit theorem, all the $\gamma$ functions (partial discrete correlation functions) in (66) can be approximated by normal random variables.

## APPENDIX E
## DERIVATION OF MEAN AND VARIANCE OF THE INPUT TO THE DETECTOR

Given that the input to the detector is

$$\begin{aligned} z_{j,+1}(mT_b) =& \beta T_c \Big(\gamma\left[\hat{x}^{(j)},\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &+\sum_{\substack{i=1\\ i\neq j}}^{N}\gamma\left[y^{(m,i)},\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &+\gamma\left[\xi,\hat{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &-\gamma\left[\hat{x}^{(j)},\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &-\sum_{\substack{i=1\\ i\neq j}}^{N}\gamma\left[y^{(m,i)},\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right] \\ &-\gamma\left[\xi,\breve{x}^{(j)},(m-1)\beta,(m-1)\beta,\beta\right]\Big). \end{aligned} \tag{67}$$

$$\begin{aligned} \overline{E_b} =& T_b \lim_{T\to\infty}\frac{1}{T}\int_0^T\left(\sum_{m=1}^{\infty}v^{(m)}(t)\right)^2 dt \\ =& T_b \lim_{M\to\infty}\frac{1}{MT_b}\int_0^{MT_b}\sum_{m=1}^{M}\left(v^{(m)}(t)\right)^2 dt \\ =& T_b \lim_{M\to\infty}\frac{1}{MT_b}\sum_{m=1}^{M}\int_{(m-1)T_b}^{mT_b}\left(\sum_{k=0}^{\beta-1}y_{k+(m-1)\beta}^{(m)}r\left[t-(kT_c+(m-1)T_b)\right]\right)^2 dt \\ =& T_b \lim_{M\to\infty}\frac{1}{MT_b}\sum_{m=1}^{M}\sum_{k=0}^{\beta-1}\left(y_{k+(m-1)\beta}^{(m)}\right)^2 T_c \\ =& T_c\left(\beta\bar{\gamma}(\hat{x},\beta)P^+ + \beta\bar{\gamma}(\breve{x},\beta)P^-\right) \end{aligned} \tag{65}$$

All the $\gamma$ functions in (67) can be approximated by normal random variables (see Appendix D). Assume that the random variables are independent of each other. The mean value of $z_{j,+1}(mT_b)$ is

$$\begin{aligned}\overline{z_{j,+1}(mT_b)} =& \beta T_c \Big(\overline{\gamma\left[\hat{x}^{(j)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &+ \overline{\sum_{\substack{i=1\\ i\neq j}}^{N} \gamma\left[y^{(m,i)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &+ \overline{\gamma\left[\xi, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &- \overline{\gamma\left[\hat{x}^{(j)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &- \overline{\sum_{\substack{i=1\\ i\neq j}}^{N} \gamma\left[y^{(m,i)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &- \overline{\gamma\left[\xi, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]}\Big) \\ =& \beta T_c \Big(\overline{\gamma\left[\hat{x}^{(j)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &+ \sum_{\substack{i=1\\ i\neq j}}^{N} \overline{\gamma\left[y^{(m,i)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &+ \overline{\gamma\left[\xi, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &- \overline{\gamma\left[\hat{x}^{(j)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &- \sum_{\substack{i=1\\ i\neq j}}^{N} \overline{\gamma\left[y^{(m,i)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]} \\ &- \overline{\gamma\left[\xi, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]}\Big) \end{aligned} \tag{68}$$

and the variance equals (69) shown at the bottom of the page.

Using the results in Table I, the values of $\overline{z_{j,+1}(mT_b)}$ and var$[z_{j,+1}(mT_b)]$ can now be found. For $\beta = 100$,

$$\begin{aligned}\overline{z_{j,+1}(mT_b)} =& \beta T_c \left(0.5 + \sum_{\substack{i=1\\ i\neq j}}^{N}(0) + 0 - 0 - \sum_{\substack{i=1\\ i\neq j}}^{N}(0) - 0\right) \\ =& 0.5\beta T_c \end{aligned} \tag{70}$$

and

$$\begin{aligned}\text{var}\left[z_{j,+1}(mT_b)\right] =& \beta^2 T_c^2 \Bigg(1.24\times 10^{-3} + \sum_{\substack{i=1\\ i\neq j}}^{N} 2.50\times 10^{-3} \\ &+ 5.0\times 10^{-3}\sigma_{n'}^2 + 2.50\times 10^{-3} \\ &+ \sum_{i=1\, i\neq j}^{N} 2.50\times 10^{-3} + 5.0\times 10^{-3}\sigma_{n'}^2\Bigg) \\ =& \beta^2 T_c^2 \Bigg(1.24\times 10^{-3} + 2.5\times 10^{-3}(2N-1) \\ &+ 1.0\times 10^{-2}\frac{N_0}{2T_c}\Bigg). \end{aligned} \tag{71}$$

For $\beta = 1000$

$$\begin{aligned}\overline{z_{j,+1}(mT_b)} =& \beta T_c \left(0.5 + \sum_{\substack{i=1\\ i\neq j}}^{N}(0) + 0 - 0 - \sum_{\substack{i=1\\ i\neq j}}^{N}(0) - 0\right) \\ =& 0.5\beta T_c \end{aligned} \tag{72}$$

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$$\begin{aligned}\text{var}\left[z_{j,+1}(mT_b)\right] =& \beta^2 T_c^2 \Bigg(\text{var}\left[\gamma\left[\hat{x}^{(j)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] \text{var}\left[\sum_{\substack{i=1\\ i\neq j}}^{N} \gamma\left[y^{(m,i)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] \\ &+ \text{var}\left[\gamma\left[\xi, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] + \text{var}\left[\gamma\left[\hat{x}^{(j)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] \\ &+ \text{var}\left[\sum_{\substack{i=1\\ i\neq j}}^{N} \gamma\left[y^{(m,i)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] + \text{var}\left[\gamma\left[\xi, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right]\Bigg) \\ =& \beta^2 T_c^2 \Bigg(\text{var}\left[\gamma\left[\hat{x}^{(j)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] + \sum_{\substack{i=1\\ i\neq j}}^{N} \text{var}\left[\gamma\left[y^{(m,i)}, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] \\ &+ \text{var}\left[\gamma\left[\xi, \hat{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] + \text{var}\left[\gamma\left[\hat{x}^{(j)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] \\ &+ \sum_{\substack{i=1\\ i\neq j}}^{N} \text{var}\left[\gamma\left[y^{(m,i)}, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right] + \text{var}\left[\gamma\left[\xi, \breve{x}^{(j)}, (m-1)\beta, (m-1)\beta, \beta\right]\right]\Bigg) \end{aligned} \tag{69}$$

and

$$\begin{aligned}\mathrm{var}[z_{j,+1}(mT_b)] =& \beta^2 T_c^2 \left( 1.24 \times 10^{-4} + \sum_{\substack{i=1\\ i\neq j}}^{N} 2.50 \times 10^{-4} \right.\\ & + 5.0 \times 10^{-4} \sigma_{n'}^2 + 2.50 \times 10^{-4} \\ & \left. + \sum_{i=1_{i\neq j}}^{N} 2.50 \times 10^{-4} + 5.0 \times 10^{-4} \sigma_{n'}^2 \right) \\ =& \beta^2 T_c^2 \Big( 1.24 \times 10^{-4} + 2.5 \times 10^{-4} (2N-1) \\ & + 1.0 \times 10^{-3} \frac{N_0}{2T_c} \Big). \end{aligned} \tag{73}$$

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