Time-Variant Structural Parameter Identification

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Abstract
A new method based on windowed measured dynamic response is proposed for model updating of a time-variant structural system with unknown initial structural responses. A two phase identification algorithm is presented to identify both the initial structural responses and the time-variant structural parameter in each small time interval. Tikhonov regularization method is applied for the former while a modified adaptive regularization method is proposed to identify the structural parameter. The second method takes care of the initial model errors in updating the structural parameters. A multi-storey linear shear frame structure with and without nonlinear seismic isolators subject to seismic ground motion is used for the numerical study. A normally distributed initial model error of the structure is included. The proposed time-variant parameter identification method is found capable of identifying the time-variant parameters fairly accurately even with 10% measurement noise.

Key words: Sensitivity, Time-variant, Parameter Identification, L-curve, Adaptive Regularization, Nonlinearity

1. Introduction

Structural parameter identification and model updating have been actively investigated in the last few decades. A structure may suffer from abrupt damages when under severe earthquake and some structural components may perform nonlinearly. It is therefore important to evaluate the condition of structural components and the load-bearing capacity of the structural system after the earthquake. However, it is difficult to identify when and where damage occurs based on the measurements from the structural system. It is also difficult to assess the severity of the damage with measurement polluted by noise. Furthermore, the analysis results are commonly influenced by model error.

A lot of methods have been developed for the linear structural condition assessment in the past. There are mainly three categories of methods for structural parameter identification: time domain methods, frequency domain methods and methods in the time-frequency domain. Kerschen et al. investigated the time-variant structural parameter identification which can be used for linear or nonlinear structures(1). The Kalman filter is an effective mean of system parameter identification and input estimation for a linear or nonlinear structure. Haykin et al. presented two forms of the extended recursive least-squares algorithm(2) which were considered for the identification of system parameter and the tracking of a chirped sinusoid with additive noise. There are also some other time-variant parameter identification methods, such as, the online identification of nonlinear hysteretic structure with an adaptive tracking techniques(3),(4) based on least-squares estimation by Yang et al., nonlinear normal mode analysis which considered the nonlinearity of structural system(5) proposed by Kerschen et al., an online sequential weighted least-squares support vector machine technique[6] illustrated by Peeters et al, and the dynamic response sensitivity method[7] with a moving time window. These methods do not have the assumption that the time of occurrence of the anomalies is known a priori. Hence, these methods could be applied to conduct the structural condition assessment online. However, most existing methods for time-variant parameter identification do not consider the uncertainties in the structural parameters or measurements.

Sensitivity methods in time domain have been investigated and applied extensively for parameter identification of linear structures. The sensitivity matrix of response with respect to the structural parameters has been used to locate and quantify the damage[8] or damping ratio[7]. Time-variant damping ratio identification has been presented with Chebyshev polynomial[9] or a moving time window[7]. However, these literatures did not consider the effect of non-zero initial structural response or the nonlinearity of structure, both of which would influence the identification result.

A new method for the time-variant parameter identification based on windowed measured data is presented in this study. The time history of measured acceleration is divided into different short non-overlapping time segments. The initial structural responses in each time segment are unknown and the structural parameters are assumed to be invariant in each of these short time intervals. A two phase identification strategy is applied to ensure the physical meaning and convergence of the identified results with the identification algorithm. In the first phase, the initial structural response in each time interval is identified with the Tikhonov regularization method. In the second phase, the structural parameter is identified with a modified adaptive regularization method. Three types of structures subject to seismic ground motion are investigated to demonstrate the proposed method, i.e. a shear frame with abrupt damage, a shear frame with nonlinear base isolation on the first floor and a shear frame with seismic resisting bracing on each floor. The results of identification are shown to be accurate even with 10% measurement noise.

2. Methodology

The equation of motion of an N dofs damped structural system subject to ground excitation is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{G}\ddot{x}_g \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the structural system respectively. $\ddot{\mathbf{x}}_g$ is the ground acceleration, $\mathbf{G}$ is the location matrix of the seismic force. $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ are respectively the vectors of acceleration, velocity and displacement responses of the structural system. Rayleigh damping model is assumed and damping ratios of the first two modes are taken to be 0.01 and 0.015 respectively.

2.1 Existing time response sensitivity method

Time response sensitivity method for model updating and structural parameter identification commonly assumes time-invariant structural parameters and zero initial response of the structure. If the damage extent of the i th element in the structure is represented as a reduction factor, α_i, a change in the global stiffness matrix of the structure can be described as

$$\Delta\mathbf{K} = \sum_{i}^{Ne} \alpha_i \mathbf{K}_i \tag{2}$$

where Ne denotes the total number of finite elements of the structure. Performing differentiation to both sides of Eq. (1) with respect to the structural parameters α_i, existing sensitivity method would give

$$\mathbf{M}\frac{\partial \ddot{\mathbf{x}}}{\partial \alpha_i} + \mathbf{C}\frac{\partial \dot{\mathbf{x}}}{\partial \alpha_i} + \mathbf{K}\frac{\partial \mathbf{x}}{\partial \alpha_i} = -\frac{\partial \mathbf{K}}{\partial \alpha_i}\mathbf{x} - a_2\frac{\partial \mathbf{K}}{\partial \alpha_i}\dot{\mathbf{x}} \tag{3}$$

The responses $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ are obtained by the step-by-step time integration method from Eq. (1). They are then substituted into Eq. (3), and the sensitivity matrices

$\partial \ddot{\mathbf{x}}_s / \partial \alpha_i, \partial \dot{\mathbf{x}}_s / \partial \alpha_i, \partial \mathbf{x}_s / \partial \alpha_i$ can then be solved from Eq. (3) using the time integration method. The local change of stiffness can be found with different optimization tools. In the following studies, the "measured" response, $\ddot{\mathbf{x}}_m$, is calculated as the solution of Eq. (1) from the finite element model of the structure with the time-invariant parameters. Taking acceleration as the measured information, the Taylor series expansion on the difference between the "measured" response, $\ddot{\mathbf{x}}_m$, and the updated response, $\ddot{\mathbf{x}}$, from Eq. (1) can be represented as

$$\ddot{\mathbf{x}}_{\mathrm{m}} - \ddot{\mathbf{x}} = \frac{\partial \ddot{\mathbf{x}}}{\partial \boldsymbol{\alpha}} \cdot \boldsymbol{\alpha} + o(\boldsymbol{\alpha}^2) \tag{4}$$

The stiffness reduction vector $\boldsymbol{\alpha}$ can be calculated from Eq. (4) with an optimization method. However, the time-variant parameter identification cannot be conducted accurately without knowledge of the initial structural responses.

2.2 Time-variant parameter identification

The structural parameter is time-variant during a seismic event or under strong wind conditions. When the time-variant parameter is included, Eq. (1) can be rewritten as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}(t)\mathbf{x} = -\mathbf{MG}\ddot{x}_g \tag{5}$$

where the stiffness matrix **K**(t) is time-variant. When the stiffness is also nonlinear, Eq. (5) could be rewritten as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}(t, \mathbf{x})\mathbf{x} = -\mathbf{MG}\ddot{x}_g \tag{6}$$

The equation of motion of the structural system can be further discretized as

$$\mathbf{M}\ddot{\mathbf{x}}(t)_i + \mathbf{C}\dot{\mathbf{x}}(t)_i + \mathbf{K}(t, x)_i \mathbf{x}(t)_i = -\mathbf{MG}\ddot{x}_g(t)_i \tag{7}$$

where subscript i denotes the i th time instant.

A general method is presented in this section to identify the initial structural responses and parameter in each time window. The structural parameter is assumed constant in each short time interval. The time history responses **Y** is a function of the initial structural response $\mathbf{Y}_0$, external force **F** and structural parameter $\boldsymbol{\alpha}$. The response vector can therefore be represented as

$$\mathbf{Y} = f(\mathbf{Y}_0, \mathbf{F}, \boldsymbol{\alpha}) \tag{8}$$

The responses of the structure can be considered as the summation of free vibration due to the non-zero initial responses and the forced vibration in each time duration. Eq. (8) can be rewritten as

$$\mathbf{Y}_{\mathrm{m}} = \mathbf{Y}_{\mathrm{fo}} + \mathbf{Y}_{\mathrm{fr}} = h(\mathbf{F}, \boldsymbol{\alpha}) + g(\mathbf{Y}_0, \boldsymbol{\alpha}) \tag{9}$$

where subscript m denotes the measured response, $\mathbf{Y}_{\mathrm{fo}} = h(\mathbf{F}, \boldsymbol{\alpha})$ and $\mathbf{Y}_{\mathrm{fr}} = g(\mathbf{Y}_0, \boldsymbol{\alpha})$ are respectively the forced and free vibration responses. Considering free vibration only, the initial structural response could be represented as the summation of all mode shapes of the structure as

$$\mathbf{Y}_0 = \begin{bmatrix} \boldsymbol{\Phi} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi} \end{bmatrix} \boldsymbol{\beta} \tag{10}$$

where $\boldsymbol{\Phi}$ is the normalized mode shape matrix of the structure and $\boldsymbol{\beta}$ is a $(2\times Ndof)\times 1$ vector of contribution coefficients of the vibration modes. $Ndof$ is the number of dofs of the structural system. Initial structural response of $\mathbf{Y}_0$ has also dimensions $(2\times Ndof)\times 1$. When the structural model error or local damage is also included, the measured response as shown in Eq. (9) can be extended to

$$\mathbf{Y}_{\mathrm{m}} = \mathbf{Y}_{\mathrm{fo}} + \mathbf{Y}_{\mathrm{ini}}\boldsymbol{\beta} + \mathbf{S}\Delta\boldsymbol{\alpha} \tag{11}$$

where $\mathbf{Y}_{\mathrm{ini}}$ is the free vibration response vector of the structure arising from the vector of initial response at all dofs of the structure. Equation (11) can be written as

$$\mathbf{Y}_{\mathrm{m}} - \mathbf{Y}_{\mathrm{fo}} = \begin{bmatrix} \mathbf{Y}_{\mathrm{ini}} & \mathbf{S} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta} \\ \Delta\boldsymbol{\alpha} \end{bmatrix} \tag{12}$$

It is noted that the unknown vector on the right-hand-side of Eq. (12) consists of the coefficient vector $\boldsymbol{\beta}$ and the stiffness change coefficients. A two phase algorithm is proposed in next section for the identification of these unknown parameters.

3. Identification with modified adaptive regularization method

Iterative regularization methods[(10),(11)] are usually adopted in practical inverse problems, such as model updating and force identification. The problem in Eq. (12) could be directly solved by iterative Tikhonov regularization with the following objective function

$$J(\Delta\boldsymbol{\alpha}^{k+1},\boldsymbol{\beta}^{k+1},\lambda)=\left\|\mathbf{Y}_{\mathrm{fr}}^{k}\boldsymbol{\beta}^{k+1}+\mathbf{S}^{k}\Delta\boldsymbol{\alpha}^{k+1}-\Delta\ddot{\mathbf{x}}^{k}\right\|^{2}+\lambda^{2}\left\|\begin{bmatrix}\boldsymbol{\beta}^{k+1} & \Delta\boldsymbol{\alpha}^{k+1}\end{bmatrix}^{T}\right\|^{2} \tag{13}$$

where $\mathbf{S}$ is the sensitivity matrix calculated from Eq. (3) and k denotes the k th iteration of the identification. Inverse problem is always ill-posed and measurement noise may have adverse effect in the process of model updating. But the convergence and physical meaning of structural parameters cannot be guaranteed due to the adverse influence of measurement noise.

A two phase identification algorithm is described as follows. The initial structural response is identified in the first phase while the structural parameter is identified in the second phase. Therefore, the iterative Tikhonov regularization method is directly applied to the objective function as

$$J(\Delta\boldsymbol{\beta}^{k+1},\lambda)=\left\|\mathbf{Y}_{\mathrm{fr}}^{k}\boldsymbol{\beta}^{k+1}-\Delta\ddot{\mathbf{x}}^{k}\right\|^{2}+\lambda^{2}\left\|\boldsymbol{\beta}^{k+1}\right\|^{2} \tag{14}$$

A modified adaptive regularization method is utilized in the second phase to ensure that the physical meaning of the identified results is met. An adaptive limit[(12)] on the summation of the identified changes will be computed based on results from last iteration step. The objective function of optimization is expressed as

$$J(\Delta\boldsymbol{\alpha}^{k+1},\lambda)=\left\|\mathbf{S}^{k}\Delta\boldsymbol{\alpha}^{k+1}-\Delta\ddot{\mathbf{x}}^{k}\right\|+\lambda^{2}\left\|\sum_{i=1}^{k+1}\Delta\boldsymbol{\alpha}^{i}-\boldsymbol{\alpha}^{k,*}\right\| \tag{15}$$

where $\boldsymbol{\alpha}^{k,*}$ is a value to coordinate the constraint of the solution in the i th iteration in the damage detection process. Parameter $\boldsymbol{\alpha}^{k,*}$ can be defined as

$$(\alpha^{k,*})_j=\begin{cases}0 & \text{if } (\sum_{i=1}^{k}\Delta\alpha^{k})_j>0\\ (\sum_{i=1}^{k}\Delta\alpha^{k})_j & \text{if } (\sum_{i=1}^{k}\Delta\alpha^{k})_j<0\end{cases} \tag{16}$$

where the subscript j denotes the j th element of the structure. $(\sum_{i=1}^{k}\Delta\alpha^{k})_j$ is the cumulative identified change of stiffness. The local damage can then be detected iteratively with the obtained optimal regularization parameter λ_a as

$$\Delta\boldsymbol{\alpha}^{k+1}=((\frac{\partial\ddot{\mathbf{x}}}{\partial\boldsymbol{\alpha}^{k}})^{T}\frac{\partial\ddot{\mathbf{x}}}{\partial\boldsymbol{\alpha}^{k}}+\lambda_{a}^{2}\mathbf{I}_{a})^{-1}(\frac{\partial\ddot{\mathbf{x}}}{\partial\boldsymbol{\alpha}^{k}})^{T}(\ddot{\mathbf{x}}_{\mathrm{m}}^{k}-\ddot{\mathbf{x}}^{k})$$
$$\boldsymbol{\alpha}_{k+1}=\boldsymbol{\alpha}_{k}+\Delta\boldsymbol{\alpha}^{k} \tag{17}$$

However, this method could only detect the structural damage with a fairly accurate initial analytical model as noted in Equations (16) and (17), and it fails when there is model error which may be positive or negative. This adaptive regularization method is modified in this study to take care of the effects of model errors. The initial stiffness of the structural elements is increased by a factor of 1.3 such that the identified stiffness change will always take up a negative value. The adaptive regularization method could then be applied for damage detection via model updating with different values of initial model errors.

The Young's modulus of material in each finite element is assumed to exhibit a normal distribution, and the parameter can be represented as

$$\mathbf{E}=\mu(\mathbf{E})+\delta\cdot\mu(\mathbf{E})\cdot\mathbf{Random} \tag{18}$$

where $\mathbf{E}$ is the vector of modulus for the structural elements, $\mu(\mathbf{E})$ denotes the mean value, δ is the coefficient of variation (COV). **Random** is a Gaussian random variable with zero mean and unit standard deviation.

4. Implementation Procedure

Step 1: Obtain the mass, damping and stiffness matrices of the initial structural model, which may be inaccurate with model errors.

Step 2: Conduct measurement on the structure.

Step 3: Divide the measurement time history into different non-overlapping short time segments numbered i from 1 to n.

Step 4: Identify initial structural response of the i th time segment with Tikhonov regularization method based on the initial finite element model in the first iteration step with the i th segment, or based on the updated finite element model in other subsequent iterations.

Step 5: Identify structural parameter in the i th time segment with the proposed modified adaptive regularization method.

Step 6: Update the FEM of the structure and calculate difference between the updated structural response and the measured structural responses.

Step 7: Set i=i+1 and repeat Steps 4 to 7 until the following convergence criteria are met.

$$\left\| \frac{\Delta\boldsymbol{\alpha}_{k+1} - \Delta\boldsymbol{\alpha}_k}{\Delta\boldsymbol{\alpha}_{k+1}} \right\| \le Tol_1 \quad \text{and} \quad \left\| \frac{\boldsymbol{\beta}_{k+1} - \boldsymbol{\beta}_k}{\boldsymbol{\beta}_{k+1}} \right\| \le Tol_2 \tag{19}$$

5. Numerical Simulation Studies

Three cases of time-variant parameter identification will be reported in this paper. The main structure is assumed linear with the mass of each storey equals 4×10^5 kg and the storey stiffness of each floor equals 2×10^8 N/m. The base excitation is the N-S El-Centro (1940) earthquake ground motion with the peak ground acceleration scaled to 0.3g. The sampling rate of measurement is 2000Hz. Each time segment of the response lasts for 0.4s with 800 data points in each time window, and 75 time segments in 30s duration are included in the identification.

A 0.05 COV in the Young's modulus of material in the finite element model is assumed to simulate initial model errors. When there is noise in the "measured" response, the polluted response is simulated by adding a normal random term to the unpolluted structural responses as

$$\ddot{\mathbf{x}}_{\mathrm{m}} = \ddot{\mathbf{x}} + E_{\mathrm{p}} N_{\mathrm{noise}} \sigma(\ddot{\mathbf{x}}) \tag{20}$$

where E_{p} is the percentage noise level, N_{noise} is a standard normal distribution term with zero mean and unit standard deviation, $\sigma(\ddot{\mathbf{x}})$ is the standard deviation of the "measured" response.

5.1 Case 1 - Shear Frame with abrupt damage

The frame structure described above has 15-storeys with rigid base connection as shown in Fig. 1. A numerical simulation study with 20% abrupt reduction of storey stiffness in the 2^{nd} and 5^{th} floor is conducted. The abrupt stiffness reduction is assumed to occur at 2s from the beginning of the excitation. The horizontal accelerations at the 3^{rd}, 6^{th} 10^{th} and 15^{th} floor floors are taken as the "measured" responses. Figures 2(a) and 2(b) show the comparison of the real storey stiffness and the identified storey stiffness time histories of the 2^{nd} and 5^{th} floor. The time of occurrence, location and severity of the abrupt damage could be identified accurately without noise in the measurement. The identified storey stiffness of the structure at the end of the 30s duration as shown in Figure 3 has very accurate results.

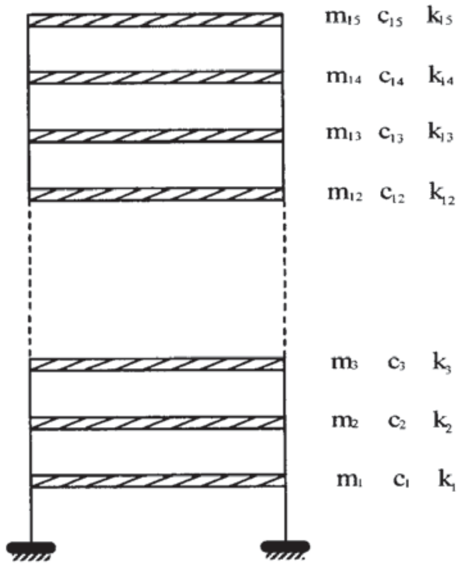


Fig. 1 Fifteen-storey shear frame

(a)

(b)

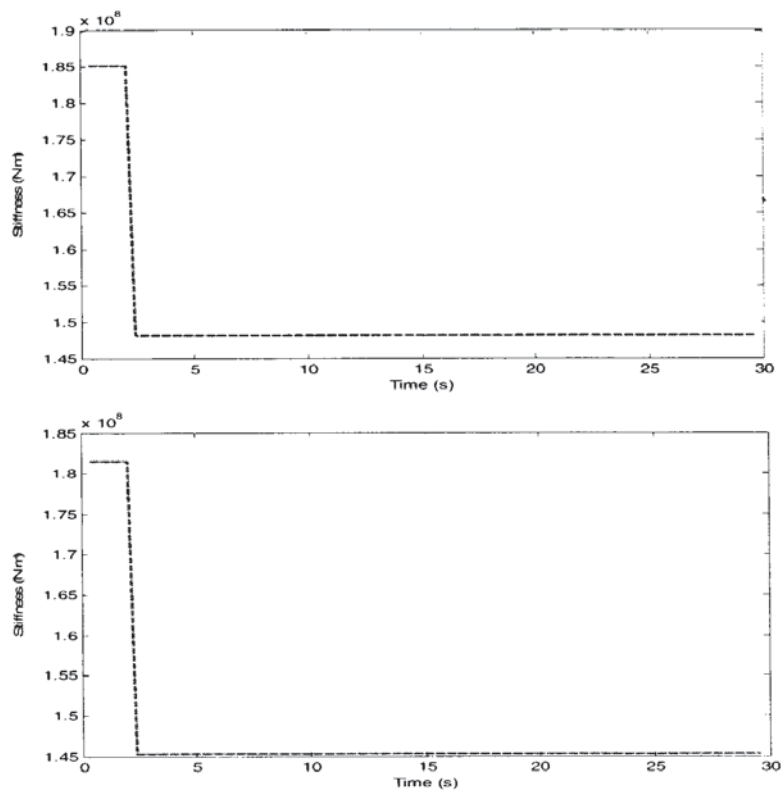


Fig. 2 – Time-variant stiffness identification result – Case 1 (without noise) (a) The stiffness of the 2nd floor, and (b) the stiffness of the 5th floor.

Figure 4 gives the identified storey stiffness time histories of the 2nd and 5th floor when there is 10% measurement noise. The identified damage extent and location are acceptable although there is a large error at the beginning of the time histories as well as some small fluctuations in the time histories. Figure 5 shows the identified storey stiffness at the end of the 30s duration. The error of identification is noted a little bit larger than that obtained from measurement without noise.

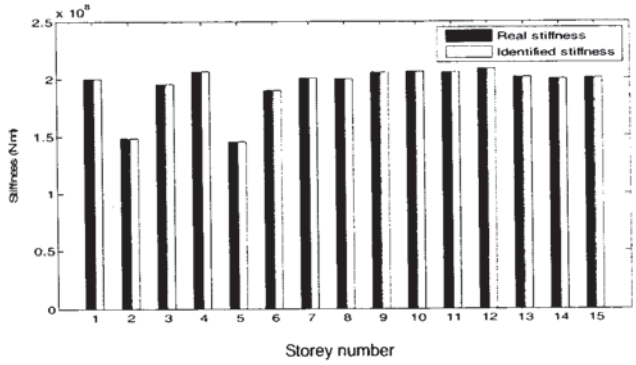
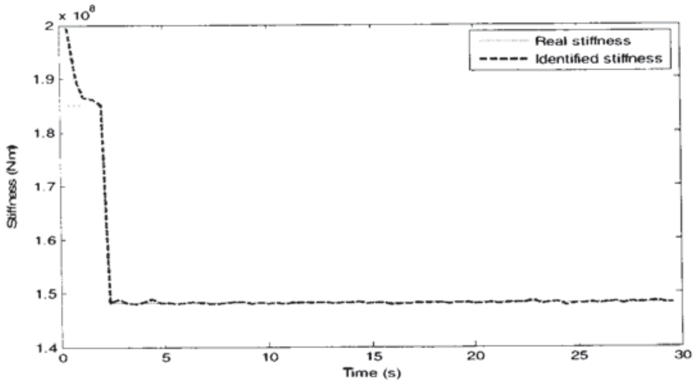


Fig. 3 - Final time-variant stiffness identification result – Case 1 (without noise)

(a)



(b)

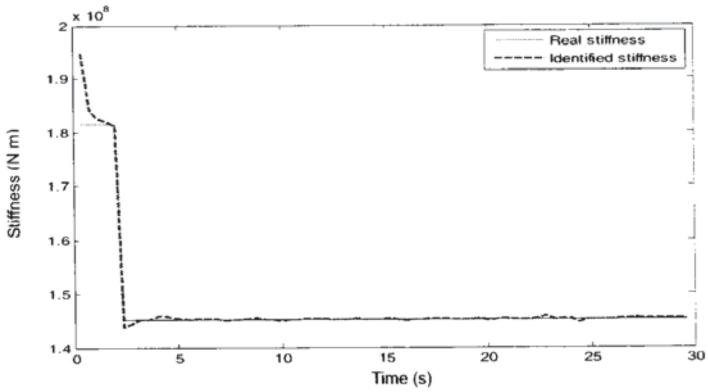


Fig. 4 – Time-variant stiffness identification result – Case 1 (with 10% noise) (a) The stiffness of the 2nd floor, and (b) The stiffness of the 5th floor.

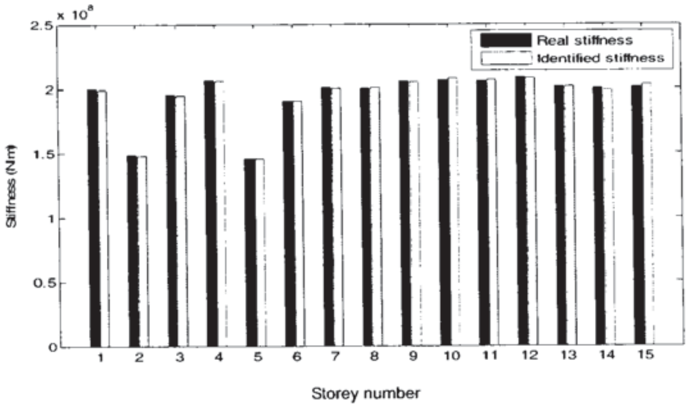


Fig. 5 – Final time-variant stiffness identification result - Case 1 (with 10% noise)

5.2 Case 2 - Shear frame with nonlinear seismic isolators

The same frame structure described earlier but with 10-storeys and additional base isolation to the first floor as shown in Fig. 6 is studied. The base isolation is simulated with a bilinear stiffness model with the restoring force and horizontal displacement relationship as shown in Fig. 7. The pre-yield elastic stiffness is defined by K_E, α_b=0.1 is the ratio of the post-yield stiffness to pre-yield elastic stiffness, and d_y is the displacement at yield. The horizontal restoring force of the isolation is defined as

$$F_b = \alpha_b K_E x_b + (1-\alpha_b) K_E z_b \tag{21}$$

where subscript b denotes the base isolation, x_b is the horizontal deformation of the isolator, and z_b is the horizontal elastic storey drift between ground floor and first floor with K_E =2 $\times 10^7$ N/m and d_y =0.01 m.

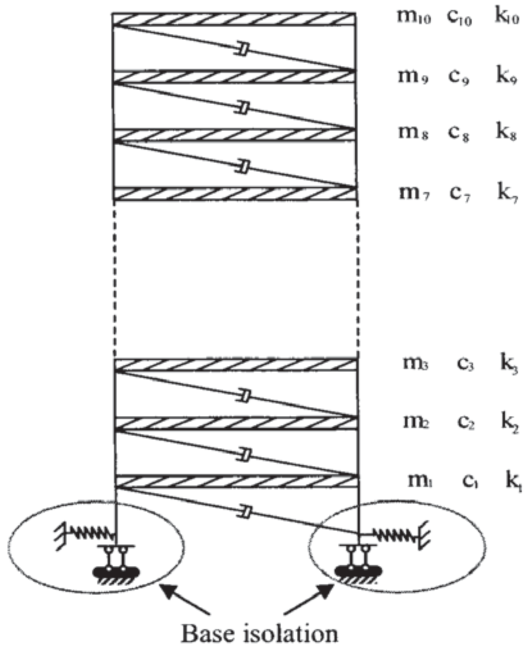


Fig. 6 - shear frame with nonlinear base isolation

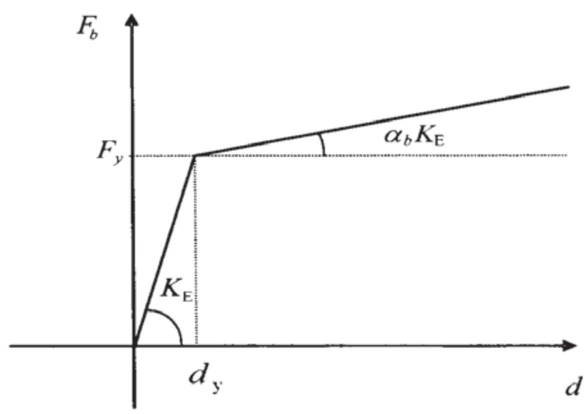


Fig. 7 - Relationship between force and displacement of bilinear restoring force model

The horizontal acceleration responses at the 3rd, 6th and 10th floor are taken as 'measured' response. The model error has a COF=0.05 but there is no stiffness reduction in the main structure above the base isolation. Again 30s of measured data divided into 75 short time segments is used for the identification. Fig. 8 shows that the time history of the nonlinear storey stiffness of the first floor could be identified accurately without measurement noise. But there are some small errors when there is softening effect of the isolation. Fig. 9 gives the identified results when there is 10% measurement noise. There are notable yet small fluctuations in the identified stiffness time history as well as large errors at the beginning of the time history. This may be due to the effect of the unknown initial responses which is transient affecting results in the first few time intervals. However the identification result is fairly accurate and acceptable.

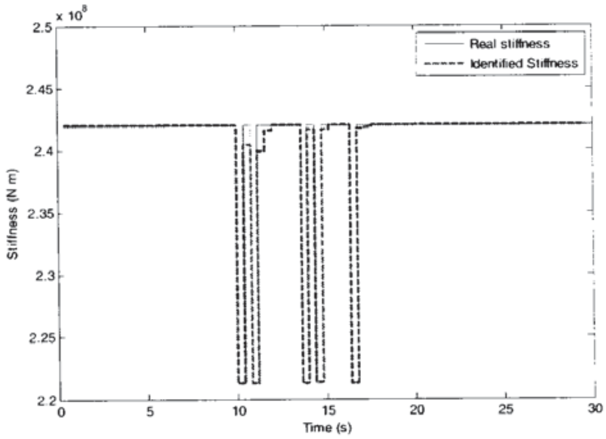


Fig. 8 - Nonlinear time-variant stiffness identification result – Case 2 (without noise)

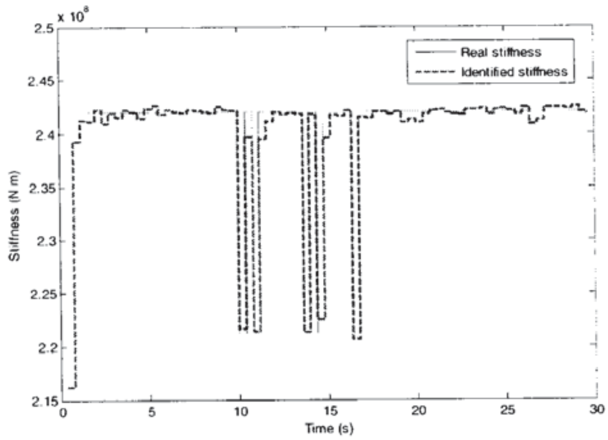


Fig. 9 - Nonlinear time-variant stiffness identification result – Case 2 (with 10% noise)

6. Discussions

In the first phase of identification in Case 1, the number of unknowns is 30 denoting the number of unknown initial displacement and velocity at all storeys of the structure. The number of measured data is 4×800 which is also the number of equations for the identification in each time segment. The size of matrix $\mathbf{Y}_{ini}$ is 3200×30. In the second phase, the number of unknown is 15 which is the number of unknown storey stiffness, and the size of the sensitivity matrix **S** is 3200×15 matrix.

In Cases 2, the number of the measured data is also 4×800 for the first phase of identification. The size of matrix $\mathbf{Y}_{ini}$ is 3200×20 and the size of **S** is 3200×10. In this case, the number of equations is always much larger than the number of unknowns and it is also over-determined. The numerical simulations show that the time-variant parameters could be identified successfully with the problem linearized in a small time interval.

7. Conclusions

A time-variant parameter identification method is developed with the problem linearized in a short time interval. Exact knowledge or assumption on the initial structural responses is not necessary. A two phase identification algorithm is presented to conduct the identification in each time segment. In the first phase, the initial structural response is identified with iterative Tikhonov regularization method while the structural model is updated with a modified adaptive regularization method in the second phase. The time of occurrence, location and severity of local change in the storey stiffness can be identified with acceptable results even when the measurement is polluted with noise. This linearized approach within a short time interval not only could identify the linear abrupt loss of stiffness but it could also identify the bilinear stiffness change in bracings and isolators. These performances enable a possible on-line structural condition assessment of a structure in the event of a severe earthquake.

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Acknowledgements

The work described in this paper was supported by a research grant from the Hong Kong Polytechnic University and the Niche Area research grant project of the Hong Kong Polytechnic University.