Damage Detection for Structures under Ambient Vibration via Consistent Regularization

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Abstract

A consistent regularization technique is proposed for the inverse identification of local damages in a structure under ambient vibration. One new technique involved in the regularization method is the introduction of a new side condition and the other technique restricts the range of variation of the regularization parameter and consistently choosing the optimal point on the L-curve. Both techniques fully make use of the information from results obtained in previous iteration steps. The covariance of covariance matrix which are formed from the auto/cross-correlation function of acceleration responses of a structure under white noise ambient excitation are used for damage detection in this paper. The components of the covariance matrix are proved to be function of the modal parameters (modal frequency, mode shape and damping parameter) of the structure. The number of vibration modes of the structure associated with the components is only limited by the sampling frequency. A simply supported thirty-one bar plane truss structure is studied where a multiple damage scenario with different noise levels is identified. Numerical results show that the proposed consistent regularization method is very effective in improving the results in the inverse problem with ill-condition phenomenon compared with the Tikhonov regularization.

Key words: Damage detection, Ambient vibration, Consistent regularization, Model updating

1. Introduction

A number of methods have been developed for damage detection in the last two decades. The sensitivity approach via model updating technique is commonly accepted and applied extensively in the engineering industry. However this type of method is weak in accommodating the influence of measurement errors leading to ill-conditioned problems as demonstrated in Friswell et al. (2001) and Humar et al. (2006). Investigations have since been conducted to deal with the ill-conditioning problems in model updating. Hansen (1992; 1998) and Vogel (2002) have proposed regularization methods for obtaining a solution of the inverse problem. It is recognized in the regularization theory that the conventional output error can be made unrealistically small if the process of damage identification is allowed to behave "badly", such that the variable has arbitrarily large deviations from the true set of parameter change. Recently Titurus and Friswell (2008) presented the sensitivity-based model updating method with an additional regularization criterion and computed the solutions based on the generalized singular value decomposition (GSVD). Specific features of the parameter and response paths when the regularization parameter varies are explored. Weber et al., (2008) applied the Tikhonov regularization and truncated singular value decomposition consistently to a nonlinear updating problem. Line search and stopping criteria known from numerical optimization are adapted to the regularized problem.

In this paper, a new covariance matrix is formed from the auto/cross-correlation function of the acceleration responses of a structure under ambient white noise excitation. The components of the covariance matrix are adopted to identify damage by model updating, during which, a consistent regularization is applied to improve the identified results due to the measurement error. One technique, which proposed a new side condition which classified the elements as possible damaged elements and undamaged elements which will be treated differently later on, is involved in the consistent regularization. A simply supported thirty-one bar plane truss structure is numerically studied. Numerical results show that the proposed consistent regularization method is very effective in improving the results in the inverse problem with ill-conditioning compared with the conventional Tikhonov regularization.

2. Covariance of covariance matrix of acceleration response

The equation of motion of a N degrees-of-freedom (DOFs) viscous damped structural system under support excitation is given as,

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=-\mathbf{M}\times\mathbf{L}\cdot\ddot{x}_s(t) \qquad (1)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the $N\times N$ mass, damping and stiffness matrices respectively. Matrix $\mathbf{L}$ with a size of $N\times 1$ is the mapping vector relating the support DOFs with the corresponding DOFs of the system. $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$ are the $N\times 1$ displacement, velocity, acceleration vectors respectively.

For ambient vibration, $\ddot{x}_s$ is assumed to be of ideal white noise distribution, the autocorrelation function of $\ddot{x}_s$ is[8]

$$E(\ddot{x}_s(\sigma_1)\ddot{x}_s(\sigma_2))=S\delta(\sigma_1-\sigma_2) \qquad (2)$$

where S is a constant defining the magnitude of excitation of $\ddot{x}_s$ when σ1=σ2, and $\delta(t)$ is the Dirac delta function.

Then the covariance $R_{pl}(\tau)$ can be written as, the detail can be seen in Ref. (Li and Law, 2010)

$$\begin{aligned}R_{pl}(\tau)&=S\sum_j\frac{\Phi_{lj}\phi_{fj}}{\omega_{dj}}e^{-\xi_j\omega_j\tau}\sum_i\frac{\Phi_{pi}\phi_{fi}}{\omega_{di}}[A_{ij}\cos(\omega_{dj}\tau)+B_{ij}\sin(\omega_{dj}\tau)]\\&=S\sum_j\frac{\Phi_{lj}\phi_{fj}}{\omega_{dj}}e^{-\xi_j\omega_j\tau}\{(\sum_i\frac{\Phi_{pi}\phi_{fi}}{\omega_{di}}A_{ij})\cos(\omega_{dj}\tau)+(\sum_i\frac{\Phi_{pi}\phi_{fi}}{\omega_{di}}B_{ij})\sin(\omega_{dj}\tau)\}\\&=S\sum_j\frac{\Phi_{lj}\phi_{fj}}{\omega_{dj}}e^{-\xi_j\omega_j\tau}\{G_j\cos(\omega_{dj}\tau)+H_j\sin(\omega_{dj}\tau)\}\end{aligned} \qquad (3)$$

where $G_j=\sum_i\frac{\Phi_{pi}\phi_{fi}}{\omega_{di}}A_{ij}$ and $H_j=\sum_i\frac{\Phi_{pi}\phi_{fi}}{\omega_{di}}B_{ij}$,

The covariance of covariance matrix is defined as (see Ref. [Li and Law, 2010]),

$$\begin{aligned}\mathbf{T}_p &= \mathbf{R}_p \cdot \mathbf{R}_p^T = \mathbf{\Phi u}(p)\cdot(\mathbf{\Phi u}(p))^T \\ &= \mathbf{\Phi u}(p)\cdot(\mathbf{u}(p))^T\mathbf{\Phi}^T \\ &= \mathbf{\Phi}(\mathbf{u}(p)\cdot(\mathbf{u}(p))^T)\mathbf{\Phi}^T\end{aligned} \tag{4}$$

The matrix $\mathbf{u}(p)\cdot(\mathbf{u}(p))^T$ in Equation (4) can be computed as,

$$\begin{aligned}[\mathbf{u}(p)\cdot(\mathbf{u}(p))^T]_{i\times j} = \frac{S^2}{\Delta t}\frac{\phi_{fi}}{\omega_{di}}\frac{\phi_{fj}}{\omega_{dj}}\{G_iG_j\frac{1}{2}[&\frac{s_i+s_j}{(s_i+s_j)^2+(\omega_{di}+\omega_{dj})^2}+\frac{s_i+s_j}{(s_i+s_j)^2+(\omega_{di}-\omega_{dj})^2}] \\ +G_iH_j\frac{1}{2}[&\frac{\omega_{di}+\omega_{dj}}{(s_i+s_j)^2+(\omega_{di}+\omega_{dj})^2}-\frac{\omega_{di}-\omega_{dj}}{(s_i+s_j)^2+(\omega_{di}-\omega_{dj})^2}] \\ +H_iG_j\frac{1}{2}[&\frac{\omega_{di}+\omega_{dj}}{(s_i+s_j)^2+(\omega_{di}+\omega_{dj})^2}+\frac{\omega_{di}-\omega_{dj}}{(s_i+s_j)^2+(\omega_{di}-\omega_{dj})^2}] \\ -H_iH_j\frac{1}{2}[&\frac{s_i+s_j}{(s_i+s_j)^2+(\omega_{di}+\omega_{dj})^2}-\frac{s_i+s_j}{(s_i+s_j)^2+(\omega_{di}-\omega_{dj})^2}]\}\end{aligned} \tag{5}$$

Equation (5) shows that matrix $\mathbf{u}(p)\cdot(\mathbf{u}(p))^T$ is only related to the mode shape, modal frequency and damping ratio of a structure. Then matrix Tp in Equation (4) is noted to be a function of the modal parameters of the structure. The change in the structural parameters is therefore related to the modal parameters and subsequently to the covariance of covariance matrix $\mathbf{T}_p$.

3. Consistent regularization

When the components in Equation (4) is used for damage identification via sensitivity analysis and model updating, the solution is often ill-conditioned. Regularization techniques are needed to provide bounds to the solution. The Tikhonov regularization expressions usually used for model updating are based on engineering assumptions on the parameter variations during iterations. The most frequently used conditions, (Friswell and Mottershead, 1995), are: (a) $\boldsymbol{\alpha}\to 0$, i.e. that the parameter values will be small; (b) $\boldsymbol{\alpha}\to\boldsymbol{\alpha}^0$, i.e. that the total parameter changes with respect to the reference model will be small and (c) $\Delta\boldsymbol{\alpha}^{k+1}\to 0$, i.e. that the parameter increment between iterations will be small. The incremental forms of these conditions are: (a) $\Delta\boldsymbol{\alpha}^{k+1}\to-\boldsymbol{\alpha}^k$ and (b) $\Delta\boldsymbol{\alpha}^{k+1}\to\boldsymbol{\alpha}^0-\boldsymbol{\alpha}^k$ while condition (c) is already in such form. Condition (a), and to some extent condition (b), represent physical assumptions, while condition (c) acts mainly as a stabilizing condition in highly non-linear problems.

In the above side conditions, the parameter variations or the updated parameters are bounded with respect to a fixed reference vector (for example a null or $\boldsymbol{\alpha}^0$) for all iterations. In fact, a parameter increment can be obtained from every iteration step and the structural parameter vector is updated. Some characteristics can be found among the updated parameter vectors between iterations. For example, some elements have large values, some have small values and some elements have fluctuating values around zero in all the iterations. These characteristics can imply some elements are possibly damaged and some elements undamaged. In order to improve the solutions in the ill-conditioned problems, these characteristics are included into the side conditions in the present study.

A new side condition is proposed similar to the side condition (b) above as follows

$$\Delta\boldsymbol{\alpha}^{k+1}\to\boldsymbol{\alpha}^*-\boldsymbol{\alpha}^k \tag{6}$$

which implies that the total parameter change with respect to a set of reference values determined from Equation (13) below is small. Here the reference vector $\boldsymbol{\alpha}^{*}$ is varying according to the results from the previous iteration steps. According to the physical definition, $\boldsymbol{\alpha}^{k}=\boldsymbol{\alpha}^{0}+\sum_{i=1}^{k}\Delta\boldsymbol{\alpha}^{i}$, then $\boldsymbol{\alpha}^{*}$ can be similarly be written as

$$\boldsymbol{\alpha}^{*}=\boldsymbol{\alpha}^{0}+\boldsymbol{\alpha}^{k,*} \tag{7}$$

Since vector $\boldsymbol{\alpha}^{0}$ is the constant set of initial analytical values, it will disappear when substituting $\boldsymbol{\alpha}^{*}$ and $\boldsymbol{\alpha}^{k}$ into Equation (10). $\boldsymbol{\alpha}^{k}$ and $\boldsymbol{\alpha}^{*}$ are written as $\boldsymbol{\alpha}^{k}=\sum_{i=1}^{k}\Delta\boldsymbol{\alpha}^{i}$, $\boldsymbol{\alpha}^{*}=\boldsymbol{\alpha}^{k,*}$ in the followings for simplicity and Equation (10) is rewritten as,

$$\Delta\boldsymbol{\alpha}^{k+1}\rightarrow\boldsymbol{\alpha}^{k,*}-\sum_{i=1}^{k}\Delta\boldsymbol{\alpha}^{i} \tag{8}$$

and $\boldsymbol{\alpha}^{k,*}$ is defined according to the following criteria:

The conditions may be unified into the cost functions which contain the residual function and a penalty function with the updated parameter vector as,

$$J(\Delta\boldsymbol{\alpha}^{k+1},\lambda)=\left\|\mathbf{S}_{k}\cdot\Delta\boldsymbol{\alpha}^{k+1}-(\Delta\ddot{\mathbf{h}}_{l}^{DWT})_{k}\right\|_{2}^{2}+\lambda^{2}\left\|\Delta\boldsymbol{\alpha}^{k+1}-(\boldsymbol{\alpha}^{k,*}-\boldsymbol{\alpha}^{k})\right\|_{2}^{2} \tag{9}$$

where $\boldsymbol{\alpha}^{k,*}$ is a quantity determined from results from previous k iterations shown in Equation (13). $\lambda\geq 0$ is the regularization parameter. The parameter λ controls the extent to which regularization is applied to the problem.

The regularization solution from minimizing the function in Equation (14) can be written in the following form as,

$$\Delta\boldsymbol{\alpha}^{k+1}=(\mathbf{S}_{k}^{T}\mathbf{S}_{k}+\lambda^{2}\mathbf{I})^{-1}(\mathbf{S}_{k}^{T}(\Delta\ddot{\mathbf{h}}_{l}^{DWT})_{k}-\lambda^{2}(\boldsymbol{\alpha}^{k}-\boldsymbol{\alpha}^{k,*})) \tag{10}$$

where the superscripts (-1) and T denote the inversion and the transpose of the matrix and $\mathbf{I}$ is the identity matrix.

4. Numerical Simulation

A simply supported plane truss structure as shown in Figure 1 serves for the simulation study. It is modeled using thirty-one planar truss finite elements without internal nodes giving 28 degrees-of-freedom. The cross-sectional area of the bar is 0.0025 m^2. Damage in the structure is introduced as a reduction in the axial stiffness of individual bars, but with the inertial properties unchanged.

Both the vertical and horizontal translational restraints at the supports are represented by large stiffness of 1.0×10^{10} *kN/m*. Rayleigh damping is adopted for the system with $c_1=0.01$ and $c_2=0.005$. The damping ratio for each vibration mode can be computed by uncoupling the damping matrix with the mode shape matrix as $\frac{\boldsymbol{\Phi}^{T}\mathbf{C}\boldsymbol{\Phi}}{2\boldsymbol{\omega}}$, where $\boldsymbol{\omega}$ is the modal frequency vector. The first 12 natural frequencies of the structure are 36.415, 75.839, 133.608, 222.904, 249.323, 358.011, 372.509, 441.722, 477.834, 507.943, 538.125 and 547.393 Hz. White noise excitation with an amplitude of 1 m^2/s^4 is assumed acting at the supports of the structure in the vertical direction. The vertical acceleration measurements at the selected nodes are recorded for a duration of 1800s for the damage identification. The sampling frequency is 2000 Hz which is high enough to capture much information from the vibration modes of the structure which are below 1000 Hz and the response data collected is considered sufficient to obtain the accurate experimental CoC matrix.

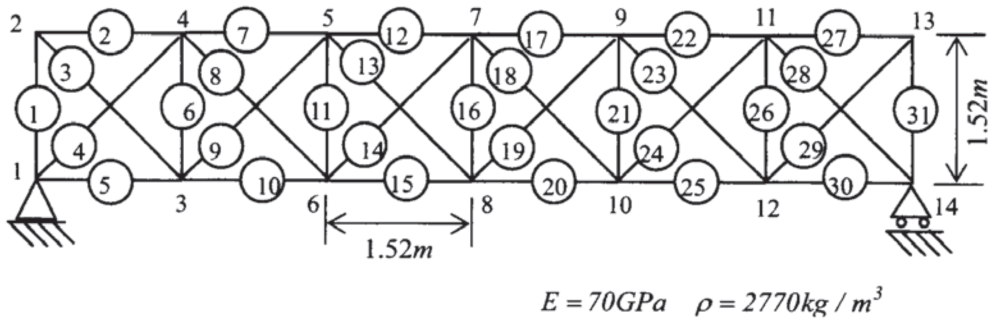

Fig. 1 - Thirty-one bar plane truss structure

The damage scenario with the modulus of elasticity of material of elements 18, and 19 reduced by 10% and that in element 20 reduced by 15% including different levels of random noise in the structure is studied.

If simulated acceleration responses from the undamaged and the damaged state are used instead of measurement, the damage index vectors $\mathbf{D}^u$ and $\mathbf{D}^d$ are computed by Equation (4). If the time duration is long enough, the damage index vectors will be accurate even with a high noise level in the acceleration responses. To simulate the noise effect in this study, $\mathbf{D}^u$ and $\mathbf{D}^d$ are computed analytically using Equations (4,5) with normally distributed random noise added to the damage index as,

$$\mathbf{D}^u_{measured} = \mathbf{D}^u \times (1 + Ep \times \mathbf{N}_{oise}), \quad \mathbf{D}^d_{measured} = \mathbf{D}^d \times (1 + Ep \times \mathbf{N}_{oise}) \qquad (11)$$

where $\mathbf{D}^u_{measured}$ and $\mathbf{D}^d_{measured}$ are the polluted damage index vectors; Ep is the noise level ; $\mathbf{N}_{oise}$ is a standard normal distribution vector with zero mean and unit standard deviation.

4.1 Damage Detection

Damage detection making use of the CoC matrix is studied. The damage scenario with different noise level using different measurement sensor sets is considered. Four accelerometers are assumed installed at Nodes 2, 3, 4 and 5 in the vertical direction. When the reference DOF p is 4 at Node 2, the CoC matrix $\mathbf{T}_4$ has a size of 4×4. When the reference DOF p is at Node 3, 4 and 5, the CoC matrices are $\mathbf{T}_6$, $\mathbf{T}_8$ and $\mathbf{T}_{10}$ with a size of 4×4 respectively. All the components from the four matrices $\mathbf{T}_4$, $\mathbf{T}_6$, $\mathbf{T}_8$ and $\mathbf{T}_{10}$ are used for the damage detection, and the damage index vector $\mathbf{D}$ has a size of 40 (=$4 \times (1+4) \times 4/2$) which is larger than 31 (the number of the stiffness parameters in all the finite elements). 1%, 2.5% and 5% white noise is added separately into the damage index vectors $\mathbf{D}^u$ and $\mathbf{D}^d$ from the undamaged and damaged structure as Equation (11) to get $\mathbf{D}^u_{measured}$ and $\mathbf{D}^d_{measured}$. The initial analytical model is assumed accurate for the damage detection. The identified results are shown in Figure 2(a) using Tihonov regularization and (b) using consistent regularization. The damage locations are identified accurately and the damage extents are also identified satisfactory without false positive in other elements using consistent regularization. However, when the noise is increased to 2.5% and 5%, there are false positive in undamaged elements and the damage extent are not accurate using Tikhonov regularization method. It shows that the consistent regularization has better ability for tolerating noise than Tikhonov regularization.

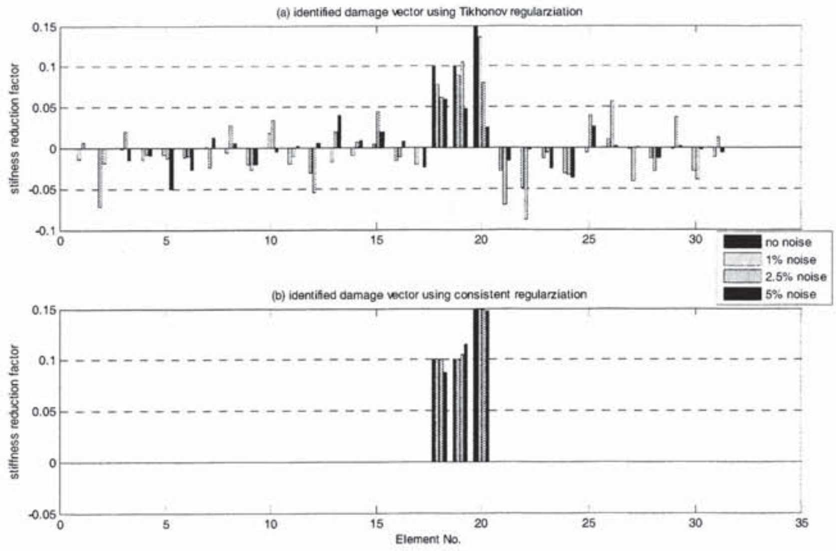

Fig. 2 – the identified damage vectors using Tikhonov regularization and consistent regularization with different level noise.

The evolution of the identified damage vectors using consistent regularization are shown in Figure 3 and Figure 4. It can be seen that when no noise and 1% noise, the values for three damaged elements converge to the accurate damage value and those for undamaged elements converge to zero quickly. When noise is increase to 2.5% and 5%, the values for damage elements converge to accurate value after 80 iterations. In spite of slow convergence, the damage location and extent are identified satisfactorily. However, the evolution using Tikhonov regularization will diverge if the computation does not stop because the ratio of signal to noise will become smaller with the iteration procedure (The figure does not shown here). It again shows that consistent regularization can improve the identified results from the model updating procedure.

5. Conclusions

In this paper, a new side condition is proposed which classified the identified elements as possible damaged elements and undamaged elements from results obtained from previous iterations. The possible damage elements are improved with small steps between iterations and the parameters for the undamaged elements are required to approach to zeros. These measures can make the updating procedure converge to the real values more quickly and accurately. It can be seen from the numerical studies that the proposed method using the proposed technique can give very accurate results in an ill-conditioned inverse problem with different level of noise. Whereas with the conventional Tikhonov regularization method, the identified parameters in the undamaged elements fluctuates greatly around the zero and incorrect optimal point on the L-curve may be chosen in some iterations. Results can be obtained only in the first several iterations and they can not be improved gradually in the following steps. The accuracy of the results is also affected by noise and the computation stops when the results fall outside their physical limits.

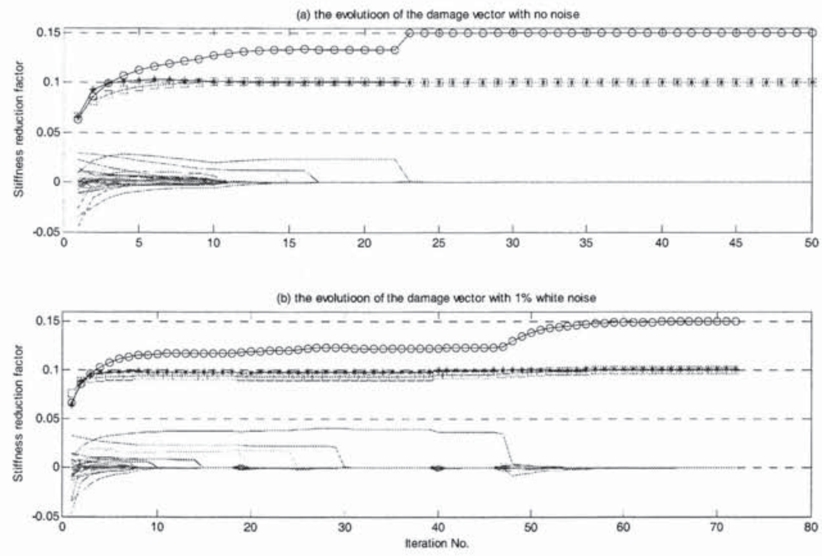

Fig.3- the evolution of the identified damage vectors with no noise and 1% noise

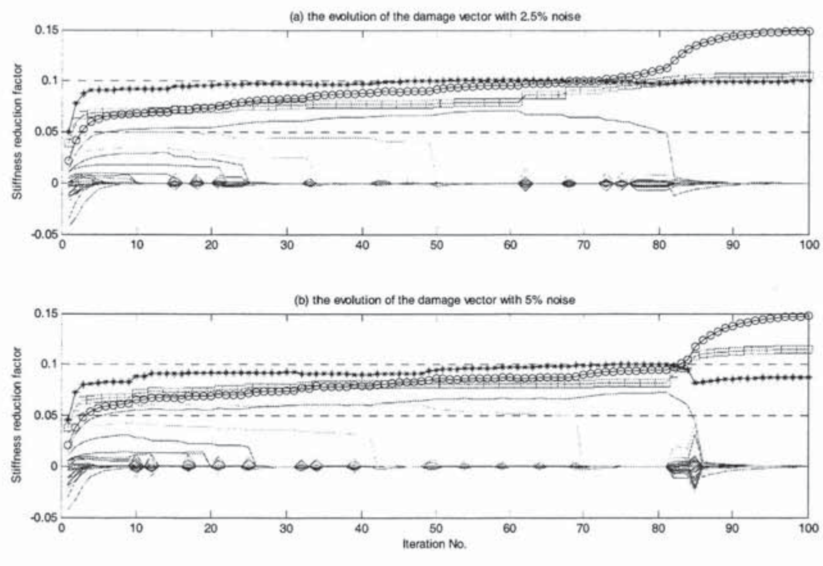

Fig.4- the evolution of the identified damage vectors with 2.5% and 5% noise

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