Damping Device through Autoparametric Effect

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Abstract

This paper presents a new design of nonlinear dynamic absorber (NDA) using the phenomenon of autoparametric interaction between the first symmetric mode and the first anti-symmetric mode of a curved beam to reduce the resonance vibration of a primary structure with a controllable operational frequency range. For a typical autoparametric theory of a curved beam, the lowest excitation force required to initiate an energy transfer from the first symmetric mode (q_1) to the first anti-symmetric mode (q_2) involves tuning the ratio of the resonance frequencies of the first symmetric mode (ω_1) and first anti-symmetric mode (ω_2) to close to 2.
The resonance frequency of the first anti-symmetric mode (ω_2) can be altered to control the operational frequency range. The autoparametric vibration response can be used to create an energy-dissipative region to achieve a controllable bandwidth. It is also possible to create a non-dissipative frequency region in between two dissipative frequency regions. This is useful for providing damping with a conventional mass damper. Numerical calculations indicate that the resonance vibration of a primary structure can be successfully reduced using this approach.

Key words: nonlinear, absorber, autoparametric.

1. Introduction

The autoparametric vibration absorber and the nonlinear energy sink are two recently developed passive nonlinear vibration absorbers. The former absorber dissipates the vibration from the primary system by energy transfer between two modes. The two quadratically coupled systems are subjected to primary excitation, and possess a 2:1 internal resonance [1]. Vyas and Bajai [2] developed an array of pendulums with slightly different natural frequencies to widen the effective bandwidth of the damper.

Vakakis and Gendelman [3-5] investigated the passive reduction of the vibration in a linear system (discrete or continuous) by the one-way irreversible transfer of energy from the linear system to an attached nonlinear damper, which is known as the energy pumping phenomenon. Viguié and Kerschen [6] studied the vibration mitigation of nonlinear mechanical systems using nonlinear dynamical absorbers and validated the approach using numerical simulations. Cochelin et al. [7] conducted experimental tests of the energy transfer between the first acoustic mode in a tube and a thin visco-elastic membrane to examine the energy pumping phenomenon in an acoustic medium coupled with an essentially nonlinear oscillator. They found that the use of a thin membrane to dissipate energy required a larger amplitude of vibration.

This paper introduces the application of a curved beam as a nonlinear vibration absorber to reduce the vibration of a linear primary structure by internal energy transfer from the first symmetric mode to the first anti-symmetric mode. The resonance frequency of the first anti-symmetric mode (ω_2) can be altered to control the operational frequency range.

2. Formulation for the vibration of a curved beam

2.1 Governing differential equation

The system under investigation is shown in Fig. 1. The initially curved beam is clamped on opposite sides and subjected to uniform excitation. The flexural bending along its width is assumed to be negligible.

Consider a beam of width B and length L in the x-direction with a thickness h that is

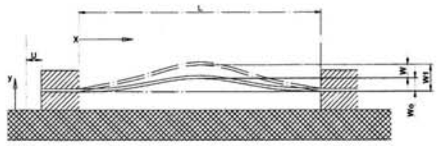

Fig. 1: Illustration of a clamped-clamped beam model.

subjected to transverse motion. The end moves only during the initial compression process and is fixed for dynamic loading. According to Hamilton's principle, the governing differential equation of the clamped beam subjected to a uniaxial static load P and a transverse harmonic support motion y is

$$m\left(\frac{\partial^2 (w(x,t)+w_o(x))}{\partial t^2}+\frac{\partial^2 y(t)}{\partial t^2}\right)+c\frac{\partial (w(x,t)+w_o(x))}{\partial t}+EI\frac{\partial^4 (w(x,t)+w_o(x))}{\partial x^4}$$
$$+P\frac{\partial^2 (w(x,t)+w_o(x))}{\partial x^2}-\frac{EA}{2L}\frac{\partial^2 (w(x,t)+w_o(x))}{\partial x^2}\int_0^L\left(\frac{\partial (w(x,t)+w_o(x))}{\partial x}\right)^2 dx=0 \quad , (1)$$

$$P=\frac{4\pi^2 EI}{L^2}+\frac{EA}{2L}\int_0^L\left(\frac{\partial w_o(x)}{\partial x}\right)^2 dx \quad , \quad (2)$$

where m is the mass per unit length, w(x,t) is the transverse displacement of the beam, w_o is the initial static deflection of the beam, E is the Young's modulus, A is the cross sectional area of the beam, I is the moment of inertia of bending, c is the damping coefficient, $F = \rho Az$ is the base excitation amplitude, z is the base excitation acceleration, ρ is the beam density, ω is the circular excitation frequency, and t is the time.

The transverse displacement is expressed in terms of the beam mode shapes as

$$w(x,t)=\sum_{i=1}^{n} q_i(t)\phi_i(x) \quad , \quad (3)$$

where q_i is the modal amplitude of the ith mode, ϕ_i is the ith mode shape function, which is normalized so that the maximum value of each mode shape is equal to 1, i is the mode number, and n is the number of modes considered.

2.2 Analytical prediction of the autoparametric interaction of a curved beam

The motion of the first symmetric mode q_1 and first anti-symmetric mode q_2 of a clamped curved beam in a two-degrees-of-freedom equation can be formulated from Eq. (1) as follows.

$$\ddot{q}_1+2\omega_1\xi_1\dot{q}_1+\omega_1^{\,2}q_1+K_{122}(a_0 q_2^{\,2})=\mathrm{Q}_1\sin(\omega t) \quad , \quad (4a)$$

$$\ddot{q}_2+2\omega_2\xi_2\dot{q}_2+\omega_2^{\,2}q_2+K_{112}w_0 q_1 q_2=0 \quad , \quad (4b)$$

where K_{122} and K_{112} are nonlinear coefficients, ξ_1 and ξ_2 are the modal damping coefficients, Q_1 is the modal force, and ω_1 and ω_2 are the circular resonant frequencies of the first symmetric mode and first anti-symmetric mode, respectively. Rearranging equation (4b) gives

$$\ddot{q}_2 + 2\omega_2\xi_2\dot{q}_2 + \omega_2^{\ 2}q_2 = -K_{112}a_0q_1q_2 \,. \tag{4c}$$

The excitation of the first anti-symmetric mode q_2 should be due to the time-dependent coefficient q_1 q_2. This system is said to be parametrically excited because the solution to $q_1(t)$ in Eq. 4a acts as a parametric excitation in the $q_1(t)q_2(t)$ term in Eq. 4b.

2.2.1 Transition curve for the symmetric mode

Letting $\tau = n_1\Omega t$, Eq. 4a can be simplified as

$$\ddot{q}_1 + \zeta_1\dot{q}_1 + q_1 + k_1q_2^{\ 2} = d\sin\eta\tau \,, \tag{5a}$$

where $\zeta_1 = 2\xi_1$; $k_1 = \frac{C_{12}}{(n_1\Omega)^2}$ is the nonlinearity coefficient, $d = \frac{Q_1}{(n_1\Omega)^2}$ is the amplitude of the external excitation, $\eta = \frac{\omega}{\eta_1\Omega} = \frac{\omega}{\omega_1}$, where $n_1 = \frac{\omega_1}{\Omega}$, Ω is the linear circular frequency of the flat beam, and $C_{12} = K_{122}w_o$.

Eq. 4b can be re-written as

$$\ddot{q}_2 + \zeta_2\dot{q}_2 + n_{21}^{\ 2}q_2 + k_2q_1q_2 = 0 \,, \tag{5b}$$

where $n_{21} = \frac{n_2}{n_1} = \frac{\omega_2}{\omega_1}; \zeta_2 = 2\xi_2 n_{21}$, $n_2 = \frac{\omega_2}{\Omega}$, $C_{21} = 2K_{112}w_0$, and $k_2 = \frac{C_{21}}{(n_1\Omega)^2}$.

2.2.2 Nonlinear response curve for the anti-symmetric mode

Letting $q_1 = A_1\cos(\eta\tau + \psi_1)$ and $q_2 = A_2\cos(\frac{1}{2}n\tau + \psi_2)$, where A_1 and A_2 are the maximum amplitudes of the dynamic responses of q_1 and q_2, these terms can be substituted into equations 5a and 5b.

A $4\pi/\eta$-periodic solution can then be found by using the Poincaré-Lindstedt method . This method leads to the following system of conditions.

$$\sigma_1A_1 + \frac{1}{2}k_1A_2^{\ 2}\cos(\psi_1 - 2\psi_2) - d\cos\psi_1 = 0 \,, \tag{6a}$$

$$\zeta_1A_1 - \frac{1}{2}k_1A_2^{\ 2}\sin(\psi_1 - 2\psi_2) - d\sin\psi_1 = 0 \,, \tag{6b}$$

$$\sigma_2A_2 + \frac{1}{2}k_2A_1A_2\cos(\psi_1 - 2\psi_2) = 0 \,, \tag{6c}$$

$$-\zeta_2A_2 - \frac{1}{2}k_2A_1A_2\sin(\psi_1 - 2\psi_2) = 0 \,. \tag{6d}$$

The amplitude of the q_1 coordinate follows from Eqs 6a to 6d, and the result is expressed in Eq. 7. A_1 is related to the stability boundary, and is called the transition curve.

$$A_1 = \frac{2}{k_2}(\sigma_2^{\ 2} + \zeta_2^{\ 2})^{0.5} \,. \tag{7}$$

According to the response-oriented approach, the response of the first symmetric mode of a curved beam without autoparametric vibration can be expressed in the following exact solution.

$$A_O = \frac{d}{(\sigma_1^{\ 2} + \zeta_1^{\ 2})^{\frac{1}{2}}} \,, \tag{8}$$

where

$$\sigma_2 = \left[1 - \left(\frac{\omega}{2\omega_2}\right)^2\right]n_{21}^{\ 2}; \sigma_1 = 1 - \frac{\omega^2}{\omega_1^{\ 2}}$$

An investigation of the curves of the linear first symmetric mode response equation (8) and the transition curve equation (7) allows the estimation of the characteristic autoparametric vibration between the resonances of the first symmetric mode q_1 and the first anti-symmetric mode q_2. The quadratic equation for the magnitude of vibration of the q_2 coordinate is obtained from Eqs 6a and 6b as follows.

$$z^2 + Mz + T = 0 \ , \quad z = \frac{1}{2}k_1 k_2 A_2^{\,2} \ , \tag{9a}$$

Where $A_2^{\,2} = \dfrac{1}{k_1 k_2}\left[-M \pm \sqrt{N}\right]$, $\quad T = k_2^{\,2} d^2 \left[\left(\dfrac{A_1}{A_0}\right)^2 - 1\right]$, (9b, 9c)

$M = 4(\zeta_1\zeta_2 - \sigma_1\sigma_2)$, Let $N = M^2 - 4T = 16\left[\dfrac{1}{4}k_2^{\,2}d^2 - (\sigma_1\zeta_2 + \sigma_2\zeta_1)^2\right]$. (9d, 9e)

The graphs for the first symmetric mode without autoparametric interaction A_0 and the transition curve A_1 derived from the Mathieu equation under a certain level of excitation are plotted in Fig. 2a.

The nonlinear (softening and hardening) vibration response curves for the first anti-symmetric mode with autoparametric vibration under free vibration A_{2r}, forced vibration without damping A_{2f}, and forced vibration with damping A_{2fd} are shown in Fig. 2b.

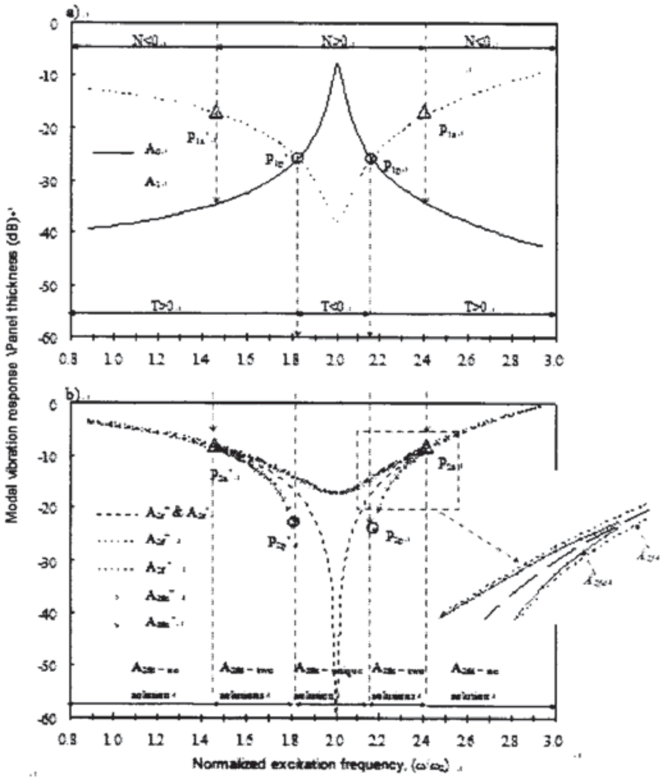

Fig. 2. (a) Vibration response of first symmetric mode of a curved beam without autoparametric vibration - w_o (Eq.6) and transition curve for autoparametric response of first symmetric mode- A_1 (Eq.7). (b) Nonlinear (hardening and softening) vibration response curve for first anti-symmetric mode with autoparaemtric vibration under free vibration - A_{2r} ; forced vibration without damping - A_{2fd} and forced vibration with damping - A_{2fd}.

2.2.2.1 Free vibration

This part is focused on the explanations and mechanisms behind the phenomenon mentioned in the previous part. Under the condition of free vibration, the amplitude of the external excitation d and the damping coefficients ζ_1 and ζ_2 equal 0, and thus N becomes 0. From Eqs 7 and 9, $A_{2r}^{2} = \frac{4\sigma_1\sigma_2}{k_1k_2}$ and $A_1 = \frac{2}{k_2}\sigma_2$ can be obtained. Thus, A_{2r} represents the free vibration behavior of a hardening and softening spring.

2.2.2.2 Forced vibration without damping

Under the condition of forced vibration without damping, the damping coefficients ζ_1 and ζ_2 equal 0. The nonlinear (softening and hardening) vibration response curves for the first anti-symmetric mode can be expressed as $A_{2f} = \sqrt{A_{2r}^{2} \pm \frac{2d}{k_1}}$, and the difference between the vibration response A_2 with and without an excitation force is only $\frac{2d}{k_1}$. From Eq. 8 $A_O = \frac{d}{\sigma_1}$, and thus $A_{2f} = \sqrt{\frac{2d}{k_1}(\frac{A_1}{A_0} \pm 1)}$. In the section T<0 ($A_1 > A_0$), there is a unique solution to A_{2f}^{+}. As presented in Fig. 2, in the two regions T>0, there are two solutions A_{2f}^{+} and A_{2f}^{-} for the magnitude of the first anti-symmetric mode. However, the negative solution $A_{2f}^{-} = \sqrt{\frac{2d}{k_1}(\frac{A_1}{A_0} - 1)}$ is unstable when $\frac{A_1}{A_0} - 1 < 0$. Thus, the condition for the activation of the autoparametric interaction of the symmetric and anti-symmetric modes is $A_0 > A_1$.

The locations of P_{1p} and P_{2p}, which is marked with a circle in Fig. 2a and Fig. 2b, respectively, corresponds to two the intersection point of A_0 and A_1. As the damping coefficients ζ_1 and ζ_2 equal 0, the normalized excitation frequency and magnitude of vibration points can be derived as follows.

Normalized excitation frequency at $P_{1p} = P_{2p} = \sqrt{1 + \left[4\left(\frac{dk_2}{8}\right)\right]^2}$. (10a)

Normalized excitation frequency at $P_{1p}' = P_{2p}' = \sqrt{1 - \left[4\left(\frac{dk_2}{8}\right)\right]^2}$.

Modal vibration magnitude/beam thickness at $P_{2p} = P_{2p}' = 20\log\left(\frac{2d}{k_1 h}\right)$. (10b)

2.2.2.3 Forced vibration with damping

For forced vibration with damping, there are three conditions for the solutions, as presented in Figs. 2a and 2b. There is a unique solution for the first condition of $T<0$ and N>0, which corresponds to $A_1 < A_0$ (Eq. 9c) and $N > M^2$ (Eq. 9e), that is, $A_{2fd}^{+} = \left(\frac{-M + \sqrt{N}}{k_1 k_2}\right)^{\frac{1}{2}}$ (see Fig. 2b).

There are two solutions for the second condition of T>0 and N>0, which corresponds to $N<M^2$, that is, $A_{2fd}^{+} = \left(\frac{-M + \sqrt{N}}{k_1 k_2}\right)^{\frac{1}{2}}$ and $A_{2fd}^{-} = \left(\frac{-M - \sqrt{N}}{k_1 k_2}\right)^{\frac{1}{2}}$ (Fig. 2b).

Damping limits the region of autoparametric interaction. N=0 is the boundary of the autoparametric interaction between the vibration of the first symmetric mode and the first anti-symmetric mode.

This boundary is marked by P_{2a} in Fig. 2b. The location of P_{2a} corresponds to the intersection of the positive and negative values of N, which means that N=0. In this situation,

$\sigma_2 = \dfrac{2k_2 d}{(4\zeta_2 + \zeta_1)}$. The normalized excitation frequency and magnitude of vibration points P_{2a} and P_{2a}' can then be derived as follows.

Normalized excitation frequency at $P_{2a}=P_{1a} = \sqrt{1+4\left(\dfrac{2k_2 d}{(4\zeta_2+\zeta_1)}\right)^2}$. (11a)

Normalized excitation frequency at $P_{2a}' = P_{1a}' = \sqrt{1-4\left(\dfrac{2k_2 d}{(4\zeta_2+\zeta_1)}\right)^2}$.

Modal vibration magnitude/beam thickness at $P_{2a} = P_{2a}' = 20\log\left(\dfrac{-4(\sigma_2)}{(k_1 k_2)^{\frac{1}{2}} h}\right)$. (11b)

There is no solution for the third condition of $T > 0$ and $N < 0$.

The peak magnitude of vibration for the first symmetric mode, which is marked with P_{1a} and corresponds to N=0, can be obtained by substituting $\sigma_2 = \dfrac{2k_2 d}{(4\zeta_2+\zeta_1)}$ into Eq. 7. The peak magnitude of vibration P_{1a} can be further simplified as

A_1 as given by $\dfrac{d}{\left(\zeta_2 + \dfrac{\zeta_1}{4}\right)}$. (12)

Thus, $\zeta_2 + \dfrac{\zeta_1}{4}$ is the effective damping ζ'. As shown in Eq. 7, in the absence of autoparametric vibration, the first symmetric mode depends on the damping of the first symmetric mode $A_1 = \dfrac{d}{\zeta_1}$. Note that in the presence of autoparametric interaction, the effective damping of the first symmetric mode is dominated by the damping of the first anti-symmetric mode q_2

3. Application of a curved beam as a nonlinear dynamic absorber

A linear dynamic absorber is a well-known passive vibration reduction device. The phenomenon of autoparametric interaction due to the first symmetric mode and the first anti-symmetric mode of a curved beam, as studied in section 2, is applied to reduce the symmetric mode vibration. In the conventional tuned mass damper system, the vibration of the primary structure is transferred to the symmetric mode of the attached absorber, and most of the energy must then be dissipated by additional damping material. For the proposed nonlinear vibration absorber, the energy can be effectively dissipated by the inherent material and joint damping through the motion of the anti-symmetric mode of the beam.

Fig. 3 presents the theoretical response of a primary structure fitted with a conventional linear vibration absorber with different damping coefficients. When ξ_1=0.001, the highest peak occurs in region 1, a lower peak occurs in region 3, and a large drop occurs in region 2. When the damping coefficient increases, the peaks in regions 1 and 3 become lower, but the vibration response in region 2 increases. This indicates that damping reduces the vibration reduction performance in region 2, but increases the vibration reduction performance in regions 1 and 3. Note that the peak in region 1 is always higher than that in region 3, which means that greater damping is required to dissipate the energy in region 1. A dynamic absorber must thus exert different damping effects in regions 1, 2, and 3.

The damping effect of the proposed nonlinear vibration absorber can be controlled with the resonance frequency of the anti-symmetric mode, as shown in Fig. 4. The frequency of the dip point of the transition curve ($2\omega_2$) is designed to correspond to the peak in region 1. The peaks in both regions 1 and 3 are intersected by the transition curve, and can thus be reduced by autoparametric interaction (i.e., the transfer of vibration of the first symmetric mode to the first anti-symmetric mode of the curved beam). The vibration in the region between 1 and 2 is not affected.

The theoretical curved beam vibrations of the symmetric and anti-symmetric modes are presented in Fig. 5a. The vibration peaks are dissipated by energy transfer to the anti-symmetric mode. The vibration of the primary structure is also successfully reduced, as shown in Fig. 5b.

The resonant peak of the primary structure at around $\omega/\omega_2 = 2.36$ is significantly decreased. The peak at around $\omega/\omega_2 = 2.76$ is insignificant because of its low amplitude. It should be noted that no additional damping material is required to achieve a sufficient reduction in vibration. Furthermore, there is no increase in vibration in region 2. The nonlinear dynamic absorber thus provides a controllable bandwidth for the damping effects through autoparametric interaction.

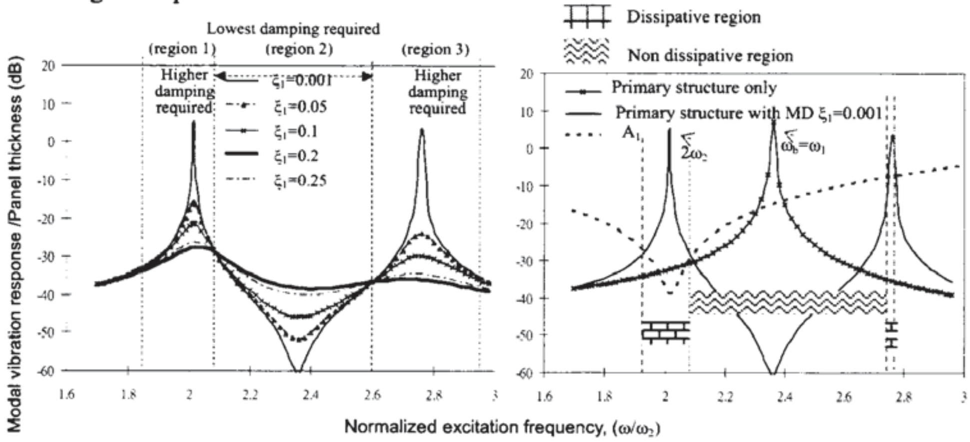

Fig. 3. Vibration of primary structure with the effect of conventional tuned mass damper. (MD)(w_o=8.4mm;$\omega_2=2\pi$ x25radian/sec;$\omega_1=2\pi$ x59 radian/sec; ω_1/ω_2=2.36;z=16g;$\xi_1=\xi_2$=0.025)

Fig. 4. The design resonant frequencies of symmetric and anti-symmetric mode for a curved beam used as an autoparametric vibration absorber.

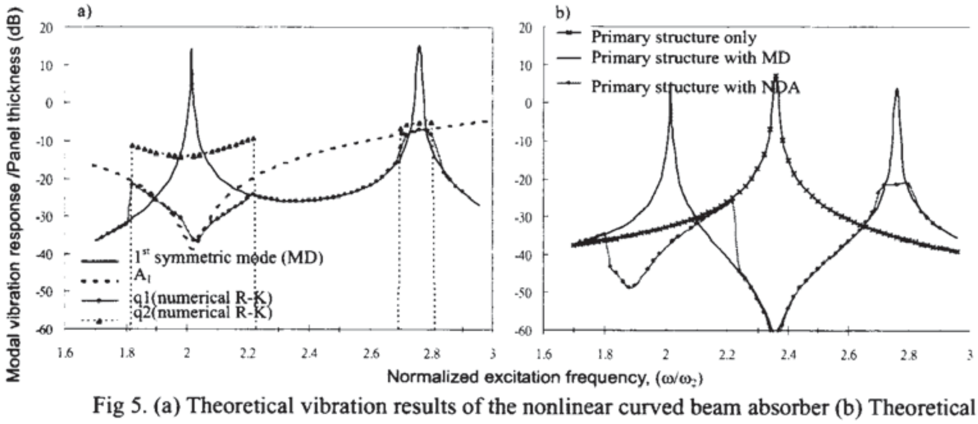

Fig 5. (a) Theoretical vibration results of the nonlinear curved beam absorber (b) Theoretical vibration results of primary structure

4. Conclusion

Based on the phenomenon of autoparametric interaction, a curved beam can serve as a nonlinear dynamic absorber, and can be used to control the operational frequency range. This is achieved through the appropriate design of the resonance frequency of the anti-symmetric mode. The two vibration peaks of the primary system are dissipated by energy transfer from the symmetric mode to the anti-symmetric mode of the secondary system. The vibration at the frequencies between the two peaks is not affected, and there is no increase in vibration compared with the scenario without a mass damper. Importantly, the use of the autoparametric effect with a nonlinear dynamic absorber means that no additional damping material is required to achieve a sufficient reduction in vibration. The autoparametric vibration response can be used to create an energy-dissipative region for a controllable bandwidth. It is also possible to create a non-dissipative frequency region between two dissipative frequency regions. Unlike the nonlinear energy sink, this approach does not require a higher amplitude of vibration. Furthermore, it can maintain a reliable performance in hot and corrosive environments in which damping material does not usually perform well.

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