The Definition and Measurement of the Probability Density Function in Lamb Wave Damage Detection Based on Data Fusion

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Abstract

The type and parameters of the probability density function (PDF) of damage occurrence were discussed. Based on the new definition of the PDF of the presence of damage given in this paper, the parameters of PDF, such as its expectation and variance, were obtained by hypothesis testing of mathematical statistics theory, which refers to the process of choosing between competing hypotheses about a probability distribution based on observed data from the distribution. In the present measurement method based on mathematical statistics theory, both the type and characteristics of the PDF can be determined according to scientific analysis of the experiment data. A numerical simulation was carried out to demonstrate this hypothesis testing-based measurement method. Compared with the existing empirical formula method, the present method can be more reliable in application. And with the present method, the Lamb waves damage detection method based on data fusion will be theoretically completed.

Key words: probability density function of damage occurrence, damage identification, probabilistic algorithm, hypothesis testing

1. Introduction

It has been proved that data fusion can improve the robust character of Lamb wave-based damage detection[1],[2],[3],[4]. The probability density function (PDF) of the presence of damage is the key part of Lamb wave damage detection based on data fusion. However, its type and characteristics were determined mainly by experience in existing work. A theoretical analysis about the PDF was necessary and will be helpful for the development of this area.

The Lamb wave damage detection method, especially the time-of-flight (ToF) method, was briefly reviewed in this paper. The reason and advantage of introducing the concept of data fusion in ToF method were discussed. The nature of PDF was revealed by using mathematical statistics theory in this discussion. A new definition of PDF was given. Therefore, a hypothesis testing-based method of finding the parameters of PDF, such as its expectation and variance, was presented and demonstrated by using a numerical simulation.

2. Lamb wave damage detection based on data fusion

2.1 ToF method

Lamb waves are a kind of elastic waves propagating in solid plates. With a high susceptibility to interference on a propagation path, e.g. damage or a boundary, Lamb waves can travel over a long distance even in materials with a high attenuation ratio, and thus a broad area can be quickly examined. Therefore, Lamb waves are considered as a prominent structural damage detection tool [5],[6].

ToF, defined as the time lag from the moment when a sensor catches the damage-reflected signal to the moment when the same sensor catches the incident signal, was widely used to locate damage [7],[8].

Consider a sensor network consisting of N piezoelectric wafers denoted by s_i ($i=1,2,...,N$). For convenience of discussion, s_m-s_n hereinafter stands for the sensing path in which s_m serves as the actuator and s_n as the sensor. The center of the damage, if any, is presumed to be (x,y) in coordinate system. Then the ToF can be defined in Eq.(1) as T_{i-j}:

$$\frac{L_{A-D}}{V_{S_0}} + \frac{L_{D-S}}{V_{S_{damage}}} - \frac{L_{A-S}}{V_{S_0}} = T_{i-j} \tag{1}$$

In which L_{A-D}, L_{D-S} and L_{A-S} represent the distance between the actuator s_i and the damage, the damage and the sensor s_j, and the actuator s_i and the sensor s_j, respectively. $V_{S_{damage}}$ and V_{S0} are velocities of the damage-converted S mode and the incipient S_0 mode, respectively.

Because there are two unknown damage parameters, (x,y), in Eq.(1), the solution of Eq.(1) will be a root loci, which implies the possible locations of the damage for a certain ToF value. In traditional approaches [9],[10], the damage location is given by seeking the intersections of two or more loci. As shown in Figures 1(a), the point with which three loci intersect was considered as the location of damage, while the points with which only two loci intersect were neglected.

There is a prerequisite in the traditional approach. That is all of the measured ToF values were perfect. However, the error in any experiment is inevitable. Therefore, as shown in Figures 1(b), there is no point where all three loci intersect with when the ToF results have error. It is suggested that the damage location can be found as the area in which the intersections of two loci concentrate. However, a scientific approach is required to give
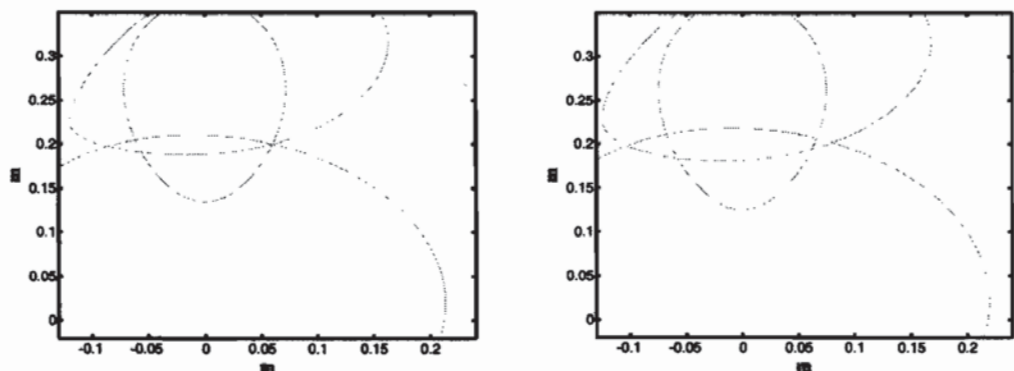
the precise result based on the intensity of intersections.

(a) Locus based on perfect ToF   (b) Locus based on ToF with error

Fig. 1 Damage localization using ToF method in a plate

2.2 ToF method combined with data fusion

The concept of data fusion was introduced by Xiaoliang Zhao et al. to improve traditional ToF method [1],[2],[3]. In traditional ToF method, only the points in loci are

considered. In data fusion method, the points absent in the loci are also considered as possible damage location because of the error in measured ToF value. The main frame of data fusion based method can be divided into two steps:

> The inspection area of the structure was evenly meshed. For a certain measured ToF, each mesh node will be evaluated its possibility for the presence of damage by using a probability density function.

> All evaluated results for each measured ToF were combined to give the detection result in a matrix form. Each element of the matrix represents the probability of the presence of damage for one mesh node.

The damage location can be determined as the mesh point which has the max probability value. The damage location can also be given in the contour form as shown in Figure. 2.
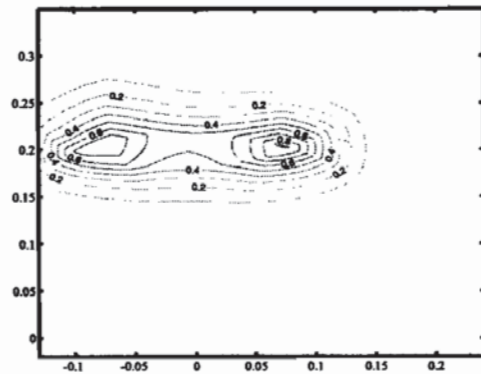

Fig. 2 Damage detection using ToF method based on data fusion.

It is clear that the PDF of damage occurrence is the key part of data fusion-based method. Zhongqing SU et al. suggest the PDF can be quantified in relation to the loci:

$$f\left(z_{ij}\right) = \left(1/\sigma_{ij}\sqrt{2\pi}\right)\exp\left[-z_{ij}^2/2\sigma_{ij}^2\right] \tag{2}$$

where $f(z_{ij})$ is the Gaussian distribution function, representing the PDF of damage occurrence at node L_i ($i=1,...,K \times K$ for the structure that is comprised of $K \times K$ mesh nodes), perceived by a sensor, s_j ($j=1,...,N$ for the sensor network consisting of N sensors). σ_{ij} is the standard variance and

$$z_{ij} = \left\|\chi_i - \mu_{ij}\right\| \tag{3}$$

where χ_i is the location vector of node Li and μ_{ij} is the location vector of the point on the locus provided by sensor s_j that has the shortest distance to node L_i.

Satisfied results have been obtained by using this kind of PDF. But it should be noticed that the standard variance σ_{ij} was selected dependable on experience.

The concept of data fusion has also adopted in some other Lamb waves-based damage detection methods rather than ToF method. Dong Wang et al. combine the concept of data fusion with virtual sensing paths method. The PDF in their work is an empirical formula and the parameter was given by experience.

3. Definition of the PDF of damage occurrence and measurement of its parameters

3.1 The nature of PDF of damage occurrence

Once the damage occurrence in position (x,y), denoted as $D(x,y)$, was detected, a corresponding observed damage index will be recorded. For ToF method, the damage index is ToF, denoted as T_{ij}.

$$D(x,y) \mapsto T_{ij} \tag{4}$$

If there were no noise in the detecting process, for a certain $D(x,y)$, T_{ij} should be constant. However, there was actually always noise in experiment. The value of T_{ij} could be

different in each measurement. Therefore, the relationship between $D(x,y)$ and observed ToF value, denoted as OT_{ij}, will be:

$$D(x,y) \rightarrow OT_{ij} \tag{5}$$

The value of OT_{ij} is random variable and cannot be predicted. But according to probability theory, a function that describes the relative likelihood for this random variable to occur at a given point can be revealed.

$$P(OT_{ij}) = f(t) \tag{6}$$

where f is a probability distribution function, t is the difference between the real ToF value T_{ij} and observed ToF value OT_{ij}:

$$t = T_{ij} - OT_{ij} \tag{7}$$

Damage localization is a inverse process compared with the above discussed relationship of $D(x,y)$ and T_{ij}.

The ToF value T_{ij} cannot be given through observed OT_{ij} due to the existence of noise. But their relationship can be known as:

$$P(T_{ij}) = f'(t') \tag{8}$$

where f' is a probability distribution function, t' is the difference between the real ToF value T_{i-j} and observed ToF value OT_{ij}:

$$t' = OT_{ij} - T_{ij} \tag{9}$$

Based on probability theory, it is learned that probability distribution function f and f' should be different only in its expectation.

3.2 Definition of PDF and measurement of parameters

From above discussion, it is learned that when a ToF value OT_{ij} is observed in experiment, the probability of damage occurrence in a point $D(x,y)$ just equals to the probability of T_{ij} corresponding to $D(x,y)$ was measured as $OT_{i,j}$ due to the noise.

Therefore, the PDF of damage occurrence can be defined as

$$P(D(x,y)) = f'(t') = f(t) \tag{10}$$

Probability theory gives a method called as hypothesis testing to determine a probability distribution function. It will be used in this paper to study the type and parameters of PDF of damage occurrence.

4. Numerical simulation

Feasibility of using the measurement method present in this paper in thin plate structures was demonstrated via FE simulation. Six PZT wafers were surface-installed at an aluminium plate. The aluminium plate was 600 mm × 600 mm × 1.5 mm in size, supported with all its four edges. The thin plate was three-dimensionally modeled using eight-node brick solid elements. To insure simulation precision, the largest dimension of FE elements was less than 1 mm and the plate was divided into multi-layer in thickness, guaranteeing that at least ten elements were allocated per wavelength of the incident diagnostic wave, which has been demonstrated sufficiently to portray the characteristics of elastic waves in the thin plate[9]. A through-thickness hole of 16 mm in diameter was assumed in the plate, 115 mm and 282 mm away from the left and upper edges of the plate, respectively, as seen in Figure 3. The S_0 mode of Lamb waves was used to detect damage. Five-cycle *Hanning* window-modulated sinusoid tone bursts at a central frequency of 300 kHz were activated as the incident diagnostic wave signal. The speed of S_0 mode is 5086.7m/s in this simulation.

The six PZT wafers were divided into three sensor pairs. For each sensor pair, the ToF was measured 10 times, while in order to simulate the measurement error in real experiment, white noise was intentionally added into the captured signals. The measured result for sensor pair 1-5 was listed in Tab 1. The PDF of damage occurrence for each sensor pair was determined by the analysis of these data in three steps:

(1) Determine the type of the PDF of $OT_{i,j}$.

With the data-set of observed ToF, the type of the PDF of $OT_{i,j}$ can be acquired using hypothesis testing of probability theory. However, the central limit theorem (CLT) states conditions under which the mean of a sufficiently large number of independent random variables, each with finite mean and variance, will be approximately normally distributed. The conditions of most experiment were close to the states of CLT. So, without loss of generality, the type of the PDF of $OT_{i,j}$ was considered as normal distribution.

(2) Calculate the parameters of the PDF of $OT_{i,j}$. Since the type has been considered as normal distribution, the parameters need to be calculated were μ and σ^2.

Table 1. Observed ToF by sensor pair 1-5

sequence number	Value(1e-6)
1	3.46
2	3.44
3	3.56
4	3.57
5	3.38
6	3.52
7	3.47
8	3.46
9	3.54
10	3.56

$$\hat{\mu}_{ij} = \frac{1}{n}\sum_{k=1}^{n} OT_{ij,k} \qquad (11)$$

$$\hat{\sigma}_{ij}^2 = \frac{1}{n-1}\sum_{k=1}^{n}\left(OT_{ij,k} - \hat{\mu}\right)^2 \qquad (12)$$

where n is the number of samples, that is the measurement time number of OT_{ij}. For the samples shown in Table 1, μ and σ^2 are 3.4970e-6 and 3.6899e-15, respectively.

(3) The PDF of damage occurrence for the sensor pair i-j will be:

$$f\left(t_{ij}\right) = \frac{1}{\sqrt{2\pi\hat{\sigma}_{ij}^2}} e^{-\frac{\left(t_{ij} - \hat{\mu}\right)^2}{2\hat{\sigma}_{ij}^2}} \qquad (13)$$

Once the PDF of damage occurrence for each sensor pair was calculated, the data fusion can be carried on and the result shown in Figure 4. It can be seen that because the variance given in above analysis is very small, the suggested damage location area reduced to a very small area. This numerical simulation demonstrates only the schedule of present method. Further investigation is needed to reveal the type and character of PDF for the sensors in real experiment environment.
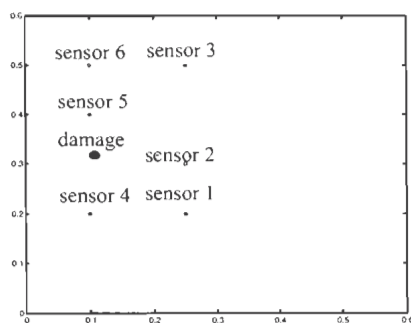

Figure 3 Schematic diagram of the aluminum plate for damage identification
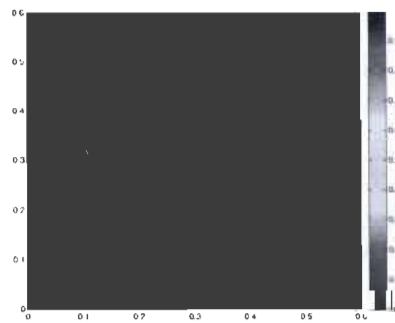
Figure 4 Probability of the presence of damage

5. Conclusion

A scientific method of selecting the type and parameters of the PDF of damage occurrence was presented. With the presented method, there will be no more empirical parts in the Lamb waves damage detection method based on data fusion. It will make the Lamb waves damage detection method based on data fusion completely in theory.

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