# Sensor Placement Strategy for Structural Damage Detection from Modal Strain Energy Change

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**Abstract**
This paper presents a sensor placement optimization strategy with the purpose of damage detection from modal strain energy change. Firstly, the damage detection method is introduced, in which damage is localized from the change of the modal strain energy before and after the occurrence of the damage. Then, the sensor placement configuration is selected as the one that contributes most to the modal strain energy change. The Pareto optimal solution theory is applied to find the optimal solution for a multi-objective optimization problem where more than one member or DOF are considered critical and need monitoring. The proposed sensor placement strategy is in line with the damage detection strategy, so that the sensor placement configuration is capable of providing the as much useful data as possible and the effectiveness of the damage detection can be maximized. The method is eventually applied to a reduced-order finite element model of the Canton Tower to find the optimal sensor placement.

***Key words***: Sensor placement, Modal strain energy change, Damage detection, Pareto optimal solution.

## 1. Introduction

Structural health monitoring (SHM) and damage detection currently plays an important role in civil engineering, since it follows the needs to evaluate the health state of the structures that undergo severe natural disasters, and the deterioration of the structures that are approaching their design lifespan. It is worth noting that the process of monitoring is directly based on the measured data from the sensors deployed on the structure. The sensor placement problem, as the first step of SHM and damage detection, is a crucial issue. A robust sensor placement is able to provide sufficient information on structural condition for further assessment. Otherwise, it would restrict the ability of SHM system for successfully detecting damage. Sensors should be placed judiciously so as to cover the largest amount of structural condition information.

In the past two decades, strategies addressing the issue of optimal sensor placement with the purpose of modal identification have been intensively investigated(1-2). They can be classified into three categories: information-based methods(3-4), energy-based methods(5), and information entropy-based methods(6-7). Some researchers have addressed the adaption of these methods for sensor placement strategy aiming at damage detection(8-10).

In this paper, a sensor placement approach is presented with the purpose of structural damage detection from modal strain energy change. Firstly, a damage indicator that is based on modal strain energy change is introduced. Damage regions can be localized utilizing the underlying premise that the modal strain energy change of the damaged element is much larger than the others. Then, the sensor positions are selected according to their contribution to the modal strain energy change. The Pareto optimal solution theory is applied to deal with the situation that multiple elements are vulnerable to damage and need to be taken into account in the SHM strategy.

## 2. Structural Damage Detection from Modal Strain Energy Change

Structural damage often causes loss of stiffness in one or more elements, which would change the structural dynamic properties such as natural frequencies and mode shapes. In view of the fact that modal strain energy is sensitive to local damage, modal strain energy change is often used as a damage indicator to locate the damaged elements by comparing the modal strain energy before and after the occurrence of damage.

Shi et al.(11) proposed a structural damage detection method on the basis of the change of modal strain energy before and after damage. In this approach, it is assumed that structural damage causes loss of stiffness properties of a structure, and subsequently causes perturbation in its dynamic features. Thus, the stiffness matrix $\mathbf{K}^d$, the $i$th modal eigenvalue $\lambda_i^d$, and the $i$th mode shape $\phi_i^d$ of the damaged structure can be expressed as

$$\mathbf{K}^d = \mathbf{K} + \sum_{j=1}^{L} \alpha_j \mathbf{K}_j \quad (-1 \le \alpha_j \le 0) \tag{1}$$

$$\lambda_i^d = \lambda_i + \Delta\lambda_i \tag{2}$$

$$\varphi_i^d = \varphi_i + \Delta\varphi_i \tag{3}$$

where the damage in a structure is represented as the summation of each elemental stiffness matrix multiplied by a damage coefficient; the superscript $d$ denotes the damage case; and $\alpha_j$ is a damage coefficient for the $j$th element.

The modal strain energy change (MSEC) for the $j$th element by use of $i$th mode shape can be expressed in terms of the small changes of the measured experimental mode shapes $\Delta\phi_i$ as

$$\mathrm{MSEC}_{ij} = 2\phi_i^T \mathbf{K}_j \Delta\phi_i \tag{4}$$

For a small change in an undamped $n$-degree-of-freedom ($n$-DOFs) dynamic system, the governing equations of motion become

$$[(\mathbf{K} + \Delta\mathbf{K}) - (\lambda_i + \Delta\lambda_i)\mathbf{M}](\phi_i + \Delta\phi_i) = 0 \tag{5}$$

Ignoring the second-order terms, and applying the orthogonal relationship $\phi_i^T \mathbf{M} \phi_i = \mathbf{I}$, the change of the $i$th mode shape can be rewritten as

$$\Delta\phi_i = \sum_{r=1}^{n} -\frac{\phi_r^T \Delta\mathbf{K} \phi_i}{\lambda_r - \lambda_i} \phi_r \quad (r \ne i) \tag{6}$$

Suppose that only single damage exists in the structure in the element $p$, the modal strain energy change for the element $p$ becomes

$$\mathrm{MSEC}_p = \sum_{p=1}^{L} -2\alpha_p \phi_i^T \mathbf{K}_j \sum_{r=1}^{n} \frac{\phi_r^T \Delta\mathbf{K} \phi_i}{\lambda_r - \lambda_i} \phi_r \quad (r \ne i) \tag{7}$$

where $\phi_i \mathbf{K}_p$ and $\mathbf{K}_j \phi_i$ are two vectors with most of their elements equal to zero except a few elements associated with those DOFs of the elemental stiffness matrix $\mathbf{K}_p$ or $\mathbf{K}_j$. Besides, the MSEC value of the damaged element is much larger than that of any other undamaged element. In view of these, the MSEC can be used as a damage indicator for identifying the damaged elements.

## 3. Sensor Placement with Modal Strain Energy Change

For a large-scale and complicated structure, since the members and the DOFs are very extensive, sensors are favorably placed on the critical members or DOFs. However, the sensors used are usually limited, even compared to the number of the critical members or DOFs. This largely restricts the ability of the SHM system for successfully detecting structural damage.

Let $N_0$ be the number of the sensors used. The measured mode shape obtained from the sensor layout has the form

$$\varphi_i = \mathbf{L}\ \phi_i \tag{8}$$

where $\varphi_i$ is the measured mode shape obtained from the sensor placement, and $\phi_i$ is the complete mode shape of the structure; and the matrix **L** is the observation matrix composed of zeros and ones that maps the structural DOFs to the measured DOFs.

In recognizing that the modal strain energy change is a preferable indicator of structural damage, once a member or a DOF is considered critical, sensors are favorably to be placed on the DOFs that contribute the most to the MSEC value for the specified member or DOF. The optimal sensor placement for $N_0$ sensors can be obtained from the objective function

$$\delta_{opt} = \arg\max_{\delta} \mathrm{MSEC}_p \tag{9}$$

where $\delta$ is the sensor placement vector that satisfies the relationship $\mathbf{L}^T\mathbf{L}=\mathrm{diag}(\delta)$. To reduce the computational efforts, the optimal sensor placement can be achieved by use of a forward sequential sensor placement (FSSP) algorithm. The positions of $N_0$ sensors are determined sequentially by placing one sensor at a time in the structure at a position that results in the least reduction in MSEC.

The optimization problem in Eq. (9) is single-objective problem in which only one member or DOF is considered as critical. However, there are usually more than one critical members or DOFs in the practical case. The sensors are required to be placed in a fashion where all the critical members or DOFs are looked after by the monitoring system. Hence, the problem devolves into a multi-objective optimization problem[12-13] of finding the sensor locations that simultaneously maximize the MSEC value for all the critical members and DOFs.

For brevity, we use the set $\{1, 2, \ldots, N_p\}$ to number all critical members and DOFs in the structure that need monitoring. Define modal strain energy change ratio for each critical case as follows

$$\mathrm{MSECR}_k = \frac{|\,\mathrm{MSEC}_k - \mathrm{MSEC}_{k,worst}\,|}{\mathrm{MSEC}_{k,best} - \mathrm{MSEC}_{k,worst}} \tag{10}$$

where $\mathrm{MSEC}_{k,worst}$ and $\mathrm{MSEC}_{k,best}$ are the modal strain energy changes computed for the two reference sensor placement configurations, that is, the optimal $\delta_{best}$ and worst $\delta_{worst}$ sensor placement configurations for $N_0$ sensors, respectively.

Let $J_k(\delta)=\mathrm{MSECR}_k(\delta)$ be the effectiveness of a sensor placement configuration $\delta$ for the $k$th critical case. The problem of identifying the optimal sensor placement that maximizes all the modal strain energy changes is formulated as a multi-objective optimization problem. The optimal sensor placement simultaneously maximizes the objectives

$$\mathbf{y} = \mathbf{J}(\delta) = \left(J_1(\delta), J_2(\delta), \ldots, J_{N_p}(\delta)\right) \tag{11}$$

For multiple objectives $J_1(\delta)$, $J_2(\delta)$,…, $J_{Np}(\delta)$, there is no single optimal solution, but rather a set of alternative solutions which are optimal in the sense that no other solutions in the search space are superior to them when all objectives are considered. Such alternative solutions, trading-off the modal strain energy changes for different critical cases, are known as multi-objective optimization as Pareto optimal solutions.

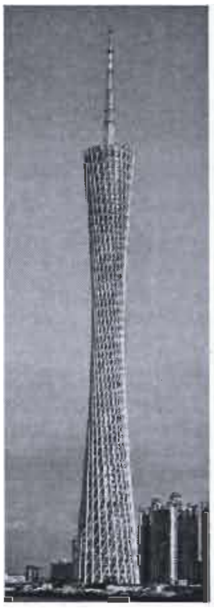
In order to find the Pareto optimal solutions, we first introduce the following statement: a decision vector $\mathbf{a}\in\delta$ is said to dominate a decision vector $\mathbf{b}\in\delta$ if and only if

$$J_k(\mathbf{a}) \le J_k(\mathbf{b}) \ \forall k \in \{1,2,...,N_p\} \text{ and } \exists m \in \{1,2,...,n\}: \\ J_m(\mathbf{a}) < J_m(\mathbf{b}) \tag{12}$$

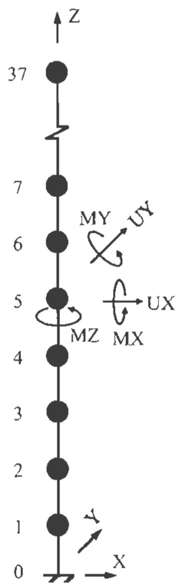
According to the above statement, non-dominated and Pareto optimal solutions can be defined as follows. A decision vector $\mathbf{a}\in\delta$ is said to be non-dominated regarding a set $\delta$ if and only if there is no vector in $\delta$ which dominates $\mathbf{a}$, that is, there is no vector $\mathbf{a}'\in\delta$ such that $J_k(\mathbf{a}')<J_k(\mathbf{a})$. All non-dominated vectors constitute admissible optimal solutions known in multi-objective optimization terminology as Pareto optimal solutions. The Pareto optimal solutions are that they cannot be improved in any objective without causing degradation in at least one other objective.

## 4. Case Study

The proposed method is applied to sensor placement problem for 16 accelerometers deployed on the instrumented Canton Tower as shown in Figure 1(a).



*a.* Canton Tower




*b.* Finite element model

Fig. 1 Canton Tower and its reduced-order finite element model

The Canton Tower (formerly named Guangzhou New TV Tower), soaring to 610 meters in height, consists of a 454-meter-high main tower and a 156- meter-high antennary mast. The structure is geometrically sophisticated that a concrete core is wrapped by triangle lattice comprising structural steel, concrete-filled columns, rings and diagonal tubes. To ensure safety and serviceability of this landmark structure, a long-term SHM system has been designed and implemented for real-time monitoring of the Canton Tower(14-15).

To proceed sensor placement optimization on the Canton Tower, a reduced-order finite element model(16) has been developed from the full-scale finite element model of the Canton Tower together with the field monitoring data. It is a 3D cantilever beam with 37 beam elements and 38 nodes as shown in Figure 1(*b*). Each node has 5 DOFs, and the whole model has 185 DOFs in total. Of these, the UX and UY DOFs account for translational deflection in the *x* and *y* directions, and the MX, MY and MZ DOFs account for twist about the *x*-, *y*- and *z*-axes, respectively. Since the vertical deflection of each node is not allowed, the UZ DOFs are discarded.

In this case study, 16 accelerometers are placed on the translational DOFs, i.e. the UX and UY DOFs. It is assumed that all 37 members are considered critical and vulnerable to damage. During the formulation of modal strain energy change, only the first mode is taken into account. Hence, the sensor placement problem is a 37-objective optimization problem. By applying the Pareto optimal solution theory, the optimal sensor placement for the 16 accelerometers is determined as shown in Table 1. The sensors are mainly located at the middle and the top of the model, which are corresponding to the waist and the antennary mast of the Canton Tower.

Table 1. Optimized placement for 16 accelerometers

| Node number | Observed direction | Node number | Observed direction |
|---|---|---|---|
| 12 | X-axis | 16 | X-axis |
| 12 | Y-axis | 16 | Y-axis |
| 13 | X-axis | 32 | X-axis |
| 13 | Y-axis | 32 | Y-axis |
| 14 | X-axis | 35 | X-axis |
| 14 | Y-axis | 35 | Y-axis |
| 15 | X-axis | 37 | X-axis |
| 15 | Y-axis | 37 | Y-axis |

## 5. Concluding Remarks

A sensor placement optimization strategy targeting for structural damage detection has been proposed and applied to optimal sensor placement for 16 accelerometers deployed on the instrumented Canton Tower of 610 m high. The optimal sensor placement is configured in line with the damage detection strategy, so that the sensor placement configuration is capable of providing the dedicated information which maximizes the effectiveness of the damage detection strategy. The case study illustrates that the optimally placed sensors are mainly located at the waist and the antennary mast of the Canton Tower.

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## Acknowledgements

The work described in this paper was supported in part by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. PolyU 5280/09E) and partially by a grant from The Hong Kong Polytechnic University through the Development of Niche Areas Programme (Project No. 1-BB68).