Using Fiber Bragg Grating (FBG) Sensors for Vertical Displacement Measurement of Bridges

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Abstract

In many bridges, vertical displacements are one of the most relevant parameters for structural health monitoring in both the short and long terms. Bridge managers around the globe are always looking for a simple way to measure vertical displacements of bridges. However, it is difficult to carry out such measurements. On the other hand, in recent years, with the advancement of fiber-optic technologies, fiber Bragg grating (FBG) sensors are more commonly used in structural health monitoring due to their outstanding advantages including multiplexing capability, immunity of electromagnetic interference as well as high resolution and accuracy. For these reasons, using FBG sensors is proposed to develop a simple, inexpensive and practical method to measure vertical displacements of bridges. A curvature approach for vertical displacement measurement using curvature measurements is proposed. In addition, with the successful development of a FBG tilt sensors, an inclination approach is also proposed using inclination measurements. A series of simulation tests of a full-scale bridge was conducted. It shows that both the approaches can be implemented to determine vertical displacements for bridges with various support conditions, varying stiffness (EI) along the spans and without any prior known loading. These approaches can thus measure vertical displacements for most of slab-on-girder and box-girder bridges. Moreover, with the advantages of FBG sensors, they can be implemented to monitor bridge behavior remotely and in real time. Further recommendations of these approaches for developments will also be discussed at the end of the paper.

Key words: Vertical displacement, structural health monitoring (SHM), bridge, Fiber Bragg Grating (FBG) sensors

1. Background

In many bridges, vertical displacements are one of the most relevant parameters for structural health monitoring in both the short and long terms. Bridge managers around the world are always looking for a simple way to measure vertical displacements of bridges. However, it is difficult to carry out such measurements. Linear variable differential transform (LVDT) displacement transducers are commonly used for displacement measurement. As they provide high accuracy and high resolution, it is generally acknowledged that they can be used as a reliable displacement measurement. However, it is unsuitable to be used for vertical displacement measurement of bridges because they require a stationary reference which is often impractical for over-water bridges. Leveling and trigonometrical leveling are the surveying methods to obtain the elevation and coordinates of points respectively. Although vertical displacements of bridges can be determined by the elevation change, these methods are unsuitable for bridges under motion due to traffic or wind gusts. Global positioning system (GPS) is designed as a navigation system for real-time positioning by military and civilian users. Now, it is also an emerging tool for measuring and monitoring both the static and dynamic displacement responses of bridges (1, 2). The accuracies of dynamic displacement measurement are at a sub-centimeter to millimeter level (2, 3). However, the accuracy of GPS measurement depends on many factors such as data sampling rate, satellite coverage, atmospheric effect, multipath effect, and GPS data processing methods (4). That causes uncertainty in accuracy level to apply the vertical displacement measurements for further application such as damage assessment. Therefore, a simple and practical method for vertical displacement measurement of bridges is desired.

In recent years, with the advancement of fiber-optic technologies, fiber Bragg grating (FBG) sensors are more commonly used in structural health monitoring (5-7) due to their outstanding advantages including multiplexing capabilities, high sample rate, small size, electro-magnetic interference (EMI) immunity, remote control as well as high resolution and precision. Besides, they are simple to fabricate, to interrogate/demodulate and easy to install. For these reasons, it is proposed to use FBG sensors to develop a simple, inexpensive and practical method to measure vertical displacements of bridges.

2. Theory of vertical displacement measurement of bridges using FBG technology

2.1 Relationships between vertical displacement, slope and curvature

For a simply supported beam under a uniformly distributed load, the beam deforms into a curve as described in Fig. 1.

According to the geometric relationship, the curvature and slope can be expressed respectively as

$$\kappa = \frac{1}{R} = \frac{d\theta}{ds} \quad and \quad v' = \frac{dv}{dx} = tan\theta \tag{1a, b}$$

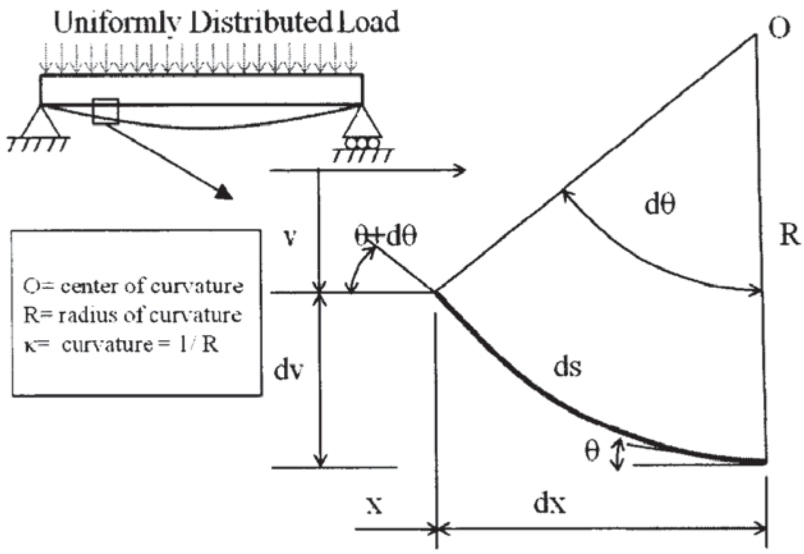


Fig. 1 The vertical displacement curve of a simply supported beam under a uniformly distributed load

Since the vertical displacement curve has a very small displacement and angle of rotation, $ds \approx dx$ and $tan\theta \approx \theta$. Hence, the curvature and slope in Eq. (1a,b) can be expressed respectively as

$$\kappa = \frac{1}{R} = \frac{d\theta}{dx} \quad \text{and} \quad \theta = \frac{dv}{dx} \tag{2a, b}$$

The first derivative of θ with respect to x in Eq. (2b) is expressed as

$$\frac{d\theta}{dx} = \frac{d^2v}{dx^2} \tag{3}$$

Combining it with Eq. (2a), the relationships between the curvature, slope and vertical displacement can be expressed as Eq. (4) and the summary of their relationships is given in Fig. 2.

$$\kappa = \frac{d\theta}{dx} = \frac{d^2v}{dx^2} \tag{4}$$

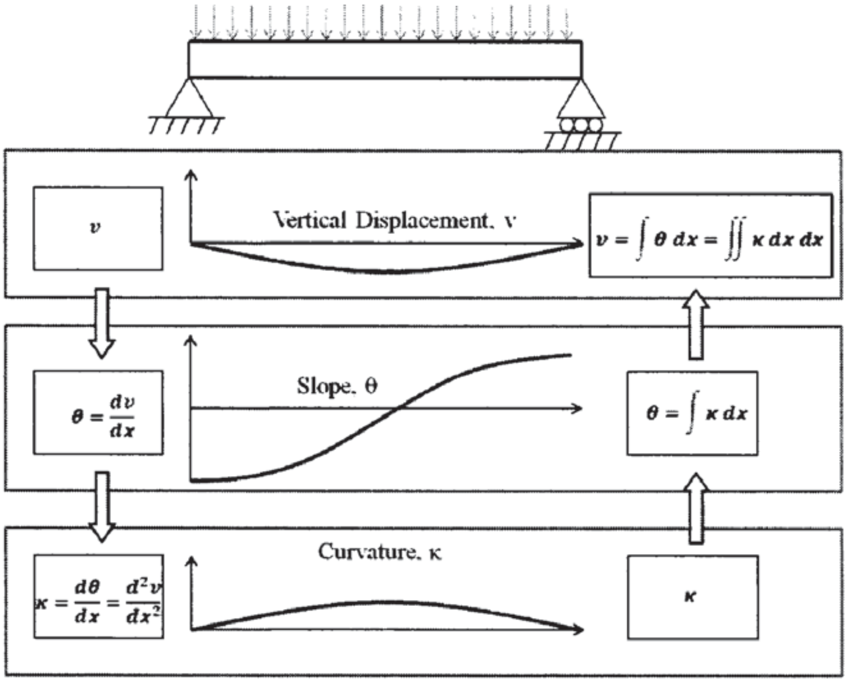


Fig. 2 The relationships between vertical displacement, slope and curvature

2.2 Curvature approach

This section presents a curvature approach to determine vertical displacements using curvature measurements based on the geometric relationship between curvature and vertical displacement. The procedure of the curvature approach is given in Fig. 3(a).

2.2.1 Curvature measurement

The relationship between curvature and strain can be expressed as

$$\kappa_i = \frac{-\varepsilon_i}{y} \tag{5}$$

where i is ith longitudinal location, ε is longitudinal strain and y is distance from neutral axis of cross section. The curvature can be determined by longitudinal strain measurements parallel to the neutral axis. However, the neutral axis may be shifted if damage of structure or temperature variation occurs. Two strain sensors placing at different distance parallel to the neutral axis can be used to eliminate the effect of the shift of the neutral axis. For two sensors at corresponding longitudinal location, the curvature is expressed as

$$\kappa_i = \frac{\varepsilon_i^b - \varepsilon_i^t}{h} \tag{6}$$

where ε^b and ε^t are the bottom and top strain and h is the distance between the sensors.

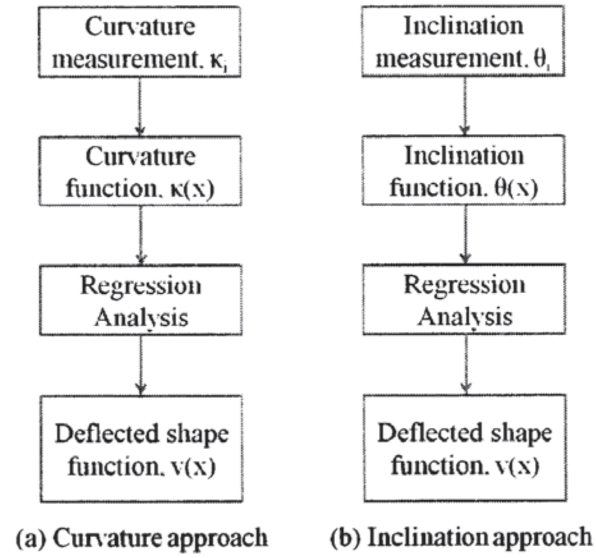


Fig. 3 Flow chat of the (a) curvature approach and (b) inclination approach of vertical displacement measurement

2.2.2 Curvature function

For measuring the vertical displacements of bridges in real time, it is necessary to consider the curvature curve is influenced by varied loading conditions. As the applied load is unknown, conducting a regression analysis is necessary to retrieve the exact curvature function that can be expressed in an *n*th order polynomial as

$$\kappa_i = c_0 + c_1 x_i + c_2 {x_i}^2 + \cdots + c_n {x_i}^n \tag{7}$$

where $c_0, c_1, c_2, \ldots, c_n$ are the coefficients of the curvature function that are obtained by the curvature measurements; x is curvilinear abscissa along the beam.

2.2.3 Deflected shape function

The deflected shape function can be determined by double integrating the curvature in Eq. (4) as

$$v(x) = \iint \kappa(x) dx dx \tag{8}$$

For a simply supported beam under a uniformly distributed load, the curvature function is a second degree polynomial as

$$\kappa_i = c_0 + c_1 x_i + c_2 {x_i}^2 \tag{9}$$

Substituting it into Eq. (8)

$$v(x) = \iint (c_0 + c_1 x + c_2 x^2) dx dx \tag{10}$$

$$v(x) = \frac{c_2 x^4}{12} + \frac{c_1 x^3}{6} + \frac{c_0 x^2}{2} + C_1 x + C_2 \tag{11}$$

where C_1 and C_2 are integration constants that can be obtained by applying boundary conditions such as zero displacement at supports or using inclination measurements. For example of assuming zero displacement at both the supports ($\mathrm{v(0)} = 0$ and $\mathrm{v(L)} = 0$ where L is length of the span),

$$C_1 = -\left(\frac{c_2 L^3}{12} + \frac{c_1 L^2}{6} + \frac{c_0 L}{2}\right) \tag{12}$$

The deflected shape function is determined as

$$v(x) = \frac{c_2x^4}{12} + \frac{c_1x^3}{6} + \frac{c_0x^2}{2} - \left(\frac{c_2L^3}{12} + \frac{c_1L^2}{6} + \frac{c_0L}{2}\right)x \quad (13)$$

2.3 Inclination approach

With the development of a FBG inclinometer [(8)] which has all advantages attributed to FBG sensors, high accuracy inclination measurement can be implemented for bridge vertical displacement measurement. As the measurements are only relevant to the geometric of deformed shapes, they will not be affected by the changes of internal deformations. That can increase the practicability of measurement for bridges. Hence, an inclination approach of vertical displacement measurement is proposed.

2.3.1 Slope function

The procedure of the inclination approach is given in Fig. 3(b). From Eq. (4), the slope function can be integrated with respective to x as

$$\theta = \int \kappa \, dx \quad (14)$$

Combining with Eq. (7), the slope function is expressed as

$$\theta_i = c_0x + \frac{c_1x_i^2}{2} + \frac{c_2x_i^3}{3} + \cdots + \frac{c_nx_i^{n+1}}{n+1} + C_1 \quad (15)$$

where C_1 is a integration constant that is defined in Eq. (12).

2.3.2 Deflected shape function

The slope of the deflection curve (θ) is the first derivative of the deflected curve (see Eq. (2a)). The deflected shape function can be determined by integrating the slope function as

$$v = \int \theta \, dx \quad (16)$$

From the example of a simply supported beam under a uniformly distributed load in 2.2.3, the curvature function is a second degree polynomial (Eq. (9)). Combining Eq. (12) and (15), the slope function can be expressed as

$$\theta_i = \frac{c_2x_i^3}{3} + \frac{c_1x_i^2}{2} + c_0x - \left(\frac{c_2L^3}{12} + \frac{c_1L^2}{6} + \frac{c_0L}{2}\right) \quad (17)$$

Then, the vertical displacement function is expressed as the same as Eq. (13).

3. Implementation for the curvature and inclination approaches to determine vertical displacements using simulated data

This section describes a series of numerical simulation tests to study the implementation of the curvature and inclination approaches. A full-scale bridge model with a span of 27.4 m under a uniformly distributed load was set up using finite element modeling as described in Fig. 4. To implement the approaches, nine curvature and nine inclination sensors data at 2.74 m center to center along the span were generated from the finite element method. The deflection shape functions were then determined by these approaches respectively. The curvature and slope functions were selected as a second and third degree polynomial, respectively. The assumption of zero displacement at both the supports was applied to the boundary condition. The deflection shape function is given in Eq. (17).

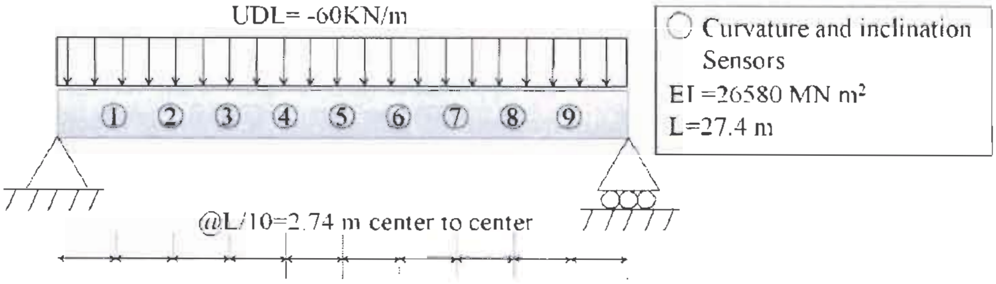


Fig. 4 Description of the bridge model- case 1

The bridge model is simply supported with nine curvature and nine inclination sensors along the span. The results are given in Fig. 5, where k0, s0 and d0 that are respectively, the curvature, slope and vertical displacement, are determined for reference. k3 and s2 are the simulated curvature and slope data that are used for implementing the curvature and inclination approaches, respectively. The curvature (K3) and slope (S2) curves that are determined by the curvature and slope approaches respectively are identical to the references (k0 and s0). The curves are exactly retrieved. The assumptions of the second degree polynomial for the curvature function and the third degree polynomial for the slope function have been verified. The vertical displacement (D3 and D2) curves are then determined and they are identical to the references (d0). It is then proven that both approaches could successfully retrieve the vertical displacement functions.

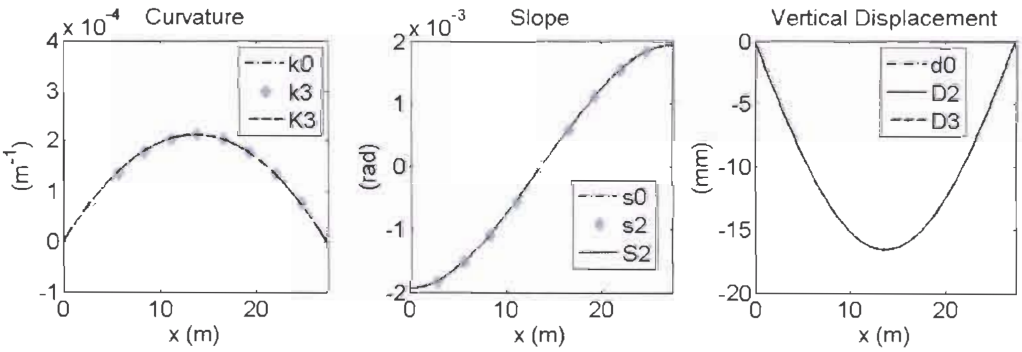


Fig. 5 The results of the curvature, slope and vertical displacement

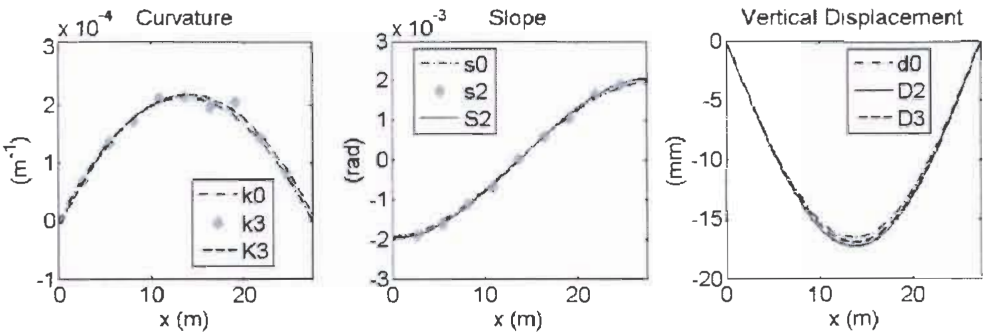
Table 1 The simulation cases

Cases	Number of sensors	Support conditions	Stiffness (EI) along the span
		Various support conditions	
1	9	Simply supported	Constant
2	9	Fixed (left) ;roller (right)	Constant
3	9	Both Fixed	Constant
		Various number of sensors	
4	8	Simply supported	Constant
5	7	Simply supported	Constant
6	5	Simply supported	Constant
7	4	Simply supported	Constant
8	3	Simply supported	Constant
		Varying Stiffness (EI)	
9	9	Simply supported	Varying
10	9	Simply supported	Varying

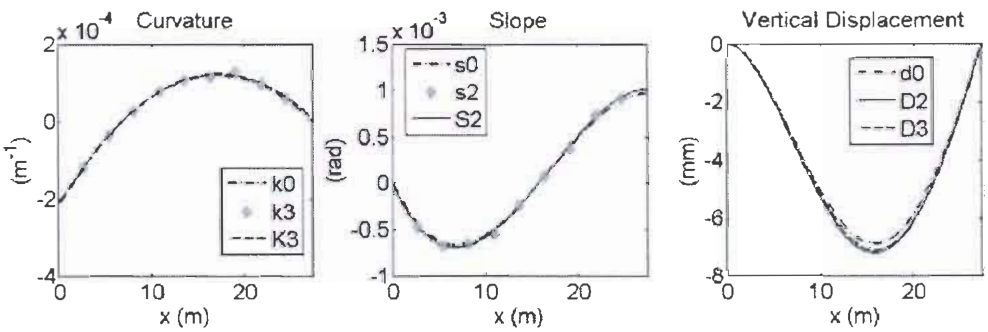
The curvature and inclination measurements in the field usually contain measurement noise. Therefore, noise of 5% noise-to-signal-ratio was added to the generated input data using the 'awgn' function in MATLAB [(9)] to simulate measurement noise interferences experienced during experimental testing. The simulation cases as described in Table 1 were tested. The bridge models of cases 1 to 3 are supported with various support conditions, aiming to verify the feasibility of the approaches for bridges with various support conditions. In cases 4 to 8, various numbers of sensors are selectively deactivated to simulate the sensors that are faulty, aiming to verify the self compensation capacity of the approaches. In cases 9 and 10, the stiffness (EI) of the bridge models varies along the spans, aiming to verify the feasibility of the approaches for various cross sections along the spans.

3.1 Implementation of the approaches with various support conditions

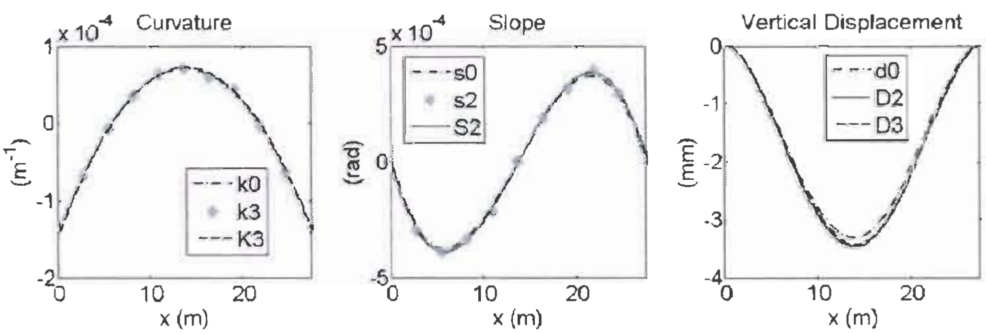
Since these approaches are based on the geometric relationships between vertical displacement, slope and curvature, support conditions have not been considered. The deflected shapes with various support conditions are directly reflected in the curvature and inclination measurements. In cases 1 to 3, these tests are to verify the feasibility of these approaches for bridges with various support conditions. These bridge models are pin-roller, fix-roller and fix-fix supports, respectively. The curvature, slope and vertical displacement curves of these cases are plotted in Fig. 6. The discrepanies of the determined vertical displacements by curvature and inclination approaches are summarized in Table 2. It is demonstrated that both the approaches can be implemented to determine the vertical displacements of bridges with various support conditions.



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 6 The curvature, slope and vertical dispalcement of cases 1 to 3

Table 2 The discrepancies of the vertical displacements between both the approaches and the finite element model

Case	Curvature approach (D3) mm (%)	Inclination approach(D2) mm (%)	Case	Curvature approach (D3) mm(%)	Inclination approach(D2) mm(%)
1	-0.43 (2.9)	-0.72 (4.4)	6	0.11(-1.0)	-0.43(4.2)
2	-0.23 (3.5)	-0.31 (4.6)	7	0.36(-2.3)	0.40(-2.5)
3	-0.14 (5.0)	-0.16 (4.9)	8	0.25(-1.5)	-0.55(3.3)
4	-0.29 (2.4)	-0.72 (5.1)	9	-0.59(3.4)	-0.75(4.3)
5	-0.32 (3.0)	-0.35 (2.1)	10	-0.34(2.6)	-0.63(4.1)

3.2 Implementation of the self compensation capacity

For these models, both the approaches require only three sensors to retrieve the curvature and slope functions. Hence, if some sensors are faulty, the redundant sensors data can compensate the data loss. In cases 4 to 8, some curvature and inclination sensors are neglected to simulate these sensors are faulty aiming to verify the self-compensation capacity. The vertical displacement curves are shown in Fig. 7. The discrepancies of the vertical displacements are summarized in Table 2. In both cases, the discrepancies of the vertical displacements of both the approaches are less than 0.72 mm. In cases 7 and 8, it is observed that the curvature and slope curve are not fitted to the reference if the sensors are not placed uniformly distributed along the spans. It is because using only three or four sensors may not give enough data to exactly retrieve the curvature and slope functions. Although only three curvature and inclination sensors can theoretically retrieve the curvature and slope functions, using five or more sensors uniformly distributed along the spans can have a higher accuracy of results and have a self-compensation capacity.

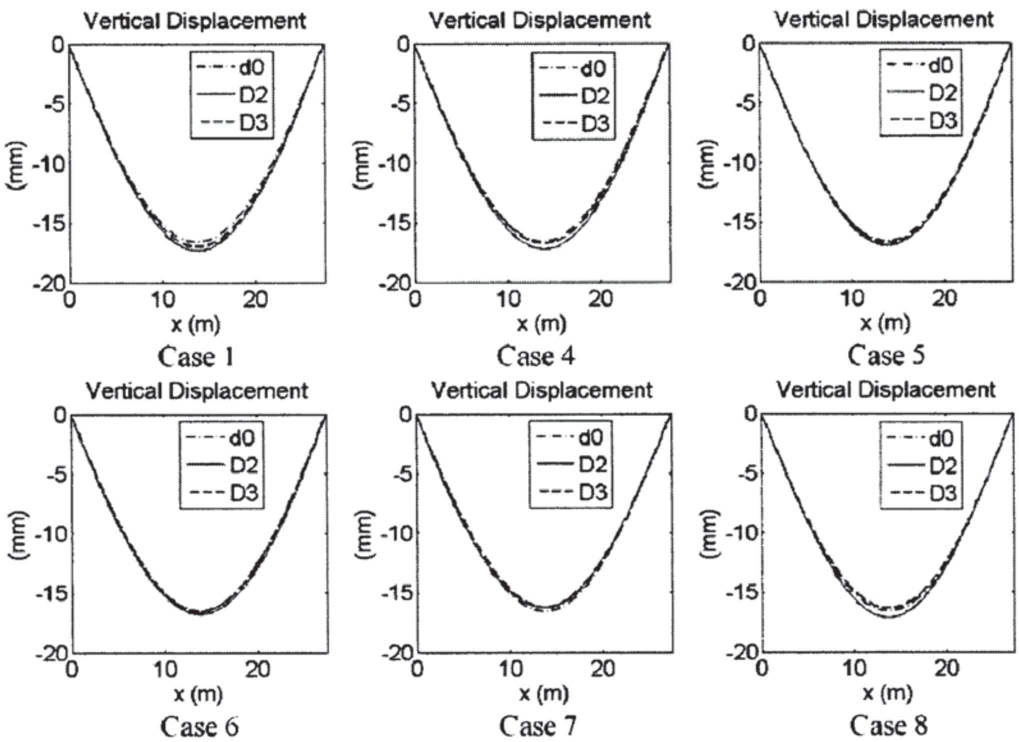


Fig. 7 The deflected shapes of cases 1 and 4 to 8

3.3 Implementation for the bridges with various stiffness(EI) along the spans

In cases 9 and 10, these tests are to study the implementation of the approaches for bridges with varying stiffness along the spans. In cases 9 and 10, the stiffnesses of the bridge models are defined in Eq. (18) and the models are described in Fig. 8.

$$EI' = \alpha \times EI \tag{18}$$

where the α is a coefficient of the stiffness. The vertical displacement curves are shown in Fig. 9. The discrepancies of the vertical displacements are summarized in Table 2. In both cases, the percentages of the discrepancies are less than 4.3 %. It is demonstrated that both the approaches are feasible to be implemented for bridges with varying stiffness along the spans.

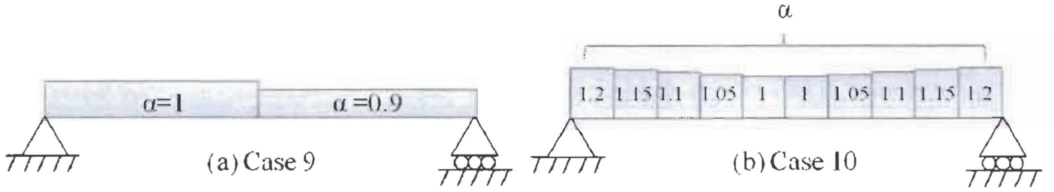


Fig. 8 The bridge models of cases 9 and 10

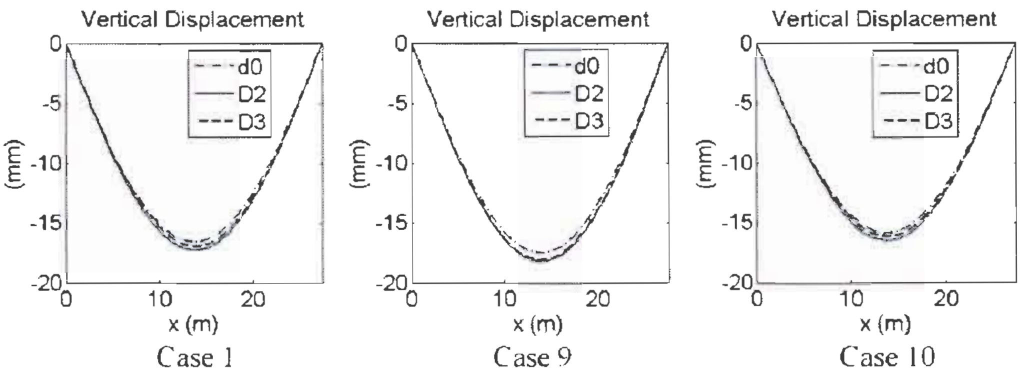


Fig. 9 The deflected shapes of cases 1, 9 and 10

4. Conclusions and recommendations

4.1 Conclusions

While it is difficult to measure vertical displacements of bridges, they are one of the most important indicators of structural behaviour. This prompts the need to develop a simple, inexpensive and practical method to measure vertical displacements of bridges. With advances in fiber-optic technologies, fiber Bragg grating (FBG) sensors have been widely used in structural health monitoring. For these reasons, a curvature approach and an inclination approach using FBG sensors are proposed. These approaches possess the advantages of FBG sensors including high sample rate, remote control and extreme weather resistance. They are appropriate for outdoor application. Since the approaches are derived by the geometric relationships between vertical displacement, slope and curvature; applied loads and structural properties such as size and elastic modulus have not been considered. In other words, the approaches do not required any prior known loading and material properties. Hence, they can be implemented for bridges along with varying cross sections. These approaches can theoretically use only three curvature and inclination sensors to determine the vertical displacements, respectively. If more than three sensors are used, these approaches have a self-compensation capacity that has been demonstrated in the simulation tests. If one or more sensors are faulty, other sensors can compensate the data loss. However, the tests have showed that using only three or four sensors may not provide enough data to exactly retrieve the curvature and slope functions. It is recommended that five or more sensors be used along spans to enhance the accuracy of the results. In most situations, the boundary conditions can be considered zero displacement at both the end supports. These approaches have also been demonstrated as feasible to be implemented for bridges with various support conditions and with varying stiffness (EI) along the spans. Hence, these approaches can be applied to most of the slab-on-girder and box-girder bridges. Since FBG sensors have high sample rate and the approaches do not required any prior known loading, the approaches can be implemented in real time measurement in service conditions under normal traffic and wind gusts.

4.2 Recommendations of vertical displacement measurement applications

Vertical displacements can be used to verify the design assumptions of bridges and update the finite element models of bridges. The methods proposed in this paper can be applied to obtain real time vertical displacements of bridges that can be further developed into a real time monitoring system. They can be used to indicate an approximate amount of loading applied on the bridges. During extreme events like typhoons, an unexpected vertical displacement can be measured in real time. If the vertical displacements exceed certain percentages of the design allowance, it can alert bridge authorities to take necessary actions. In the long term, the mean vertical displacements of bridges can be determined which can indicate the load capacity changes of bridges. A long term performance monitoring system can be developed accordingly. In these approaches, the curvature and slope of bridges are measured and the vertical displacements are determined. As they are sensitivity to the stiffness change of the bridge structures, their change can indicate the stiffness change for damage assessment. Hence, a displacement-based damage detection method can be further developed using both the measurands.

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