

# Statistical Structural Damage Detection with Consideration of Temperature Variation

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## Abstract

Structural responses vary with changing environmental conditions, particularly temperature. The variation in structural responses caused by temperature changes may mask the variation caused by structure damages. A data fusion-based damage detection approach under varying temperature conditions is presented in the paper. First, the temperature effect on the vibration properties is eliminated based on a linear regression model without losing damage information. Second, vibration properties are used to detect the structural damages with the Bayesian technique. Different sets of data measured at different times may lead to inconsistent monitoring results due to the uncertainties involved. The Dempster-Shafer evidence theory is employed to integrate the individual damage detection results and obtain a consistent one. An experiment on a two-story portal frame is investigated to demonstrate the effectiveness of the proposed method with consideration of the model uncertainty, measurement noise, and temperature effect. It shows that the damage detection results obtained by combining the damage basic probability assignments from each set of test data are more accurate than those obtained from each test data separately, and elimination of the temperature effect on the vibration properties can increase the damage detection accuracy.

**Key words:** Structural damage detection; temperature effect; data fusion; Dempster-Shafer evidence theory; Bayesian technique

## 1. Introduction

A number of damage identification methods have been developed during the last decades<sup>(1)</sup>. Most of these methods are based on the changes in signal-based or model-based structural vibration data. However, the inevitable measurement noise, the inaccurate finite element model, and changing environment limit the successful application of these methods to large-scale civil structures<sup>(2)(3)</sup>. In particular, the variations in the vibration properties caused by changing environment can be larger than those caused by structural damages<sup>(4)-(7)</sup>. Neglect of the environmental effects may lead to false damage detection. Therefore, these factors should be considered appropriately in damage detection and other structural health monitoring applications.

Bao et al.<sup>(8)</sup> proposed a Dempster-Shafer (D-S) evidence theory-based damage identification method integrating the Bayesian method, in which the measurement noise from multiple sets of data and the modeling errors are included. However, the variation in temperatures has not been considered. The current paper extends the study and presents a

data fusion-based damage detection method under varying temperature conditions. The linear relation between the frequencies and the structural temperature is investigated and used to eliminate the temperature effect on the vibration properties. The vibration data measured under different conditions are transferred to those under the same reference temperature. Subsequently, the previously developed D-S data fusion theory is employed for damage detection or condition assessment. Effectiveness of the method is verified through application to a laboratory-tested two-storey frame.

## 2. The Experiment

The experimental model is a two-storey steel frame structure, as shown in Fig. 1. The frame is modelled using 20 Euler-Bernoulli beam elements.

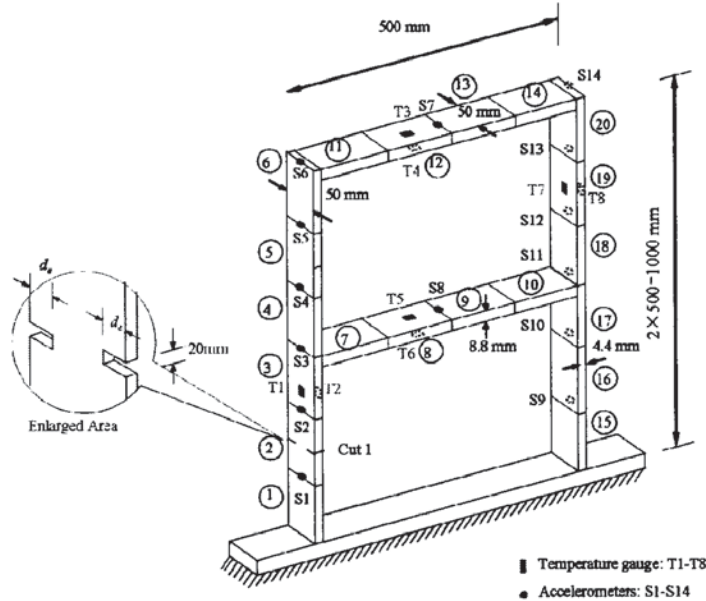


Fig. 1 Configuration of the experimental model.

The frame was exposed to ambient for one whole day. The temperatures of the beams and columns were measured every 30 seconds using 8 thermocouples (T1–T8). The frame was excited using an instrumented hammer, and the vibration responses were recorded by 14 accelerometers (S1–S14). A total of 30,720 data points were collected at a sampling rate of 2,048 Hz for each hammer impact. The vibration tests were performed 5 times every 20 minutes. A total of  $5 \times 28 = 140$  sets of vibration data were recorded throughout the day.

Three damage scenarios were then introduced by a cut length of 20 mm and depth  $d_e = 5, 10, \text{ and } 15$  mm, respectively. In each damage scenario, a one-day vibration testing was conducted, and the temperature and vibration data were measured similarly, resulting in 140 sets of vibration data for each damage case.

The relation between the first four modal frequencies and the temperature of the frame in all damage cases are shown in Fig. 2, in which the temperature is the mean value of the eight temperature gauges. The figure demonstrates that all the five frequencies decrease with a linear increase in temperature. For most of the modes, the frequencies decrease with the increase in damage severity under the same temperature. In addition, the frequencies of the undamaged structure at a higher temperature are smaller than the frequencies of the slightly damaged structure ( $d_e = 5$  mm) at a lower temperature, indicating that temperature has a more significant effect on the frequencies than do slight damage. In more severe damage cases of  $d_e = 10$  and 15 mm, the changes in frequencies caused by the damage are larger than those caused by temperature change.

The modal assurance criterion (MAC) is also investigated under different temperature conditions. No clear relation between the MAC values and the temperature can be observed.

Consequently, only the temperature effect on frequencies is eliminated in the damage detection.

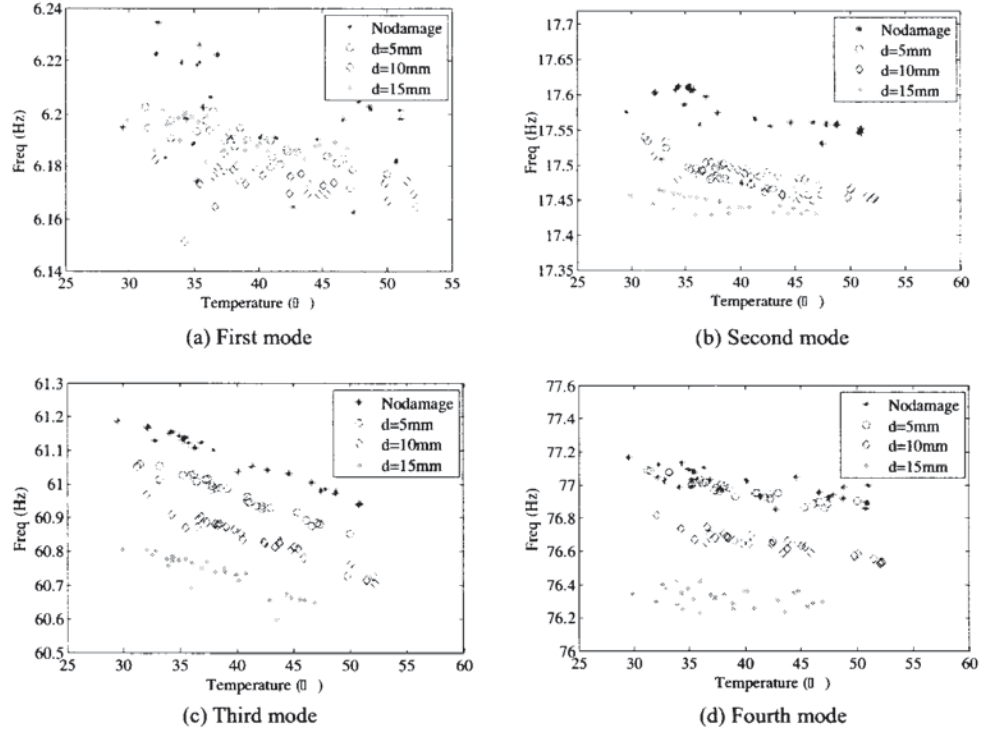


Fig. 2 The measured frequencies versus temperature.

### 3. Elimination of Temperature Effect on Frequencies

As the measured frequencies show a linear relation with temperature, a linear regression model between the frequency and temperature is established as

$$\hat{\omega}_r(\theta, T) = \beta_0^{(r)} + \beta_T^{(r)}T + \varepsilon_{\hat{\omega}_r}, \quad (1)$$

where  $\beta_0^{(r)}$  and  $\beta_T^{(r)}$  are regression coefficients, and  $\varepsilon_{\hat{\omega}_r}$  is the error of the  $r$ -th frequency. The regression coefficients can be obtained using least-squares fitting.

Next, a reference temperature  $T_0$  is selected. The measured frequencies at temperature  $T$  can be transformed into those at the reference temperature by

$$\begin{aligned} \hat{\omega}_r^e(\theta, T) &= \hat{\omega}_r(\theta, T) - \Delta\hat{\omega}_r(\theta, \Delta T), \\ &= \hat{\omega}_r(\theta, T) - \beta_T^{(r)}(T - T_0) \end{aligned} \quad (2)$$

where  $\hat{\omega}_r(\theta, T)$  is the measured frequency, and  $\hat{\omega}_r^e(\theta, T)$  is the transformed frequency at temperature  $T_0$ .

### 4. D-S Evidence Theory-based Data Fusion

D-S evidence theory is a mathematical theory of evidence, which uses the basic probability assignment function (BPA), the belief function and the plausibility function to quantify evidence and its uncertainty<sup>(9)</sup>.

For a finite set of mutually exclusive and exhaustive propositions  $\Omega$ , also known as a frame of discernment, a power set  $2^\Omega$  is the set of all the subsets of  $\Omega$  including itself and a null set  $\phi$ . The BPA (represented by  $m$ ) is a function  $m: 2^\Omega \rightarrow [0, 1]$  such that:

$$m(\phi) = 0 \quad (1)$$

$$\sum_{A \in 2^\Omega} m(A) = 1 \quad (2)$$

The measures of evidence (i.e. BPA) from different resources can be combined using



Dempster's rule of combination. Let  $m_1(B)$  and  $m_2(C)$  be basic probability assignment given by two different sources. The combination rule is:

$$m(A) = (1-k)^{-1} \sum_{B \cap C = A} m_1(B)m_2(C) \quad (3)$$

$$k = \sum_{B \cap C = \emptyset} m_1(B)m_2(C) \quad (4)$$

where  $k$  is the basic probability mass associated with conflict; and  $(1-k)$  is used to compensate for the loss of non-zero probability assignments to non-intersecting subsets and ensure that the probability assignments of resultant BPA sum to 1.

## 5. Bayesian-based Damage Detection

### 5.1 Basic Principal

A system stiffness matrix  $\mathbf{K}$  can be expressed as an assembly of element stiffness matrices. For a model with  $N_a$  elements or substructures, the global stiffness matrix is represented as follows:

$$\mathbf{K} = \sum_{i=1}^{N_a} \alpha_i \mathbf{K}_i \quad (5)$$

where  $\mathbf{K}_i$  is the  $i$ th element stiffness matrix and  $\alpha_i$  ( $0 \leq \alpha_i \leq 1$ ) is a non-dimensional parameter which represents the contribution of the  $i$ th element.

Modal data sets are collected and estimated from repeated or continuous vibration tests. The modal data for the  $n$ th test is referred to as  $\hat{\Lambda}_n$ .

$$\hat{\Lambda}_n = [\hat{\omega}_1^{(n)}, \dots, \hat{\omega}_{N_m}^{(n)}, \hat{\phi}_1^{(n)T}, \dots, \hat{\phi}_{N_m}^{(n)T}]^T \quad (6)$$

where  $\hat{\omega}_r^{(n)}$  and  $\hat{\phi}_r^{(n)}$  denote the  $r$ th estimated frequency and mode shape in the  $n$ th data set, respectively;  $N_m$  is the number of measured modes; subscript  $T$  denotes matrix or vector transpose. When vibration tests are repeated  $N_s$  times, the total collection of modal data set is denoted by  $\hat{D}_t = \{\hat{\Lambda}_1, \hat{\Lambda}_2, \dots, \hat{\Lambda}_{N_s}\}$ .

In damage detection, let  $H_{i,d}$  and  $H_{i,ud}$  denote the damaged and undamaged states of the  $i$ th element, respectively. The frame of discernment is composed of two exhaustive and exclusive hypotheses as follows:

$$\Omega_i = \{H_{i,d}, H_{i,ud}\} \quad (7)$$

The power set is composed with four propositions of  $\Omega_i$

$$2^{\Omega_i} = \{\emptyset, \{H_{i,d}\}, \{H_{i,ud}\}, \Omega_i\} \quad (8)$$

BPAs of the four propositions will be calculated from the experimental data as follows.

### 5.2 Calculation of BPA

According to the Bayesian probabilistic damage detection methodology proposed by Vanik et al. (2000), the uncertainties in the values of the parameters  $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_{N_a}]^T$  are quantified by PDFs that are obtained using the dynamic test data sets  $\hat{D}_t$ .

$$p(\alpha | \hat{D}_t) = c \exp\left[-\frac{1}{2} J(\alpha)\right] \quad (9)$$

where

$$J(\alpha) = (\alpha - \alpha_0)^T \mathbf{S}^{-1} (\alpha - \alpha_0) + \sum_{r=1}^{N_m} J_r(\alpha) \quad (10)$$

$\mathbf{S}$  is a diagonal matrix of variances of  $\alpha$ , which reflects the level of uncertainty in the nominal model;  $\alpha_0$  is generally chosen as  $\alpha_0 = [1, 1, \dots, 1]^T$  to reflect that the nominal structural model is the most probable model in the absence of any data;  $J_r(\alpha)$  is

$$J_r(\alpha) = \sum_{n=1}^{N_s} \left[ \frac{(\hat{\omega}_r^{(n)2} - \omega_r^{(n)2})^2}{\delta_{\omega r}^2} + \frac{\varphi_r^T (\mathbf{I} - \hat{\varphi}_r^{(n)} \hat{\varphi}_r^{(n)T}) \varphi_r}{\delta_{\varphi r}^2 \|\varphi_r\|^2} \right] \quad (11)$$

The marginal PDFs for  $\alpha_i$  can be obtained by integrating the joint PDF in Eq. (9) using Laplace's method for asymptotic expansion:

$$p(\alpha_i | \hat{D}_i) \approx \varphi \left( \frac{\alpha_i - \hat{\alpha}_i}{\hat{\sigma}_i} \right) \quad (12)$$

where  $\varphi$  is the standard Gaussian PDF;  $\hat{\alpha}_i$  is the most probable model obtained by minimizing  $J(\alpha)$  in Eq. (10);  $\hat{\sigma}_i^2$  is the  $i$ th diagonal element of  $L(\hat{\alpha})^{-1}$ , and  $L(\hat{\alpha})^{-1}$  is the Hessian matrix of  $J(\alpha)$  evaluated at  $\hat{\alpha}$ .

The approximate calculation of probabilistic damage measure of each element is given by<sup>(10)</sup>:

$$P^{(i)}(d_i) \approx \Phi \left( \frac{(1-d_i)\hat{\alpha}_i^{ud} - \hat{\alpha}_i^{pd}}{\sqrt{(1-d_i)^2(\hat{\sigma}_i^{ud})^2 + (\hat{\sigma}_i^{pd})^2}} \right) \quad (13)$$

where  $d_i \in [0, 1]$  is the damage threshold for the  $i$ th element, and  $P^{(i)}$  is the probabilistic damage measure.

In the experiment,  $N_t$  groups of  $\hat{D}_t$  ( $t=1, 2, \dots, N_t$ ) are extracted from the monitoring data. The  $n$ th BPAs  $m_t^{(i)}(\phi_i)$ ,  $m_t^{(i)}(H_{i,d})$ ,  $m_t^{(i)}(H_{i,ud})$  and  $m_t^{(i)}(\Omega_i)$  from data  $\hat{D}_t$  are defined as follows:

$$m_t^{(i)}(\phi_i) = 0 \quad (14a)$$

$$m_t^{(i)}(H_{i,d}) = \eta \times P_t^{(i)}(d_i) \quad (14b)$$

$$m_t^{(i)}(H_{i,ud}) = 1 - \eta \times P_t^{(i)}(d_i) \quad (14c)$$

$$m_t^{(i)}(\Omega_i) = 1 - m_t^{(i)}(H_{i,d}) - m_t^{(i)}(H_{i,ud}) = 1 - \eta \quad (14d)$$

where  $t=1, 2, \dots, N_t$ ,  $\eta \in [0, 1]$  is the weighting coefficient reflecting the reliability of damage decision using the Bayesian method. If there is no any data available at the initial stage, the prior BPAs are defined as follows:

$$m_0^{(i)}(\phi_i) = 0; \quad m_0^{(i)}(H_{i,d}) = 0; \quad m_0^{(i)}(H_{i,ud}) = 0; \quad m_0^{(i)}(\Omega_i) = 1 \quad (15)$$

where  $m_0^{(i)}(\Omega_i) = 1$  implies that the damage state of the structure has the maximum uncertainty.

### 5.3 Combination of BPA

The BPAs of  $m_0^{(i)}$  to  $m_t^{(i)}$  are then combined recursively by the D-S combination rule, similar to that shown in Eqs. (3) and (4):

$$m_{012 \dots t}^{(i)}(\phi_i) = 0 \quad (16a)$$

$$m_{012 \dots t}^{(i)}(H_{i,d}) = \frac{1}{1 - k_t^{(i)}} \left[ m_{012 \dots (t-1)}^{(i)}(H_{i,d}) m_t^{(i)}(H_{i,d}) + m_{012 \dots (t-1)}^{(i)}(H_{i,d}) m_t^{(i)}(\Omega_i) + m_t^{(i)}(H_{i,d}) m_{012 \dots (t-1)}^{(i)}(\Omega_i) \right] \quad (16b)$$

$$m_{012 \dots t}^{(i)}(H_{i,ud}) = \frac{1}{1 - k_t^{(i)}} \left[ m_{012 \dots (t-1)}^{(i)}(H_{i,ud}) m_t^{(i)}(H_{i,ud}) + m_{012 \dots (t-1)}^{(i)}(H_{i,ud}) m_t^{(i)}(\Omega_i) + m_t^{(i)}(H_{i,ud}) m_{012 \dots (t-1)}^{(i)}(\Omega_i) \right] \quad (16c)$$

$$m_{012 \dots t}^{(i)}(\Omega_i) = \frac{1}{1 - k_t^{(i)}} m_{012 \dots (t-1)}^{(i)}(\Omega_i) m_t^{(i)}(\Omega_i) \quad (16d)$$

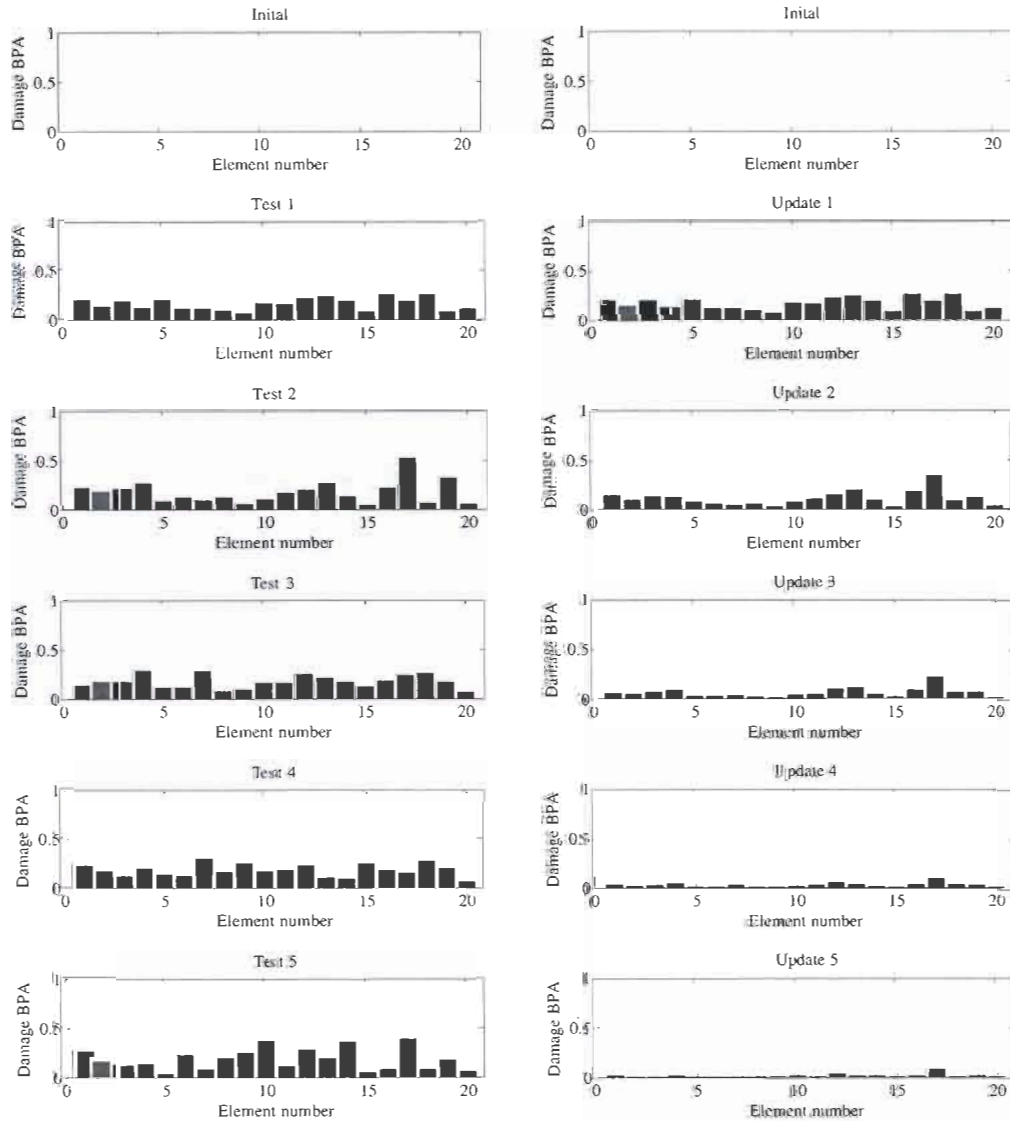
$$k_t^{(i)} = m_{012 \dots (t-1)}^{(i)}(H_{i,d}) m_t^{(i)}(H_{i,ud}) + m_{012 \dots (t-1)}^{(i)}(H_{i,ud}) m_t^{(i)}(H_{i,d}) \quad (16e)$$

where  $i = 1, 2, \dots, N_t$ ,  $j = 1, 2, \dots, N_\theta$ , and  $m_{012\dots i}^{(i)}$  is the BPA combining  $m_0^{(i)}$  through  $m_i^{(i)}$ .

#### 5.4 The Frame Structure

A total of 140 sets of vibration measurement data in each damage case are assorted into  $N_t = 7$  groups, each with  $N_s = 20$  sets of data. The BPAs in each group ( $m_0^{(i)}, m_1^{(i)}, m_2^{(i)}, \dots, m_{N_t}^{(i)}$ ,  $i = 1, 2, \dots, N_\theta$ ) can be obtained. The weighting coefficient is estimated as  $\eta = 0.85$  based on engineering experience. The BPAs from the seven groups of data are combined using the D-S combination rule,

Fig. 3 shows the damage detection results using each group of data and the data fusion approach without eliminating the temperature effect for damage cases  $d_c = 5$  mm. The BPAs in the left column of the figures are those using the individual group of data; the right column shows the evolution of the BPAs when individual groups of data are combined. In particular, the BPAs of “Update 1” are the same as those of “Test 1” (the first group); the BPAs of “Update 2” combine the BPAs of “Test 1” and “Test 2” (the second group) using the D-S combination rule; and “Update 7” gives the final BPAs integrating all seven groups of data. Fig. 3 indicates that this small damage cannot be identified even if all groups of data are combined.



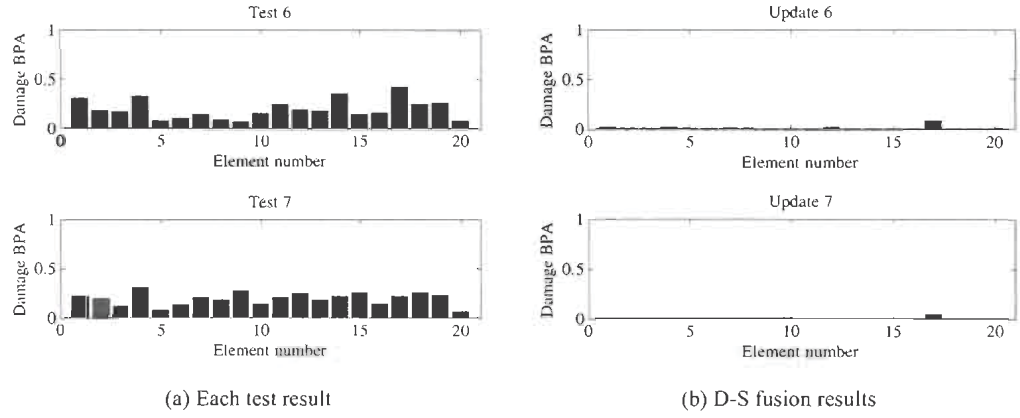
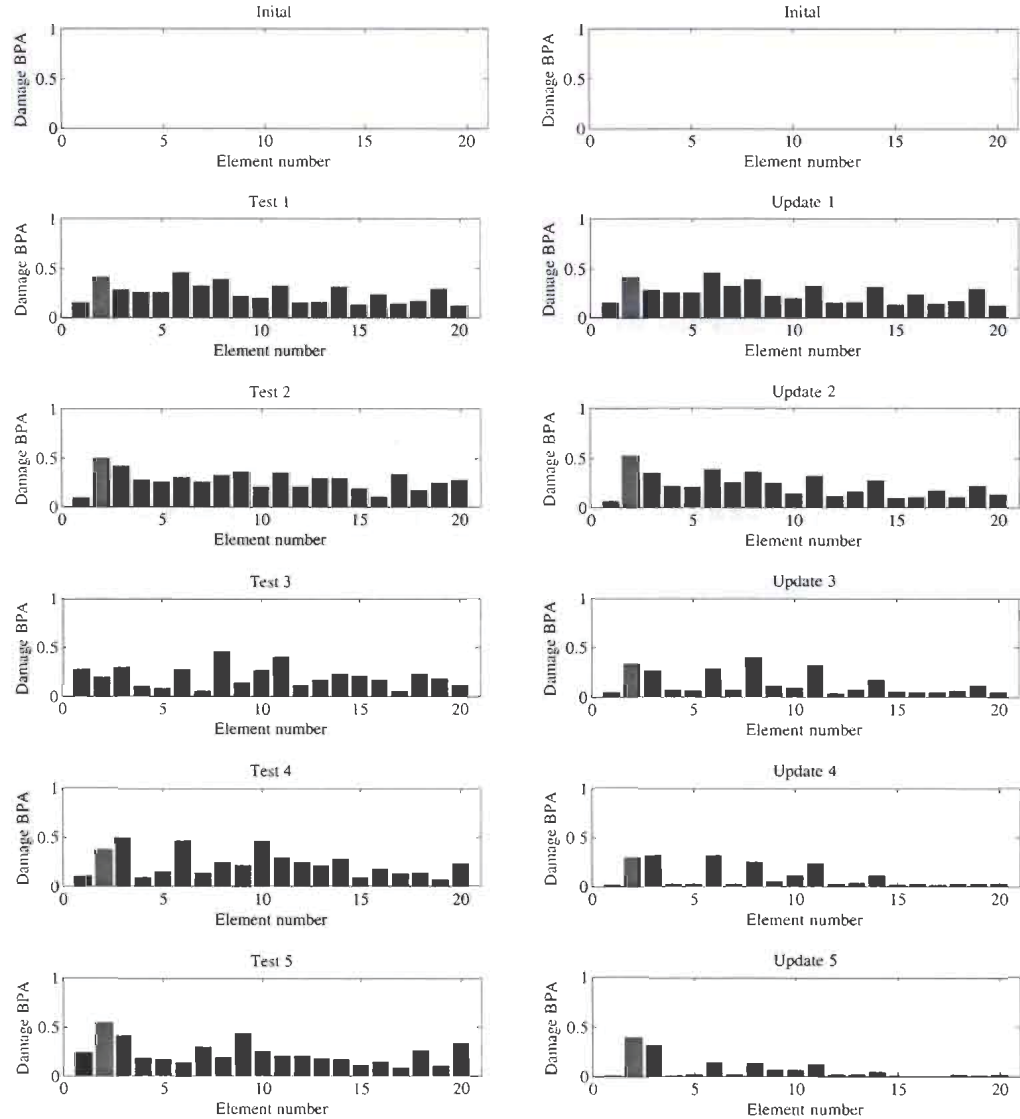


Fig. 3 Damage detection results of damage case  $d_e=5$  mm disregarding the temperature effect.

These results verify that the slightest damage cannot be identified correctly without considering the variation in temperatures, although the uncertainties in the measurement data and the modeling are considered using the Bayesian method. The detection of the damage with the proposed elimination of the temperature effect on frequencies is applied. Fig. 4 shows that the slightest damage ( $d_e = 5$  mm) can be identified successfully with the D-S fusion.



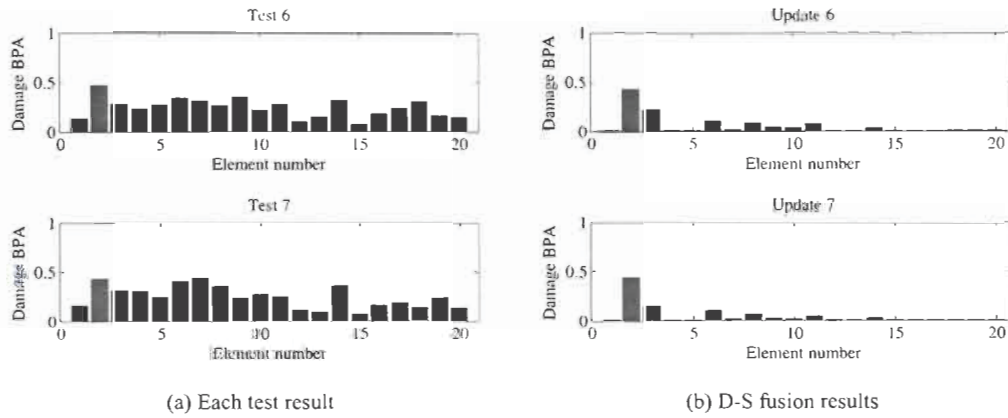


Fig. 4 Damage detection results of damage case  $d_e = 5$  mm when the temperature effect is eliminated.

The damage detection results of damage case  $d_e = 10$  mm with and without eliminating the temperature effect are shown in Fig. 5, in which only some D-S fusion results are shown for brevity. The true damaged element can be detected correctly, and the probability of a false identification is low. The results with consideration of temperature effect are more accurate than those the temperature effect is not eliminated. These observations demonstrate that the elimination of the temperature effect on frequencies can improve the damage detection results, especially for slight damages. Fig. 6 shows the damage detection results of damage case  $d_e = 15$  mm, in which the damage is correctly located, and false identification has lower probability than when the temperature effect is neglected.

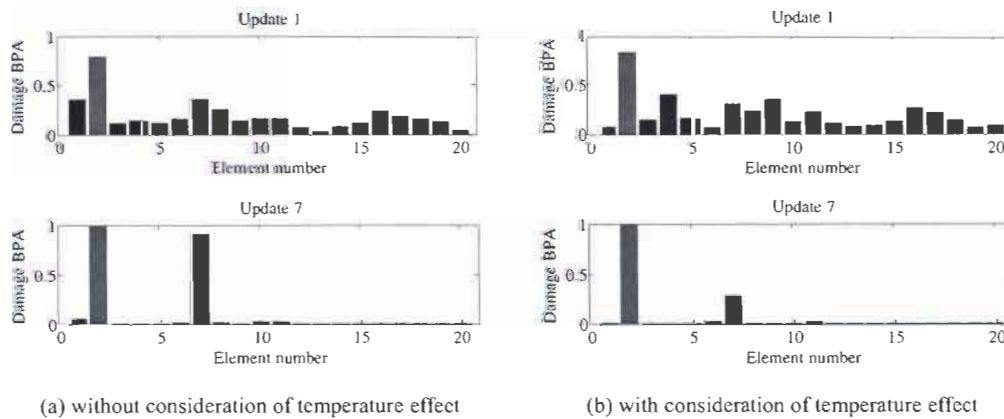


Fig. 5 Damage detection results of damage case  $d_e = 10$  mm.

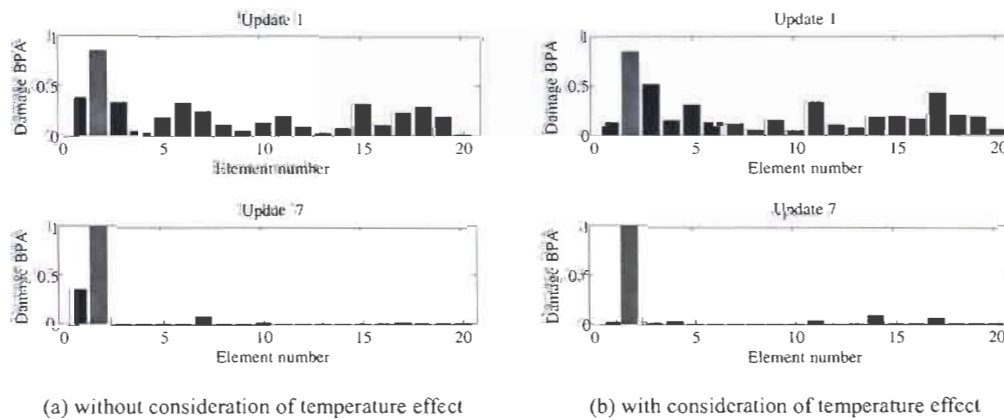


Fig. 6 Damage detection results of damage case  $d_e = 15$  mm.



## 6. Conclusions

A data fusion-based damage detection method, with consideration of the temperature effect, is proposed. A series of experiments is carried out on a two-storey steel frame under different temperature conditions to verify the effectiveness of the proposed method in damage detection. The results show that:

- 1) The temperature has a significant effect on the vibration frequencies in a linear manner approximately, whereas it has little effect on mode shapes.
- 2) After elimination of temperature effect on frequencies, the damage is still not identifiable from each group of data. Combination of the BPAs from different sets of data using the D-S approach can reduce the false identification induced by these uncertainties in the measurement noise and finite element modeling.
- 3) If the temperature effect is not considered, combination of the BPAs from different sets of data cannot detect slight damage. The damage detection results, with the elimination of the temperature effect, are improved, especially for slight damage.

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