Damage Detection in a Bridge Structure under Traffic Loads

S. S. LAW** and Jun. LI**
**Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hunghom, Kowloon, Hong Kong
E-mail: cesslaw@polyu.edu.hk

Abstract

This paper proposes a damage detection approach for a structure under moving vehicular loads. The dynamic response reconstruction technique in the wavelet domain is extended in the scenario where a structure is subject to moving vehicular loads. The transmissibility matrix between two sets of response vectors of the structure is formulated using the unit impulse response function in the wavelet domain with the moving loads at different locations. Measured acceleration responses from the structure in the damaged state are used for the identification and the damage detection procedure is conducted without the knowledge of the time-histories of the moving loads and properties of the moving vehicle. A dynamic response sensitivity-based method is used for the structural damage identification and the adaptive Tikhonov regularization technique is adopted to improve the identification results when noise effect is included in the measurements. Numerical studies on a three-dimensional box-section girder subject to a two-axle three-dimensional vehicle are conducted to validate the proposed approach of damage identification of a structure under moving loads. The simulated damage can be effectively identified even with 10% noise in the measurements.

Key words: Damage identification, Response reconstruction, Unit impulse response function, Wavelet, Moving loads, Bridge

1. Introduction

Traffic excitations are usually mixed with other ambient excitation sources, such as ground motions, wind loading and temperature effect for bridge structures in real situations. The response due to the moving vehicular loads is generally far larger than that under ambient vibrations especially for short- and medium-span concrete bridge decks. Therefore damage identification could also be conducted in the time domain using measured dynamic responses directly instead of the modal information in the frequency domain. Majumder and Manohar(1) developed a time-domain approach to detect damages in a beam using vibration data under the passage of a moving oscillator. The study combines finite element modeling for the vehicle-bridge system with a time-domain formulation to detect changes in the structural parameters. The structural properties and motion of the moving vehicle are assumed to be known. Park et al.(2) proposed a method to identify the distribution of stiffness reductions in a damaged reinforced concrete slab bridge under moving loads by using a modified bivariate Gaussian distribution function. The information of moving loads is assumed available in this study. A method for simultaneous identification of moving masses and structural local damage from measured responses has been presented(3). The masses and damage extents are taken as the optimization variables. The mass model may not accurately represent the moving vehicle and the bridge-vehicle interaction effect. In practical applications, the properties of the moving vehicle and the road surface roughness are not easy to obtain accurately and thus they are usually assumed as unknown. Therefore the interaction forces induced by the moving vehicle should be treated as unknown moving load time-histories.

It is desirable to conduct the system identification based only on the system output (vibration responses of the bridge) because the system input (traffic excitations) is difficult to measure. With the aid of high computation capacity of digital computers, it is possible to analyze the bridge-vehicle interaction problem with more sophisticated bridge and vehicle models(4). Zhu and Law(5) proposed a method for identification of the time-histories of interaction forces and structural damage iteratively using a two-step identification procedure. Later, the structural condition assessment problem is studied in a three-span box-section concrete bridge deck subject to a three-dimensional moving vehicle by identifying the time-histories of the interaction forces and system parameters in an iterative manner(6). The effect of bridge-vehicle system interaction and road surface roughness profile are implicitly taken into account by identifying the moving interaction forces using measurements from the bridge structure. It is found that a sufficient number of sensors may be required to make sure that the identification equation for the identification of the interaction forces and system parameters is over-determined. It is noted that the accuracy of the identified moving loads may have a large influence on the identification accuracy of the structural damage.

Most existing methods assume that the properties of the vehicle are available or the vehicle-bridge interaction load is needed to be identified from the measured responses of the structure. This paper proposes a damage identification approach where knowledge of the moving vehicle is not required and there is no need to identify these moving loads in the damage detection process. The dynamic response reconstruction technique in the wavelet domain(7) is further developed for a structure subject to moving vehicular loads.

2. Response Reconstruction in a Structure under Moving Vehicular Loads

2.1 Dynamic Response of a Structure under Moving Vehicular Loads

The dynamic equation of motion of a damped structural system with N degrees-of-freedom (DOFs) subject to moving vehicular loads can be written as,

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{R_l(t)\}\{P_{\text{int}}(t)\} \tag{1}$$

where M, C and K are the $N\times N$ mass, damping and stiffness matrices of the structure respectively; $\ddot{x}$, $\dot{x}$ and x are respectively the acceleration, velocity and displacement response vectors of the structure; $\{P_{\text{int}}(t)\}$ is the bridge-vehicle interaction force vector acting on the bridge structure. $\{R_l(t)\}\{P_{\text{int}}(t)\}$ is the equivalent nodal load vector applied on the structure at location l at time instant t with the mapping vector $R_l(t)$. The vector $R_l(t)$ is time-varying and it can be represented by the shape function to compute the equivalent nodal loads(8). Rayleigh damping $[C]=a_1[M]+a_2[K]$ is assumed, where a_1 and a_2 are the Rayleigh damping coefficients. The dynamic responses of the structure can be obtained from Equation (1) using the Newmark-β method(9).

2.2 Unit Impulse Response Function in Wavelet Domain under Moving Loads

It should be noted that the mapping vector $\{R_l(t)\}$ in Equation (1) is time-varying when the structure is subject to moving vehicular loads. The impulse response function with the moving loads at different locations is developed in the following paragraphs to formulate the input-output relationship for the structure when subject to the interaction forces $\{P_{\text{int}}(t)\}$.

The equation of motion of the damped structural system under the unit impulse interaction force at location l at a specific time instant t is,

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{R_l(t)\}\delta(t) \tag{2}$$

where, $R_l(t)$ denotes the shape function mapping the interaction force at location l at time instant t to the associated DOFs of the structure. The impulse response function with the moving loads at location l can be obtained using the Newmark-β method by solving the following equation of motion and initial conditions,

$$\begin{cases}[M]\ddot{h}_l(t)+[C]\dot{h}_l(t)+[K]h_l(t)=0\\ h_l(0)=0, \quad \dot{h}_l(0)=M^{-1}R_l(t)\end{cases} \tag{3}$$

where, h_l, $\dot{h}_l$ and $\ddot{h}_l$ are the unit impulse displacement, velocity and acceleration vectors with the moving loads at location l, respectively.

When the structural system is subject to the moving load $P_{\text{int}}(t)$ with zero initial conditions, the acceleration response $\ddot{x}_s(t)$ from sensor location s at time instant t can be obtained as,

$$\ddot{x}_s(t)=\int_0^t \ddot{h}_{s,l_\tau}(t-\tau)P_{\text{int}}(\tau)d\tau \tag{4}$$

in which, $\ddot{h}_{s,l_\tau}(t)$ is the unit impulse response function with the moving loads at location l_τ for sensor location s. It is noted that $\ddot{h}_{s,l_\tau}(t)$ can be obtained from Equation (3) with the moving loads placed at different locations one time step at a time. Then the impulse response function with the moving loads at different locations will be used in Equation (4) to formulate the input-output relationship. The vectors $\ddot{h}_{s,l_\tau}(t-\tau)$ and $P_{\text{int}}(\tau)$ can be expanded in terms of the discrete wavelet transform (DWT) as[10],

$$\ddot{h}_{s,l_\tau}(t-\tau)=h_{s,0}^{DWT}+\sum_j\sum_k h_{s,2^j+k}^{DWT}\psi(2^j\tau-k) \tag{5}$$

$$P_{\text{int}}(\tau)=P_0^{DWT}+\sum_j\sum_k P_{2^j+k}^{DWT}\psi(2^j\tau-k) \tag{6}$$

where $\psi(2^j\tau-k)$ is the wavelet basis function, $h_{s,2^j+k}^{DWT}$ and $P_{2^j+k}^{DWT}$ are the expansion coefficients for the impulse response function and moving force vectors respectively. Substituting Equations (5) and (6) into the convolution integral in Equation (4), and using the orthogonal conditions of the wavelet basis functions[11] as follows,

$$\int_0^t \psi(2^j\tau-k)d\tau=0 \tag{7}$$

$$\int_0^t \psi(2^j\tau-k)\psi(2^r\tau-s)d\tau=\begin{cases}1/2^j & \text{when } r=j \text{ and } s=k\\ 0 & \text{otherwise}\end{cases} \tag{8}$$

The following formula can then be derived as

$$\ddot{x}_s(t)=\ddot{h}_s^{DWT}(t)P_{\text{int}}^{DWT} \tag{9}$$

in which, $\ddot{h}_s^{DWT}(t)$ and P_{int}^{DWT} are the discrete wavelet transforms of $\ddot{h}_{s,l_\tau}(t-\tau)$ and $P_{\text{int}}(\tau)$, respectively and they are given as,

$$P_{\text{int}}^{DWT}=\begin{bmatrix}P_0^{DWT} & P_1^{DWT} & \cdots & P_{2^j+k}^{DWT}\end{bmatrix}^T$$

$$\ddot{h}_s^{DWT}(t)=[\ddot{h}_{s,0}^{DWT}(t) \quad \ddot{h}_{s,1}^{DWT}(t) \quad \cdots \quad \ddot{h}_{s,2^j+k}^{DWT}(t)/2^j]$$

For the entire time history data, for example, $\ddot{x}_s=[\ddot{x}_s(t_1) \quad \ddot{x}_s(t_2) \quad \cdots \quad \ddot{x}_s(t_n)]^T$, the system input-output relationship for the structure subject to moving loads can be expressed as,

$$\ddot{x}_{s(n\times 1)} = \ddot{h}_s^{DWT}{}_{(n\times ru)} P_{\text{int}}^{DWT}{}_{(ru\times 1)} \tag{10}$$

in which,

$$\ddot{h}_s^{DWT} = \begin{bmatrix} \ddot{h}_s^{DWT}(t_1) \\ \ddot{h}_s^{DWT}(t_2) \\ \vdots \\ \ddot{h}_s^{DWT}(t_n) \end{bmatrix}$$

where n, r and u are the number of sampled data in the response data, the number of moving loads and the number of wavelet coefficients in the discrete wavelet transform, respectively.

2.3 Response Reconstruction in a Structure under Moving Loads

The measured responses from the structure subject to moving loads are divided into two sets, noted as the First-set response vector $\ddot{x}_1(t)$ and the Second-set response vector $\ddot{x}_2(t)$ respectively. They are represented in the wavelet domain from Equation (13) as follows,

$$\begin{cases} \ddot{x}_1(t)_{(mn\times 1)} = \ddot{h}_1^{DWT}{}_{(mn\times ru)} P_{\text{int}}^{DWT}{}_{(ru\times 1)} \\ \ddot{x}_2(t)_{(qn\times 1)} = \ddot{h}_2^{DWT}{}_{(qn\times ru)} P_{\text{int}}^{DWT}{}_{(ru\times 1)} \end{cases} \tag{11}$$

in which, m and q are the number of measurements in the First-set response vector and the number of measurements in the Second-set response vector, respectively.

When the number of measurements in the First-set response vector is at least equal or larger than the number of moving loads on the structure, the pseudo-inverse $\left(h_1^{DWT}\right)^+$ exists[12] and the following Equation can be obtained from the first row of Equation (11),

$$P_{\text{int}}^{DWT} = \left(\ddot{h}_1^{DWT}\right)^+ \ddot{x}_1(t) \tag{12}$$

Substituting Equation (12) into the second row of Equation (11), we have,

$$\ddot{x}_{2r}(t) = T_{12}\ddot{x}_1(t) \tag{13}$$

where,

$$T_{12} = \ddot{h}_2^{DWT}\left(\ddot{h}_1^{DWT}\right)^+ \tag{14}$$

The Second-set response vector $\ddot{x}_{2r}(t)$ can be reconstructed from the First-set response vector $\ddot{x}_1(t)$ of the structure from Equation (13).

Moreover, Equation (14) defines the transmissibility matrix in the wavelet domain between two sets of time-domain response vectors from the structure and the presented response reconstruction technique for a bridge structure subject to moving loads can be applied for the structural damage detection.

3. Structural Damage Detection

In some existing condition assessment approaches where an initial analytical finite element model of the structure is needed, the parametric model updating method for damage identification is popular because it keeps the structural connectivity and the physical meaning of the updated stiffness matrix is clear. The initial structural finite element model is updated to match the predicted and measured vibration properties or vibration responses as closely as possible. In this study, a sensitivity-based finite element model updating method is used for structural damage identification. The damage is assumed only related to a stiffness reduction such as a change in the elastic modulus of a specific element. The mass matrix is assumed to be unchanged before and after the damage. The elemental stiffness factors in the initial intact structural finite element model are iteratively updated to minimize the difference vector $\{\Delta\ddot{x}\}$ between the reconstructed acceleration responses and the measured acceleration responses from the damaged structure.

3.1 Damage Model

The initially linear-elastic structure is assumed to remain linear-elastic after the occurrence of small local damage. The system stiffness matrix K_d of the damaged structure can be expressed as,

$$K_d = \sum_{i=1}^{n} \alpha_i K_i = \sum_{i=1}^{n} (1 + \Delta\alpha_i) K_i \tag{15}$$

where, K_i, α_i are the ith elemental stiffness matrix in the intact state and the ith elemental stiffness factor in the damage state, respectively. Therefore, $\Delta\alpha_i$ represents the extent of stiffness reduction of the ith element with $0.0 \le \alpha_i \le 1.0$.

3.2 Damage Detection Algorithm

The objective function of the damage detection algorithm is defined as the difference between two sets of response vectors

$$f_{obj} = \left\| \ddot{x}_{2m}(t) - \ddot{x}_{2r}(t) \right\|_2 \tag{16}$$

where, $\ddot{x}_{2m}(t)$ is the measured Second-set response vector from the damaged structure subject to moving loads. $\ddot{x}_{2r}(t)$ is the reconstructed Second-set response vector from Equation (13) with the measured First-set response vector $\ddot{x}_1(t)$ in the damaged state. The transmissibility matrix T_{12} in Equation (14) is obtained by using the impulse response function matrix from Equation (9). The vector α of structural elemental stiffness factors is then iteratively updated by minimizing the objective function in Equation (16) such that the reconstructed response vector $\ddot{x}_{2r}(t)$ can match the measured response vector $\ddot{x}_{2m}(t)$ well.

The dynamic response sensitivity-based model updating method(13) without considering the second- and higher-order effects is adopted here with

$$[S]\{\Delta\alpha\} = \{\Delta\ddot{x}\} = \{\ddot{x}_{2m}\} - \{\ddot{x}_{2r}\} \tag{17}$$

where, $\Delta\alpha$ is the perturbation of the vector of structural elemental stiffness factors, $[S]$ is the sensitivity matrix of the response $\ddot{x}_{2r}(t)$ with respect to the structural elemental stiffness factors. The objective function in Equation (16) is an implicit function with respect to structural elemental stiffness factors. It has been verified that the numerical sensitivity matrix can also be used for model updating effectively(14), and thus the sensitivity matrix $[S]$ is obtained using numerical finite difference method(15). It is noted that the number of equations $q \times n$ should be larger than the number of unknown elemental stiffness parameters to make sure that the identification in Equation (17) is over-determined.

3.3 Adaptive Tikhonov Regularization

The adaptive Tikhonov regularization method(16) has been proposed to improve the model updating results by categorizing all the structural elements to be assessed into two groups of possible damaged elements and intact elements from results obtained in previous iteration. The perturbation of elemental stiffness reduction factors of the possible damaged elements in each iteration is limited to a small range and the reduction factors of other elements are restrained close to zeros. It has been shown that the adaptive Tikhonov regularization has obvious advantage over the traditional Tikhonov regularization with less false positives and false negatives especially when relatively high noise level exists in the measurements. On the other hand, adaptive Tikhonov regularization can give results without divergence but with a slower convergence speed. The adaptive Tikhonov regularization technique is used in this study to obtain the solution vector $\Delta\alpha$ from Equation (17).

4. Numerical Studies

Numerical studies on a simply-supported box-section girder bridge structure are conducted to illustrate the accuracy and effectiveness of the proposed structural damage identification approach. The total length of the box-section bridge deck is 30m. The plan view and cross-section of the bridge deck model are shown in Figures 1(a) and 1(b), respectively. The Young's modulus and mass density are respectively $2.6\times10^4\,MPa$ and $2500kg/m^3$.

The finite element model of the bridge deck consists of 66 nodes and 60 flat shell elements(17) with six DOFs at each node. The numberings of nodes and elements of the finite element model are shown in Figure 1. The structural system has 396 DOFs in total. The bridge deck is simply-supported at nodes 5, 6, 65 and 66 at two ends of the deck, and the translational restraints at the supports are represented by a large stiffness of 3×10^9 kN/m. The first ten intact structural natural frequencies are from 4.44 to 21.61 Hz. Rayleigh damping is assumed in this study and the damping ratios for the first two modes are taken as $\xi=0.012$.

4.1 Dynamic Analysis of the Bridge-Vehicle System

The vehicle is according to H20-44 truck in AASHTO(18) with a two-axle three-dimensional vehicle model with seven DOFs as shown in Figure 2. The specific parameters of the vehicle are from reference(19) with a mass of 17, 000kg. The dynamic responses of the bridge structure are obtained by solving the coupled bridge-vehicle system equation of motion(20).

The two-axle three-dimensional vehicle crosses the bridge along the travelling path as shown in Figure 1(a). Seven sensors are assumed distributed on the deck in this case to measure the acceleration responses from the damaged bridge deck. The measurements are divided into two sets of responses and they are shown in Table 1. The number of measurements in the First-set response vector is equal to five and it is greater than the number of interaction forces induced by the moving vehicle which is four. The velocity of the moving force is 20 m/s and the sampling rate is 100Hz. Class *C* road surface roughness(21), corresponding to the average road pavement condition, is included in the bridge-vehicle system analysis. The acceleration response data within the first 3 seconds are used except otherwise stated.

4.2 Forward Response Reconstruction in Wavelet Domain

10% damage in both the 28th and 29th elements is simulated in the web of the bridge structure in the form of a reduction in the elastic modulus of these elements as shown in Figure 1. The simulated local damages are introduced in the structure and responses are obtained at the First-set and Second-set sensor locations in the damaged state. The reconstructed Second-set response vector is obtained from Equation (13) and is compared with the true Second-set response. It should be noticed that no noise is added to the measurements. The comparisons of forward response reconstruction results are shown in Figure 3. Figures 3(a) and 3(c) show the true and reconstructed responses at the sensor locations in the Second-set response vector. The difference vectors $(\ddot{x}_{true}(t)-\ddot{x}_{ur}(t))$ between the true and reconstructed responses of these two sensors in the Second-set response vector are shown in Figures 3(b) and 3(d). The relative errors are 1.74×10^{-11} and

1.39×10^{-11}, respectively. These results indicated that the proposed response reconstruction method in the structure subject to moving vehicular loads is very accurate.

4.3 Damage Identification Results

Damage identification is performed with the two-axle three-dimensional vehicle crossing the bridge along the travelling path as shown in Figure 1(a). The acceleration responses are obtained from the damaged bridge structure subject to the moving vehicle and they are taken as the simulated "measured" responses. 10% noise effect is included in the acceleration measurements.

Acceleration measurements with and without noise effect are used for the damage identification. Table 2 gives the associated information on convergence of the iterative procedure. The computation of matrix T_{12} in this case becomes intensive since four interaction forces from the moving vehicle are applied on the bridge structure. It should be noticed that approximately 6 hours are required for one iteration with a Intel Core 2 Quad 2.4G PC with 8G memory in the bridge-vehicle system analysis and in the process of computing sensitivity matrix for the structure subject to moving loads.

Figure 4 shows the damage identification results. For the noise-free case, the dynamic response data within the first 3 seconds are used for damage identification. The damage locations and extents are identified accurately with 9.9996% and 9.9987% stiffness reductions in 28th and 29th element respectively. This indicates that the proposed approach for damage identification in the structure under moving vehicle loads is correct. For the case with 10% noise, response data in the first 0.8s and 1.5 to 2.2s are used for the identification since the responses in these periods are much larger and could be less sensitive to the noise effect. The damages can be identified effectively with 10.41% and 11.84% stiffness reduction in 28th and 29th elements respectively when 10% noise effect is included in the measurements with very small false positives and false negatives similar to the observations in Figure 4. In addition, it should be noticed that the sensor selections in the First-set and Second-set response vectors would influence the damage identification results especially for the case with noisy measurements. However, the issue on optimal placement of sensor in the First-set and Second-set response vectors is not examined in this study.

Table 1. Sensor placement configuration

Sensor placement configuration	Sensor locations
First-set	Node 8(z), 20(z), 21(z), 45(z), 56(z)
Second-set	Node 14(z), 51(z)

Note: "Node 8(z)" denotes that the sensor is placed along the z-direction at Node 8.

Table 2. Information on convergence

	No noise	10% noise
Required iterations	6	22
Error of convergence	2.01 $\times10^{-5}$	9.85$\times$ 10^{-5}

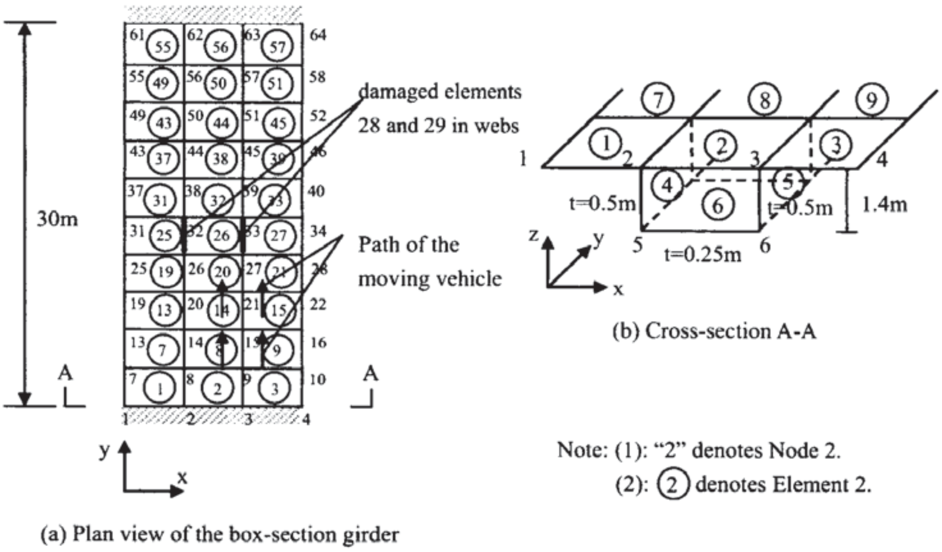

Fig. 1 Finite element model of the box-section girder structure

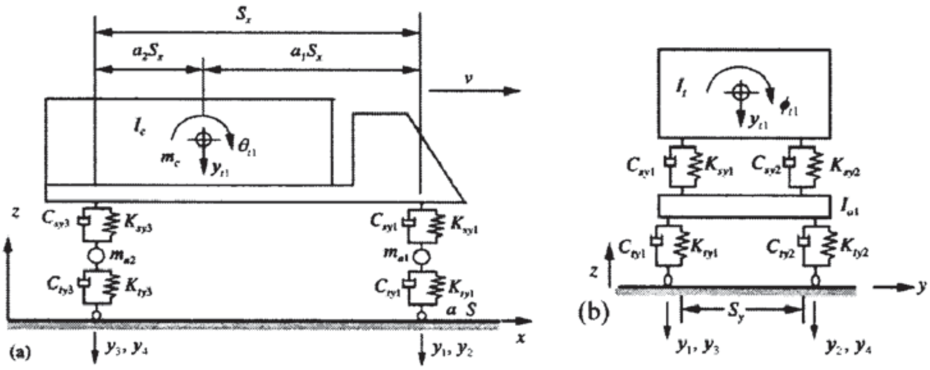

Fig. 2 A two-axle three-dimensional vehicle with seven DOFs

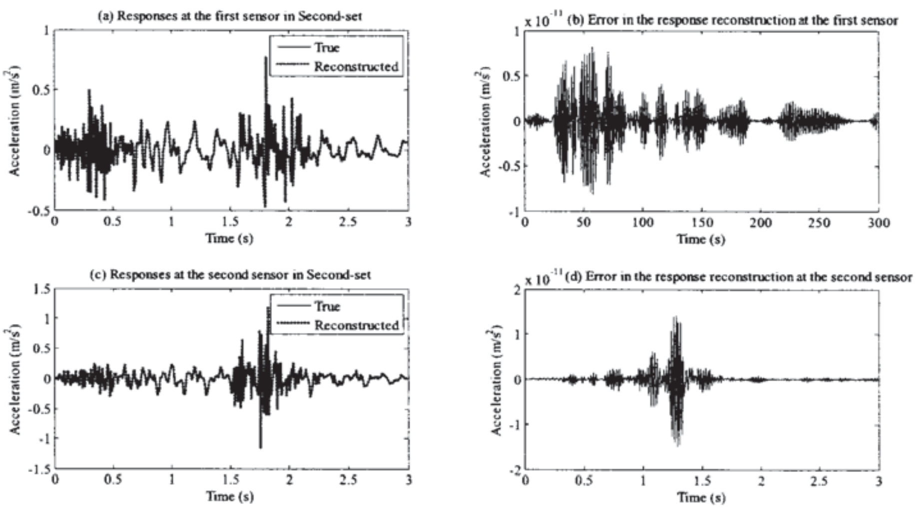

Fig. 3 True and reconstructed responses in the Second-set response vector

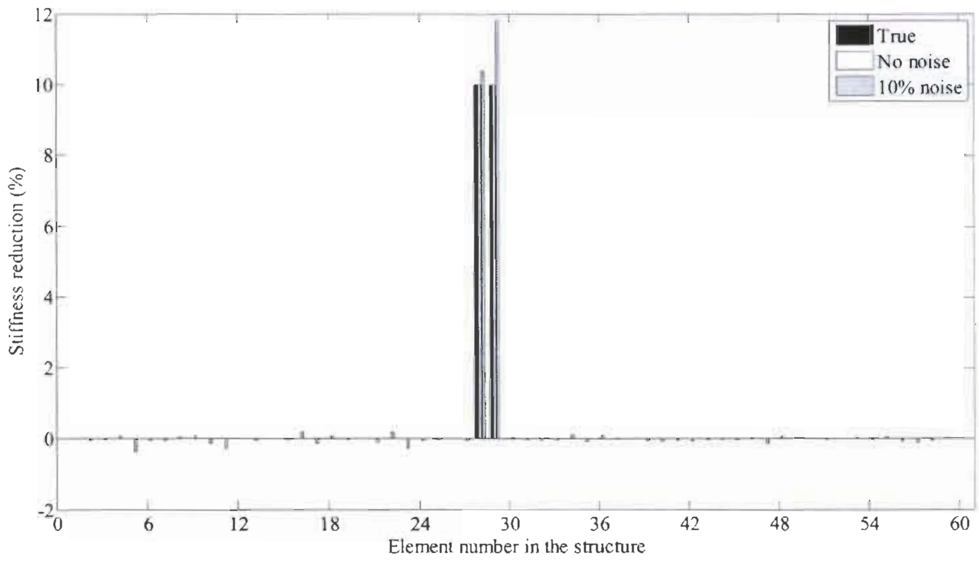

Fig. 4 Damage identification results

5. Discussions and Conclusions

A structural damage identification approach is proposed for bridge structures under moving vehicular loads based on the dynamic response reconstruction in the wavelet domain. The relationship between two sets of time-domain response vectors from the structure is formulated using the unit impulse response function with the moving loads at different locations. Acceleration responses from the damaged structure are used for the identification without the need to identify the time-histories of the moving loads and the properties of moving vehicle are not required. A dynamic response sensitivity-based method is used for the structural damage identification with the local damage modelled as a change in the elemental stiffness factors. The adaptive Tikhonov regularization technique is adopted to improve the identification results when noise effect is included in the measurements. Numerical studies on a three-dimensional box-section bridge deck subject to a two-axle three-dimensional vehicle are studied to validate the proposed approach. The simulated damage can be identified even with 10% noise in the measurements.

The proposed approach for structural damage identification is verified numerically with simulated "measured" responses without and with noise effect. Additional model errors due to complex environmental effects, such as wind, temperature effect and other random sources would arise in the filed-testing. Therefore, further studies are required to demonstrate the performance of the proposed damage identification approach with in-field testing data.

References

(1) L. Majumder, C. S. Manohar, A time-domain approach for damage detection in beam structures using vibration data with a moving oscillator as an excitation source, Journal of Sound and Vibration, Volume 268(4), 2003, pp. 699-716.

(2) T. Park, M. Noh, S. Lee, G. Z. Voyiadjis, Identification of a distribution of stiffness reduction in reinforced concrete slab bridges subjected to moving loads, ASCE Journal of Bridge Engineering, Volume 14(5), 2009, pp. 355-365.

(3) Q. Zhang, Ł. Jankowski, Z. D. Duan, Simultaneous identification of moving masses and structural damage, Structural and Multidisciplinary Optimization, Volume 42(6), 2010, pp. 907-922.

(4) F. T. K. Au, Y. S. Cheng and Y. K. Cheung, Vibration analysis of bridges under moving

vehicles and trains: an overview, Progress in Structural Engineering Materials, Volume 3(3), 2001, pp. 299-304.

(5) X. Q. Zhu, S. S. Law, Damage detection in simply supported concrete bridge structure under moving vehicular loads, ASME Journal of Vibration Acoustics, Volume 129(1), 2007, pp. 58-65.

(6) S. S. Law, J. Li, Updating the reliability of a concrete bridge structure based on condition assessment with uncertainties, Engineering Structures, Volume 32(1), 2010, pp. 286-296.

(7) J. Li, S. S. Law, Substructural response reconstruction in wavelet domain, ASME Journal of Applied Mechanics, Volume 78(4), 2011, 041010.

(8) S. S. Law, J. Q. Bu, X. Q. Zhu, S. L. Chan, Vehicle axle loads identification using finite element method, Engineering Structures, Volume 26(8), 2004, pp. 1143-1153.

(9) N. W. Newmark, A method of computation for structural dynamics, ASCE Journal of the Engineering Mechanics Division, Volume 85(3), 1959, pp. 67-94.

(10) D. E. Newland, An Introduction to Random Vibrations, Spectral and Wavelet Analysis, third ed., Longman Group Limited, England, 1993.

(11) I. Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, PA, 1992.

(12) R. Penrose, A generalized inverse for matrices, Mathematical Proceedings of the Cambridge Philosophical Society, Volume 51(3), 1955, pp. 406-413.

(13) Z. R. Lu, S. S. Law, Features of dynamic response sensitivity and its application in damage detection, Journal of Sound and Vibration, Volume 303(1-2), 2007, pp. 305-329.

(14) S. Zivanovic, A. Pavic, P. Reynolds, Finite element modeling and updating of a lively footbridge: the complete process, Journal of Sound and Vibration, Volume 301(1-2), 2007, pp. 126-145.

(15) K. W. Morton, D. F. Mayers, Numerical Solution of Partial Differential Equations, An Introduction, second ed., Cambridge University Press, 2005.

(16) X. Y. Li, S. S. Law, Adaptive Tikhonov regularization for damage detection based on nonlinear model updating, Mechanical System and Signal Processing, Volume 24(6), 2010, pp. 1646-1664.

(17) Y. W. Kwon, H. C. Bang, The Finite Element Method using MATLAB, second ed., USA: CRC Press LLC, 2000.

(18) AASHTO LRFD bridge design specifications. American Association of State Highway and Transportation Officials. Washington, DC. 2007.

(19) X. Q. Zhu, S. S. Law, Dynamic load on continuous multi-lane bridge deck from moving vehicles, Journal of Sound and Vibration, Volume 251(4), 2002, pp. 697-716.

(20) S. S. Law, J. Li, S. Q. Wu, Prestress identification in box-girder bridge deck under traffic load, Proceedings of the 3rd International Symposium on Environment Vibrations: Prediction, Monitoring, Mitigation and Evaluation, Taipei, TAIWAN, 2007, pp. 631-638.

(21) ISO 8606: 1995(E). Mechanical vibration – road surface profiles – reporting of measured data.

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