PLAYING WITH THE RULES AND MAKING MISLEADING STATEMENTS: A RESPONSE TO LUO, HODGES, WINSHIP, AND POWERS[1]

In "The Sensitivity of the Intrinsic Estimator to Coding Schemes: A Comment on Yang, Schulhofer-Wohl, Fu, and Land," Luo, Hodges, Winship, and Powers (2016) commence by noting that, in a series of articles, Fu, Yang, and Land described the intrinsic estimator (IE) and proposed that it is a general-purpose, robust, reliable, and useful tool for estimating age-period-cohort (APC) and similar models, in which an exact linear dependence among the explanatory variables makes identification and estimation problematic (Fu 2000, 2008; Yang, Fu, and Land 2004; Yang, Schulhofer-Wohl, Fu, and Land 2008; Yang 2008). Luo et al. then claim to raise "concerns about the robustness" and thus usefulness of the IE by showing that IE estimates can be "highly sensitive" to a researcher's choice of coding scheme or model parameterization. In this response, we find these "concerns" to be based on misinterpretations, misunderstandings, and misrepresentations of the IE and, accordingly, misleading. We conclude with comments and suggestions for additional research on APC models.

DEPENDENCE OF THE IE ON THE DESIGN MATRIX

In the classical APC accounting/multiple classification/fixed effects linear model or generalized linear model for an age-by-time period table of population means, rates, or proportions in which the age groups and time period intervals are equal and constant in length, there is a vector of outcomes (e.g., population rates or proportions or transformation [logarithmic, logistic] thereof), **Y**, with expected value or mean **Xb**, where **X** is the design matrix and **b** is the parameter vector (Yang et al. 2008). The design matrix **X** has one row for each observation (i.e., for each element in the vector **Y**) and one column for each element in **b**. The parameter vector **b** has one element for an intercept, $a - 1$ elements for the age effect, $p - 1$ elements for the period effect, and $a + p - 2$ elements for the cohort effects. Thus **b** has $2(a + p) - 3$ elements.

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Luo et al. (2016) make two statements about the dependence of the IE on the design matrix. First, for a given coding scheme (parameterization) of the effect coefficients in the **b** vector, the constraint used by the IE to identify the APC accounting model depends on the number of age and period (and thus cohort) categories (Luo et al. 2016, p. 6, citing Kupper et al. 1985).[2] We agree with this statement. This property follows from the fact that the IE is estimated by the Moore-Penrose generalized inverse matrix of the model, and this matrix depends on the specification of the age and period categories.

Second, Luo et al. state that "even with a fixed number of age and period categories, the IE depends on the design matrix through the coding scheme that is used." Luo et al. relatedly state,

> [Yang et al. 2008] argue that the essential purpose of the IE is to remove the influence of the coding scheme, or in equivalent terms, the design matrix (p. 1707). Below we show that this is not the case and show in detail that the IE is in fact sensitive to the coding scheme, sometimes dramatically so. As such, there is no basis to Yang et al.'s claim, critical in the assessment of its robustness and desirability, that the IE removes the effect of the design matrix or, given this, that it provides good estimates of the parameters that have generated the data. (p. XX)

The text of the Luo et al. comment illustrates these claims by presentation of divergent trends of estimated age, period, and cohort coefficients for three different empirical data sets under three different coding schemes: (1) that the respective sets of estimated age, period, and cohort coefficients sum to zero (the sum-to-zero coding), (2) that the coefficients of the first categories of the respective age, period, and cohort coefficients are set equal to zero (the $\beta_{\text{first}} = 0$ coding), and (3) that the coefficients of the last categories of the respective age, period, and cohort coefficients are set equal to zero (the $\beta_{\text{last}} = 0$ coding). Luo et al. then present a corresponding algebraic analysis in their appendix.

Here is the problem with this analysis: It is based on a misinterpretation and misrepresentation of the IE. Specifically, as explicitly stated by Yang and Land (2013*a*, p. 79):[3]

[2] Luo et al. refer to the identifying constraint on the APC accounting model coefficient vector **b** used by the IE as "implicit," citing the O'Brien (2011) and Luo (2013) commentaries. However, turnabout is fair play—Luo et al. do not cite the responses to O'Brien (2011) by Fu, Land, and Yang (2011) and to Luo (2013) by Yang and Land (2013*b*). These rejoinders make it clear that the constraint used by the IE is far from "implicit." For instance, Yang and Land (2013*b*) state that the "implicit" constraint on the age, period, and cohort effects that Luo (2013) claims is assumed by the IE is, in fact, an alternative characterization of the constraint that has been explicitly stated in all of the prior work on the IE.

[3] The primary focus of Luo et al. is Yang et al. (2008), which was not, however, our first, nor our last, publication with respect to studies of statistical methods for APC analysis and empirical applications thereof. At various points in this response, we reference other related publications.

> Instead of using reference categories, the IE uses the "usual ANOVA-type constraints" that the sums of the respective A, P, and C coefficients equal zero, termed *effect coding*. The computational algorithm used by the IE estimates the resulting *effect coefficients* for each of the $a - 1$, $p - 1$, and $a + p - 2$ A, P, and C categories, respectively, which is consistent with the definition of the parameter vector [of the APC fixed effects generalized regression model]. Then the IE uses the zero-sum constraints to obtain the numerical values of the deleted A, P, and C categories. (P. X)

In other words, while the numerical and algebraic demonstrations in Luo et al. of the divergence of the estimated trends of the age, period, and cohort coefficients of the $\beta_{\text{first}} = 0$ and the $\beta_{\text{last}} = 0$ from those of the sum-to-zero may be interesting in and of themselves, they are misleading and irrelevant with respect to the IE—because only the sum-to-zero coding is used to define and estimate the IE.[4] These demonstrations could only be derived by "playing with the rules" by which the IE is defined and estimated and constructing an associated misrepresentation thereof.

Why is ANOVA-type effect coding of the model coefficients used in the specification of the IE? And why has this coding been used for the APC accounting model at least since Fienberg and Mason (1978)? Note that an age-by-time period table of population rates or proportions is *balance*d with respect to age groups and time periods in the sense that, for each age group, there is a full set of time periods of observed rates, and, for each time period, there is a full set of age groups of observed rates. For models not of full rank applied to balanced data, sum-to-zero constraints on the coefficients long have been applied (Searle 1971).[5]

After making this erroneous statement about the dependence of the IE on the design matrix through the coding scheme that is used, Luo et al. assert that this statement "directly contradicts Yang et al.'s (2008) critical asser-

[4] The fact that the IE "uses the ANOVA normalization" of the coefficients in the **b** vector of the APC accounting model "so that the estimated effects for age (and period and cohort) sum to zero" was noted long ago (Smith 2004, p. 115).

[5] Fienberg and Mason (1978, p. 8) note that age-by-time period arrays of population rate or proportion data are unbalanced with respect to cohorts in the sense that there is only one observation corresponding to the first and last cohorts, two observations corresponding to the second and next to last cohorts, and so forth, and that the sum-to-zero constraint on the cohort coefficients could, therefore, be replaced by a weighted sum-to-zero constraint. See also Masters et al. (2016), who cite Kupper et al.'s (1985) analysis of ANOVA-type effect coding as compared to a dummy-variable design (that imposes a reference effect of zero for each of the APC factors) in which it is argued that the latter can lead to misleading patterns in estimated coefficients. Masters et al. (2016) add to the Kupper et al. (1985) analysis the point that a given dummy-variable design will strongly privilege a particular solution in the solution space, which is what accounts for the sensitivity to the coding schemes in Luo et al.

tion that the IE is invariant to the choice of the design matrix." Again, Luo et al. are playing with the rules in making this statement. The specific "assertion" made in Yang et al. (2008, p. 1708) is that "the IE satisfies a condition for estimability of linear functions of the parameter vector **b** that was established by Kupper et al. (1985, app. B) and recently further elaborated by Fu (2008). Estimable functions are invariant with respect to whatever solution . . . to the normal equations is obtained." The context of Yang et al. (2008) in which this statement was made is one in which the sum-to-zero ANOVA-type constraint is applied to the A, P, and C effect coefficients. This sum-to-zero constraint combined with the satisfaction of the Kupper et al. (1985) condition for estimability ensures that the IE is estimable, and thus its invariance property as stated by Yang et al. (2008).

GENERALIZED INVERSES, INVARIANCE, AND STATISTICAL PROPERTIES OF THE IE

To further elaborate, we note that mathematical and statistical models are wonderful cognitive devices for the study of empirical phenomena. In all cases, however, such models, when unfettered by research norms and guidelines and uninformed by substantive knowledge relevant to a particular empirical analysis, can lead to possibilities that have little relationship to their research applications. In the case of the APC accounting model, the IE follows the sum-to-zero normalization of the age, time period, and cohort coefficients for reasons just noted, a normalization that is conventional in this line of research.

Within this context, the IE has certain desirable statistical properties that either are ignored or misrepresented by Luo et al. To state these, note that, since the design matrix **X** of this model is one less than full column rank, the parameter space of the unconstrained APC linear model coefficient vector **b** can be decomposed into the direct sum of two linear subspaces that are perpendicular to each other (Yang et al. 2008, p. 1704). One subspace corresponds to the unique zero eigenvalue of the matrix $\mathbf{X}^{\mathrm{T}}\mathbf{X}$ and is of dimension 1; it is termed the null subspace of the design matrix **X**. The other, nonnull subspace is the complement subspace orthogonal to the null space. Because of this orthogonal decomposition of the parameter space, each of the infinite number of solutions of the normal equations for a conventional normal errors regression model specification of the APC accounting model (or generalized estimating equations for generalized linear model versions thereof) for an estimator $\hat{\boldsymbol{b}}$ of the coefficient vector **b** can be written as

$$\hat{\boldsymbol{b}} = \boldsymbol{B} + \boldsymbol{s}\boldsymbol{B}_0, \qquad (1)$$

where **s** is a scalar/real number corresponding to a specific solution and $\mathbf{B_0}$ is a unique eigenvector of Euclidean norm or length 1. The eigenvector

$\mathbf{B}_0$ does not depend on the observed rates **Y**, only on the design matrix **X**, and thus is completely determined by the numbers of age groups and period groups—regardless of the event rates.

Another way of characterizing equation (1) is to state that each of the infinite set of solutions of the unconstrained APC accounting model for $\hat{\boldsymbol{b}}$ corresponds to a specific generalized inverse matrix where, for a particular real number **s**, $\mathbf{sB}_0$ is the part contributed by the null subspace of the parameter space of the coefficient vector **b**. Among this infinite set of generalized inverses/solutions, the IE **B** is obtained by setting $\mathbf{s} = \mathbf{0}$, that is, by reducing the projection of the estimator $\hat{\boldsymbol{b}}$ onto the null subspace to zero. This is the algebraic basis of the invariance property stated by Yang et al. (2008) that was cited above. Again, the context of that invariance property is a design matrix **X** with a fixed number of rows and columns to which a sum-to-zero constraint on the age, period, and cohort coefficients has been applied.

Geometrically, the IE corresponds to the "point on the solution line that is closest to the origin" as characterized by O'Brien (2011) and as emphasized by Luo et al. (2016). In the algebra of generalized inverses, the IE corresponds to the estimated coefficient vector obtained by application of the Moore-Penrose generalized inverse solution of the estimating equations of the less than full rank design matrix of the APC accounting model. This solution has been characterized as the one generalized inverse among the infinite set of generalized inverses that is "well defined" (Girosi and King 2008, p. 237) and as the "most representative" among the infinite set of possible solutions corresponding to all possible just-identifying equality constraints (O'Brien 2011). In addition, if an APC accounting model for a specific age-by-time period array of data is just-identified by a classical approach, namely, by the imposition of an *equality constraint* (i.e., setting two of the age, period, or cohort coefficients equal) and if this equality constraint actually holds true for the age-by-time period data being modeled, then the resulting estimated coefficient vector $\hat{\boldsymbol{b}}$ will either be identical to the IE **B** or within sampling error thereof and this property can be used to construct a corresponding asymptotic *t*-test of *statistical estimability* of the equality constraint in the data (Yang et al. 2008, p. 1729).

ROBUSTNESS STUDIES OF THE IE

Since all approaches to estimation of APC accounting models must impose an identifying restriction on the elements of the **b** coefficient vector, all are based on restricted models. The IE is no exception, as we have stated in all of our publications using this estimator, including Yang et al. (2008). Given this, robustness studies—that is, studies of the sensitivity of numerical estimates of the age, period, and cohort effect coefficients to model specifications—are important. Luo et al. is a useful contribution to this line of robustness studies in the sense that it demonstrates the inconsistent results that can be obtained when incorrect normalizations are applied to the elements of the **b** vector.

It is important to note, however, that robustness studies of the IE that are not based on incorrect model specifications have been reported in prior publications. For instance, comparisons of IE estimates of **b** with those obtained by various equality constraints for both empirical and simulated data were reported in Yang et al. (2004) and Yang et al. (2008). The latter publication also compared estimates of the age, period, and cohort coefficients from application of the IE to a table of aggregated-population-level rates with estimates from the application of a mixed (fixed and random) effects hierarchical APC model to the individual-level repeated sample survey data from which the rates were constructed. The IE also was shown by Fu et al. (2011) to reproduce the empirical findings from another approach to identification of the APC accounting model—the age-period-cohort-characteristic (APCC) model of O'Brien (2000). Methodologically, the asymptotic convergence of APC estimators to the IE has been studied (Fu and Hall 2006; Fu 2008), and Xu and Powers (2015) showed that a Bayesian ridge model with a common prior for the ridge parameter yields estimates of age, period, and cohort effects similar to those based on the IE and to those based on a ridge estimator.[6]

REACHING TOO FAR—QUALIFYING A LUO ET AL. STATEMENT

In addition to their misrepresentations of the IE and the misleading conclusions to which they lead, Luo et al. (p. X) state that the sensitivity of the IE estimates of the APC accounting model "is even true, as shown in the mathematical appendix, with sum-to-zero coding schemes that have different omitted categories." In their appendix they include some algebra and a simulation study of a data set with three age groups, three time periods, and five cohorts "using two different sum-to-zero coding schemes, namely, the sum-to-zero coding with the last category of each effects omitted and the same coding with the first category omitted. . . . The resulting two sets of IE estimates are different." The implication from the context of this statement is

[6] Luo et al. (n.2) appear exasperated by the confidence that these studies have given to researchers who have used the IE in empirical studies, stating, "Some users of the IE appear to believe that it gives unbiased estimates of the true or data-generating parameters . . . This is false." We think otherwise based both on the cited robustness studies and on our personal experiences with simulations studies and empirical applications of the IE.

that the two sets of estimates are both numerically different and lead to divergent inferences about the trends across categories in the effect coefficients. Is that a valid general statement? Or is it the result of the particular algebraic form used in the simulation and a failure to recognize the statistical properties of the IE estimates?

To explore this, we conducted simulations based on variations of the algebraic model used by Luo et al. We found that, for certain parameter values, this mathematical construction reduces the dimensionality of the underlying APC accounting model from three to two temporal dimensions, usually A and P. Since the IE assumes that all three temporal dimensions are operative in a particular empirical application (see below), the IE should never be applied in such cases.[7]

To further explore the generalizability of the Luo et al. statement, we applied the IE to the table of U.S. female adult mortality rates analyzed in Yang et al. (2008) with 16 five-year age groups (from ages 20–24 to 95+), eight five-year time periods, 1960–99, and 26 five-year cohorts (1865–69 to 1975–79) using the two different sum-to-zero coding schemes that have different (first and last) omitted categories. The estimates of the age, period, and cohort effect coefficients are shown graphically in figure 1.

If the assertions of Luo et al. are correct, the IE estimates under the two different coding schemes should both be numerically different and lead to divergent inferences about the age, period, and cohort effects. In fact, however, in figure 1 the IE estimates of the age, time period, and cohort effect coefficients omitting the first age category are practically on top of the IE estimates of the corresponding estimated effect coefficients omitting the last age category—and the 95% confidence intervals also overlap very substantially.[8] From the confidence intervals, it can be seen that all corresponding values of both sets of estimates are within sampling errors of each other. In other words, to qualify the assertions of Luo et al., while the resulting two sets of IE estimates show small numerical differences, these differences are not statistically significant and do not lead to corresponding divergent in-

[7] Other simulations we have conducted using variations on the Luo et al. simulation model for parameter values that correspond to three operative temporal dimensions do not demonstrate the pathologies Luo et al. claim. Luo previously has made the mistake of applying and analyzing the IE to models/data in which only two temporal dimensions are operative and independent; see Yang and Land's (2013b) response to Luo (2013).

[8] In typical APC analyses of age-by-time period tables of population statistics, the age and cohort categories are substantially more numerous than the number of time periods. In such circumstances, the IE estimates of the time period effect coefficients generally will show more sensitivity than the estimates of the age or cohort coefficients to the omission of the first or last category. However, our empirical and simulation studies generally support the conclusion that the numerical differences in these two sets of estimated period effect coefficients are within the bounds of stochastic uncertainty of the estimates.

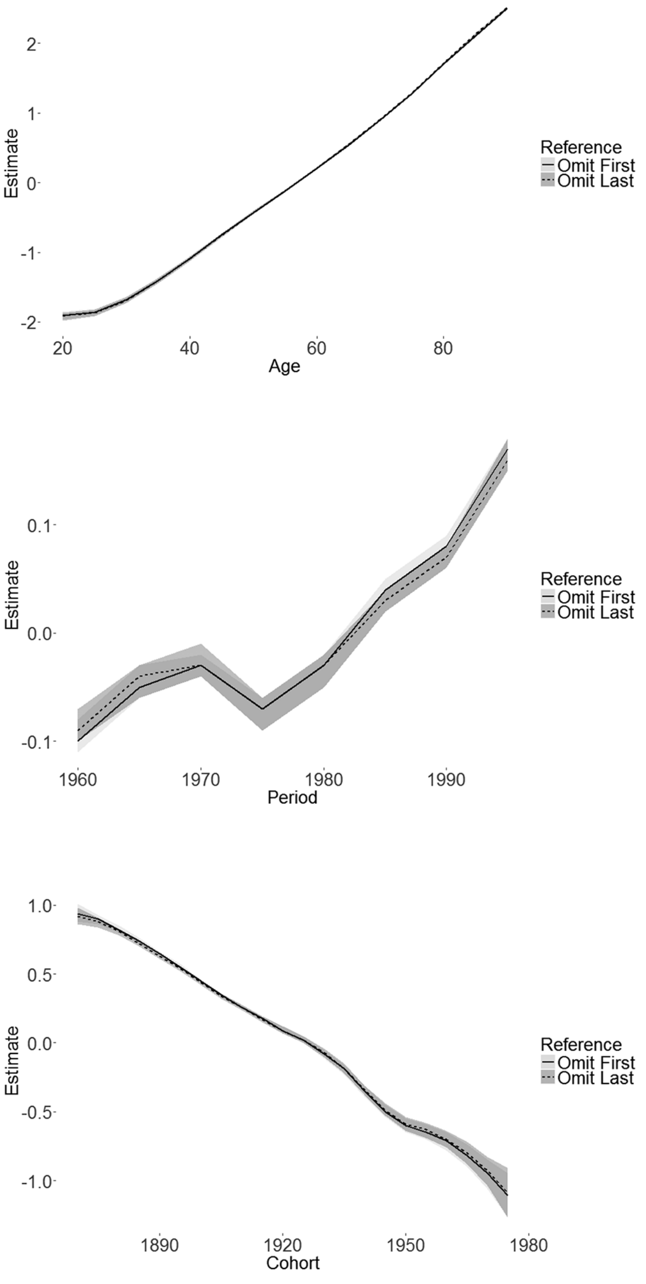

FIG. 1.—IE Estimates of U.S. female adult mortality, 1960–99, under two different sum-to-zero coding schemes, with 95 confidence intervals

ferences. To be sure, artificial numerical examples of APC models or tables can be constructed for which estimated age, period, and cohort effects are quite different, as in Luo et al.'s appendix. But this is not a general sensitivity of the IE estimates to the coding scheme, as claimed there.

GUIDELINES FOR EMPIRICAL APPLICATIONS OF THE IE

Given the various commentaries on the intrinsic estimator that have been published in recent years (O'Brien 2011; Luo 2013; Luo et al. 2016), it is clear that the initial development of this approach to estimation of the APC accounting model (Fu 2000) and its elaboration (Fu 2008; Fu et al. 2011; Yang et al. 2004, 2008) have been an attention-catching innovation. Unfortunately, however, misinterpretations, misunderstandings, misrepresentations, and corresponding misapplications of the IE have occurred. In view of this, we emphasize two guidelines for the empirical application of this estimator.

First, as emphasized by Yang et al. (2008) and Yang and Land (2013*a*, 2013*b*), any empirical application of the IE to the APC accounting model should follow a three-step procedure: Step 1 is to conduct descriptive data analyses using graphics, with the objective being to provide a qualitative understanding of patterns of temporal variations in an age-by-time period array of data to be modeled. Step 2 is model fitting and calculation of model selection statistics, such as the Bayesian information criterion (BIC). The objective is to ascertain whether the data are sufficiently well described by any single-factor or two-factor model of age (A), time period (P), and cohort (C) effects for which there is no identification problem. Only when these analyses suggest that all three dimensions are operative should one proceed with step 3: estimation of a three-factor APC model to which a constrained estimator can be applied to identify the A, P, and C effects. By revisiting Glenn's (2005) numerical example, Yang and Land (2013*a*, p. 109) emphasized that "imposition of a full APC model on data when a reduced model fits the data equally well or better constitutes a model misspecification and should be avoided." Empirical examples of chronic disease mortality in Yang (2008) and cancer mortality in Yang and Land (2013*a*) showed the necessity of all three steps, whereas those of cancer incidence for certain sites in the latter show that the first two steps suffice. A blind application of the IE, or any other constrained estimator, of the full three-factor APC model is never recommended.

Second, as illustrated well by Luo et al., if the IE is applied to estimate the **b** coefficient vector of the APC accounting model for a specific set of empirical data, the correct coefficient normalization of the coefficients should be utilized. Otherwise, misleading estimates can be produced.

CONCLUSION

The APC accounting/multiple classification model is not identified unless we impose one additional constraint whose validity cannot be tested with any data. Just as there are infinitely many generalized inverse matrices to calculate the coefficient vector of this model, there are infinitely many corresponding possible constraints. The IE imposes one such constraint and applies one particular normalization constraint and coding rule to the effect coefficients, which produces estimators with certain desirable numerical and statistical properties. If, erroneously, the effect coefficients are subjected to different coding/normalizations, then there are infinitely many possible pseudo-IE estimators, as illustrated by Luo et al.'s comment. Bayesian models can also be interpreted as imposing constraints, though in a less transparent way. Because there are limits to testability of the empirical validity of any of these constraints in any conceivable data, it is in some sense hopeless to debate which constraint is better. Given this, researchers should not use the IE, or any other constraint such as equality-of-coefficients or a cohort characteristic proxy constraint, without careful thought about whether it is reasonable in their particular context. In this regard, Luo et al.'s target is a utopian method, and they criticize how this utopian method deviates from the reality faced by methodologists and, in general, social scientists. If the IE is such a utopian method, we could not agree more with Luo et al.'s critique. Yet, our original claim was that, as we have also repeatedly illustrated in a series of publications, there is no utopian method of APC analysis. Every APC method including the IE has its own strength and weakness, and its mathematical assumptions should be subject to the investigation of researchers. Ignoring mathematical assumptions adopted by a specific APC method, or playing with the rules, leads to reasonable critiques of a utopian method but misleading critiques of that specific APC method per se.

Future research and the academic debate should focus on other issues.

1. *Methodology*: To assess the empirical applicability of any of the constraints, we have to judge them by whether they give reliable results in practice. What is a good definition of "reliable"? In what ways does reliability depend on the context? How should we judge the appropriateness of a constraint in a particular context? Relatively little is known about these issues. And the application of relatively recent developments in statistical methodology such as regression shrinkage methods and copula-based or kernel-based semiparametric estimation of age, period, and cohort effects should be explored.
2. *Substance*: The purpose of methodology is to help answer underlying social scientific questions. Are there methods that allow us to answer such questions in a way that is independent of the particular APC constraints we choose? An example is the model of continuously accumu-

lating cohort effects recently developed by Schulhofer-Wohl and Yang (2015). But more such methods are needed.

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