compared with the case using $\mathbf{w}_{lin}$. However, the difference maybe too small for practical interest.

5. Conclusions

The pdf of EEPN are analytically derived and simulations are conducted to investigate the effect of EEPN on coherent communication systems operating at 100Gb/s and beyond. The effect of EEPN induced phase noise is approximately twice as large as EEPN induced amplitude noise. The contributions to the overall EEPN by transmitter lasers is shown to be negligible for a link with residual dispersion of 700 ps/nm or above. Optimized linear MMSE equalizers in presence of EEPN are derived but only marginal performance improvements are obtained. In addition, the effects of EEPN on cycle slip probabilities of QPSK systems using the Viterbi-Viterbi algorithm for carrier phase recovery are studied. Further investigations on the effect of EEPN on other carrier recovery techniques and experimental characterizations of EEPN in coherent systems will be topics for future research.

APPENDIX A: Pdf of EEPN

In the absence of ASE noise and Tx laser phase noise, (11) becomes

$$y(t)=\left(\left[\sum_n x_n g\left(t-nT_0\right)\right]e^{j\phi_r(t)}\right)*p(t)\approx\left(\sum_n x_n q\left(t-nT_0\right)\right)e^{j\phi_r(t)}$$

where the validity of the approximation is explained in Appendix B. The received samples are then

$$\begin{aligned}y_m&=\left[\sum_n x_n q\left(mT-nT_0\right)\right]e^{j\phi_r(mT)}\approx\left[\sum_n x_n q\left(mT-nT_0\right)\right]\left[1+j\phi_r\left(mT\right)\right]\\&=\sum_n x_n q\left(mT-nT_0\right)+j\phi_r\left(mT\right)\sum_n x_n q\left(mT-nT_0\right)\\&=Q_m+Q_m j\phi_r\left(mT\right).\end{aligned}$$

Let $\Delta_{m-m'}=\phi_r(mT)-\phi_r(m'T)$ and $\mathbf{w}_{lin}=\left[w_L\ w_{L-1}\ \cdots\ w_{-L+1}\ w_{-L}\right]^T$. In this case, the estimation error for the k^{th} transmitted symbol is given by

$$\begin{aligned}\varepsilon&=\hat{x}_k-x_k=\mathbf{w}_{lin}^T\mathbf{y}\cdot\left(e^{-j\phi_r(kT_0)}\right)-x_k\\&=\left[jQ_{Sk+L}\Delta_L\ \ jQ_{Sk+L-1}\Delta_{L-1}\cdots jQ_{Sk-L+1}\Delta_{-L+1}\ \ jQ_{Sk-L}\Delta_{-L}\right]^T\mathbf{w}_{lin}\\&=jw_LQ_{Sk+L}\Delta_L+jw_{L-1}Q_{Sk+L-1}\Delta_{L-1}+\cdots jw_{-L+1}Q_{Sk-L+1}\Delta_{-L+1}+jw_{-L}Q_{Sk-L}\Delta_{-L}\\&=\sum_{i=-L}^{-1}r_i\Delta_i+0+\sum_{i=1}^{L}r_i\Delta_i\\&=\sum_{i=-L}^{-1}\left[\left(\Delta_i-\Delta_{i+1}\right)\sum_{m=-L}^{i}r_m\right]+\sum_{i=1}^{L}\left[\left(\Delta_i-\Delta_{i-1}\right)\sum_{m=i}^{L}r_m\right]\end{aligned}$$

where $r_i=jw_iQ_{Sk+i}$, and $\Delta_i-\Delta_{i\pm1}$ are independent Gaussian random variables with zero mean and variance $2\pi\Delta\upsilon T$. For a given x_k, $r_{-L},\ r_{-L+1},\ \cdots,\ r_{L-1},\ r_L$ are functions of the neighboring symbols $\mathbf{X}=\left[\cdots x_{k-2},x_{k-1},x_{k+1},x_{k+2}\cdots\right]$. Therefore, for a given neighboring symbol sequence $\mathbf{X}$, ε will be a complex Gaussian random variable with zero mean and covariance matrix

$2\pi\Delta\upsilon T\Lambda(\mathbf{X})$ where

$$\Lambda(\mathbf{X})=\begin{bmatrix} \sum_{i=-L}^{-1}\left(\sum_{m=-L}^{i}\mathrm{Re}\{r_m\}\right)^2+\sum_{i=1}^{L}\left(\sum_{m=i}^{L}\mathrm{Re}\{r_m\}\right)^2 & \sum_{i=-L}^{-1}\left(\sum_{m=-L}^{i}\mathrm{Re}\{r_m\}\sum_{m=-L}^{i}\mathrm{Im}\{r_m\}\right)+\sum_{i=1}^{L}\left(\sum_{m=i}^{L}\mathrm{Re}\{r_m\}\sum_{m=i}^{L}\mathrm{Im}\{r_m\}\right) \\ \sum_{i=-L}^{-1}\left(\sum_{m=-L}^{i}\mathrm{Re}\{r_m\}\sum_{m=-L}^{i}\mathrm{Im}\{r_m\}\right)+\sum_{i=1}^{L}\left(\sum_{m=i}^{L}\mathrm{Re}\{r_m\}\sum_{m=i}^{L}\mathrm{Im}\{r_m\}\right) & \sum_{i=-L}^{-1}\left(\sum_{m=-L}^{i}\mathrm{Im}\{r_m\}\right)^2+\sum_{i=1}^{L}\left(\sum_{m=i}^{L}\mathrm{Im}\{r_m\}\right)^2 \end{bmatrix}.$$

Therefore, for a given x_k, the pdf of EEPN is given by

$$f\left(\varepsilon \mid x_k\right)=\frac{1}{|C|^L}\cdot\sum_{\mathbf{X}\in C^L}\frac{1}{4\pi^2\Delta\upsilon T\,|\Lambda(\mathbf{X})|}\exp\left(-\frac{1}{4\pi\Delta\upsilon T}\left[\mathrm{Re}\{\varepsilon\}\ \mathrm{Im}\{\varepsilon\}\right]\Lambda(\mathbf{X})^{-1}\left[\mathrm{Re}\{\varepsilon\}\ \mathrm{Im}\{\varepsilon\}\right]^T\right)$$

where *C* is the set {1, -1} for BPSK and {1, -1, *j*, *-j*} for QPSK formats etc. and | *C* | denotes the cardinality of *C*. Note that in presence of ASE noise with variance σ^2 per quadrature, $\Lambda(\mathbf{X})$ can be replaced by $\Lambda(\mathbf{X})+\sigma^2\mathbf{I}$.

APPENDIX B: Proof of $\left(g(t)\cdot e^{j\phi_r(t)}\right)*p(t)\approx q(t)e^{j\phi_r(t)}$

Denote $u(t)=\left(g(t)\cdot e^{j\phi_r(t)}\right)*p(t)$, the Fourier Transform of $u(t)$ is given by

$$U(f)=\left(G(f)*\Phi(f)\right)\cdot P(f)$$

where $\Phi(f)$ is the Fourier Transform of $e^{j\phi_r(t)}$. Now,

$$U(f)=P(f)\int_{-\infty}^{\infty}\Phi(f_1)G(f-f_1)df_1\approx P(f)\int_{-\Delta\upsilon/2}^{\Delta\upsilon/2}\Phi(f_1)G(f-f_1)df_1$$

as the bandwidth of $e^{j\phi_r(t)}$ is approximately $\Delta\upsilon$. Since the bandwidth of the low-pass filter $P(f)$ is in the order of 30 GHz for a symbol rate of 27.59 GSym/s and $\Delta\upsilon$ is at most several MHz in practice,

$$P(f)\approx P(f-f_1)\quad \text{for}\quad \left|f_1\right|\le\Delta\upsilon/2. \tag{B.1}$$

In this case,

$$\begin{aligned} U(f)&\approx P(f)\int_{-\Delta\upsilon/2}^{\Delta\upsilon/2}\Phi(f_1)G(f-f_1)df_1 \\ &\approx\int_{-\Delta\upsilon/2}^{\Delta\upsilon/2}\Phi(f_1)P(f-f_1)G(f-f_1)df_1 \\ &=\int_{-\Delta\upsilon/2}^{\Delta\upsilon/2}\Phi(f_1)Q(f-f_1)df_1=Q(f)*\Phi(f). \end{aligned} \tag{B.2}$$

Therefore, by taking the inverse Fourier Transform, we obtain $u(t)\approx q(t)\cdot e^{j\phi_r(t)}$. Note that this is an alternative way to illustrate the origins of EEPN. Consider replacing $P(f)$ with $H^{-1}(f)=e^{j4\pi^2f^2\beta_2L/2}$ that characterize compensation of accumulated CD in a transmission link. For any given $\Delta\upsilon$,

$$H^{-1}(f)\ne H^{-1}(f-f_1)\quad \text{for}\quad \left|f_1\right|\le\Delta\upsilon/2$$

whenever $\beta_2 L$ is large enough and hence (B.2) will not follow and multiplication and convolution will not commute. Therefore, the effect of EEPN will always scale up with accumulated CD independent of laser linewidth.

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