# Correcting cross-media instrument metamerism for reflectance estimation in multispectral imaging

Hui-Liang Shen,<sup>1,\*</sup> Chen-Wei Weng,<sup>1</sup> Hui-Jiang Wan,<sup>1</sup> and John H. Xin<sup>2</sup>

<sup>1</sup>Department of Information and Electronic Engineering, Zhejiang University, Hangzhou 310027, China <sup>2</sup>Institute of Textiles and Clothing, The Hong Kong Polytechnic University, Hong Kong, China \*Corresponding author: shenhl@zju.edu.cn

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In multispectral imaging, the color accuracy of spectral reflectance estimation degrades significantly if the medium of test samples is different from that of calibration samples. This occurs mainly for two reasons, i.e., the different characteristics of spectral reflectances and the different measurement principles between an imaging system and a spectrophotometer. In this paper, this problem is referred to as cross-media instrument metamerism. We propose to correct it by using calibration samples from a standard color chart and a limited number of tuning samples with a target medium as *a priori* knowledge. The reflectance transform is computed by using both calibration and tuning samples, and the metamerism transform is calculated by modeling the correlation of camera responses between neighboring imaging channels. Experimental results show that the proposed method produces satisfactory spectral and colorimetric accuracy in reflectance estimation. The method could be deployed in practical applications when the available samples of certain media are inadequate for accurate reflectance estimation in a multispectral imaging system. © 2011 Optical Society of America

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## **1. INTRODUCTION**

The multispectral imaging technique has recently attracted much interest for its ability in estimating spectral reflectances at the spatial resolution of camera pixels. With the estimated reflectances, the color appearance of imaged objects can be reproduced under any illumination conditions [1]. In textile and other industries, the reproduced colors benefit the subjective and objective color quality evaluation of products.

The accurate estimation of spectral reflectance from multichannel camera responses is an essential problem in multispectral color imaging. In recent years, many methods have been introduced, among which Wiener estimation [2–4] and pseudoinverse [1,5] were widely deployed in the literature. We note that, in many previous studies, the calibration samples and test samples are of identical medium such as the standard ColorChecker charts [6]. Under this circumstance, the spectral estimation accuracy can be quite satisfactory.

In practice, however, two causes will degrade the estimation accuracy. First, the reflectance characteristics of the calibration samples may be different to those of the test samples. For example, Shimano and Hironaga [3] recently verified that the selection of training samples is crucial to accurate reflectance estimation. Second, the measurement principle of the imaging system is quite different to that of a spectrophotometer. This means that the light beam received by the optical detector of the spectrophotometer is not the same light beam captured by the camera. Because of these two reasons, crossmedia instrument metamerism will occur when the calibration and test samples are of different media. Figure 1 further illustrates the second reason. Assume that two samples,  $\mathcal{A}$  and  $\mathcal{B}$ . are made of different media and their spectral reflectances measured by the spectrophotometer are exactly the same. When imaged by the multispectral system, the camera responses of these two samples are different. Intuitively, sample  $\mathcal{A}$  reflects less light than sample  $\mathcal{B}$  at the optical path of camera, and thus produces lower camera responses.

## A. Previous Studies

A few studies have noticed or addressed the instrument metamerism issue. Hong *et al.* [7] found that the cross-media eye-camera metamerism was obvious in the colorimetric characterization of an RGB camera. Shimano [2] deployed the Wiener estimation technique to recover spectral reflectance without prior knowledge of imaging noise and objects being imaged, yielding the colorimetric accuracy around 2.5 to 3.5 color difference units. Other work, such as [8,9], also mentioned the cross-media instrument metamerism issue, but did not conduct in-depth studies.

The method introduced by Chung *et al.* [10] is the most relevant in instrument metamerism correction. For two different spectrophotometers, due to the difference in optical and mechanical design, the disagreement between spectral measurements is quite large. The authors used 12 standard ceramic tiles to improve interinstrumental agreement based on the so-called R-model. For its relevance to this work, we briefly review that method.

Let  $\lambda$  denote the wavelength in the visible spectrum ranging from 400 to 700 nm, at a sampling interval of 10 nm. Suppose  $s(\lambda)$  is the reflectance measured by the standard (reference) spectrophotometer, and  $r(\lambda)$  is the reflectance measured by the spectrophotometer to be calibrated. Based on the narrowband nature of a spectrophotometer measurement, the Rmodel exploits the correlation between these two reflectances in neighboring wavelengths as

$$s(\lambda) = a_1 r(\lambda - 1) + a_2 r(\lambda) + a_3 r(\lambda + 1) + a_4, \tag{1}$$



Fig. 1. (Color online) Two samples with different media but identical reflectances produce different camera responses. (a) Spectral reflectance. (b) Multichannel camera responses.

where the coefficients  $a_i$   $(1 \le i \le 4)$  are determined for each individual wavelength. Note that at the two end wavelength points (400 and 700 nm), only three coefficients are needed.

## B. Motivation and Overview of This Work

As previously mentioned, the instrument metamerism between a multispectral imaging system and a spectrophotometer is caused by their differences in geometry design and measurement principle. The spectral calibration process of the imaging system will be unavoidably affected by the reflective property of the calibration samples. To estimate the reflectances of another medium accurately, in our opinion, prior knowledge of that medium is necessary.

The most straightforward way, of course, is to calibrate the imaging system by using calibration samples with the same medium to test samples. However, in industrial applications, the number of available samples of that medium may be insufficient for system calibration. Motivated by this, we propose to correct cross-media instrument metamerism by using the calibration set  $\mathcal{A}$ , as well as a small tuning set  $\mathcal{T}$ , that is of the same medium to test set  $\mathcal{B}$ .

Figure 2 shows the flowchart of the proposed method. The reflectance transform W is initially calculated from the reflectances and camera responses of the calibration set; the metamerism transform D is computed from the camera responses of calibration and tuning samples in neighboring imaging channels. Then these two transforms are optimized in an iterative procedure.



Fig. 2. Flowchart of the proposed instrument metamerism correction method.

# 2. IMAGING MODEL AND REFLECTIVE CHARACTERISTICS

In this work, the spectrophotometer used for spectral reflectance measurement is model GretagMacBeth 7000A. It adopts the CIE recommended geometry diffuse/8°, with the diffuse illumination produced by an integrating sphere [6]. The multispectral imaging system consists of a monochrome camera and K(=16) narrowband filters. The filters are installed on a filter wheel that connects to the lens. The lighting direction is 45° with respect to the object surface normal, and the viewing angle is along the normal direction. This configuration is the CIE recommended 45°/normal geometry, which is suitable to imaging systems.

We give an intuitive description of reflective characteristics of object surfaces. The bidirectional reflectance distribution function (BRDF) [11] is usually deployed to quantify the reflective property of an object surface; it is a function of five dimensions, with one for wavelength, two for incident angles, and two for reflective angles. When two samples are of different media, their BRDFs are not identical even if these two samples are dyed with the same colorants. As the light intensities captured by camera are different, the spectrally calibrated multispectral imaging system by calibration medium  $\mathcal{A}$  is not reliable to estimate the reflectance of test medium  $\mathcal{B}$ .

In the following, we give the imaging model of the multispectral system. For simplicity, we omit the geometric factors in formulas like other studies [1,2]; this would not cause any confusion in understanding this work. Let  $m_k(\lambda)$  be the spectral responsivity of the *k*th channel  $(1 \le k \le K)$  of the imaging system. The camera response of that channel is formulated as

$$u_k = \int_{\lambda} m_k(\lambda) r(\lambda) \mathrm{d}\lambda + b_k, \qquad (2)$$

where  $b_k$  is the response caused by the dark current.

To facilitate representation and computation, Eq. (2) can be formulated in vector-matrix notation as

$$\mathbf{u} = \mathbf{M}\mathbf{r} + \mathbf{b},\tag{3}$$

where  $\mathbf{u} \in \mathbb{R}^{K}$  and  $\mathbf{b} \in \mathbb{R}^{K}$  are the column vectors of the camera responses of the imaged sample and the dark current, respectively;  $\mathbf{M} \in \mathbb{R}^{K \times N}$  is the matrix of spectral responsivity, and  $\mathbf{r} \in \mathbb{R}^{N}$  is the column vector of spectral reflectance.

## **3. PROPOSED METHOD**

Reflectance estimation is the reverse procedure of imaging process specified by Eq. (3). More specifically, the objective of spectral calibration is to find the reflectance transform  $\mathbf{W} \in \mathbb{R}^{N \times (K+1)}$  so that reflectance can be calculated from responses as

$$\mathbf{r} = \mathbf{W}\tilde{\mathbf{u}},\tag{4}$$

where  $\tilde{\mathbf{u}} \in \mathbb{R}^{K+1}$  is an augmented column vector

$$\tilde{\mathbf{u}} = \begin{pmatrix} \mathbf{u} \\ 1 \end{pmatrix}. \tag{5}$$

We use  $\tilde{\mathbf{u}}$  instead of  $\mathbf{u}$  because of the additive dark current response in the image process described in Eq. (3). Actually, we found that, even if the dark current response is removed in the preprocess step, adding a constant in  $\tilde{\mathbf{u}}$  always yields an improved accuracy in reflectance estimation. This is consistent with the polynomial regression for colorimetric characterization where the constant one is always adopted [7].

## A. Precalculation of Reflectance Transform

To calculate the reflectance transform  $\mathbf{W}$ , an adequate number, say  $L_A$ , of calibration samples  $\mathcal{A}$  are needed. We stack the reflectance vectors and augmented response vectors into the matrices  $\mathbf{R} \in \mathbb{R}^{N \times L_A}$  and  $\tilde{\mathbf{U}} \in \mathbb{R}^{(K+1) \times L_A}$ , respectively:

$$\mathbf{R} = (\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_{L_A}), \tag{6}$$

$$\tilde{\mathbf{U}} = (\tilde{\mathbf{u}}_1, \tilde{\mathbf{u}}_2, \dots, \tilde{\mathbf{u}}_{L_A}). \tag{7}$$

To calibrate the multispectral imaging system for reflectance estimation, we need to find the reflectance transform matrix  $\mathbf{W}$  in

$$\mathbf{R} = \mathbf{W}\tilde{\mathbf{U}},\tag{8}$$

such that the spectral error term  $||\mathbf{R} - \mathbf{W}\tilde{\mathbf{U}}||^2$  is minimized. When the spectral responsivity matrix  $\mathbf{M}$  and the statistics of system noise are known,  $\mathbf{W}$  can be computed by Wiener estimation [2,12]. Considering these data are unknown in this work, we calculate  $\mathbf{W}$  under the least-squares sense as

$$\mathbf{W} = \mathbf{R}\tilde{\mathbf{U}}^+,\tag{9}$$

where superscript + denotes pseudoinverse (PINV).

To facilitate the following computation, we decompose the form of W into a matrix  $\mathbf{W}_s \in \mathbb{R}^{N \times K}$  and a column vector  $\mathbf{w} \in \mathbb{R}^N$  as

$$\mathbf{W} = (\mathbf{W}_s, \mathbf{w}). \tag{10}$$

From Eqs. (4) and (5), it is clear that  $W_s$  and w are in correspondence to response vector  $\mathbf{u}$  and constant one in reflectance computation, respectively.

#### **B.** Calculation of Metamerism Transform

Similar to the case of calibration samples, the reflectance matrix and augmented response matrix of the tuning samples are assembled as

$$\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_{L\tau}),\tag{11}$$

and

$$\tilde{\mathbf{V}} = (\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2, \dots, \tilde{\mathbf{v}}_{L_{\mathcal{T}}}), \tag{12}$$

where  $\mathbf{s}_i \in \mathbb{R}^N$  and  $\tilde{\mathbf{v}}_i \in \mathbb{R}^{K+1}$   $(1 \le i \le L_T)$  are the reflectance vectors and augmented response vectors of the tuning samples, respectively.

If we treat the calibrated multispectral imaging system as a spectrophotometer, then the instrument metamerism can be modeled in the reflectance space according to Eq. (1). In this way, the calculation of reflectance transform W is only related to calibration set  $\mathcal{A}$ . However, we argue that the estimation of W can be further improved if the prior knowledge of tuning samples is incorporated. Under this consideration, we model the correlation in response space instead of reflectance space as

$$\mathbf{S} = \mathbf{W} \mathbf{D} \mathbf{V},\tag{13}$$

where the metamerism transform matrix  $\mathbf{D} \in \mathbb{R}^{(K+1) \times (K+1)}$  is defined as

$$\mathbf{D} = \begin{pmatrix} d_{11} & d_{12} & 0 & 0 & 0 & \dots & 0 & 0 & \bar{d}_1 \\ d_{21} & d_{22} & d_{23} & 0 & 0 & \dots & 0 & 0 & \bar{d}_2 \\ 0 & d_{32} & d_{33} & d_{34} & 0 & \dots & 0 & 0 & 0 & \bar{d}_3 \\ & \dots & & \dots & & \dots & & \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & d_{K,K-1} & d_{K,K} & \bar{d}_K \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$(14)$$

Similar to Eq. (1), matrix **D** only has nonzero elements in the neighboring imaging channels and the last column of the constant bias. We consider this matrix form sufficient to model the camera responses disagreement between different media; one example of response disagreement is illustrated in Fig. 1(b).

For simplicity, we use  $\mathbf{D}_s \in \mathbb{R}^{K \times K}$  to denote the top-left submatrix and  $\mathbf{d} \in \mathbb{R}^K$  to denote the top-right subvector in **D**, respectively. Then, Eq. (14) becomes

$$\mathbf{D} = \begin{pmatrix} \mathbf{D}_s & \mathbf{d} \\ \mathbf{0}^\mathsf{T} & \mathbf{1} \end{pmatrix},\tag{15}$$

where  $\mathbf{0} \in \mathbb{R}^{K}$  is the column vector of zeros.

The objective of instrument metamerism correction is to determine  $\mathbf{D}_s$  and  $\mathbf{d}$ . Substituting Eqs. (10) and (15) into Eq. (13) and after some arrangement, we get

$$(\mathbf{D}_s, \mathbf{d})\tilde{\mathbf{V}} = \mathbf{W}_s^+ (\mathbf{S} - \mathbf{w}\mathbf{1}^\mathsf{T}), \tag{16}$$

where  $\mathbf{1} \in \mathbb{R}^{L_{\mathcal{T}}}$  is the column vector of ones. Then, the nonzero elements in each row of **D** can be solved by the leastsquares.

# C. Optimization of Reflectance and Metamerism Transforms

As mentioned above, the reflectance transform W calculated in Eq. (4) depends on the calibration set  $\mathcal{A}$  only. In order to obtain optimized W and D, we calculate W on both calibration set  $\mathcal{A}$  and tuning set  $\mathcal{T}$  as

$$(\mathbf{R}, \eta \mathbf{S}) = \mathbf{W}(\tilde{\mathbf{U}}, \eta \mathbf{D}\tilde{\mathbf{V}}), \tag{17}$$

where  $\eta$  is a constant controlling the estimation error of the tuning samples. To balance the errors of these two sample sets, we define  $\eta$  as

Table 1.Specification of the Calibration, Tuning,<br/>and Test Samples Used in the Experiment

	Calibration $(\mathcal{A})$	Tuning $(\mathcal{T})$	Test $(\mathcal{B})$	
Medium Sample numbers	ColorChecker 96	Pantone Specified	Pantone 460	

$$\eta = \left(\frac{L_A}{L_T}\right)^{1/2}.$$
(18)

Then, W is calculated as

$$\mathbf{W} = (\mathbf{R}, \eta \mathbf{S})(\tilde{\mathbf{U}}, \eta \mathbf{D}\tilde{\mathbf{V}})^+.$$
(19)

With this updated **W**, we can recalculate **D** according to Eq. (16). Based on the recalculated **D**, we can again update **W** using Eq. (19). This iterative computation is outlined in the following algorithm.

#### **Algorithm 1. Instrument Metamerism Correction**

Input: Reflectance data R,S and response data  $\tilde{U},\tilde{V}$  of the calibration and tuning samples

 Initialize reflectance transform W using solely calibration samples [Eq. (9)]

#### Repeat

Calculate metamerism transform D for tuning samples [Eq. (16)]
 Compute W using both calibration and tuning samples [Eq. (19)]
 End
 Output: W and D

The stop criterion of the algorithm can either be controlling color accuracy in two adjacent iterations or controlling the maximum number of iterations. Our investigation found that only two iterations were needed to get the optimal W and D. Under these two transforms, the reflectance of a test sample with response  $\tilde{\mathbf{v}}$  is estimated as

$$\hat{\mathbf{s}} = \mathbf{W} \mathbf{D} \tilde{\mathbf{v}}.$$
 (20)

# 4. EXPERIMENT

In the experiment, we used the 96 colors (excluding the peripherally surrounding ones) on the X-Rite Digital Color-



Fig. 4. (Color online) Distributions of spectral rms error with respect to the number of tuning samples  $L_T$ . (a) 9-channel system. (b) 16-channel system.

Checker SG as the calibration sample set  $\mathcal{A}$ . We also uniformly picked 560 Pantone fabric patches from a Pantone book. These 560 Pantone patches are randomly divided into two parts with unequal sizes. The first part, which contains 460 samples, serves as the test set  $\mathcal{B}$ ; the other part, which has 100 samples, is used as the sample pool for the extraction of tuning set  $\mathcal{T}$ . This specification is shown in Table 1. Figure 3 shows the CIE  $a^*b^*$  distributions of the calibration, tuning, and test samples. The media of the ColorChecker and Pantone samples are quite different. The ColorChecker chart is made of colored papers, while the Pantone samples are textile fabrics dyed with various colorants. This means that the



Fig. 3. (Color online) CIE  $a^*b^*$  distributions of the (a) calibration samples, (b) tuning sample pool, and (c) test samples.

Table 2. Spectral and Colorimetric Errors of the Proposed Method, Compared with Baseline Methods

		Spectral rms		$\Delta E_{00}$ under D65		$\Delta E_{00}$ under F2				
Channel Number	Method	Mean	Median	Max	Mean	Median	Max	Mean	Median	Max
9	PINV	0.0174	0.0141	0.0659	1.91	1.47	23.8	2.00	1.58	23.5
	R-model	0.0175	0.0148	0.0726	1.43	1.22	5.60	1.63	1.39	6.41
	Proposed	0.0128	0.0118	0.0659	1.21	1.09	4.29	1.19	1.07	3.87
16	PINV	0.0210	0.0153	0.1041	2.21	1.79	16.2	2.19	1.81	16.1
	R-model	0.0113	0.0099	0.0661	1.20	1.11	3.86	1.30	1.16	4.20
	Proposed	0.0104	0.0091	0.0597	1.06	0.99	2.65	1.05	0.97	2.71

reflectance characteristics of these two media are different. The reflectance data of all ColorChecker and Pantone samples were measured by the spectrophotometer Gretag-MacBeth 7000A.

Our multispectral imaging system contains 16 narrowband filters, whose central wavelengths are 400, 420, ..., and 700 nm. In additional to the 16-channel system, we also evaluated the proposed method on a 9-channel system, with the 400, 440, ..., 680, and 700 nm filters. In the 9-channel system, we only need to model the correlations between the corresponding single channels, due to the narrowband nature of the filters. For practical application, we did not further consider the configuration of channel numbers less than 9; configuration is appropriate for imaging systems with broadband filters, not narrowband ones.

In this study, the color accuracy of reflectance estimation is evaluated in terms of both spectral and colorimetric error metrics. The spectral rms error between the actual reflectance s and estimated reflectance  $\hat{s}$  is computed as

$$\operatorname{rms} = \left(\frac{(\mathbf{s} - \hat{\mathbf{s}})^{\mathsf{T}}(\mathbf{s} - \hat{\mathbf{s}})}{N}\right)^{1/2}.$$
 (21)

The colorimetric error, denoted as  $\Delta E_{00}$ , is calculated according to the CIEDE2000 color difference formula [13] under various CIE standard illumination such as D65 and F2.

The proposed method was compared with two baseline methods, i.e., direct-PINV and the R-model. The R-model corrects instrument metamerism in the reflectance domain. For the PINV method, when set A was used for calibration and set B for test, the average spectral rms errors were around 0.05 in



Fig. 5. (Color online) Estimated reflectances of a Pantone test sample by different methods on the 16-channel system.

both 9-channel and 16-channel systems. This large errors clearly indicate the material metamerism problem. For fair comparison, the direct-PINV method computes reflectance transform W on the tuning set  $\mathcal{T}$  so that prior knowledge of test medium is included, as in the R-model and the proposed method.

### A. Influence of Tuning Sample Number

The tuning samples were sequentially selected from the Pantone tuning pool. Figure 4 shows the distributions of spectral rms error with respect to the  $L_T$  values. The proposed method always performs better than the R-model. The spectral rms error of the proposed method is lower than that of the direct-PINV method when  $L_T < 36$  in the 9-channel system, and when  $L_T < 48$  in the 16-channel system. Actually, the direct-PINV method becomes overfitted when  $L_T$  values are small. This clearly indicates that the proposed method is of value when there are inadequate tuning samples in practical applications.

### **B.** Color Accuracy

Without loss of generality, we compare the color accuracy of these three methods in case of  $L_T = 16$ . The results are shown in Table 2. As expected, the direct-PINV method performs worst, verifying that 16 colors are obviously inadequate to estimate the reflectance transform. The proposed method performs best; the average spectral rms errors are 0.0128 and 0.0104 in the 9-channel and 16-channel systems, respectively. Figure 5 shows the estimated reflectances of a test sample by three methods. The reflectance recovered by the proposed method matches the actual measurement most, while that by direct-PINV exhibits obvious deviations from the actual one.

# 5. CONCLUSION

We have proposed a method to correct cross-media instrument metamerism for accurate reflectance estimation in multispectral imaging. This work is inspired by the fact that, when the imaging system is spectrally calibrated by standard samples of one medium, it is usually not accurate to estimate the reflectance of samples with a different medium. The imaging system can be recalibrated, but the samples of that medium may be temporarily insufficient in practical applications.

The proposed method deals with this problem by using standard calibration samples and a small number of tuning samples as prior knowledge. The reflectance transform is initially computed from the calibration samples, and then optimized together with the metamerism transform based on both calibration and tuning samples. Experimental results validate that the proposed method is of value when the number of tuning samples is limited in practical applications.

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