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Microphone configuration for beamformer design using the Taguchi method

Kit Yan Chan,* Cedric Ka Fai Yiu† and Sven Nordholm ‡

Abstract

In beamformer design, the microphone configurations which represent microphone number and positions are necessary to be optimized in order to improve the effectiveness of speech enhancement. Determination of microphone configuration, number of elements and positions is a nonlinear and non-convex NP-hard optimization problem which was not specified before. However, this is a nonlinear and non-convex NP-hard optimization problem. Gradient-based optimization methods can only converge to suboptimal solutions. Although the recently developed heuristic methods may find better configurations, they require long convergence time. In this paper, we study the effectiveness of using Taguchi method to determine microphone configuration. The Taguchi method is a robust and systematic optimization approach for designing reliable and high-quality models. The method conducts systematic trials based on an orthogonal array which represents a subset of representative configurations. It determines the configurations based on the experimental trials, while the heuristic methods determine the configurations by searching through the configuration domain until no better configuration can be found. A case study based on a common office environment is used as an example to illustrate the effectiveness of the Taguchi method and the commonly used heuristic methods. The numerical results demonstrate that the method is capable to develop the microphone configurations with similar performance compared with the heuristic methods when short computational time is only available. Hence, the method is a strong candidate to design microphone configurations when short development time is only available.

Keywords: Beamformer design, microphone configuration, speech enhancement, Taguchi method, orthogonal array, evolutionary algorithms

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1 Introduction

Multichannel beamformers play an important role in speech enhancement [1–3] particularly in teleconferencing, hands-free communications, speech recognition and hearing aids. They are effective to reduce localized and ambient noise from a desired direction via spatial filtering [4–6].

Many model-based approaches have been developed to determine filter coefficients of multichannel beamformers in order to improve speech enhancement effectiveness [7–12]. An appropriate microphone configuration is essential to be determined to do the speech enhancement, as beamformer performance changes significantly when different microphone placements are used. When more microphones are used, more freedom of configurations can be set up. The searching freedom is larger and the better beamforming performance is likely to be obtained. However, using more microphones increases the cost, weight, power consumption and heat dissipation of the beamformers. When the number of microphones is too small, the radiation pattern outside the main beam can be lost. Therefore, it is necessary to optimize both the number of microphones and the beamformer performance [13].

For the microphone configuration design, we need to test 2^n configurations in order to find the optimal one, when n feasible locations are available. Hence, it is almost impossible to test all microphone configurations, when n is large. The optimal microphone configurations cannot be determined effectively by gradient-based methods as this is a NP-hard non-convex problem. Similar to the design of microphone configurations, heuristic methods including evolutionary programming [14, 15], genetic algorithm [16, 17], simulated annealing algorithm [18–20] and pattern search algorithm [21] have been developed in order to determine the optimal sparse antenna array configurations. A hybrid descent method [22] has been developed to determine optimal solutions for microphone placements. However, these heuristic methods suffer the limitations that they require long computational time when the cost function is computationally complicated and also the termination condition can only be defined implicitly.

As this is time consuming to determine the optimal configuration based on the full factorial set, it is more desirable to determine an appropriate configuration based on a representative subset. In quality control, the Taguchi method has been successfully used to design reliable and high-quality products based on a subset of design configurations [23, 24], where the number of design factors of products is huge and it is almost impossible to conduct all experiments for all design configurations. The Taguchi method uses the combination from orthogonal arrays to determine the representative subset of configurations. It has been successfully used to design reliable and high-quality products at low cost for various products such as automobiles and consumer electronics [25–29].

Similar to design of high quality products, microphone configuration design is involved with a large number of design factors [30]. In this paper, we propose the Taguchi method to develop microphone configuration for beamformer design. The Taguchi method conducts systematic experiments based on orthogonal arrays to study the microphone configuration, and then it determines the appropriate microphone configurations with effective beamforming performance and small numbers of microphones. The Taguchi method attempts to overcome the limitations of the heuristic method [22], which encounters slow convergence rate and non-deterministic convergence solution.

A case study is conducted based on a common office [31] in order to evaluate the effectiveness of the Taguchi method. The numerical results obtained by the Taguchi method are compared with those obtained by the two commonly used heuristic methods including genetic algorithm (GA) [32–34] and particle swarm optimization algorithm (PSO) [35] for determining microphone configurations. These two heuristic methods are also commonly used on solving multi-objective problems and also they have been used on designing microphone configurations [36–41]. The comparison indicates that the Taguchi method is capable to develop the microphone configurations with similar beamforming performance compared with the two heuristic methods while shorter computational time is required on the Taguchi method. Hence, the Taguchi method is a good candidate on microphone configuration design when a quick but reasonable solution is desired. Noting that array configurations need to change depending on the signal scenario.

The rest of this paper is organized as follows. In Section 2, we formulate the microphone configuration problem which aims to optimize the beamformer performance using a small number of microphones. In Section 3, a Taguchi method is developed to solving this microphone configuration problem. In Section 4, a case study is used to demonstrate the effectiveness of the Taguchi method by comparing with the two commonly used heuristic methods. In Section 5, a conclusion is given.

2 Optimal microphone configuration for beamformer design

For the beamformer design, we assume that only M feasible locations are available. As illustrated in Figure 1, r_i with $i = 1,2,..,M$ are the M feasible locations of the microphones of which the transfer function of the $i-$th microphone is given as:

$$A(r_i, f) = \frac{1}{||r_s - r_i||} e^{\frac{-j2\pi f_s ||r_s - r_i||}{c}}, \tag{1}$$

where r_s is the location of the sound source and c is the sound speed under the atmosphere. Although the heuristic method [22] has been developed to solve (2), it is based on a set of random but repeatingly evaluating the cost function in order to search for a better sub-optimal solution. The computational time of the heuristic method is too long and the convergence solution is not deterministic. When a sampling rate, f_s, is used by the microphones to capture the signal, the frequency response of i-th finite impulse response (FIR) filter can be represented by (2) with respect to the

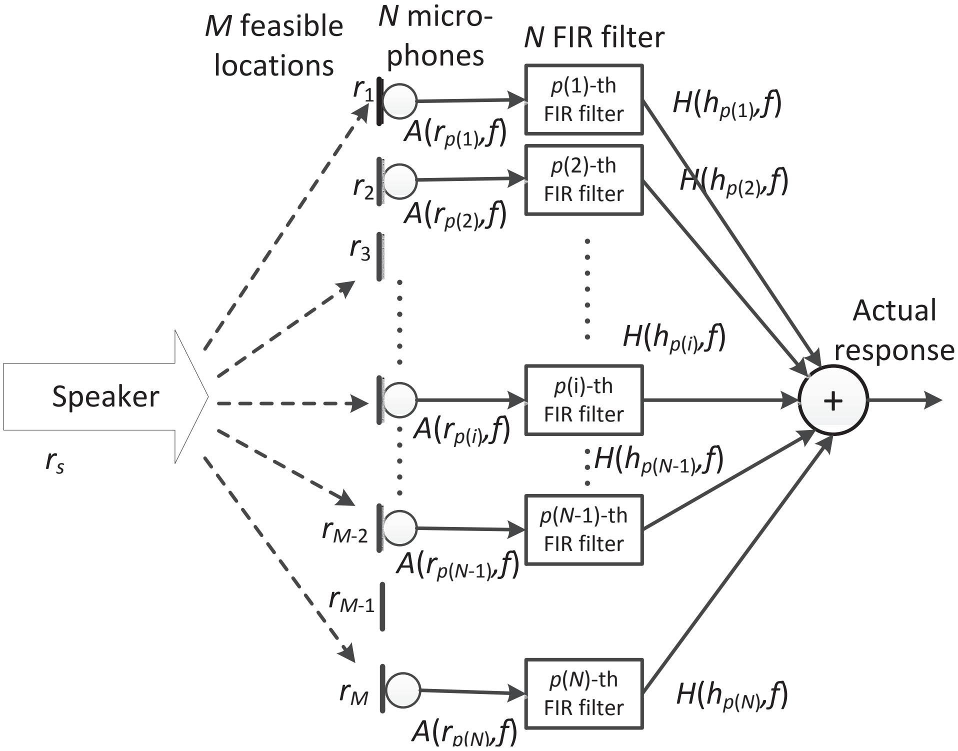

Figure 1: The beamformer design with M feasible locations

i-th microphone as

$$H(h_i, f) = h_i^T d_0(f), \tag{2}$$

where h_i represents the coefficients of the i-th FIR filter with the filter length L and $d_0(f)$ is the Kronecker delta frequency response. h_i and $d_0(f)$ are given respectively as

$$h_i = [h_i(0), h_i(1), ..., h_i(L-1)]^T, \text{ and}$$

$$d_0(f) = [1, e^{\frac{-j2\pi f}{f_s}}, ..., e^{\frac{-j2\pi f}{f_s}(L-1)}]^T.$$

Based on the transfer function of the i-th microphone, $A(r_i, f)$, and the i-th frequency response, $H(h_i, f)$, given in (1) and (2) respectively, the actual response can be generated by the beamformer as

$$G(\overline{r_{\bar{p}}}, \overline{h_{\bar{p}}}, f) = \sum_{i=1}^{N} H(h_{p(i)}, f) A(r_{p(i)}, f), \tag{3}$$

when N microphones are configured on the beamformer and they are installed on N of the M feasible locations; $\bar{p}$ is the index vector of the microphone configuration. It illustrates the feasible locations of the N microphones can be installed. $\bar{p}$ is given by $\bar{p} = [p(1), ..., p(N)]$ with $p(i) \neq p(j) \in [1, 2..., M]$, and $i \neq j \in [1, 2..., N]$; $\overline{h_{\bar{p}}}$ is the set of coefficients for the N FIR filters with respect to the N microphones. $\overline{h_{\bar{p}}}$ is given by $\overline{h_{\bar{p}}} = [h_{p(1)}, h_{p(2)}, ..., h_{p(N)}]^T$; and $\overline{r_{\bar{p}}}$ represents the locations of the N microphones. $\overline{r_{\bar{p}}}$ is given by $\overline{r_{\bar{p}}} = (r_{p(1)}, r_{p(2)}, ..., r_{p(N)})$.

Assume that the desired response is $G_d(r_s, f)$, the beamformer attempts to minimize the error between $G(\overline{r_{\bar{p}}}, \overline{h_{\bar{p}}}, f)$ and $G_d(r_s, f)$, by using the optimal microphone configuration indexed with $\bar{p}$ and the optimal set of FIR coefficients, $\overline{h_{\bar{p}}}$. Here we adopt the l_2-norm criterion to evaluate the error between $G(\overline{r_{\bar{p}}}, \overline{h_{\bar{p}}}, f)$ and $G_d(r_s, f)$, as the other objective criteria such as the $l_\infty-$norm [42, 43] and the l_1-norm minimax [12, 44] are more computationally expensive. Based on l_2-norm, the objective function (4) is formulated in order to evaluate the effectiveness of the beamformer which is embedded with the FIR filters with the coefficient set, $\overline{h_{\bar{p}}}$, and is configurated with the microphones with the indexes, $\bar{p}$.

$$E(\overline{r_{\bar{p}}}, \overline{h_{\bar{p}}}) = \frac{1}{||\Omega||} \int_{\Omega} \rho(r_s, f) ||G(\overline{r_{\bar{p}}}, \overline{h_{\bar{p}}}, f) - G_d(r_s, f)||^2 df, \tag{4}$$

where Ω is a specified spatial-frequency domain as the definition field of $G_d(r_s, f)$, and $\rho(r_s, f)$ is a positive weighting function. Usually, the domain, $\Omega = \Omega_p \bigcup \Omega_s$, consists of the passband region,

Ω_p, and stopband region, Ω_s.

In order to obtain the optimal microphone configuration, the beamformer design aims to optimize two criterions: i) the error, $E(\bar{r}_{\bar{p}}, \bar{h}_{\bar{p}})$, between the actual response and the desired response, and ii) the number of microphones, N, installed on the beamformer, which can be represented by the index vector length, $|\bar{p}|$. The optimal microphone configuration problem is formulated by (5) as a multi-objective optimization problem.

$$\min_{\bar{p}\in\Lambda(M), \bar{h}_{\bar{p}}\in\mathbb{R}^{L\times|\bar{p}|}} \left(E(\bar{r}_{\bar{p}}, \bar{h}_{\bar{p}}), |\bar{p}|\right) \tag{5}$$

where $\Lambda(M)$ is a set of permutation vectors of which each permutation vector consists of unidentical elements and the length of each permutation vector is less than the number of feasible microphones, M. $\Lambda(M)$ is given by

$$\begin{aligned} \Lambda(M) = \{\bar{p} = [p(1), \ldots, p(N)] | \forall p(i) \in [1, \ldots, M], \text{with } N \leq M; \\ \text{but } p(i) \neq p(j) \text{ for all } i \neq j \in [1, \ldots, N]\ \}. \end{aligned}.$$

The microphone configuration optimization problem (5) can be simplified as (6), as the set of optimal filter coefficients, $\bar{h}^*_{\bar{p}}$, can be determined using the two-stage based interior point method [45] when the microphone locations, $\bar{r}_{\bar{p}}$, are given.

$$\min_{\bar{p}\in\Lambda(M)} \left(E(\bar{r}_{\bar{p}}, \bar{h}^*_{\bar{p}}), |\bar{p}|\right). \tag{6}$$

The existing heuristic method [22] has been used to minimize the error E in (5) but it cannot minimize the $|\bar{p}|$ which represents the number of microphones used in the array. (5) is formulated to minimize both E and $|\bar{p}|$. Although solving (6) is simpler than solving (5), (6) is still NP-hard multi-objective optimization problem. When the brute-force search is used to determine the optimal solution of (6) by all possible in $\Lambda(M)$, the computational complexity of $O(2^M)$ is required. However, testing all microphone configurations is impractical. For example, when there are only 25 feasible locations for the multichannel beamformer design, more than 33.5 millions configurations are required to be simulated. To determine the microphone configuration in a shorter time, a systematical and effective method, namely the Taguchi method [24, 46], is proposed, where the Taguchi method has been widely used to maximize the quality characteristics and robustness of products. The Taguchi method uses a representative subset of microphone configurations in order to determine the most appropriate one. The operations of the Taguchi method for determining the multichannel beamformer configuration are detailed in Section 3.

3 Determination of microphone configuration

In this section, the Taguchi method is proposed to determine the optimal microphone configuration. First, an orthogonal array with respect to the feasible locations of the microphones is generated. Experiments are conducted based on the microphone configurations represented by the combinations of the orthogonal array. Based on the analysis for Pareto-optimal set, the optimal combination is determined with respect to the two citations defined in (5) for the microphone configuration design.

3.1 Development of orthogonal array

In order to determine the optimal microphone configuration with M feasible locations, all the possible combinations of microphone configurations are required to be tested. Instead of testing each combination of microphone configuration, we can use the Taguchi method to select only a subset of these combinations while minimizing the number of experimental trials [23, 24].

The Taguchi method is developed with a series of orthogonal arrays namely $L_n(2^M)$, where $L_n(2^M)$ is a $n \times M$ matrix denoted by $[a_{i,j}]_{n\times M}$ and each element, $a_{i,j}$, is either 0 or 1. Each $L_n(2^M)$ consists of n rows and each row represents a combination for a microphone configuration. For the i-th combination, the j-th feasible location is installed with a microphone when $a_{i,j} = 1$. When $a_{i,j} = 0$, the j-th feasible location is not installed. As an illustration, two orthogonal arrays namely $L_4(2^3)$ and $L_8(2^4)$ are considered as:

$$L_4(2^3) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_8(2^4) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$L_4(2^3)$ can be used to determine the appropriate microphone configuration with three feasible locations using four experiments. The 1-st row represents the combination for the 1-st experiment of which each of the three feasible locations is installed with a microphone. Hence, the microphone configuration for the 1-st experiment is indexed with $\bar{p}(1) = [1,2,3]$. The 2-nd row represents the combination for the 2-nd experiment that the first feasible location is installed with a microphone and the other two feasible locations are not installed. Hence, the microphone configuration for the 2-nd experiment is indexed with $\bar{p}(2) = [1]$. The 3-rd row represents the combination for the 3-rd experiment that the second feasible location is installed with a microphone and the other two feasible locations are not installed. Hence, the microphone configuration for the 3-rd experiment is indexed with $\bar{p}(3) = [2]$. For $L_n(2^M)$, the microphone configuration for the i-th experiment with $i = 1, 2, ..., n$ is indexed with

$$\bar{p}(i) = [j, \text{ if } a_{i,j} = 1 \text{ with } j = 1, 2, ..., M] \tag{7}$$

Similarly, $L_8(2^4)$ is used to determine the appropriate microphone configuration with four feasible locations using eight experiments. When the full factorial design is used for the multichannel beamformer design, 16 (i.e. $2^4 = 16$) experiments are required. When $L_8(2^4)$ is used, only 8 experiments are required. Hence, 8 (i.e. $16 - 8$) experiments can be saved when comparing with the full factorial design.

All $L_n(2^M)$ have two orthogonal properties: **a)** the numbers of 0s and 1s exist $M/2$ times in all columns; and **b)** every combination of 1 and 0 exists $M/4$ times in any two columns. Therefore, the elements in each combination of $L_n(2^M)$ are orthogonal. Based on the combinations of $L_n(2^M)$, an appropriate models can be obtained for an additive or an quadratic system as the combinations of $L_n(2^M)$ are scatted uniformly over the space of all possible combinations [47]. This orthogonal property exists in all combinations of $L_n(2^M)$, whereby every state of a feasible location occurs the same number of times for all experiments. It minimizes the number of required experiments when the orthogonal property is retained.

Algorithm 1: GenOA

Input: M *% number of feasible locations*

Output: $[a_{i,j}]_{n\times M}$ *% orthogonal array* $L_n(2^M)$

Step 1: Define a parameter M_2, where $M_2 = (2^{i+1}-1)$ with $2^i - 1 \leq M \leq 2^{i+1}-1$ for an integer $i \geq 1$.

Step 2: Define two parameters J and n, where $J = \log_2(M_2+1)$ and $n = 2^J$.

Step 3: Construct the elements for the basic columns as:

$$a_{i,j} = \left\lfloor \frac{i-1}{2^{J-k}} \right\rfloor \operatorname{mod}(2)$$

where $j = 2^{k-1}$; $k = 1,2,\ldots,J$; and $i = 1,2,\ldots,n$.

Step 4: Construct the nonbasic columns as:

$$a_{j+s-1} = (a_s + a_j) \operatorname{mod}(2)$$

where $s = 1,2,\ldots,J$ and $j = 2^1, 2^2, \ldots, 2^{J-1}$.

Step 5: Change the value of $a_{i,j}$ by:

$$a_{i,j} = |a_{i,j} - 1|$$

where $i = 1,2,\ldots,M$ and $j = 1,2,,\ldots,n$.

Algorithm 1 namely Orthogonal array generator (GenOA) is used to generate $L_n(2^M)$ (or $[a_{i,j}]_{n\times M}$) for a feasible number of locations, M, where $n = 2^{i+1}$ with $2^i - 1 \leq M \leq 2^{i+1}-1$ for an integer $i \geq 1$. In $L_n(2^M)$, the j-th column is denoted by a_j, where a_j with $j = 2^{k-1}$ and $k = 1,2,\ldots,\log_2(n)$ are the basic columns and the other columns are the nonbasic columns. GenOA generates the basic columns first and then it generates the nonbasic columns.

Based on GenOA, the orthogonal array, $L_n(2^M)$, can be generated to determine the optimal microphone configuration for M feasible locations using n experiments. Hence, $2^M - n$ experiments can be saved as the full factorial design requires 2^M experiments. Table I shows the numbers of experiments required by the orthogonal arrays and the full factorial designs when different numbers feasible locations are used. It shows that significant amount of experiments can be saved when the

Table I: Experiments required by the orthogonal arrays and the full factorial designs

Number of feasible locations	**3**	**7**	**15**	**31**	**63**	**127**	**255**	**511**
Orthogonal arrays	$L_4(2^3)$	$L_8(2^7)$	$L_{16}(2^{15})$	$L_{32}(2^{31})$	$L_{64}(2^{63})$	$L_{128}(2^{127})$	$L_{256}(2^{255})$	$L_{512}(2^{511})$
Experiments required on Orthogonal arrays	4	8	16	32	64	128	256	512
Experiments required on full factorial designs	8	128	32768	2.15×10^9	9.22×10^{18}	1.70×10^{38}	5.79×10^{76}	6.70×10^{153}

orthogonal arrays are used.

3.2 Taguchi method for microphone configuration design

The Taguchi method first conducts n experiments with respect to the n microphone configurations indexed by the n combinations on $L_n(2^M)$. Assume that $E(\overline{r}_{\overline{p}(i)}, \overline{h}_{\overline{p}(i)})$ and $|\overline{p}(i)|$ with $i = 1, 2, \ldots, n$ are the two citations defined in (5) for the i-th experiment, where $\overline{p}(i)$ is the index of the i-th combination on $L_n(2^M)$; $E(\overline{r}_{\overline{p}(i)}, \overline{h}_{\overline{p}(i)})$ represents the error between the desired response and the actual response; and $|\overline{p}(i)|$ represents the number of microphones, when the i-th experiment indexed with $\overline{p}(i)$ is conducted. Based on the n experimental results, the objective coordinates namely, $\overline{f}((\overline{p}(i)))$, involved with the two citations is defined as:

$$\overline{f}((\overline{p}(i))) = [E(\overline{r}_{\overline{p}(i)}, \overline{h}_{\overline{p}(i)}), |\overline{p}(i)|] \tag{8}$$

where $i = 1, 2, \ldots, n$. Assume the universal be

$$\overline{U} = \{\overline{f}(\overline{p}(1)), \overline{f}(\overline{p}(2)), \ldots, \overline{f}(\overline{p}(n))\}, \tag{9}$$

the corresponding $\overline{f}(\overline{p}(i))$ in the Pareto-optimal set [48] namely $\overline{P}$ which is the subset of $\overline{U}$ can be determined. The corresponding $\overline{f}(\overline{p}(i))$ in $\overline{P}$ are composed of all $\overline{f}(\overline{p}(i))$ in $\overline{U}$, where the two citations of the corresponding $\overline{f}(\overline{p}(i))$ are simultaneously smaller than those $\overline{f}(\overline{p}(j))$ which are not in $\overline{P}$. Alternatively, $\overline{p}(i)$, is Pareto-optimal *if and only if* there is no $\overline{p}(j)$ for which $\overline{f}(\overline{p}(j)) = (E(\overline{r}_{\overline{p}(j)}, \overline{h}_{\overline{p}(j)}), |\overline{p}(j)|)$ dominates $\overline{f}(\overline{p}(i)) = (E(\overline{r}_{\overline{p}(i)}, \overline{h}_{\overline{p}(i)}), |\overline{p}(i)|)$. Therefore, no $\overline{f}(\overline{p}(j)) \in U$ exists such that

$$(E(\overline{r}_{\overline{p}(j)}, \overline{h}_{\overline{p}(j)}) < E(\overline{r}_{\overline{p}(i)}, \overline{h}_{\overline{p}(i)})) \wedge (|\overline{p}(j)| < |\overline{p}(i)|) \tag{10}$$

All $\overline{f}(\overline{p}(i))$ in $\overline{P}$ is the Pareto-optimal, efficient, or admissible set of the multi-objective problem (5). Alternatively, $\overline{f}(\overline{p}(i))$ are the solutions of the multi-objective problem (5). The Pareto-optimality is only a conceptual notion towards the practical solution of a multi-objective problem (5), which usually involves the choice a single compromise solution from the non-dominated set based on the preference information. Here a rank-based approach [49] is proposed where all nondominated objective coordinates are assigned as rank 1 which is illustrated in Figure 2. An objective coordinate, $\overline{f}(\overline{p}(i))$, dominated by d_i objective coordinates, is considered. The rank for $\overline{f}(\overline{p}(i))$ is given by:

$$\text{rank}(\overline{f}(\overline{p}(i))) = 1 + d_i \tag{11}$$

Algorithm 2 namely Taguchi method (TM) is used to determine the optimal microphone configuration, $\overline{p}(i)$ with $\overline{f}(\overline{p}(i)) \in \overline{P}$, using the orthogonal array when the feasible locations, r_i with $i = 1, \ldots, M$, are given. The objective coordinates, $\overline{f}(\overline{p}(i))$, with rank 1 are defined as the Pareto-optimal of which $\overline{f}(\overline{p}(i)) \in \overline{P}$.

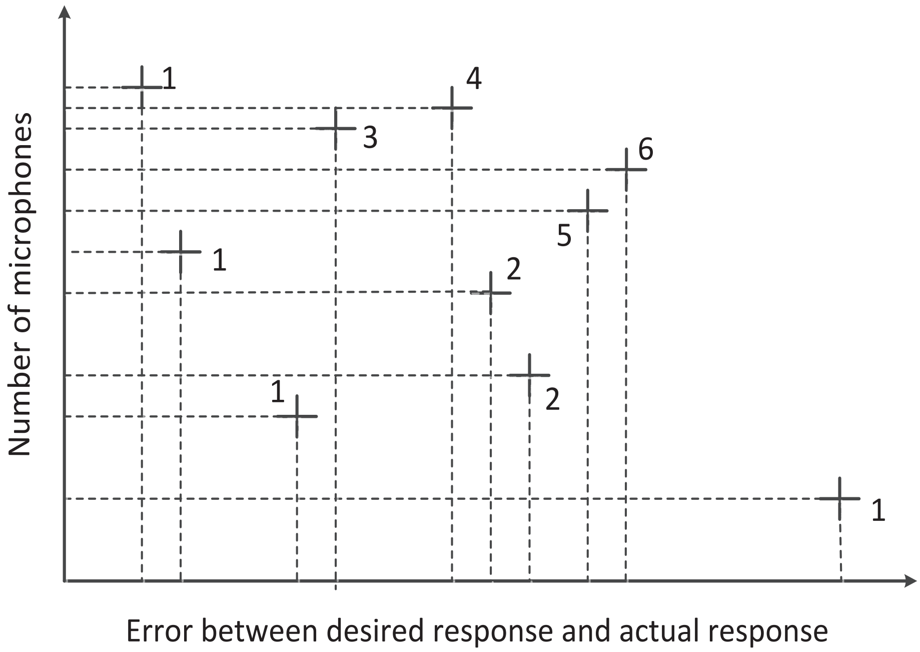

Figure 2: Ranking of the two citations for microphone configuration design.

Algorithm 2: Taguchi method (TM)

Input: r_i with $i = 1, \ldots, M$
% the M feasible locations

Output: All $\bar{p}(i)$ with $\bar{f}(\bar{p}(i)) \in \bar{P}$
% the multichannel microphone configurations

Step 1: Generate the orthogonal array, $L_n(2^M)$, with respect to the M feasible locations using **GenOA**.

Step 2: Conducts n experiments with respect to the n microphone configurations indexed by the n combinations on $L_n(2^M)$.

Step 3: Determine the objective coordinates, $\bar{f}((\bar{p}(i)))$ in (8), based on the n experimental results.

Step 4: Determine the ranks for all $\bar{f}((\bar{p}(i)))$ based on (11).

Step 5: Define the objective coordinates, $\bar{f}(\bar{p}(i))$ with rank 1, to be the Pareto-optimal of which $\bar{f}(\bar{p}(i)) \in \bar{P}$.

4 Case studies of microphone configuration design

4.1 Experimental set-up

In this section, a model for a common rectangular office [31] with the size of 8m×4m×3m is used to evaluate the effectiveness of the Taguchi method for microphone configuration design. The model simulates the characteristics of the room surface based on the image-source method [50]. It models the diffuse reverberation tail as decaying random noise. All codes for the development of the Taguchi method and the office model are based on the MATLAB platform on a PC with Intel(R), Core(TM) i5 CPU with 2.53 GHz. In this research, the desired response function is specified over a region which simulates a teleconferencing or hands free mobile phone application. This includes the frequency range of human voice, and a range of positions that microphones are directed towards. The desired response function in the passband region is given as:

$$G_d(r_s, f) = e^{-j2\pi f\left(\frac{||r_s - r_c||}{c} + \frac{L-1}{2}T\right)},$$

where c =340.9m/s is the sound speed; the sampling time, T, is set as 0.125×10^{-3} seconds; the weighting function, $\rho(\bar{r_{\bar{p}}}, f)$, in (4) is chosen as 1; and r_c is the center position of the microphones indexed with $\bar{p}$. It is given as:

$$r_c = \frac{1}{|\bar{p}|}\sum_{i=1}^{|\bar{p}|} r_{p(i)}$$

Here we consider a configuration design problem in two dimensions. The speaker is stationed on the plane with $z = 0$ so that both passband and stopband are defined on this plane. The M feasible locations for the microphones are placed in the plane with $z = 1$ which is one meter vertically away from the speaker. As the speaker is on the plane with z =0, both passband and stopband are also located on the floor. The passband region Ω_p and the stopband region Ω_s are defined in the following:

$$\Omega_p = \{(r, f) \mid ||(x,y)|| \leq 0.4\text{m}, z = 0\text{m}, 0.5\text{kHz} \leq f \leq 1.5\text{kHz}\}, \text{ and}$$

$$\Omega_s = \{(r, f) \mid ||(x,y)|| \leq 0.4\text{m}, z = 0\text{m}, 2.0\text{kHz} \leq f \leq 4.0\text{kHz}\}\ldots$$
$$\cup\{(r, f) \mid 1.8\text{m}\leq ||(x,y)|| \leq 3.0\text{m}, z = 0\text{m}, 0.5\text{kHz}\leq f \leq 1.5\text{kHz}\}\ldots$$
$$\cup\{(r, f) \mid 1.8\text{m}\leq ||(x,y)|| \leq 3.0\text{m}, z = 0\text{m}, 2.0\text{kHz}\leq f \leq 4.0\text{kHz}\}.$$

For the discretization of $\Omega = \Omega_p \bigcap \Omega_s$, each of the frequency domain regions is generated 60 points, and the spatial domain regions are taken every 0.2m. The feasible locations of the 37 microphones, r_i with $i = 1, 2, \ldots, 37$, is distributed on the circumference with the radius, 0.832m, and the center, (0,0,1), where the circumference is located horizontally on the top of the rectangle office. They are illustrated by "o" in Figure 4. This design attempts to simulate the audio appliance that only the circumference is available for installing the microphones. This design problem attempts to use minimize number of microphones in order to achieve maximum beamformer performance. $r_1 = (0, 0, 1)$, and the other feasible locations, r_i with $i = 2, 3, \ldots, 37$, are given by:

$$r_i = (0.832 \cdot \cos(\tfrac{2 \cdot \pi \cdot (i-1)}{36}), 0.832 \cdot \sin(\tfrac{2 \cdot \pi \cdot (i-1)}{36}), 1)$$

4.2 Implementation of the Taguchi method

The Taguchi method discussed in Section III is used to determine the appropriate microphone configuration. It first determines the orthogonal array based on Algorithm 1 (GenOA) in order to generate a set of the microphone configurations for the experimental purpose. As there are 21 feasible locations, the orthogonal array, $L_{32}(2^{21})$ with 21 design factors and 32 combinations of microphone configurations, was generated and is illustrated in Figure 5. Figure 5 also shows the

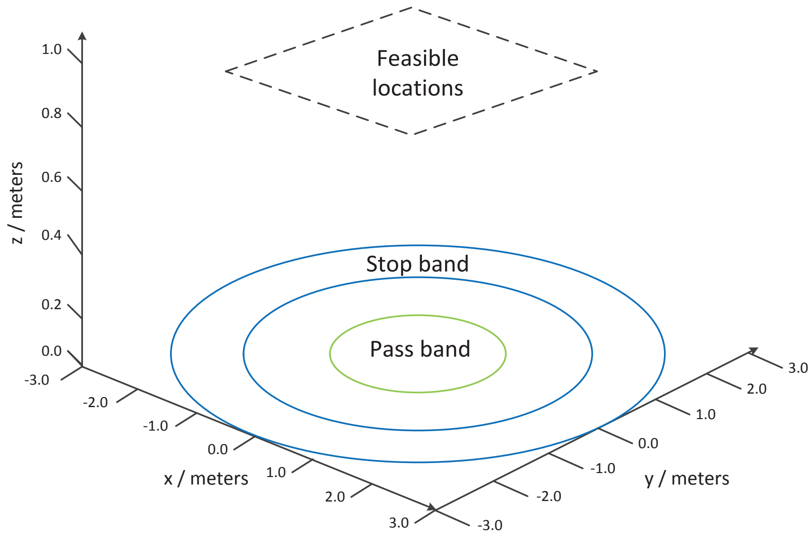

Figure 3: Problem configuration.

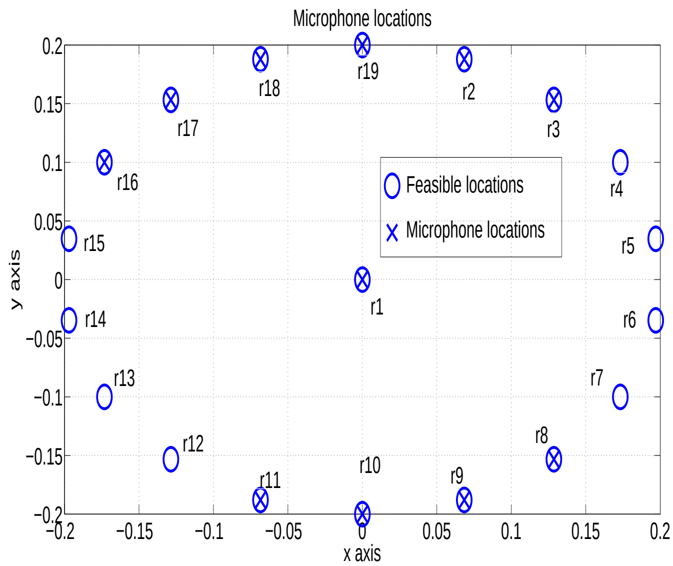

Figure 4: Feasible positions and microphone positions.

32 experimental results which illustrate the errors between the actual and desired responses and the numbers of microphones used on the 32 microphone configurations.

Based on $L_{32}(2^{21})$, 32 experiments are only required to be conducted in order to determine the appropriate microphone configuration. If the full factorial design is used, a total of 2.10×10^6 (i.e. 2^{21}) experiments are required to determine the appropriate microphone configuration as the number of feasible locations is 21. When 2 minutes are required for each experiments, 4.20×10^6 minutes (i.e. 70×10^3 hours or 2917 days) are required for the full factorial design. Using 2917 days to design a microphone configuration is not practical. When the Taguchi method is used, only 64 minutes (i.e. 32 experiments or 1.067 hours) are required to be used. Therefore, a significant amount of experimental efforts can be saved when the Taguchi method is used.

To determine the Pareto-optimal microphone configurations, the Algorithm 2, Taguchi method (TM) presented in Section III.B is used. The most right hand column of Figure 5 shows the rank for each microphone configuration with respect to (5). The 1-st, 2-nd, 4-th, 6-th and 12-th microphone configurations were graded as Rank 1. Also, the rank for each microphone configuration is shown in Figure 6. It illustrates the solutions in the Pareto-optimal set dominate by the other microphone configurations.

For the 1-st microphone configuration with rank 1, Figure 6 shows that the error is -38.96dB and the number of microphones used on the design is 19. We plot an actual response in the *xy*-plane for frequency 1400Hz in Figure 7. It shows that the higher signal to noise ratio exists in the passband (i.e. $||(x,y)|| \leq 0.4$m) and lower signal to noise ratio exists in the stopband which is in the region of 1.8m $\geq ||(x,y)|| \leq$ 3.0m. Although the error is the smallest, the number of microphones is the largest. The 5-th microphone configuration is in the middle of the Pareto-optimal set. 11 microphones were used which is much smaller than the 1-st configuration. Also, the errors obtained are generally small compared with the others. This example demonstrates that the procedures of using the Taguchi method in designing the microphone configuration. Based on the Taguchi method, experimental combinations on microphone configuration design can be advised.

Trials	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}	Error (dB)	No. microphones	Rank
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**-38.96**	**19**	**1**
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**-31.38**	**15**	**1**
3	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	-23.86	11	6
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**-20.58**	**7**	**1**
5	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	**-26.49**	**11**	**1**
6	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	-20.45	7	2
7	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	-23.86	11	7
8	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	-20.41	7	3
9	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	-24.01	9	2
10	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0	0	1	1	-23.97	9	3
11	1	0	0	1	1	0	0	0	0	1	1	0	0	1	1	1	1	0	0	-23.74	9	5
12	**1**	**0**	**0**	**1**	**1**	**0**	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**-24.13**	**9**	**1**
13	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	-23.06	9	8
14	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	1	1	-23.40	9	7
15	1	0	0	0	0	1	1	0	0	1	1	1	1	0	0	1	1	0	0	-23.55	9	6
16	1	0	0	0	0	1	1	0	0	1	1	1	1	0	0	0	0	1	1	-23.87	9	4
17	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	-20.56	9	10
18	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	1	0	1	-20.52	9	11
19	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	1	0	1	0	-20.50	9	12
20	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	-20.49	9	14
21	0	1	0	0	1	0	1	1	0	1	0	0	1	0	1	1	0	1	0	-20.46	9	16
22	0	1	0	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	1	-20.45	9	18
23	0	1	0	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	-20.50	9	13
24	0	1	0	0	1	0	1	0	1	0	1	1	0	1	0	0	1	0	1	-20.45	9	17
25	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	-20.13	9	26
26	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	1	0	-20.14	9	25
27	0	0	1	1	0	0	1	0	1	1	0	0	1	1	0	1	0	0	1	-20.39	9	21
28	0	0	1	1	0	0	1	0	1	1	0	0	1	1	0	0	1	1	0	-20.12	9	27
29	0	0	1	0	1	1	0	1	0	0	1	0	1	1	0	1	0	0	1	-20.48	9	15
30	0	0	1	0	1	1	0	1	0	0	1	0	1	1	0	0	1	1	0	-20.37	9	23
31	0	0	1	0	1	1	0	0	1	1	0	1	0	0	1	1	0	0	1	-20.33	9	24
32	0	0	1	0	1	1	0	0	1	1	0	1	0	0	1	0	1	1	0	-20.38	9	22

Figure 5: The results for the 64 microphone configurations (a) First objective: Error between the actual response and the desired response; (b) Second objective: Number of microphones; and (c) Rank with respect to the two objectives.

4.3 Evaluations of the Taguchi method

To compare the results obtained by the Taguchi method which small amount of experimental time is only available, two commonly-used heuristic methods including genetic algorithm (GA) [32] and particle swarm optimization algorithm (PSO) [35] have also been used as they have been used on designing microphone configurations [36–41]. They are commonly used on solving multi-objective problems. The comparison attempts to show the performance of the microphone configurations when small amount of experimental time is required for the configuration design. The following parameters and mechanisms were used in the two heuristic methods:

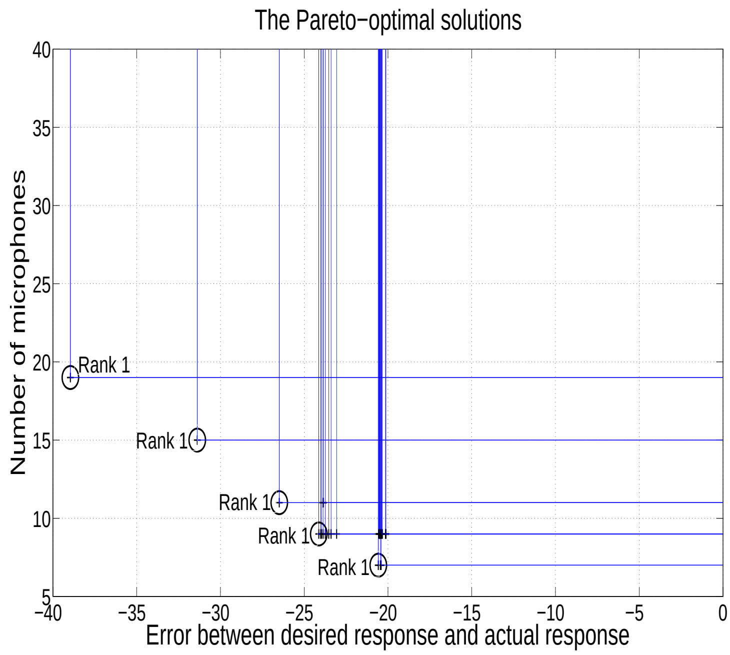

Figure 6: The Pareto-optimal set for the microphone configurations.

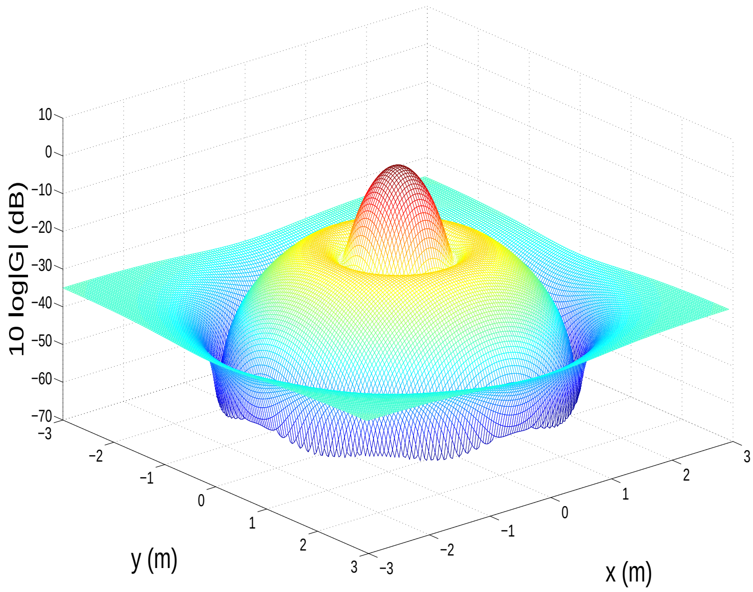

Figure 7: Magnitude of the actual response with the frequency of 1kHz.

In the GA, a population of chromosomes in binary representation with M bits is used, where the bit with '1' represents that the corresponding feasible location is installed with the microphone and that with '0' represents no microphone is installed. (5) is used as the GA fitness function which are identical to the that used in the Taguchi method. The evolutionary operations including selection, crossover and mutation discussed in [51], is used to generate a new population. The evolutionary operations repeat until the termination condition is reached, where the GA is terminated when a chromosome dominates the Pareto-optimal solution obtained by the Taguchi method. This termination condition attempts to evaluate whether the Taguchi method is more effective than the GA when small amount of computational evaluations are used. Here the two GAs, namely GA-10 and GA-20, were used, where the populations with 10 and 20 chromosomes were used in the GA-10 and the GA-20 respectively. The following parameters were used in the two GAs: crossover rate = 0.8; and mutation rate = 0.1.

In the PSO, each particle is represented by a binary string with M elements which is same as the GA chromosomes. The fitness function of the PSO is same as that used in the Taguchi method which is (5). When a particle of PSO dominates the Pareto-optimal solution obtained by the Taguchi method, the PSO is terminated. It attempts to evaluate whether the Taguchi method is more effective than the PSO when small amount of computational evaluations are used. Here the two PSOs with two swarm sizes, namely PSO-10 and PSO-20, were used, where PSO-10 and PSO-20 have 10 and 20 particles respectively. The maximum and minimum inertia weights of the two PSOs were set to 0.9 and 0.2, respectively, and the initial acceleration coefficients were set to 2.

As both the GAs and the PSOs are involved with the stochastic operations, different microphone configurations can be generated with different runs. Therefore, GA-10, GA-20, PSO-10 and PSO-20, were run 30 times, and the computational efforts for all runs were recorded. The

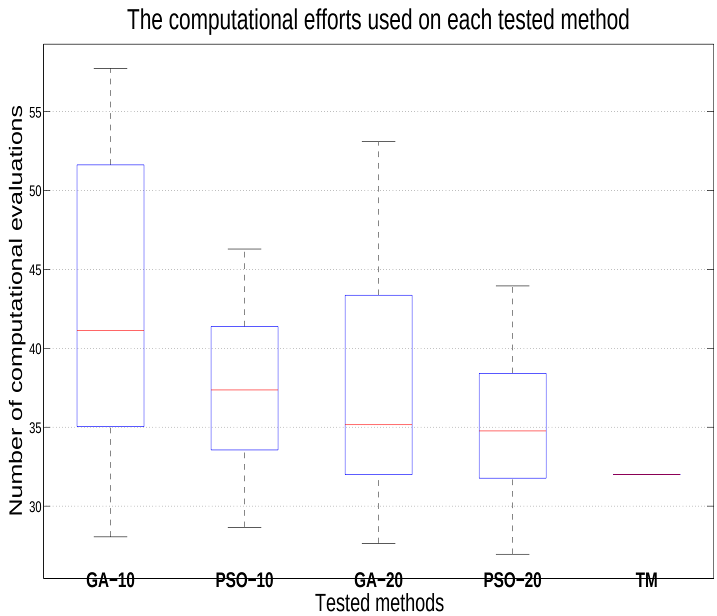

Figure 8: Computational efforts used on the proposed Taguchi method and the four tested methods.

computational efforts used for all the tested methods are shown in Figure 8. It shows that both PSO-10 and PSO-20 required about 37 and 35 computational evaluations which are more than those required by the Taguchi method which required 32. The computational evaluations used by the GA-10 and the GA-20 are about 42 and 35 which are more than that of the Taguchi method. Therefore, the Taguchi method is more effective than the two heuristic methods, GAs and PSOs, in designing microphone configurations. Also, the Taguchi method is a deterministic method of which same microphone configurations can be generated with different runs, unlike both the GAs and PSOs that different microphone configurations are generated with different runs. Hence, the advantages of the Taguchi method are indicated when small number of experiments is required.

Table II: Details of the four designs with four different numbers of feasible locations

Number of feasible locations	**16**	**36**	**64**	**100**
Orthogonal arrays	$L_{32}(2^{16})$	$L_{64}(2^{36})$	$L_{128}(2^{64})$	$L_{256}(2^{100})$
Experiments required on Orthogonal arrays	32	64	128	256
Experiments required on full factorial designs	65536	1.8447×10^{19}	3.4028×10^{38}	1.1579×10^{77}

4.4 Further Validation of the Taguchi method

We consider four microphone configuration designs with M to be 16, 36, 64, and 100, where the feasible locations of the M microphones is given by dividing the plane with $z = 1$ into $\sqrt{M} \times \sqrt{M}$ hexagonal grids on a plane with 3x3 dimension. We consider the hexagonal grids, as they are effective on arithmetic computations for many signal processing operations [52]. The orthogonal arrays, $L_{32}(2^{16})$, $L_{64}(2^{36})$, $L_{128}(2^{64})$, and $L_{256}(2^{100})$, are used for the four designs with M as 16, 36, 64 and 100 respectively, and 32, 64, 128 and 256 microphone configurations are considered in the $L_{32}(2^{16})$, $L_{64}(2^{36})$, $L_{128}(2^{64})$ and $L_{256}(2^{100})$ respectively. To compare the computational time required by the Taguchi method, GA-10, GA-20, PSO-10 and PSO-20 are used to determine the microphone configurations, where the termination condition is same as that used in Section IV.C. The numbers of experiments required on the orthogonal arrays and full factorial designs are shown in Table II.

GA-10, GA-20, PSO-10 and PSO-20, were run 30 times for the four designs, and the computational evaluations required for all runs were recorded. Figure 9 shows computational evaluations required by the Taguchi method and the other tested methods. Figure 9(a) shows that

GA-10 normally required 34 to 42 computational evaluations to obtain the solution obtained by the Taguchi method, where the Taguchi method only required 32 computational evaluations. Also the medium computational evaluations required by GA-10 is 39 which is larger than that required by the Taguchi method. Hence, Taguchi method required less computational effort than GA-10. Similar results can be found in GA-20, PSO-10 and PSO-20 of which more computational evaluations are generally required than those required by the Taguchi method. For the designs for the 36, 64 and 100 feasible locations, similar results show that the Taguchi method required less computational evaluations than GA-10, GA-20, PSO-10 and PSO-20. Therefore, the Taguchi method is generally more effective than the two heuristic methods when small numbers of computational evaluations are required.

Better results can be explained by the mechanisms of the Taguchi method and the two heuristic methods. The Taguchi method covers the microphone configuration in an orthogonal way and all combinations are balanced orthogonally. It is more likely to obtain a reasonable configuration when small number of experiments is conducted. The two heuristic methods distributes the combination in a random way and the combinations are not distributed orthogonally. When small number of experiments is required, Taguchi method can obtain similar configurations which are generated by the heuristic methods. The heuristic methods generally require more computational efforts.

5 Conclusions

In this paper, the Taguchi method was developed to design effective beamformer which uses small number of microphones. This configuration design problem is NP-hard nonlinear and non-convex, as 2^n configurations are required to be tested when n feasible locations are available for the microphone configuration design. Although heuristic methods have been developed to solve this problem, they require long computational time. The Taguchi method attempts to conduct systematical experiments based on orthogonal arrays to study the microphone configuration using a small amount of computational time, and it can roughly determine the appropriate microphone configurations using small numbers of microphones. The effectiveness of the Taguchi method is evaluated based on a case study which models a common office. The results show that the Taguchi method is capable to develop the microphone configurations with similar performance compared with the two commonly used heuristic methods when shorter computational time is only available on the design.

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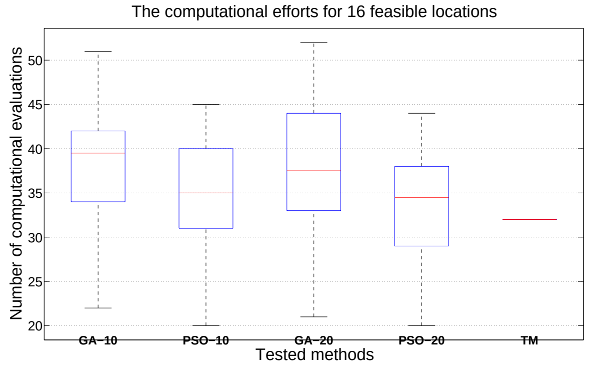
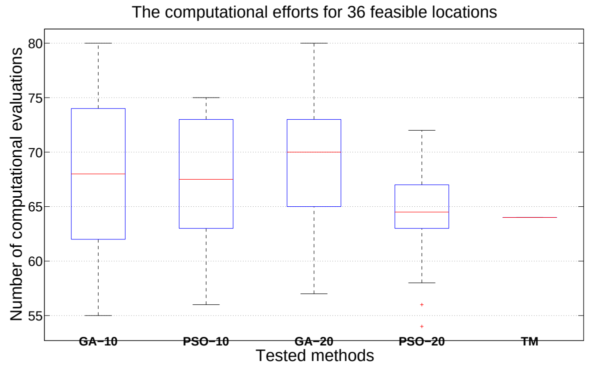
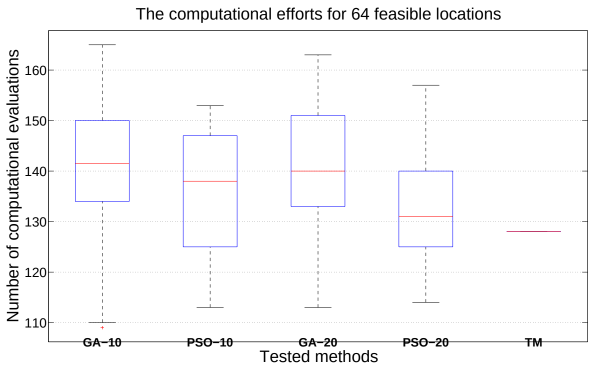


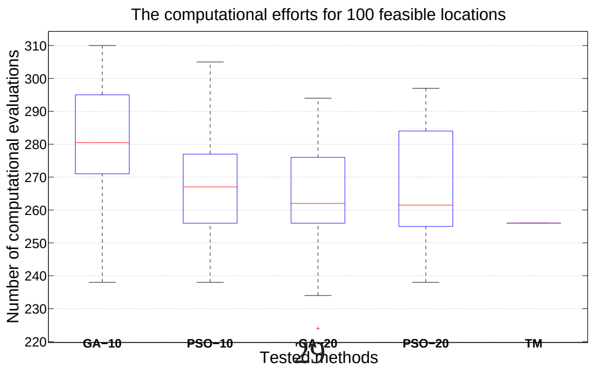
(a) Design with 16 feasible locations



(b) Design with 36 feasible locations



(c) Design with 64 feasible locations



(d) Design with 100 feasible locations

Figure 9: Computational efforts for the four designs required by the tested methods