Effects of the parabolic potential and confined phonons on the polaron in a quantum wire

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By using the Lee, Low, and Pines variational method, we have studied the electron-confined phonon interaction within a rectangular quantum wire under an additional parabolic potential. Formulas for the polaron self-energy, the electron effective mass along the wire, and the ground-state energy are derived. Numerical calculations are performed for a typical GaAs quantum wire within the mesoscopic size using the idea of Fourier decomposition of the wave function. In comparison with previous calculations, our results show that the effect of phonon confinement always reduces the magnitude of the electron-phonon interaction and the associated physical quantities, whereas the additional parabolic potential tends to enhance not only this interaction but also the ground-state energy.

I. INTRODUCTION

With the recent advances in the epitaxial techniques for the growth of compound semiconductor structures, it is possible to grow wirelike structures[1-8] in the low-nanometer scale. A great deal of research effort is currently being devoted to the study of these quasi-one-dimensional structures because of their potential applications and uncovering new phenomena. In particular, one of the interesting problems in semiconductor physics concerning those quantum-wire structures is the effect of electron-phonon interactions on the energy levels and the effective mass and their variations against the sizes of wire of electrons at low temperature. It is normally expected that the results should be quite different from those found in the quasi-two-dimensional semiconductor structures.[9,10] Although several groups of researchers have employed various models to investigate this quasi-one-dimensional electron gas, the theoretical nature of those models is still rather incomplete.

As a typical example, Degani and Hipolito[11,12] have calculated the energy shifts and the effective mass of an electron and the exciton binding energies associated with the effects of the electron(hole)–optical-phonon interaction in a quantum-well wire of GaAs surrounded by $Ga_{1-x}Al_xAs$. However, they have used the bulk-phonon approximation for the phonon system instead of the confined phonons which have been shown by Fasol *et al.*[13] to be existent with striking experimental evidence. In principle, the polaronic states should be affected by the changes in the form of the Fröhlich Hamiltonian caused by phonon confinement. Stroscio[14] has recently derived the modified Fröhlich Hamiltonian for the interaction between an electron and the confined LO-phonon modes in a quasi-one-dimensional system. Zhu and Gu have then employed this Hamiltonian to study the system of a free polaron in a rectangular quantum-well wire[15] and subsequently a polaron bound to a fixed hydrogenic impurity placed on the axis of the quantum wire.[16] However, in their former paper, the calculation was restricted to the weak-coupling system and they did not study the effective mass of the polaron due to the limitation of the perturbative method that they used. In their latter paper, they have shown that the effective potential induced by the interaction of the electron with the confined LO phonons depends not only on the electron position but also on the sizes of the wire. However, in all their work as well as most other work, the electron is assumed to be confined within an infinite square well potential. This is obviously contrary to the realistic case in which the potential experienced by the electrons (apart from the Coulombic contribution from the impurities) within the wire should not be zero or constant. Indeed, Kash *et al.*[17] have recently observed some good evidence for the existence of a parabolic potential well in a quantum wire produced by strain gradients using a patterned carbon stressor. On the other hand, Yildirim and Ercelebi[18,19] have theoretically achieved the effective reduction of dimensionality for free polarons located initially in a three-dimensional system. However, a main problem

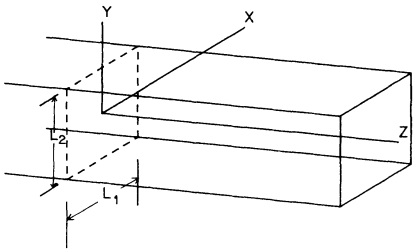

FIG. 1. Coordinates frame for the quantum wire.

concerning their approach is that their parabolic potential is so strong that it will unfortunately cause the band nonparabolicity and the loss of validity of the effective mass approximation. Besides, they have assumed the bulk-phonon approximation without noting the effect of phonon confinement. Therefore it is now very worthwhile to study the effect of a weak parabolic potential on an electron interacting with the spatially confined LO phonons.

In this paper, we shall adopt the same confined phonons assumptions as used in Refs. 15 and 16 to investigate the physical properties of a polaron under a parabolic potential in a rectangular quantum wire by means of the Lee-Low-Pines variational method recently modified by us.[20] In Sec. II, the theory of our treatments is established and then the explicit formulas for calculating the important energies and the effective mass of the electron induced by the confined LO phonons and their variations against the quantum-wire sizes and the strength of the parabolic potential are derived. In Sec. III, the numerical calculations are carried out and the results are shown in the form of several diagrams. The discussion of the results and a brief conclusion are given in Sec. IV.

II. THEORY

For our present theoretical modeling, the quantum wire are assumed to have rectangular cross sections and are made of polar semiconductor materials surrounded by a vacuum (see Fig. 1). It is also assumed that the effective mass approximation is valid so as to simplify our consideration. In the present work, the electrons are confined in infinitely hard potential wells and are affected by the parabolic potentials in the x and y directions. The total Hamiltonian of the system can be partitioned as

$$\mathcal{H}=\mathcal{H}_e=\mathcal{H}_{\rm LO}+\mathcal{H}_{e\text{-LO}} \ . \tag{1}$$

The first term $\mathcal{H}_e$ in (1) consists of the electron kinetic energy and the parabolic potentials experienced by the electron,

$$\mathcal{H}_e=\frac{1}{2m_b}(p_x^2+p_y^2+p_z^2)+V_p(x,y) \ ,$$
$$V_p(x,y)=\tfrac{1}{2}m_b\omega_1^2x^2+\tfrac{1}{2}m_b\omega_2^2y^2 \ , \tag{2}$$
$$|x|\le L_1/2 \text{ and } |y|\le L_2/2 \ ,$$

where m_b is the electron band mass, and ω_1 and ω_2 are the harmonic oscillator frequencies. It is noted that the confined longitudinal-optical phonons in our rectangular quantum wire are characterized by traveling waves in the z direction and standing waves in the x and y directions. Thus the confined LO-phonon Hamiltonian reads[15]

$$\mathcal{H}_{\rm LO}=\sum_{m_1,m_2,k_z}\hbar\omega_{\rm LO}\hat{A}^{\dagger}_{m_1p_1m_2p_2}(k_z)\hat{A}_{m_1p_1m_2p_2}(k_z) \ , \tag{3}$$

where $\hat{A}^{\dagger}_{m_1p_1m_2p_2}(k_z)$ and $\hat{A}_{m_1p_1m_2p_2}(k_z)$ are, respectively, some combinations of the creation and annihilation operators for LO phonons with frequency $\omega_{\rm LO}$ and wave vector $\mathbf{k}$. It should be noted that the confined LO-phonon wave vector $\mathbf{k}\equiv(m_1\pi/L_1,m_2\pi/L_2,k_z)$ is constrained by the Brillouin zone boundary, to wit, $m_1\le L_1/2a$ and $m_2\le L_2/2a$ where a is the lattice constant. Consequently, m_1 and m_2 can be any natural numbers lying in the range $1\le m_1\le N_1/2$ ($L_1=N_1a$) and $1\le m_2\le N_2/2$ ($L_2=N_2a$). Furthermore, p_1 and p_2 are positive (+) and negative (−) according to whether m_1 and m_2 are odd or even.

The last term in Eq. (1) is the interaction Hamiltonian of an electron with the confined LO phonons and its form is modified from Stroscio's[14] one-dimensional Fröhlich Hamiltonian as follows:

$$\mathcal{H}_{e\text{-LO}}=2\alpha'\sum_{k_z}e^{-ik_zz}\sum_{m_1,m_2}\text{csn}\left[\frac{m_1\pi x}{L_1}\right]\text{csn}\left[\frac{m_2\pi y}{L_2}\right]I(m_1,m_2,k_z)[\hat{A}_{m_1p_1m_2p_2}(k_z)-\hat{A}^{\dagger}_{m_1p_1m_2p_2}(-k_z)] \ , \tag{4}$$

where

$$\alpha'=i\left[2\pi e^2\hbar\omega_{\rm LO}\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right]\Big/V\right]^{1/2} \ ,$$

$$\text{csn}\left[\frac{m\pi x}{L}\right]=\begin{cases}\cos\left[\dfrac{m\pi x}{L}\right] & \text{for } m \text{ odd}\\ \sin\left[\dfrac{m\pi x}{L}\right] & \text{for } m \text{ even} \ ,\end{cases}$$

$$I(m_1,m_2,k_z)=\left[\left[\frac{m_1\pi}{L_1}\right]^2+\left[\frac{m_2\pi}{L_2}\right]^2+k_z^2\right]^{-1/2} \ ,$$

V is the wire volume ($V=L_1L_2L_3$, L_3 is the length of the quantum wire), and ϵ_0 and ϵ_∞ are the static and the optical dielectric constants, respectively.

Now, we adopt the variational treatment of Lee, Low, and Pines[21] in a slightly modified way[20] to cope with the Hamiltonian in (1). In essence, we need to perform two successive unitary transformations to $\mathcal{H}$ with

$$U_1=\exp\left[-iz\sum_{m_1,m_2,k_z}k_z\hat{A}^{\dagger}_{m_1p_1m_2p_2}(k_z)\hat{A}_{m_1p_1m_2p_2}(k_z)\right] \tag{5a}$$

and

$$U_2=\exp\left[\sum_{m_1,m_2,k_z}[\hat{A}^{\dagger}_{m_1p_1m_2p_2}(k_z)f_{m_1p_1m_2p_2}(k_z)-\hat{A}_{m_1p_1m_2p_2}(k_z)f^{*}_{m_1p_1m_2p_2}(k_z)]\right], \tag{5b}$$

where $f_{m_1p_1m_2p_2}(k_z)$ and $f^{*}_{m_1p_1m_2p_2}(k_z)$ are the variational parameters which will subsequently be determined by minimizing the electron energy. After a lengthy algebraic manipulation, we obtain the transformed Hamiltonian

$$\begin{aligned}\mathcal{H}^{*}&=U_2^{-1}U_1^{-1}\mathcal{H}U_1U_2\\&=\frac{1}{2m_b}(p_x^2+p_y^2+\not{p}_z^2)+V_p(x,y)+\frac{\hbar^2}{2m_b}\left[\sum_{m_1,m_2,k_z}k_z|f_{m_1p_1m_2p_2}(k_z)|^2\right]^2\\&+\sum_{m_1,m_2,k_z}|f_{m_1p_1m_2p_2}(k_z)|^2\left[\hbar\omega_{\mathrm{LO}}+\frac{\hbar^2k_z^2}{2m_b}-\frac{\hbar}{m_b}k_z\not{p}_z\right]\\&+2\alpha'\sum_{m_1,m_2,k_z}\mathrm{csn}\left[\frac{m_1\pi x}{L_1}\right]\mathrm{csn}\left[\frac{m_2\pi y}{L_2}\right]I(m_1,m_2,k_z)[f_{m_1p_1m_2p_2}(k_z)-f^{*}_{m_1p_1m_2p_2}(-k_z)]+\mathcal{H}_1^{*},\end{aligned} \tag{6}$$

where the perturbative term $\mathcal{H}_1^{*}$ contains phonon operators which yield zero contribution to the ground-state energy in the first-order approximation. Since the total momentum in the z direction

$$\not{p}_z\equiv p_z+\sum_{m_1,m_2,k_z}\hat{A}^{\dagger}_{m_1p_1m_2p_2}(k_z)\hat{A}_{m_1p_1m_2p_2}(k_z)\hbar k_z$$

commutes with $\mathcal{H}$ and so it can be reduced to a c number, $\hbar q_z$ (say) with eigenfunction $\exp(i\hbar q_z z)$.

Following the above notations, the wave function of our system can be written as

$$|\Psi(x,y,z,\mathbf{k})\rangle=\frac{1}{(L_3)^{1/2}}e^{i\hbar q_z z}|\phi(x,y)\rangle|N_{\mathbf{k}}\rangle, \tag{7}$$

where $|\phi(x,y)\rangle$ is the xy component of the electron wave function in the quantum wire and $|N_{\mathbf{k}}\rangle$ is the wave function of the phonon field in which there are N numbers of LO phonons with the wave vector $\mathbf{k}=(m_1\pi/L_1,m_2\pi/L_2,k_z)$. In the low-temperature limit, very few phonons are excited and so we may assume the vacuum state $|0\rangle$ as the phonon ground state which satisfies

$$A_{m_1p_1m_2p_2}(k_z)|0\rangle=0.$$

Correspondingly, the expectation value of $\mathcal{H}$ in such a state becomes

$$\mathcal{F}=\langle\phi(x,y)|\mathcal{H}_{\mathrm{eff}}|\phi(x,y)\rangle, \tag{8}$$

where

$$\begin{aligned}\mathcal{H}_{\mathrm{eff}}&=\frac{1}{2m_b}(p_x^2+p_y^2+\hbar^2q_z^2)+V_p(x,y)+\frac{\hbar^2}{2m_b}\left[\sum_{m_1,m_2,k_z}k_z|f_{m_1p_1m_2p_2}(k_z)|^2\right]^2\\&+\sum_{m_1,m_2,k_z}|f_{m_1p_1m_2p_z}(k_z)|^2\left[\hbar\omega_{\mathrm{LO}}+\frac{\hbar^2k_z^2}{2m_b}-\frac{\hbar^2k_z}{m_b}q_q\right]\\&+2\alpha'\sum_{m_1,m_2,k_z}\left\langle\phi\left|\mathrm{csn}\left[\frac{m_1\pi x}{L_1}\right]\mathrm{csn}\left[\frac{m_2\pi y}{L_2}\right]\right|\phi\right\rangle I(m_1,m_2,k_z)[f_{m_1p_1m_2p_2}(k_z)-f^{*}_{m_1p_1m_2p_2}(-k_z)].\end{aligned} \tag{9}$$

For our quasi-one-dimensional quantum wire, it is quite feasible to introduce a parameter η such that

$$\sum_{m_1,m_2,k_z}k_z|f_{m_1p_1m_2p_2}(k_z)|^2=\eta q_z. \tag{10}$$

Inserting this into (9) and requiring that

$$\delta\mathcal{F}/\delta f_{m_1p_1m_2p_2}(k_z)=0=\delta\mathcal{F}/\delta f^{*}_{m_1p_1m_2p_2}(k_z) \tag{11}$$

we have

$$f_{m_1p_1m_2p_2}(k_z)=\frac{2\alpha'\langle\phi|\mathrm{csn}(m_1\pi x/L_1)\mathrm{csn}(m_2\pi y/L_2)|\phi\rangle I(m_1,m_2,k_z)}{[\hbar\omega_{\mathrm{LO}}+\hbar^2k_z^2/2m_b-(1-\eta)(\hbar^2k_z/m_b)q_z]} \tag{12}$$

and $f^*_{m_1p_1m_2p_2}(k_z)$ is simply the conjugate formula of (12).

As we are interested only in the slow electrons which are always observed in experiments, so we may simply set $q_z \simeq 0$. By substituting formula (12) and its conjugate into (10) and expanding them to the first power of q_z, the parameter η is then given by

$$\eta = \frac{\alpha D}{1+\alpha D} , \tag{13}$$

where

$$\alpha = \frac{e^2}{2\hbar\omega_{\rm LO}} \left[\frac{2m_b\omega_{\rm LO}}{\hbar} \right]^{1/2} \left[\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right]$$

and

$$D = \frac{4\pi}{L_1L_2} \sum_{m_1,m_2} \left| \left\langle \phi \left| \mathrm{csn}\left[\frac{m_1\pi x}{L_1} \right] \mathrm{csn}\left[\frac{m_2\pi y}{L_2} \right] \right| \phi \right\rangle \right|^2 \times \frac{k_{m_1m_2}+3k_{\rm LO}}{(k_{m_1m_2}+k_{\rm LO})^3} , \tag{14}$$

where the reciprocal of the polaron radius is

$$k_{\rm LO} = \left[\frac{2m_b\omega_{\rm LO}}{\hbar} \right]^{1/2}$$

and

$$k_{m_1m_2} = \left[\left[\frac{m_1\pi^2}{L_1} \right]^2 + \left[\frac{m_2\pi}{L_2} \right]^2 \right]^{1/2} .$$

Now we may take the variation minimum of $\mathcal{F}$ as the ground-state energy of the polaron confined in our rectangular quantum wire. Using formulas (10)–(13), we finally obtain

$$F = \min_{ff^*}(\mathcal{F}) = \left\langle \phi \left| -\frac{\hbar^2}{2m_b} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] + V_p(x,y) \right| \phi \right\rangle + \frac{\hbar^2 q_z^2}{2m_b} \frac{1}{1+\alpha D} + E_s , \tag{15}$$

where

$$E_s = -\alpha\hbar\omega_{\rm LO} \frac{8\pi}{L_1L_2} \times \sum_{m_1,m_2} \frac{\langle\phi|\mathrm{csn}(m_1\pi x/L_1)\mathrm{csn}(m_2\pi y/L_2)|\phi\rangle^2}{k_{m_1m_2}(k_{\rm LO}+k_{m_1m_2})} . \tag{16}$$

Physically, $E_s[\phi]$ simply represents the self-energy of the polaron due to the electron-confined LO-phonon interaction when the particular $\phi_{\rm min}$ is chosen to minimize $F[\phi_{\rm min}]=E_g$, which corresponds to the ground-state energy. From (15), we note that the polaron effective mass $m_{\rm eff}$ along the wire (z direction) is given by

$$m_{\rm eff} = (1+\alpha D)m_b . \tag{17}$$

III. CALCULATIONS AND RESULTS

For our subsequent numerical calculations, it is more convenient to use the polaron units in which $k_{\rm LO}=1$ and $\hbar\omega_{\rm LO}=1$. Then (15) is reduced to

$$F[\phi] = \left\langle \phi(x,y) \left| -\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] + \tfrac{1}{4}\omega_1^2x^2 + \tfrac{1}{4}\omega_2^2x^2 \right| \phi(x,y) \right\rangle + \frac{q_z^2}{(1+\alpha D)} + E_s[\phi], \tag{18}$$

where

$$D[\phi] \equiv \frac{4\pi}{L_1L_2} \sum_{m_1,m_2} \left\langle \phi \left| \mathrm{csn}\left[\frac{m_1\pi x}{L_1} \right] \mathrm{csn}\left[\frac{m_2\pi y}{L_2} \right] \right| \phi \right\rangle^2 \frac{k_{m_1m_2}+3}{(k_{m_1m_2}+1)^3}$$

and

$$E_s[\phi] = -\frac{8\pi\alpha}{L_1L_2} \sum_{m_1,m_2} \frac{\langle\phi|\mathrm{csn}(m_1\pi x/L_1)\mathrm{csn}(m_2\pi y/L_2)|\phi\rangle^2}{k_{m_1m_2}(1+k_{m_1m_2})} .$$

To find the $\phi_{\rm min}$ which minimizes the $F[\phi]$ in (18), we shall adopt the following variational approach.[20]

First, we express the xy components of the electron wave function as

$$\phi(x,y) = \sum_l g_l\psi_l(x,y) , \tag{19}$$

where g_l are the variational parameters such that $\sum_l g_l^2=1$ and the basis functions $\psi_l(x,y)$ are the eigenfunctions of the effective Hamiltonian $\mathcal{H}^{(0)}_{\rm eff}$ embedded in the expectation term of (18) and are arranged in ascending order in terms of their energies. Such an arrangement will permit us to make a good approximation by taking a finite and small number of ψ_l for our minimization as expected from the usual perturbation theory. Second, ψ_l can further be reexpressed as a linear combination of the basis for the infinite square potential well, to wit,

$$\left\{\frac{2}{(L_1L_2)^{1/2}}\mathrm{csn}\frac{n_1\pi x}{L_1}\mathrm{csn}\frac{n_2\pi y}{L_2}\right\} \tag{20}$$

and this idea is warranted by the Fourier decomposition of wave function. Again, the basis functions are arranged in ascending order according to $n_1^2+n_2^2$ where n_1 and n_2 are natural numbers. In practical calculations for a typical GaAs quantum wire, it is found that basis functions with n_1 and n_2 up to 10 only are needed to yield quite accurate eigenfunctions ψ_l for a fairly strong parabolic potential ω_1, $\omega_2 \simeq 10\omega_{\mathrm{LO}}$. The experimental value of $\hbar\omega_1$ and $\hbar\omega_2$ given by Kash *et al.*[17] is 2.4 meV which is less than one-tenth of $\hbar\omega_{\mathrm{LO}}=36.7$ meV. Overall, the ground-state energy E_g is given by

$$E_g=\min_{\{g_l\}}(\langle\phi\{g_l\}|\mathcal{H}_{\mathrm{eff}}^{(0)}|\phi\{g_l\}\rangle+E_g[\phi\{g_l\}]) . \tag{21}$$

For the typical GaAs quantum wire with electron-phonon coupling constant $\alpha=0.39$ and lattice constant $a=0.565$ nm, we have calculated the polaron self-energy, the effective mass of electron in the z direction, and the ground-state energy for various values of the parabolic potential strength $\omega_1=\omega_2$ $(=\omega)$ and quantum-wire cross sections. The results are presented in Figs. 2–4.

IV. DISCUSSIONS AND CONCLUSIONS

In Fig. 2, our calculated $E_s/(-\alpha\hbar\omega_{\mathrm{LO}})$ is found to be always less than 1 when there is no parabolic potential applied, i.e., $\omega=0$. This result contrasts sharply with the unity value in the macroscopic system[21] and with the even greater value for the case of a confined electron interacting with bulk LO phonons.[20,12] The reason is due to the fact that phonon confinement has significantly reduced the number of phonons interacting with the electron in a quantum wire. In other words, the summation in (16) for E_s is taken over a very limited number of confined phonon modes labeled by m_1 and m_2 whereas all the previous work has taken the summation over all phonon modes in the whole crystal without any confinement. Besides, it is noted that there is a peak for small $N_1 \simeq N_2$ but this peak is gradually smearing out as N_1 and N_2 become larger. The dip in E_s for smaller N_2 is obviously due to the reduction in the number of

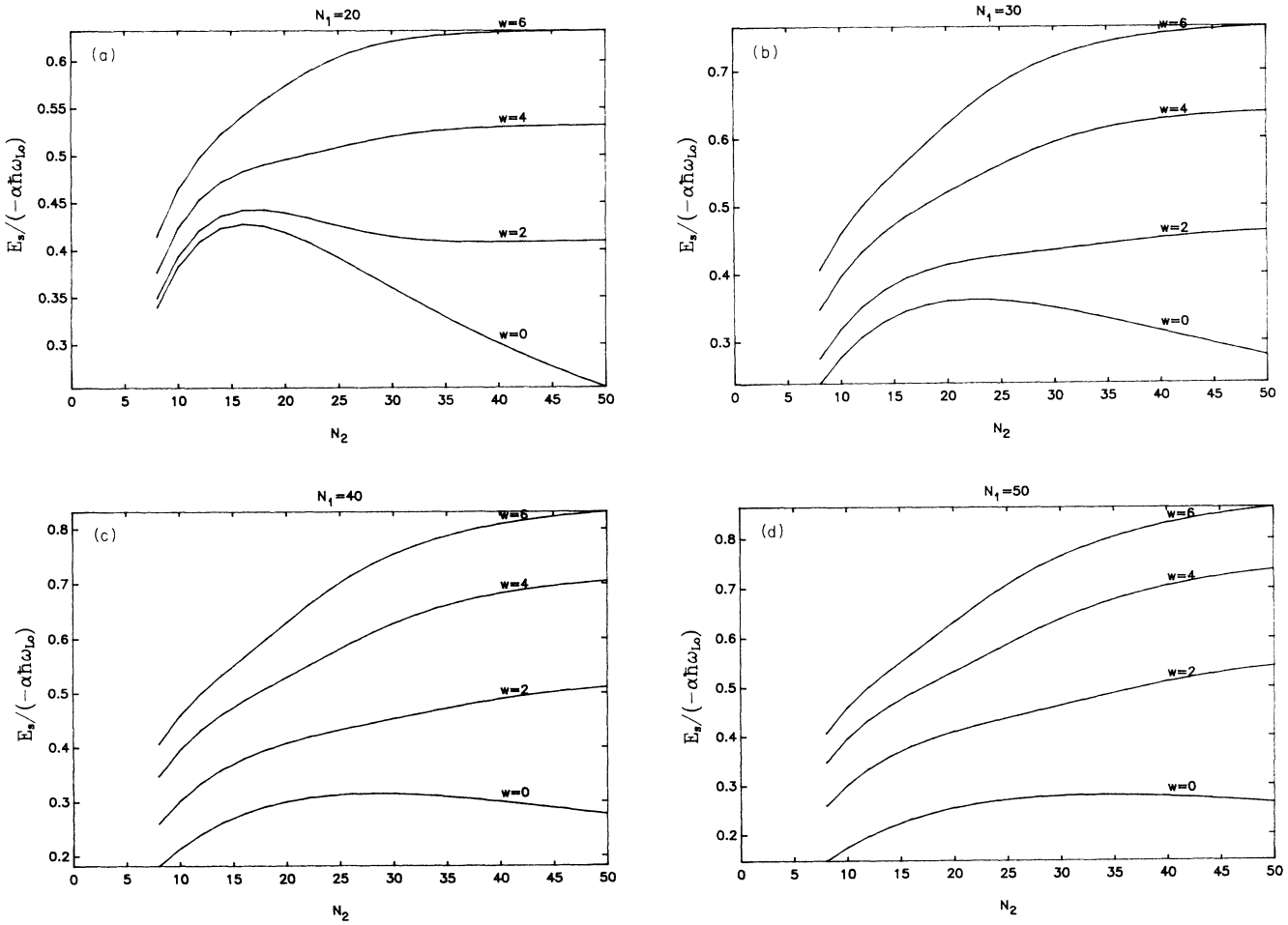

FIG. 2. Variations of the polaron self-energy E_s with one size N_2 of the quantum wire and with the parabolic potential strength ω (in polaron units) for the other wire size N_1 fixed at (a) 20, (b) 30, (c) 40, and (d) 50.

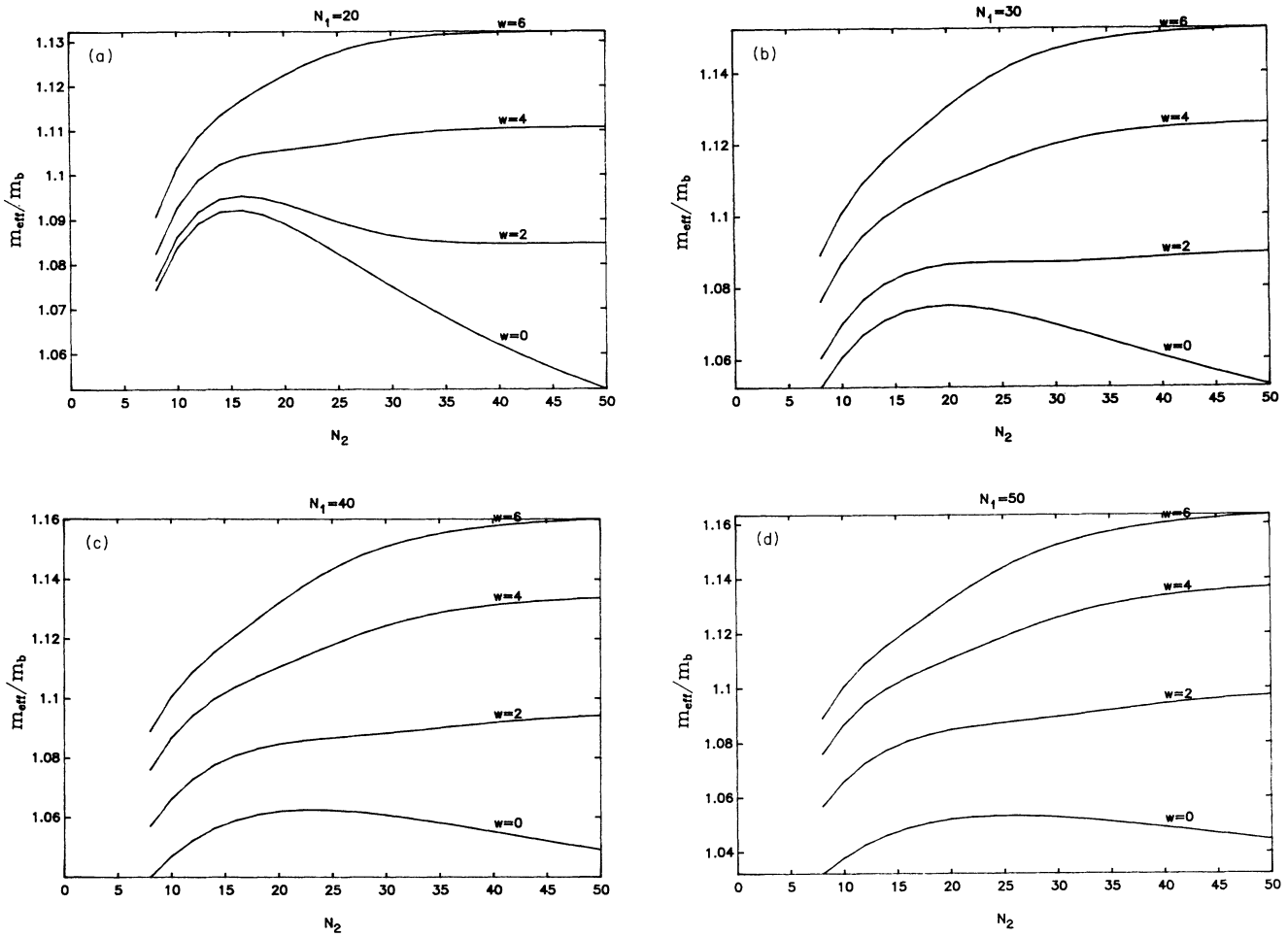

FIG. 3. Variations of the effective mass m_{eff} (relative to the band mass) of an electron along the wire with one size N_2 of the quantum wire and with the parabolic potential strength ω (in polaron units) for the other wire size N_1 fixed at (a) 20, (b) 30, (c) 40, and (d) 50.

confined phonons whereas the gradual decline for larger N_2 is most likely ascribed to the reducing matching between the electron and the phonon distributions. When the parabolic potential is added, the magnitude of the self-energy E_s is increased with increasing parabolic potential strength ω and with increasing wire sizes N_1 and N_2. The peak disappears for a sufficiently large ω in which the parabolic potential should dominate over the spatial confinement of the electron as N_1 and N_2 become larger.

In Fig. 3, our calculated effective electron mass m_{eff} along the wire is not much greater than the three-dimensional (3D) value of $(1+\alpha/6)m_b$ given in Ref. 21 and is always smaller than the corresponding previous calculations[20,12] under the bulk-phonon approximation. The reason is similar to that for E_s and so it is necessary to consider the phonon confinement in all calculations to avoid the overestimation of phonon effect. Again, there is a peak for small $N_1 \simeq N_2$ but this peak is gradually smearing out as N_1 and N_2 become larger. The effective mass m_{eff} is increased when the parabolic potential is added. Above a certain value of the parabolic potential strength ω or values of N_1 and N_2, the peak disappears because the parabolic potential effect has dominated over that of the electron spatial confinement.

In Fig. 4, the calculated ground-state energy E_g is found to be decreasing monotonically with the wire sizes N_1 and N_2 in the mesoscopic region because the main contribution to it is the electron spatial confinement energy. Besides, the E_g is found to be an increasing function of the parabolic potential strength ω but it will rapidly converge to a steady value when the wire size N_1 or N_2 increases. No peak is observed here because the contribution from electron spatial confinement is much greater than that of the electron-phonon interactions E_s, even without the parabolic potential contribution.

In summary, we have modified the variational method of Lee, Low, and Pines[21] to establish a method to study the interaction between an electron and the confined phonons within a quantum wire under an additional parabolic potential. Some formulas have also been derived for the electron self-energy, the effective mass along the wire, and the electron ground-state energy. Numerical calculations have been carried out for a typical GaAs quantum

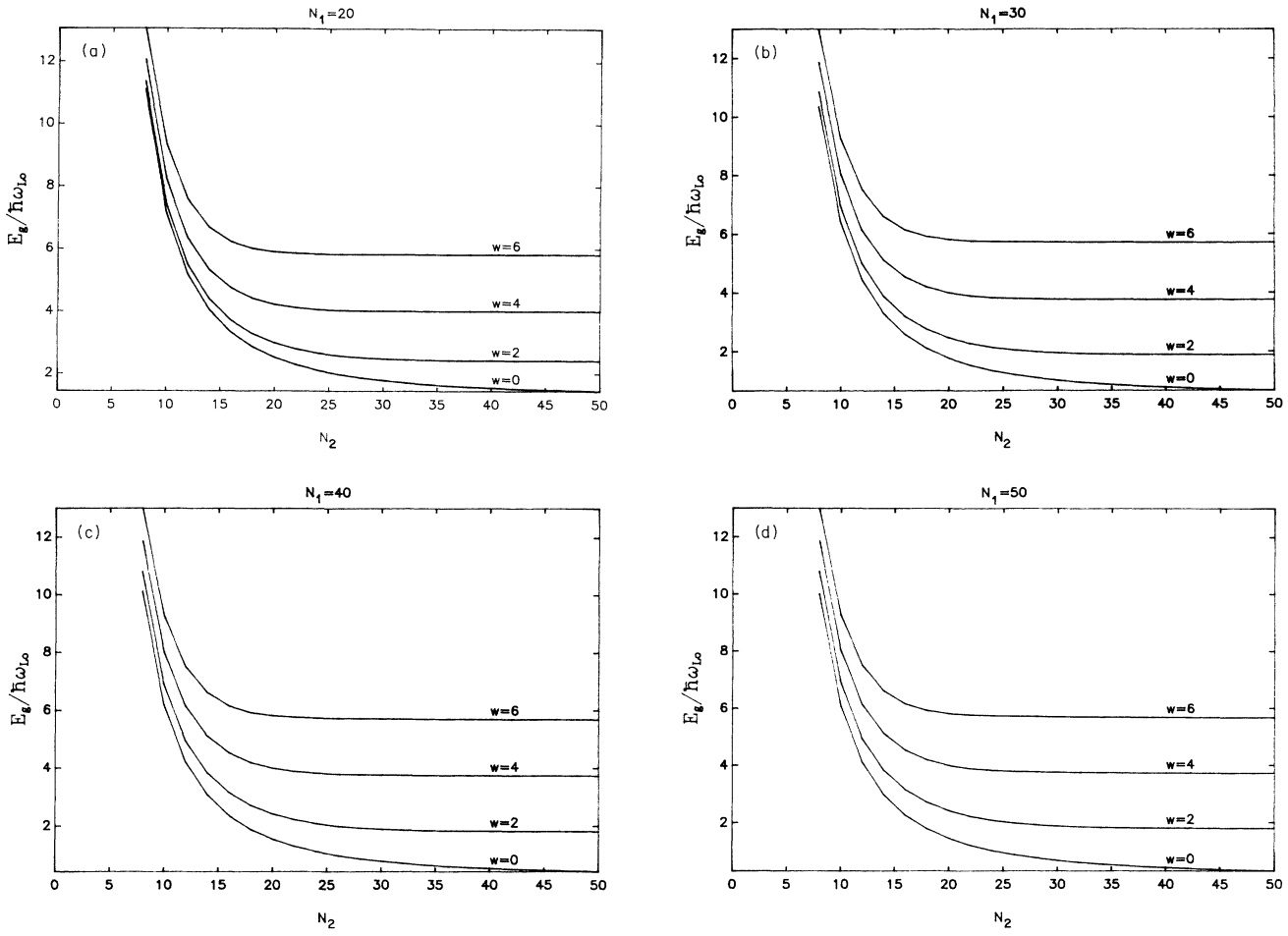

FIG. 4. Variations of the ground-state energy E_g of a confined polaron with one size N_2 of the quantum wire and with the parabolic potential strength ω for the other wire size N_1 fixed at (a) 20, (b) 30, (c) 40, and (d) 50. E_g and ω are given in polaron units.

wire and the results have then been compared with previous calculations. It is concluded that the phonon confinement and the parabolic potential have a significant but different effect on the values of the physical quantities. The phonon confinement has a reduction effect whereas the parabolic potential tends to enhance electron-phonon interaction.

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