Weak decays in the light-front quark model

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(Received 7 March 1995)

We study the form factors of heavy-to-heavy and heavy-to-light weak decays using the light-front relativistic quark model. For the heavy-to-heavy $B \to D^{(*)}$ semileptonic decays we calculate the corresponding Isgur-Wise function for the whole kinematic region. For the heavy-to-light $B \to P$ and $B \to V$ semileptonic decays we calculate the form factors at $q^2 = 0$; in particular, we have derived the dependence of the form factors on the b-quark mass in the $m_b \to \infty$ limit. This dependence cannot be produced by extrapolating the scaling behavior of the form factors at $q^2_{\rm max}$ using the single-pole assumption. This shows that the q^2 dependence of the form factors in regions far away from the zero recoil could be much more complicated than that predicted by the single-pole assumption.

PACS number(s): 13.20.He, 12.39.Hg, 12.39.Ki

I. INTRODUCTION

In the last few years, great progress has been made in understanding weak decays of hadrons containing heavy-quarks. The heavy-quark symmetry, which appears in the heavy-quark limit, can simplify many aspects of the weak decays of heavy hadrons [1]. Because of the heavy-quark symmetry all form factors in the heavy-to-heavy-type decays such as $B \to D^{(*)} e\bar{\nu}_e$ ($D^{(*)} = D$ or D^*) can be related, in the heavy-quark limit, to a single universal function called the Isgur-Wise function. The Isgur-Wise function is of nonperturbative origin and has been of great interest to both theoretical and experimental studies. In particular, the Isgur-Wise function of the $B \to D^{(*)}$ semileptonic decays has been widely studied [2–6].

The heavy-quark symmetry can also shed some light on the heavy-to-light-type weak decays. For example, one can derive the dependence of form factors (there is more than one form factor in this case) on the heavy-quark mass in the zero-recoil region, i.e., near $q^2_{\rm max}$ [7]. However, away from the zero-recoil region, one still needs a model-dependent method to understand the form factors.

In this paper we study the form factors of heavy-to-heavy and heavy-to-light weak decays using the light-front relativistic quark model. The light-front relativistic quark model was developed quite a long time ago, and there have been many successful applications [8–11]. Here we use this model to calculate the Isgur-Wise function for the heavy-to-heavy $B \to D^{(*)}$ semileptonic decays. It is known that the light-front model usually can only work at $q^2 \leq 0$. However, it is possible to use the results at $q^2 = 0$ to get the Isgur-Wise function for the whole kinematic region [12].

We also study the form factors in the heavy-to-light decays such as $B \to \pi$ and $B \to \rho$ semileptonic decays in the heavy b-quark limit. In particular, we are interested in the dependence of these form factors on the b-quark mass m_b, since the pole-dominance assumption for the form factors is usually used to move away from the zero-recoil region. Heavy-to-light weak decays are especially sensitive to the q^2 dependence of the form factors with such an assumption. The scaling behavior of these form factors at $q^2 = 0$ allows us to compare it with the pole-dominance assumption and to understand the behavior of the form factors away from $q^2_{\rm max}$.

The paper is organized as follows. In Sec. II we present a brief introduction to the light-front relativistic quark model; in Sec. III, we calculate the Isgur-Wise function for $B \to D^{(*)}$ decays; in Sec. IV we study the heavy-to-light form factors in the heavy-quark limit, and we give our conclusions in Sec. V.

II. THE LIGHT-FRONT RELATIVISTIC QUARK MODEL

The light-front relativistic model [8] has been applied recently to many aspects of heavy-meson weak decays [9–11] where more details can be found. In particular, a nice introduction can be found in [9]. Here we only give a brief description of the model.

It is known that the dynamics of a relativistic system is determined by the ten generators of the Poincaré group. In the Poincaré group there is always a kinematic subgroup of which the generators are independent of the interaction [14]. In the usual "instant" form, the instant time $x^0 = t$ is invariant under the corresponding kinematic subgroup, which has six generators. Correspondingly, in the light-front form, $x^+ = t + x^3$ is left invariant by the light-front kinematic subgroup. One of the advantages of the light-front form is that its kinematic subgroup has the maximum number of generators (seven).

In the light-front quark model [15] a ground-state meson $V(Q\bar{q})$ with spin J can be described by the state vector

$$|V(P_V,J_3,J)\rangle = \int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \delta(\mathbf{P}_V-\mathbf{P}) \times \sum_{\lambda_1,\lambda_2} \Psi^{J,J_3}(\mathbf{p}_1,\mathbf{p}_2,\lambda_1,\lambda_2) \times |Q(\lambda_1,\mathbf{p}_1)\bar{q}(\lambda_2,\mathbf{p}_2)\rangle \ . \quad (1)$$

The quark coordinates are given by

$$p_{1+}=x_1P_+\ ,\ \mathbf{p}_{1\perp}=x_1\mathbf{P}_\perp+\mathbf{k}_\perp\ ,$$
$$p_{2+}=x_2P_+\ ,\ \mathbf{p}_{2\perp}=x_2\mathbf{P}_\perp-\mathbf{k}_\perp\ ,$$
$$x_1+x_2=1\ ,0\le x_{1,2}\le 1\ ,\quad \mathbf{P}=\mathbf{p}_1+\mathbf{p}_2\ . \quad (2)$$

In the light-front convention, for $\mathbf{P}$ (and similarly for other vectors), $\mathbf{P}=(P_+,\mathbf{P}_\perp)$ where $P_+=P_0+P_z$ and $\mathbf{P}_\perp=(P_x,P_y)$. We denote *quark* Q by subscript 1 and *antiquark* $\bar{q}$ by 2.

It can be shown that the quantities $x_{1,2}$ and $\mathbf{k}_\perp$ are invariant under the light-front kinematic subgroup [9]. In the light-front quark model, the individual quarks are on mass shell ($p_1^2=m_1^2$ and $p_2^2=m_2^2$) and $\mathbf{p}_1+\mathbf{p}_2=\mathbf{P}$, but $p_{10}+p_{20}\neq P$. The sum of the four-momenta of the quarks is

$$M_0^2=(p_1+p_2)^2=\frac{m_1^2+\mathbf{k}_\perp^2}{x_1}+\frac{m_2^2+\mathbf{k}_\perp^2}{x_2}\ . \quad (3)$$

One can introduce a more intuitive quantity, the internal momentum $\mathbf{k}=(k_z,\mathbf{k}_\perp)$, where k_z is defined through

$$x_1=x=\frac{e_1+k_z}{e_1+e_2}\ ,\quad x_2=1-x=\frac{e_2-k_z}{e_1+e_2}\ ,$$
$$e_i=\sqrt{m_i^2+\mathbf{k}^2}\ \ (i=1,2)\ . \quad (4)$$

Then we have

$$M_0=e_1+e_2\ . \quad (5)$$

Obviously, $\mathbf{k}$ is invariant under the light-front kinematic subgroup because of the invariance of $x_{1,2}$ and $\mathbf{k}_\perp$.

To be invariant under the kinematic subgroup, the wave function can only be the function of $x_{1,2}$ and $\mathbf{k}_\perp$ or equivalently $\mathbf{k}$. Thus the wave function is independent of the motion of the hadron. A relativistic description of the meson can then be achieved by solving the wave function from the relativistic eigenvalue equation [9]

$$(e_1+e_2+V_{12})\ \Psi^{J,J_3}=m_V\ \Psi^{J,J_3}\ , \quad (6)$$

where V_{12} is the potential and m_V is the meson mass.

Rotational invariance of the wave function fmr states with spin J and zero orbital angular momentum requires the wave function to have the form [8, 9] (with $x=x_1$)

$$\Psi^{J,J_3}(\mathbf{p}_1,\mathbf{p}_2,\lambda_1,\lambda_2)=R^{J,J_3}(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)\phi(x,\mathbf{k}_\perp), \quad (7)$$

where $\phi(x,\mathbf{k}_\perp)$ is even in $\mathbf{k}_\perp$ and

$$R^{J,J_3}(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)=\sum_{\lambda,\lambda'}\langle\lambda_1|R_M^\dagger(x_1,\mathbf{k}_\perp,m_Q)|\lambda\rangle\langle\lambda_2|R_M^\dagger(x_2,-\mathbf{k}_\perp,m_{\bar{q}})|\lambda'\rangle C^{J,J_3}(\tfrac{1}{2},\lambda;\tfrac{1}{2},\lambda'). \quad (8)$$

In Eq. (8), $C^{J,J_3}(\frac{1}{2},\lambda;\frac{1}{2},\lambda')$ is the Clebsch–Gordan coefficient and the rotation $R_M(\mathbf{k}_\perp,m_i)$ on the quark spins is the Melosh rotation [13]:

$$R_M(x_i,\mathbf{k}_\perp,m_i)=\frac{m_i+x_iM_0-i\boldsymbol{\sigma}\cdot(\mathbf{n}\times\mathbf{k}_\perp)}{\sqrt{(m_i+x_iM_0)^2+\mathbf{k}_\perp^2}}\quad (i=1,2)\ , \quad (9)$$

where $\mathbf{n}=(0,0,1)$, $\boldsymbol{\sigma}$ are the Pauli-spin matrices, and M_0 is defined in Eq. (3). The spin-wave function $R^{J,J_3}(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)$ in (8) can also be written as

$$\begin{aligned}R^{J,J_3}(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)&=\chi_{\lambda_1}^\dagger R_M^\dagger(x_1,\mathbf{k}_\perp,m_Q)\ S^{J,J_3}\ R_M^{\dagger T}(x_2,-\mathbf{k}_\perp,m_{\bar{q}})\chi_{\lambda_2}\\&=\chi_{\lambda_1}^\dagger\ U_V^{J,J_3}(x,\mathbf{k}_\perp)\ \chi_{\lambda_2}\ ,\end{aligned} \quad (10)$$

where S^{J,J_3} is defined by

$$S^{J,J_3}=\sum_{\lambda,\lambda'}|\lambda\rangle\langle\lambda'|\ C^{J,J_3}(\tfrac{1}{2},\lambda;\tfrac{1}{2},\lambda')\ . \quad (11)$$

For the pseudoscalar and vector mesons, the nonrelativistic spin matrix is

$$S^{0,0}=\frac{i\sigma_2}{\sqrt{2}}\ ,\quad S^{1,\pm1}=\frac{1\pm\sigma_3}{2}\ ,\quad S^{1,0}=\frac{\sigma_1}{\sqrt{2}}\ . \quad (12)$$

Without the Melosh rotation, the spin of the wave function will be just S^{J,J_3}. The explicit expressions of $U^{J,J_3}(x,\mathbf{k}_\perp)$ in Eq. (10) can be found in Ref. [10].

The matrix element of the $B(b\bar{q})$ meson decaying to a meson $V(Q\bar{q})$ is

$$\langle V(p_V,J_3)|\bar{Q}\Gamma b|B(p_B)\rangle = \int dx\, d^2\mathbf{k}_\perp \sum_{\lambda_1,\lambda_2,\lambda_1'} \frac{\Psi_V^{*1,J_3}(\mathbf{p'}_1,\mathbf{p'}_2,\lambda_1',\lambda_2)\ \bar{u}_Q(\mathbf{p'}_1,\lambda_1')\Gamma u_b(\mathbf{p}_1,\lambda_1)\ \Psi_B^{0,0}(\mathbf{p}_1,\mathbf{p}_2,\lambda_1,\lambda_2)}{x}$$
$$= \int dx\, d^2\mathbf{k}_\perp \frac{\phi_V^*(x,\mathbf{k}'_\perp)\phi_B(x,\mathbf{k}_\perp)}{x}\ \mathrm{Tr}\left[U_V^{\dagger 1,J_3}(x,\mathbf{k}'_\perp)\ U_\Gamma\ U_B^{0,0}(x,\mathbf{k}_\perp)\right], \tag{13}$$

where U_Γ is defined by

$$\bar{u}_Q(\mathbf{p_1'},\lambda_1')\ \Gamma\ u_b(\mathbf{p_1},\lambda_1) = \chi_{\lambda_1'}^\dagger\ U_\Gamma\ \chi_{\lambda_1} \tag{14}$$

and

$$p_{1,2+} = x_{1,2}\ P_{B+}\ ,\ \mathbf{p_{1\perp}} = x_1\ \mathbf{P_{B\perp}} + \mathbf{k}_\perp\ ,\ \mathbf{p_{2\perp}} = x_2\ \mathbf{P_{B\perp}} - \mathbf{k}_\perp\ ,$$
$$p'_{1,2+} = x_{1,2}\ P_{V+}\ ,\ \mathbf{p'_{1\perp}} = x_1\ \mathbf{P_{V\perp}} + \mathbf{k}'_\perp\ ,\ \mathbf{p'_{2\perp}} = x_2\ \mathbf{P_{V\perp}} - \mathbf{k}'_\perp\ . \tag{15}$$

Since we use the spectator quark model, $\mathbf{p_2'} = \mathbf{p_2}$, and $\mathbf{k}_\perp$ and $\mathbf{k}'_\perp$ have the relation

$$\mathbf{k}_\perp - \mathbf{k}'_\perp = (1-x)(\mathbf{P_B} - \mathbf{P_V})_\perp = (1-x)\mathbf{q}_\perp\ . \tag{16}$$

For Eq. (13) we need to choose $q_+ = (p_B - p_V)_+ = 0$. Thus $q^2 = -\mathbf{q}_\perp^2 \leq 0$. Also, Eq. (13) is in fact expected to be valid only for good currents such as $\Gamma = \gamma_+, \gamma_+\gamma_5, \ldots$. There are contributions other than the one given in Eq. (13) if $q_+ \neq 0$ and the current is not a good current [9]. For $\Gamma = \gamma_+$ and $\gamma_+\gamma_5$, U_Γ in (14) is

$$U_{\gamma_+} = 2\ x\ P_{B+}\ ,\ \ U_{\gamma_+\gamma_5} = 2\ x\ P_{B+}\ \sigma_3\ . \tag{17}$$

We define the form factors of the $B(b\bar{q}) \to P(Q\bar{q})$ transition between two pseudo–scalar mesons by

$$\langle P(p_P)|\ \bar{Q}\gamma_\mu b\ |B(p_B)\rangle = F_1(q^2)\left(p_B + p_P - \frac{(m_B^2 - m_P^2)}{q^2}q\right)_\mu + F_0(q^2)\frac{(m_B^2 - m_P^2)}{q^2}q_\mu, \tag{18}$$

where $q = p_B - p_P$, and one has $F_0(0) = F_1(0)$. For the B meson transition to a vector meson $V(Q\bar{q})$ we define

$$\langle V(p_V,\epsilon)|\bar{Q}i\sigma_{\mu\nu}q^\nu b_R|B(p_B)\rangle = f_1(q^2)i\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}p_B^\lambda p_V^\sigma + \left[(m_B^2 - m_V^2)\epsilon_\mu^* - (\epsilon^*\cdot q)(p_B + p_V)_\mu\right] f_2(q^2)$$
$$+(\epsilon^*\cdot q)\left[q_\mu - \frac{q^2\ (p_B+p_V)_\mu}{m_B^2 - m_V^2}\right] f_3(q^2) \tag{19}$$

and

$$\langle V(p_V,\epsilon)|\ \bar{Q}\gamma_\mu(1-\gamma_5)b\ |B(p_B)\rangle = \frac{2V(q^2)}{m_B+m_V}i\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p_V^\alpha p_B^\beta - 2m_V\frac{(\epsilon^*\cdot p_B)}{q^2}q^\mu A_0(q^2)$$
$$-\left[(m_B+m_V)\epsilon^{*\mu}A_1(q^2) - \frac{(\epsilon^*\cdot p_B)}{m_B+m_V}(p_B+p_V)^\mu A_2(q^2) - 2m_V\frac{(\epsilon^*\cdot p_B)}{q^2}q^\mu A_3(q^2)\right]. \tag{20}$$

Note that the form factor $f_3(q^2)$ does not contribute to the decay $B \to K^*\gamma$. In Eqs. (19) and (20) ϵ is the polarization vector of the vector meson, and

$$A_3(q^2) = \frac{m_B+m_V}{2m_V}A_1(q^2) - \frac{m_B - m_V}{2m_V}A_2(q^2)\ ,\ A_3(0) = A_0(0)\ . \tag{21}$$

Using the good current $\Gamma = \gamma_+$ and Eq. (13), we can obtain the following expression for the form factor $F_1(0)$:

$$F_1(0) = \int dx\ d^2\mathbf{k}_\perp \phi_P^*(x,\mathbf{k}_\perp)\phi_B(x,\mathbf{k}_\perp)\frac{(\mathcal{A}_Q\mathcal{A}_b + \mathbf{k}_\perp^2)}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}}\ , \tag{22}$$

where

$$\mathcal{A}_Q = x\ m_{\bar{q}} + (1-x)m_Q\ ,\ \mathcal{A}_b = x\ m_{\bar{q}} + (1-x)m_b\ . \tag{23}$$

The form factor $F_1(0)$ can be rewritten using the internal momentum $\mathbf{k} = (k_z, \mathbf{k}_\perp)$ defined through Eq. (4) instead of the variables $(x, \mathbf{k}_\perp)$ as

$$F_1(0) = \int d^3\mathbf{k}\eta_P^*(\mathbf{k}')\eta_B(\mathbf{k})\sqrt{\frac{e_Q e_2' M_0^b}{e_b e_2 M_0^Q}}\frac{(\mathcal{A}_Q\mathcal{A}_b + \mathbf{k}_\perp^2)}{\sqrt{(A_Q^2+\mathbf{k}_\perp^2)(A_b^2+\mathbf{k}_\perp^2)}}\ , \tag{24}$$

where the momentum $\mathbf{k}$ in the B meson and $\mathbf{k}'$ in the P meson have the relation

$$\mathbf{k}_\perp = \mathbf{k}'_\perp \;,\quad \frac{e_b + k_z}{e_b + e_2} = x = \frac{e_Q + k'_z}{e_Q + e'_2} \;, \tag{25}$$

and for each meson, respectively,

$$\phi(x, \mathbf{k}_\perp) = \sqrt{\frac{d\mathbf{k}_z}{dx}}\,\eta(\mathbf{k}) \;,\quad \frac{d\mathbf{k}_z}{dx} = \frac{e_1 e_2}{x_1 x_2 M_0} \;. \tag{26}$$

In (24) M_0^b and M_0^Q represent, respectively, the quantity M_0 of (3) for mesons B and P. The formulas for the other form factors can be obtained similarly using good currents $\Gamma = \gamma_+\gamma_5, \gamma_+, \ldots$ and Eq. (13):

$$A_0(0) = \int d^3\mathbf{k}\; \eta_V^*(\mathbf{k}')\eta_B(\mathbf{k})\; \sqrt{\frac{e_Q e'_2 M_0^b}{e_b e_2 M_0^Q}}\, \frac{\mathcal{A}_Q\mathcal{A}_b + (2x-1)\mathbf{k}_\perp^2 + \frac{2(m_b+m_Q)(1-x)\mathbf{k}_\perp^2}{W_V}}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}} \;, \tag{27}$$

$$f_1(0) = \int d^3\mathbf{k}\; \eta_V^*(\mathbf{k}')\eta_B(\mathbf{k})\; \sqrt{\frac{e_Q e'_2 M_0^b}{e_b e_2 M_0^Q}}\, \frac{\mathcal{A}_Q\mathcal{A}_b + x\;\mathbf{k}_\perp^2 + \frac{(m_b+m_Q)(1-x)\mathbf{k}_\perp^2}{W_V}}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}} \;, \tag{28}$$

$$V(0) = \int d^3\mathbf{k}\; \eta_V^*(\mathbf{k}')\eta_B(\mathbf{k})\; \sqrt{\frac{e_Q e'_2 M_0^b}{e_b e_2 M_0^Q}}\, \frac{(m_B+m_V)(1-x)\left(\mathcal{A}_b + \frac{\mathbf{k}_\perp^2}{W_V} + (1-x)(m_b - m_Q)\mathbf{k}_\perp^2\Theta_V\right)}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}} \;, \tag{29}$$

$$A_2(0) = -\int d^3\mathbf{k}\; \eta_V^*(\mathbf{k}')\eta_B(\mathbf{k})\; \sqrt{\frac{e_Q e'_2 M_0^b}{e_b e_2 M_0^Q}}\;(m_B+m_V)(1-x) \tag{30}$$

$$\times \frac{(1-2x)\mathcal{A}_b + x_2[m_Q + (1-2x)m_b + 2xm_2]\Theta_V\mathbf{k}_\perp^2 + \frac{2\left[(\mathcal{B}_Q\mathcal{A}_b+\mathbf{k}_\perp^2)(1+\Theta_V\mathbf{k}_\perp^2)+\frac{1}{2}\mathbf{k}_\perp^2\right]}{W_V}}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}},$$

where

$$W_V = M_0^Q + m_Q + m_2 \;,\; \mathcal{B}_Q = x\; m_2 - (1-x)m_Q \;,$$
$$\Theta_V = \left(\frac{d\tilde{\phi}_V}{d\mathbf{k}_\perp^2}\right)/\tilde{\phi}_V \;,\; \tilde{\phi}_V = \frac{\phi_V}{\sqrt{\mathcal{A}_Q^2 + \mathbf{k}_\perp^2}} \;. \tag{31}$$

Note that one can get $A_1(0)$ from $A_0(0)$ and $A_2(0)$ using Eq. (21) and $f_1(0) = 2f_2(0)$. In [9], similar formulas have also been given.

III. THE WAVE FUNCTIONS

The meson wave function $\phi(x, \mathbf{k}_\perp)$ is model dependent and difficult to obtain; often simple forms are assumed for them. One reasonable assumption is a Gaussian-type wave function

$$\phi(x,\mathbf{k}_\perp) = \eta(\mathbf{k})\sqrt{\frac{d\mathbf{k}_z}{dx}}$$

$$\eta(\mathbf{k}) = \frac{1}{(\pi\omega^2)^{3/4}}\exp\left(-\frac{\mathbf{k}^2}{2\omega^2}\right) \;. \tag{32}$$

The parameter ω is a scale parameter and should be of the order of Λ_{QCD}. This wave function has been used in many previous applications of the light-front quark model [8, 9]. The results are generally quite successful.

A similar wave function is

$$\phi(x,\mathbf{k}_\perp) = \eta(\mathbf{k})\sqrt{\frac{d\mathbf{k}_z}{dx}} \;,\quad \eta(\mathbf{k}) = N\,\exp\left(-\frac{M_0^2}{2\omega^2}\right) \;. \tag{33}$$

Here N is the normalization constant. Equation (33), in fact, differs from Eq. (32) only when the two quarks of the meson have different masses. If they have equal mass, i.e., $m_1 = m_2 = m$, as is the case for the π and ρ, $M_0^2 = (e_1 + e_2)^2 = 4\,e_1^2 = 4\,e_2^2 = 4\,(m^2 + \mathbf{k}^2)$. The two wave functions are equivalent, since they differ only by a constant factor. The wave function (33) has been also applied for heavy mesons for which the two quarks certainly have different masses [16, 17].

Another possibility is the wave function adopted in [18]:

$$\phi(x,\mathbf{k}_\perp) = N\sqrt{x(1-x)}$$
$$\times\exp\left(-\frac{M^2}{2w^2}\left[x - \frac{1}{2} - \frac{m_Q^2 - m_{\bar{q}}^2}{2M^2}\right]^2\right)$$
$$\times\frac{\exp\left(-\frac{\mathbf{k}_\perp^2}{2w^2}\right)}{\sqrt{\pi w^2}} \;, \tag{34}$$

where M is the mass of the meson. We normalize the wave function to 1:

$$1 = \int dx\; d^2\mathbf{k}_\perp|\phi(x,\mathbf{k}_\perp)|^2 = \int d^3\mathbf{k}|\eta(\mathbf{k})|^2. \tag{35}$$

The wave function $\phi(x,\mathbf{k}_\perp)$ should also satisfy [9]

$$f_M = 2\sqrt{\frac{3}{(2\pi)^3}}\int dx d^2\mathbf{k}_\perp\ \phi(x,\mathbf{k}_\perp)\ \frac{\mathcal{A}_Q}{\sqrt{\mathcal{A}_Q^2+\mathbf{k}_\perp^2}} = 2\sqrt{\frac{3}{(2\pi)^3}}\int d^3(\mathbf{k})\ \eta(\mathbf{k})\ \sqrt{\frac{x_1x_2M_0}{e_1e_2}}\frac{\mathcal{A}_Q}{\sqrt{\mathcal{A}_Q^2+\mathbf{k}_\perp^2}}\ , \tag{36}$$

where f_M is the meson decay constant. When the decay constant is known, this condition is usually one of several ways to determine the values of parameters in the wave function. For heavy mesons such as the B meson it imposes a constraint on the wave function because the decay constant has the scaling behavior [1]

$$f_B \propto \frac{1}{\sqrt{m_b}}\ ,\quad m_b\to\infty\ . \tag{37}$$

We have not included perturbative corrections in (37).

We now consider the general behavior of the heavy-meson wave function $\phi(x,\mathbf{k}_\perp)$ in the heavy-quark limit $m_b\to\infty$. Here we take the B meson as an example. The distribution amplitude $\int d^2\mathbf{k}_\perp\phi(x,\mathbf{k}_\perp)$ of a heavy meson is known to have a peak near $x\simeq 1$. If we denote the x coordinate of the peak in $\phi(x,\mathbf{k}_\perp)$ by x_0 and the width of the peak in x by Δ_x, then $x_0\to 1$ and $\Delta_x\to 0$ as $m_b\to\infty$. Thus the wave function behaves as a δ function in x. On the other hand, $\phi(x,\mathbf{k}_\perp)$ vanishes if $\mathbf{k}_\perp^2 \gg \Lambda_{\rm QCD}^2$. To see this in some detail, consider Eq. (4). If we use $(k_z,\mathbf{k}_\perp)$ as the coordinates, then we expect that $|\mathbf{k}|$ should, in general, be of the order of $\Lambda_{\rm QCD}$. Thus from Eq. (4), it is easy to see that the x coordinate of the peak of $\phi(x,\mathbf{k}_\perp)$ behaves as

$$x = 1-\frac{\tilde\Lambda}{m_b}+\cdots\ , \tag{38}$$

where $\tilde\Lambda$ is a function of $\mathbf{k}_\perp^2$, but it is of order of $\Lambda_{\rm QCD}$.

All three wave functions listed before have the above general feature. However, as we show in the Appendix, the wave functions (32) and (34) satisfy the scaling law (37), but (33) does not. The wave function (34) is obtained assuming factorization with respect to the spin and orbital motion [12, 19], which is not the case in the light-front relativistic quark model. Thus we will only use the wave function (32) in the following sections.

IV. THE ISGUR-WISE FUNCTION OF $B\to D^{(*)}$ DECAYS

With the formalism and the wave function given in the preceding sections, it is now straightforward to calculate the form factors of the $B\to D^{(*)}$ semileptonic decays. We study the behavior of these form factors in the limit $m_b\to\infty$ and $m_c\to\infty$.

We use the wave function (32) and define $r=\frac{m_c}{m_b}$. Also, we take $m_B=m_b$, $m_D=m_c$, and set the scale parameters for the heavy mesons equal, $\omega_H\equiv\omega_B=\omega_D$. (The same ω_D holds for both D and D^* mesons.) In the integrals (24) and (27)–(30), $|\mathbf{k}|$ for the B meson is of the order of the scale parameter ω_H, and k'_z for the $D^{(*)}$ is given by

$$k'_z = \frac{1}{2}\left(r(k_z-\sqrt{m_2^2+\mathbf{k}_\perp^2+k_z^2}\ ) + \frac{(k_z+\sqrt{m_2^2+\mathbf{k}_\perp^2+k_z^2}\ )}{r}\right)+O\left(\frac{1}{m_b}\right)\ . \tag{39}$$

(Note that $\mathbf{k}'_\perp=\mathbf{k}_\perp$.) In terms of the variable x the integrand in these integrals peaks at

$$x = 1-\frac{\sqrt{m_2^2+\mathbf{k}_\perp^2+k_z^2}-k_z}{m_b}+\cdots\ , \tag{40}$$

and M_0^c and M_0^b in these integrals become

$$M_0^b\to m_b\ ,\quad M_0^c\to m_c = r\ m_b\ ,\qquad e_b\to m_b, e_c\to m_c = r\ m_b. \tag{41}$$

In (27)–(30) all terms proportional to $\frac{1}{W_V}$ can be neglected in the heavy-quark limit. Thus,

$$F_1^{BD}(0) = A_0^{BD^*}(0) = f_1^{BD^*}(0) = \int d^3\mathbf{k}\eta_D^*(\mathbf{k}')\eta_B(\mathbf{k})\sqrt{\frac{e'_2}{e_2}}\frac{(\mathcal{A}_c\mathcal{A}_b+\mathbf{k}_\perp^2)}{\sqrt{(\mathcal{A}_c^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}}\ , \tag{42}$$

$$A_2^{BD^*}(0) = V^{BD^*}(0) = \int d^3\mathbf{k}\ \eta_D^*(\mathbf{k}')\eta_B(\mathbf{k})\ \sqrt{\frac{e'_2}{e_2}}\frac{(m_B+m_{D^*})(1-x)(\mathcal{A}_b+m_b(1-r)(1-x)\mathbf{k}_\perp^2\Theta_{D^*})}{\sqrt{(\mathcal{A}_c^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}}\ . \tag{43}$$

We are left with two sets of form factors. Now we need to show that these two sets of form factors are equal, as required by the heavy-quark symmetry. For the special mass ratio $r=1$, we can show analytically that all the above form factors equal 1. However, it is not easy to show that the form factors in (42) and (43) are equal for an arbitrary ratio r. Nevertheless we can use numerical calculation to show that these form factors are indeed equal [20]. Thus there is only one independent form factor in the model, as required by the heavy-quark symmetry. This form factor does not depend on the heavy-quark masses m_b and m_c but their ratio r:

$$h = \tilde h(r,m_2,\omega_H) = F_1^{BD}(0) = A_0^{BD^*}(0) = A_1^{BD^*}(0) = A_2^{BD^*}(0) = V^{BD^*}(0) = f_1^{BD^*}(0) = \int d^3\mathbf{k}\eta_D^*(\mathbf{k}')\eta_B(\mathbf{k})\sqrt{\frac{e'_2}{e_2}}\ \frac{(\mathcal{A}_c\mathcal{A}_b+\mathbf{k}_\perp^2)}{\sqrt{(\mathcal{A}_c^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}}\ . \tag{44}$$

In [12, 19] it is argued that the knowledge of the form factor h at $q^2 = 0$ suffices to determine the Isgur-Wise function in the whole kinematic region. The basic idea is as follows. The Isgur-Wise function depends only on the velocity product $v \cdot v'$:

$$v \cdot v' = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D} \quad . \tag{45}$$

We define

$$y = v \cdot v'(q^2 = 0) = \frac{r^2+1}{2r} \; . \tag{46}$$

Keeping $q^2 = 0$ fixed, y changes as the mass ratio r changes in the interval $[1, \infty]$, which can cover the whole kinematic region in weak decay. Writing h in terms of y,

$$h = h(y, m_2, \omega_H) \; , \tag{47}$$

shows the Isgur-Wise function to be

$$\xi(y) = \sqrt{\frac{2}{y+1}}\; h(y, m_2, \omega_H) \; . \tag{48}$$

Hence we obtain the Isgur-Wise function $\xi(y)$ in the whole kinematic region, even though in the light-front quark model we can only calculate the form factors at $q^2 = 0$.

It is easy to see that the Isgur-Wise function obtained this way satisfies $\xi(1) = 1$, since when $r = 1$ then $h = 1$ and $y = 1$. Also, the Isgur-Wise function satisfies Bjorken's constraint on the derivative $\rho^2 = -\xi'(1) = 1/4 - h'(1, m_2, \omega_H) > 1/4$ because the overlap of normalized wave functions cannot be larger than one, i.e., $h'(1, m_2, \omega_H) < 0$ [12, 19].

The final Isgur-Wise function is calculated numerically. We use the light quark masses obtained by fitting f_π and f_ρ [9]: $m_2 = 0.25$ GeV. For the heavy-quark masses, $f_D \simeq 200$ MeV for $m_c = 1.6$ GeV and $\omega_D = 0.45$ GeV. Similarly, $f_B \simeq 190$ MeV for $m_b = 4.8$ GeV and $\omega_B = 0.55$ GeV. These values of f_B and f_D lie in the ranges of recent lattice results [21, 22]. We use the above masses for b and c quarks and calculate the Isgur-Wise function in the range $1 \le y \le \frac{r_0^2+1}{2r_0} \simeq 1.67$, where $r_0 = \frac{m_c}{m_b} = 0.33$. Since we work in the heavy-quark limit, we need to set $\omega_H = \omega_D = \omega_B$. To see the dependence of the Isgur-Wise function on the parameter ω_H, we use two values for ω_H: $\omega_H = 0.55$ and 0.50 GeV. We also give the result corresponding to $m_2 = 0.30$ GeV. In Fig. 1 we show the Isgur-Wise function $\xi(y)$ for the $B \to D^{(*)}$ semileptonic decays. For comparison we also show the functions $[2/(y+1)]^m$ $(m = 1, 2)$, which correspond to the single- and double-pole-like form factors in the heavy-quark limit. Our result for the slope of the Isgur-Wise function is

$$\begin{aligned} \omega_H = 0.55\ \text{GeV},\quad & \rho^2 = -\xi'(1) = 1.23, \quad m_2 = 0.25\ \text{GeV}, \\ & \rho^2 = -\xi'(1) = 1.27, \quad m_2 = 0.30\ \text{GeV}, \\ \omega_H = 0.50\ \text{GeV},\quad & \rho^2 = -\xi'(1) = 1.25, \quad m_2 = 0.25\ \text{GeV}, \\ & \rho^2 = -\xi'(1) = 1.29, \quad m_2 = 0.30\text{GeV}. \end{aligned} \tag{49}$$

One can see that for the same ω_H a larger spectator quark

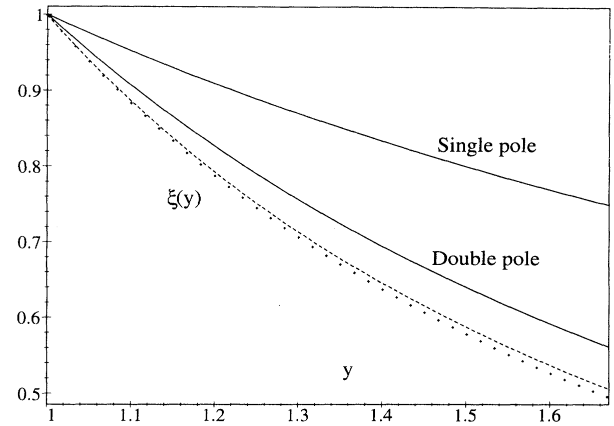

FIG. 1. The Isgur-Wise function $\xi(y)$ for the $B \to D^{(*)}$ semileptonic decays. The upper dashed curve corresponds to $\omega_H = 0.55$ GeV and $m_2 = 0.250$ GeV; the lower dashed curve corresponds to $\omega_H = 0.55$ GeV and $m_2 = 0.300$ GeV. The upper solid curve corresponds to $\omega_H = 0.50$ GeV and $m_2 = 0.250$ GeV. The lower solid curve corresponds to $\omega_H = 0.50$ GeV and $m_2 = 0.300$ GeV.

mass (m_2) gives a larger slope. The slope, in general, however, is not very sensitive to either ω_H or m_2.

For comparison, we list in Table I a number of recent calculations of ρ^2. The lattice calculations [3] and most of the relativistic quark models [2] tend to give larger values ($\rho^2 \gtrsim 1$). The values from the QCD sum-rule calculations [4] are generally somewhat smaller. The nonrelativistic quark model [5] gives $\rho^2 \simeq 0.67$ (with the large recoil effect used to fit the π electromagnetic charge radius) but a relativistic modification of this model carried out by Close and Wambach gives $\rho^2 \simeq 1.19$ [2].

TABLE I. The slope ρ^2 of the Isgur-Wise function for $B \to D^{(*)} e \bar{\nu}_e$ decaying at zero recoil.

	ρ^2 :
This work	for $\omega_H = 0.55$ GeV 1.23 ($m_2 = 0.25$ GeV) 1.27 ($m_2 = 0.30$ GeV) for $\omega_H = 0.50$ GeV 1.25 ($m_2 = 0.25$ GeV) 1.29 ($m_2 = 0.30$ GeV)
Ahmady *et al.* [2]	0.54–1.5
Bernard *et al.* [3]	$1.41 \pm 0.19 \pm 0.41$
Blok and Shifman [4]	0.5–0.8
Close and Wambach [2]	1.19 ± 0.02
El-Hady *et al.* [2]	1.28
Holdom and Sutherland [2]	1.24–1.36
Huang and Luo [4]	1.01 ± 0.02
Isgur *et al.* [5]	0.64
Ivanov and Mizutani [2]	0.42–0.82
Jin *et al.* [2]	0.97
Kiselev [2]	1.25
Kugo *et al.* [2] 37	1.8–2.0
Mannel *et al.* [6]	1.77 ± 0.74

V. FORM FACTORS OF HEAVY-TO-LIGHT DECAYS IN THE HEAVY-QUARK LIMIT

In this section we study the form factors of heavy-to-light transitions $B \to P(Q\bar{q})$ and $B \to V(Q\bar{q})$, where $m_b \to \infty$ as before, and the quark Q has a smaller finite mass but Q is a light quark. Consider the form factors of Eqs. (24) and (27)–(30). Since the wave function of the B meson peaks near $x \simeq 1$, i.e., $x \to 1$ when $m_b \to \infty$, the *integrands* usually also have a peak. The x coordinate of this peak takes, in general, the form

$$x = 1 - O\left(\frac{1}{m_b^n}\right) \quad , \quad n > 0 \; , \tag{50}$$

where n depends on the specific form of the wave functions. (With the wave function we used the integrand for the heavy–to–heavy decays has $n = 1$.) Because of Eq. (50) the quantity,

$$M_0^Q \propto m_b^{\frac{n}{2}} \to \infty \; . \tag{51}$$

Therefore, regardless of the specific form of the wave functions, terms in the expressions for the form factors proportional to $\frac{1}{W_V}$ can be ignored when $m_b \to \infty$. This is similar to what we saw in the last section for the heavy-to-heavy transitions. For heavy-to-light transitions the form factors in (27)–(30) now reduce to

$$F_1(0) = \int d^3\mathbf{k}\eta_P^*(\mathbf{k}')\eta_B(\mathbf{k})\sqrt{\frac{e_Q e_2' M_0^b}{e_b e_2 M_0^Q}}\frac{(\mathcal{A}_Q\mathcal{A}_b + \mathbf{k}_\perp^2)}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}} \; , \tag{52}$$

$$A_0(0) = f_1(0) = \int d^3\mathbf{k}\eta_V^*(\mathbf{k}')\eta_B(\mathbf{k})\sqrt{\frac{e_Q e_2' M_0^b}{e_b e_2 M_0^Q}}\frac{(\mathcal{A}_Q\mathcal{A}_b + \mathbf{k}_\perp^2)}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}} \; , \tag{53}$$

$$A_2(0) = V(0) = \int d^3\mathbf{k}\; \eta_V^*(\mathbf{k}')\eta_B(\mathbf{k})\; \sqrt{\frac{e_Q e_2' M_0^b}{e_b e_2 M_0^Q}}\frac{(m_B+m_V)(1-x)[\mathcal{A}_b + (1-x)(m_b - m_Q)\mathbf{k}_\perp^2\Theta_V]}{\sqrt{(\mathcal{A}_Q^2+\mathbf{k}_\perp^2)(\mathcal{A}_b^2+\mathbf{k}_\perp^2)}} \; , \tag{54}$$

in the limit $m_b \to \infty$. Thus in the light-front quark model we obtain two independent form factors for $B \to V$ transitions regardless of the specific form of the wave function. Because of Eq. (53) we found [23, 10, 24] that the ratio $\mathcal{I}$, which relates the decay rate of $B \to K^*\gamma$ and that of $B \to \rho e\bar{\nu}_e$ at $q^2 = 0$ is 1 in the SU(3) flavor symmetry limit.

In general there should be four independent form factors for the heavy-to-light $B \to V$ transition and, in particular, the four form factors $A_1(q^2)$, $A_2(q^2)$, $V(q^2)$ and $f_1(q^2)$ are independent of each other [25]. Here, in the light-front relativistic quark model we obtain only two independent form factors due to the vanishing of terms proportional to $\frac{1}{W_V}$. This can be traced back to the treatment of the quark spins (7)–(10), which corresponds to a weak-binding limit [26]. This is a known approximation in the light-front quark model [27]. In the matrix element for the heavy-to-light transition $B \to V$ the integrand has contributions only from $x_1 = x \to 1$ as $m_b \to \infty$. Thus the Melosh rotation (9) for the transition quarks b and Q becomes

$$R_M(\mathbf{k}_\perp, m_b) \to 1 \; , \quad R_M(\mathbf{k}_\perp, m_Q) \to 1 \; , \tag{55}$$

even though the quark Q is light. Thus the Melosh rotation affects only the spectator quarks.

We now use the wave function (32) to obtain the form factors, the details of which are given in the Appendix. The final expression for the form factors in the $m_b \to \infty$ limit is

$$\begin{aligned} f_1(0) &= V(0) = A_0(0) = A_2(0)(= A_1(0)) \\ &= \frac{4 \times 2^{11/12}}{\sqrt{3}}\, r_V^{-7/12}\left(\frac{m_2}{m_b}\right)^{2/3} \exp\left[\frac{-3\,(2r_V)^{1/3} m_2^{4/3} m_b^{2/3} + (3 + 4\, r_V)\, m_2^2 + 2\, m_Q^2}{16\, \omega_V^2}\right], \end{aligned} \tag{56}$$

where $r_V = \omega_V^2/\omega_B^2$ with ω_V and ω_B being the meson scale parameters in wave function (32). The replacement of ω_V by ω_P and r_V by $r_P = \omega_P^2/\omega_B^2$ gives $F_1(0)$ ($= F_0(0)$). It is interesting to note that though there are, in general, two sets of form factors for $B \to V$ transition, these two sets of form factors become equal when we use the wave function (32). We attribute this equality to the specific form of the wave function (32). We see no reason that this is generally true for arbitrary wave functions. Obviously, all the form factors have the following dependence on m_b:

$$\frac{\exp(-m_b^{2/3} a)}{m_b^{2/3}} \; , \tag{57}$$

where $a = 3(2r_L)^{1/3} m_2^{4/3}/16\omega_L^2$ ($L = V$ or P).

The dependence of the heavy-to-light form factors on m_b is interesting because it allows us to compare it with the prediction from the pole-dominance assumption for the heavy-to-light form factors. For example, Burdman and Donoghue [28] have pointed out the inconsistency between the scaling behavior of the heavy-to-light form factor at $q^2_{\max}$ and the single-pole-like form factors used in the Bauer-Stech-Wirbel (BSW) model [18]. One knows [7]

$$\begin{aligned} &F_0(q^2_{\max}) \propto m_b^{-1/2}\ ,\ A_1(q^2_{\max}) \propto m_b^{-1/2},\\ &F_1(q^2_{\max}) \propto m_b^{1/2}\ , V(q^2_{\max}) \propto m_b^{1/2},\\ &A_2(q^2_{\max}) \propto m_b^{1/2}, \end{aligned} \tag{58}$$

but all form factors in the BSW model behave as $\propto m_b^{-1}$ at $q^2 = 0$ when $m_b \to \infty$. Thus in BSW model the single-pole-like q^2 dependence certainly cannot produce the scaling law (58).

The light-front quark model is a relativistic quark model, which contains many important ingredients not included in the BSW model. Obviously, a single-pole-like q^2 dependence combined with (58) cannot produce our results. In our opinion, this is an indication that a single-pole-like q^2 dependence may not be correct, at least in region far away from the zero-recoil point $q^2_{\max}$, such as $q^2 = 0$. In fact, the pole-dominance assumption is generally expected to be correct only near the zero-recoil region. The actual q^2 dependence of the form factors far away from $q^2_{\max}$ could be much more complicated. When compared with (58), our result indicates that the set $F_1(q^2)$, $V(q^2)$, and $A_2(q^2)$ may have a similar type of q^2 dependence, while the set $F_0(q^2)$ and $A_1(q^2)$ also has a similar q^2 dependence but one that is different from the first set. For example, $F_1(q^2)$ may increase as q^2 faster than $F_0(q^2)$. Similarly, $V(q^2)$ and $A_2(q^2)$ may increase faster than $A_1(q^2)$.

The form factors in the heavy-to-light $B \to P$ and $B \to V$ semileptonic decays and their dependence on the b quark mass in the $m_b \to \infty$ limit have been studied by many other people [22, 29–34]. In particular, in [31, 32] the light-cone QCD sum rule gives $F_1(0) \propto m_b^{-3/2}$, which is not in agreement with a single-pole extrapolation. However, in [30] a QCD sum-rule calculation gives $F_1(0) \propto m_b^{-1/2}$, consistent with a single-pole assumption for the whole kinematic region. Though there are differences in all these studies, it seems that most people do agree that form factors $V(q^2)$ and $A_2(q^2)$ may increase faster than the form factor $A_1(q^2)$ [30, 33, 34] as q^2 increases.

Finally, we give the numerical results of the form factors at $q^2 = 0$ for the transitions $B \to \pi$, $B \to \rho$, $B \to K$, and $B \to K^*$:

$$\begin{aligned} &F_1^{B\to\pi}(0) = 0.26,\quad f_1^{B\to\rho}(0) = 0.28\ ,\quad V^{B\to\rho}(0) = 0.32,\\ &A_0^{B\to\rho}(0) = 0.30,\quad A_1^{B\to\rho}(0) = 0.21\ ,\quad A_2^{B\to\rho}(0) = 0.18;\\[1ex] &F_1^{B\to K}(0) = 0.34,\quad f_1^{B\to K^*}(0) = 0.37\ ,\\ &V^{B\to K^*}(0) = 0.42,\\[1ex] &A_0^{B\to K^*}(0) = 0.40\ ,\\ &A_1^{B\to K^*}(0) = 0.29\ ,\quad A_2^{B\to K^*}(0) = 0.24\ . \end{aligned} \tag{59}$$

For the B meson we have used $m_b = 4.8$ GeV, $\omega_B = 0.55$ GeV; for π, ρ, K, and K^*, the parameters are taken from [9]: $m_u = m_d = 0.25$ GeV, $\omega_\pi = \omega_\rho = 0.32$ GeV, $m_s = 0.37$ GeV, $\omega_K = \omega_{K^*} = 0.39$ GeV. The heavy-to-light form factors are most sensitive to the transition quark mass ratio of m_Q to m_b. Obviously, if $m_Q/m_b \to 0$, these form factors vanish. Even so, changing the mass of m_Q from 0.37 to 0.50 GeV gives the following small changes for the $B \to K$ and $B \to K^*$ form factors:

$$\begin{aligned} &F_1^{B\to K}(0) = 0.38\ ,\quad f_1^{B\to K^*}(0) = 0.40\ ,\\ &V^{B\to K^*}(0) = 0.45\ ,\\ &A_0^{B\to K^*}(0) = 0.43\ ,\\ &A_1^{B\to K^*}(0) = 0.33\ ,\quad A_2^{B\to K^*}(0) = 0.28\ . \end{aligned} \tag{60}$$

VI. CONCLUSION

In this paper we have studied the form factors of the heavy-to-heavy and heavy-to-light weak transitions in the light-front relativistic quark model. For the heavy-to-heavy $B \to D^{(*)}$ transitions we have shown that the form factors satisfy the heavy-quark symmetry relations. We have calculated the corresponding Isgur-Wise function. The slope of the Isgur-Wise function agrees with most other calculations. We have also studied the heavy-to-light $B \to P$ and $B \to V$ transitions. For the transition $B \to V$ the model produces at most two independent form factors. In general there are four independent form factors; this reduction comes from using the weak-binding limit and is independent of the choice of wave function. With a specific wave function, we have derived the dependence of the form factors (at $q^2 = 0$) on the b quark mass in the $m_b \to \infty$ limit. This dependence cannot be produced by extrapolating the scaling behavior of the form factors at $q^2_{\max}$ using the single-pole assumption. This shows that the q^2 dependence of the form factors in a regions far away from the zero-recoil could be much more complicated than that predicted by the single-pole assumption. When compared with the scaling behavior of the form factors at $q^2_{\max}$ our result suggests, for example, that $F_1(q^2)$ increases as q^2 faster than $F_0(q^2)$ and similarly, $V(q^2)$ and $A_2(q^2)$ increase faster than $A_1(q^2)$.

ACKNOWLEDGMENT

This work was in part supported by the Natural Sciences and Engineering Council of Canada.

APPENDIX

1. Wave functions and the scaling law (37)

Here we examine the wave functions (32)–(34) to see if they satisfy the scaling law (37). We first look at the wave function (32). For the B meson the wave function $\eta_B(\mathbf{k})$ in the integral (36) does not lead to any m_b dependence. Since, in the heavy quark limit,

$$x_1 = x \to 1 \ , \ M_{0B} \to m_b \ , \ e_1 \to m_b \tag{A1}$$

and

$$x_2 = 1 - x \propto m_b^{-1} \ , \ \ e_2 \propto m_b^0 \ \ , \ \ \mathcal{A}_b \propto m_b^0 \ , \tag{A2}$$

we can easily obtain the scaling law (37). Note that here $\propto m_b^0$ means independent of m_b.

Next, we look at the wave function (33). In fact we can show that wave functions of the following type for the B meson do not satisfy the scaling law (37):

$$\phi_B(x,t) = N_B \ g(x,t) \ \exp[-f(x,t)], \quad t = \mathbf{k}_\perp^2 \ , \tag{A3}$$

where

$$f(x,t) = \frac{(M_0^b)^2}{2\omega_B^2} \tag{A4}$$

and $g(x,t)$ is some polynomial, rational, or irrational function. We assume the function $\phi_B(x,t)$ peaks when $f(x,t)$ has a minimum. Suppose $f(x,t)$ is at its minimum when $x = x_0$ and $t = t_0$; then

$$x_0 = 1 - \frac{m_2}{m_b + m_2} \quad , \quad t_0 = 0 \ . \tag{A5}$$

We expand $f(x,t)$ around $x = x_0$ and $t = t_0$:

$$f(x,t) = f(x_0,t_0) + \frac{1}{2}\frac{d^2}{dx^2} f(x_0,t_0)(x-x_0)^2 + \frac{d}{dt} f(x_0,t_0) \ t + \cdots \tag{A6}$$

where

$$f(x_0,t_0) = \frac{(m_b+m_2)^2}{2\omega_B^2},$$
$$\frac{d^2}{dx^2} f(x_0,t_0) = \frac{(m_2+m_b)^4}{m_2 m_b \omega_B^2},$$
$$\frac{d}{dt} f(x_0,t_0) = \frac{(m_2+m_b)^2}{2m_2 m_b \omega_B^2}. \tag{A7}$$

We can show that higher-order terms in x can be neglected. (Obviously there is no higher-order term in t.) The derivatives $(d^2/dx^2)f(x_0,t_0)$ and $(d/dt)f(x_0,t_0)$ determine the width of the wave function $\phi(x,t)$ at (x_0,t_0). The wave function can be considered as zero when $(x - x_0)$ is of order larger than $m_b^{-3/2}$ and $t - t_0$ of order larger than m_b^{-1}. This means the width of the wave function (33) becomes zero in both x and t when $m_b \to \infty$.

Now we can determine N_B of (A3) from the normalization condition (35). We introduce δ and λ to remove the dependence of the integration variables on m_b:

$$x = x_0 + \left(\frac{m_2}{m_b}\right)^{3/2} \delta, \quad t = t_0 + \left(\frac{m_2}{m_b}\right) m_2^2 \ \lambda \ . \tag{A8}$$

With the new variables (δ,λ), the exponent $f(x,t)$ is now

$$f(x,t) = \frac{(m_2+m_b)^2}{2\omega_B^2} + \frac{m_2^2}{2\omega_B^2}(\delta^2+\lambda) + \cdots \ , \tag{A9}$$

and the wave function (A3) becomes

$$\phi_B(x,t) = g(x,t)\exp[-f(x,t)] \to m_b^j \ \bar{g}(\delta,\lambda) \ \exp\left[-\frac{(m_2+m_b)^2}{2\omega_B^2} - \frac{m_2^2}{2\omega_B^2}(\delta^2+\lambda)\right], \tag{A10}$$

where $\bar{g}(\delta,\lambda)$ has no dependence on m_b, and j is some number depending on the specific form of $g(x,t)$. For the wave function (33), which is an example of (A3),

$$\phi_B(x,t) = N_B\sqrt{\frac{d\mathbf{k}_z}{dx}} \ \exp[-f(x,t)] \to N_B \ m_b \ \exp\left[-\frac{(m_2+m_b)^2}{2\omega_B^2} - \frac{m_2^2}{2\omega_B^2}(\delta^2+\lambda)\right]. \tag{A11}$$

Note

$$x \to 1 \ , \ \ x_2 \to \frac{m_2}{m_b} \ , \ \ M_0^b \to m_b \ ,$$
$$e_b \to m_b \ , \quad e_2 \to m_2 \ . \tag{A12}$$

With the new variables (δ,λ) the normalization condition (35) becomes

$$1 = \int dx \ d^2\mathbf{k}_\perp |\phi_B(x,\mathbf{k}_\perp)|^2 = \pi \ m_2^2 \left(\frac{m_2}{m_b}\right)^{5/2} \int d\delta \ d\lambda |\phi_B(x,\mathbf{k}_\perp)|^2 \tag{A13}$$

and (36) becomes

$$f_B = 2\sqrt{\frac{3}{(2\pi)^3}} \int dx \ d^2\mathbf{k}_\perp \phi_B(x,\mathbf{k}_\perp) \frac{\mathcal{A}_b}{\sqrt{\mathcal{A}_b^2+\mathbf{k}_\perp^2}} = 2\sqrt{\frac{3}{(2\pi)^3}} \ \pi \ m_2^2 \left(\frac{m_2}{m_b}\right)^{5/2} \int d\delta \ d\lambda \ \phi_B(x,\mathbf{k}_\perp) \ . \tag{A14}$$

Note the factor $\mathcal{A}_b/\sqrt{\mathcal{A}_b^2+\mathbf{k}_\perp^2} \to 1$ in the $m_b \to \infty$ limit. In both (A13) and (A14) the factor $\pi \ m_2^2 \ (\frac{m_2}{m_b})^{5/2}$ comes from the integration-variable transformation. From Eqs. (A10) and (A13) and (A14) we get

$$f_B \propto m_b^{-5/4} \tag{A15}$$

for the wave function of type (A3). We can show that the wave function (34) satisfies the scaling law (37).

2. Heavy-to-light form factors in the heavy-quark limit

Now we use the wave function (32) to study the dependence of the heavy-to-light form factors on m_b. The integrands in (52)–(54) are all of the form $\tilde{g}(k_z,t)\exp[-\tilde{f}(k_z,t)]$ with

$$\tilde{f}(k_z,t) = \tilde{f}_B(k_z,t) + \tilde{f}_L(k_z,t), \tag{A16}$$

where

$$\tilde{f}_B(k_z,t) = \frac{\mathbf{k}^2}{2\omega_B^2} = \frac{r_L\,\mathbf{k}^2}{2\omega_L^2}\;, \quad \tilde{f}_L(k_z,t) = \frac{\mathbf{k}'^{\,2}}{2\omega_L^2}\;,$$

$$\mathbf{k}^2 = k_z^2 + t\;, \quad \mathbf{k}'^2 = k_z'^2 + t\;, \quad t = \mathbf{k}_\perp^2\;, \quad r_L = \frac{\omega_L^2}{\omega_B^2}\;, \tag{A17}$$

($L = P$ for the $B \to P$ transition and $L = V$ for the $B \to V$ transition, and $\tilde{g}(k_z,t)$ can be determined from (24) and (27)–(30). The internal momentum k_z' can be expressed in terms of k_z and t through (25).

The minimum point of $\tilde{f}(k_z,t)$ is where the integrand peaks. Suppose the coordinates of the minimum point of $\tilde{f}(k_z,t)$ are (k_{z0},t_0). One can then expand $\tilde{f}(k_z,t)$ near (k_{z0},t_0):

$$\tilde{f}(k_z,t) = \tilde{f}(k_{z0},t_0) + \frac{1}{2}\frac{d^2}{dk_z^2}\tilde{f}(k_{z0},t_0)(k_z-k_{z0})^2 + \frac{d}{dt}\tilde{f}(k_{z0},t_0)\;(t-t_0) + \cdots\;. \tag{A18}$$

One can show that higher-order terms in (A18) can be neglected. We find

$$t_0 = 0\;\;,\quad k_{z0} = -2^{-4/3} r_L^{-1/3} m_2^{2/3} m_b^{1/3}\;. \tag{A19}$$

The coefficients in the expansion (A18) are

$$\tilde{f}(k_{z0},t_0) = \left(\frac{1}{2\omega_L^2}\right)\left[\frac{3}{8}(2r_L)^{1/3} m_2^{4/3} m_b^{2/3} - \left(\frac{3+4r_L}{8}\right)m_2^2 - \frac{1}{4}m_Q^2\right], \tag{A20}$$

$$\frac{d^2}{dk_z^2}\tilde{f}(k_{z0},t_0) = \frac{3}{\omega_B^2}\;\;,\quad \frac{d}{dt}\tilde{f}(k_{z0},t_0) = \frac{1}{2^{8/3}\omega_L^{4/3}\omega_B^{2/3}}\left(\frac{m_b}{m_2}\right)^{2/3}. \tag{A21}$$

Thus the width at the peak is independent of m_b in k_z but is of order $m_b^{-2/3}$ ($\to 0$) in t. Hence, in the $m_b \to \infty$ limit, the exponential function $\exp[-\tilde{f}(k_z,t)]$ behaves as a δ function in t, and the contribution to the integrands in (52)–(54) comes only from $t_0 = 0$. In terms of the coordinate x the peak is at

$$x_0 = 1 - (2\,r_L)^{-1/3}\left(\frac{m_2}{m_b}\right)^{2/3} + \cdots\;,\quad x_{20} = 1 - x_0 = (2\;r_L)^{-1/3}\left(\frac{m_2}{m_b}\right)^{2/3} + \cdots\;. \tag{A22}$$

It is interesting to notice that though $x_0 \to 1$, as in the heavy-to-heavy decays, $x_{20} = 1 - x_0$ has a different dependence on m_b. It is not of order m_b^{-1} but $m_b^{-2/3}$. Again, in terms of x, one can show that the contribution to the integrand comes only from $x = x_0$.

Because of (A19) and (A22), the integrands in (52)–(54) become much simpler. To obtain an analytical expressions for integrals in (52)–(54) we again introduce a couple of new variables (ξ,λ) to get rid of the m_b dependence in (k_z,t):

$$k_z = k_{z0} + m_2\;\xi,\quad t = t_0 + \left(\frac{m_2}{m_b}\right)^{2/3} m_2^2\;\lambda\;. \tag{A23}$$

In terms of the new variables, $\tilde{f}(k_z,t)$ now becomes

$$\tilde{f}(k_z,t) = \tilde{f}(k_{z0},t_0) + \left(\frac{3\;r_L}{2}\frac{m_2^2}{\omega_L^2}\right)\delta^2 + \left(\frac{(2r_L)^{1/3}}{8}\frac{m_2^2}{\omega_L^2}\right)\lambda + \cdots\;. \tag{A24}$$

We substitute (A23) into the integrals (52)–(54) and keep the leading terms in the expansion in m_b. The integrals (52)–(54) then become

$$F_1(0) = \int d\delta\;d\lambda\;\exp\left[-\tilde{f}(k_{z0},t_0) - \left(\frac{3\;r_P}{2}\frac{m_2^2}{\omega_P^2}\right)\delta^2 - \left(\frac{(2r_P)^{1/3}}{8}\frac{m_2^2}{\omega_P^2}\right)\lambda\right]C,$$

$$A_0(0) = f_1(0) = \int d\delta\;d\lambda\;\exp\left[-\tilde{f}(k_{z0},t_0) - \left(\frac{3\;r_V}{2}\frac{m_2^2}{\omega_V^2}\right)\delta^2 - \left(\frac{(2r_V)^{1/3}}{8}\frac{m_2^2}{\omega_V^2}\right)\lambda\right]C,$$

$$A_2(0) = V(0) = \int d\delta\;d\lambda\;\exp\left[-\tilde{f}(k_{z0},t_0) - \left(\frac{3\;r_V}{2}\frac{m_2^2}{\omega_V^2}\right)\delta^2 - \left(\frac{(2r_V)^{1/3}}{8}\frac{m_2^2}{\omega_V^2}\right)\lambda\right]C$$
$$\times\left[\left(\frac{m_b}{m_2}\right)^{1/3}\frac{2^{8/3}\omega_V^2 - r_V^{1/3}m_2^2\lambda}{8r_V^{1/3}\omega_V^2} - \frac{2\omega_V^2\delta - 2^{-8/3}r_L^{1/3}m_2^2\lambda}{\omega_V^2}\right], \tag{A25}$$

where

$$C = \frac{2^{3/4}\, r_L^{1/4}\, m_2^{11/3}}{2\sqrt{\pi} m_b^{2/3} \omega_L^3} \quad (L = P \text{ or } V) \ . \tag{A26}$$

The final expression for the form factors in the $m_b \to \infty$ limit is

$$F_1(0) = \frac{4 \times 2^{11/12}}{\sqrt{3}}\, r_P^{-7/12} \left(\frac{m_2}{m_b}\right)^{2/3} \exp\left[\frac{-3\,(2r_P)^{1/3} m_2^{4/3} m_b^{2/3} + (3+4r_P)\, m_2^2 + 2\, m_Q^2}{16\, \omega_P^2}\right],$$

$$A_0(0) = f_1(0) = A_2(0) = V(0) = \frac{4 \times 2^{11/12}}{\sqrt{3}}\, r_V^{-7/12} \left(\frac{m_2}{m_b}\right)^{2/3} \exp\left[\frac{-3\,(2r_V)^{1/3} m_2^{4/3} m_b^{2/3} + (3+4r_V)\, m_2^2 + 2\, m_Q^2}{16\, \omega_V^2}\right]. \tag{A27}$$

The difference between $F_1(0)$ and the form factors of $B \to V$ transition comes from the scale parameter.

Finally, we give a brief explanation about why the two groups of form factors $A_0(0) = f_1(0)$ and $A_2(0) = V(0)$ are equal in our calculation. Since in terms of the coordinates x and t the contribution to the integrand comes only from x_0 and t_0 given by (A19) and (A22), we have

$$\mathcal{A}_b \to m_b\, x_{20}\ ,\ \mathcal{A}_Q \to m_2\ ,\ \mathbf{k}_\perp^2 \to 0\ . \tag{A28}$$

Thus in the integrals (52)–(54),

$$\frac{(\mathcal{A}_Q \mathcal{A}_b + \mathbf{k}_\perp^2)}{\sqrt{(\mathcal{A}_Q^2 + \mathbf{k}_\perp^2)(\mathcal{A}_b^2 + \mathbf{k}_\perp^2)}} \to 1\ ,$$

$$\frac{(m_B + m_V)(1-x)(\mathcal{A}_b + (1-x)(m_b - m_Q)\mathbf{k}_\perp^2 \Theta_V)}{\sqrt{(\mathcal{A}_Q^2 + \mathbf{k}_\perp^2)(\mathcal{A}_b^2 + \mathbf{k}_\perp^2)}}$$

$$\to \frac{[m_b\, x_{20} + m_2 + m_b\, x_{20}\, t\, \Theta_V]}{m_2}\ . \tag{A29}$$

In the square brackets of (A29) we keep the m_2 term because the other two terms, though of higher order in m_b, cancel each other, as we begin to show now. The definition of Θ_V is given in (31):

$$\Theta_V = \left(\frac{d\tilde{\phi}_V}{d\mathbf{k}_\perp^2}\right) / \tilde{\phi}_V\ ,$$

$$\tilde{\phi}_V = \frac{\phi_V}{\sqrt{\mathcal{A}_Q^2 + \mathbf{k}_\perp^2}} = G_V(k_z, t) \exp\left[-\tilde{f}_V(k_z, t)\right]\ , \tag{A30}$$

where

$$G_V(k_z, t) = \frac{\sqrt{\frac{dk_z'}{dx}}}{(\pi\omega^2)^{3/4}\sqrt{\mathcal{A}_Q^2 + \mathbf{k}_\perp^2}}\ . \tag{A31}$$

Thus

$$\Theta_V = -\frac{d}{dt}\tilde{f}_V(k_z, t) + \left(\frac{d}{dt}G_V(k_z, t)\right) \Big/ G_V(k_z, t)\ . \tag{A32}$$

One can show that the leading contribution to Θ comes from $-\frac{d}{dt}\tilde{f}_V(k_z, t)$ at (k_{z0}, t_0): i.e.,

$$\Theta_V = -\frac{d}{dt}\tilde{f}_V(k_{z0}, t_0)\ . \tag{A33}$$

The coefficient $\frac{d}{dt}\tilde{f}(k_{z0}, t_0)$ in (A21) is the sum of $\frac{d}{dt}\tilde{f}_B(k_{z0}, t_0)$ and $\frac{d}{dt}\tilde{f}_V(k_{z0}, t_0)$

$$\frac{d}{dt}\tilde{f}(k_{z0}, t_0) = \frac{d}{dt}\tilde{f}_B(k_{z0}, t_0) + \frac{d}{dt}\tilde{f}_V(k_{z0}, t_0)\ , \tag{A34}$$

but since

$$\frac{d}{dt}\tilde{f}_V(k_{z0}, t_0) = \frac{1}{2^{8/3}\omega_L^{4/3}\omega_B^{2/3}}\left(\frac{m_b}{m_2}\right)^{2/3} \gg \frac{d}{dt}\tilde{f}_B(k_{z0}, t_0) = \frac{1}{2\omega_B^2}\ , \tag{A35}$$

$\frac{d}{dt}\tilde{f}(k_{z0}, t_0)$ is equal to $\frac{d}{dt}\tilde{f}_V(k_{z0}, t_0)$,

$$\frac{d}{dt}\tilde{f}(k_{z0}, t_0) \to \frac{d}{dt}\tilde{f}_V(k_{z0}, t_0) = \frac{1}{2^{8/3}\omega_L^{4/3}\omega_B^{2/3}}\left(\frac{m_b}{m_2}\right)^{2/3}\ , \tag{A36}$$

as given in (A21). Thus in the expression (54), when being integrated over t, terms proportional to $m_b x_{20}$ and $m_b x_{20} t \Theta_V$ of (A29) become

$$\int dt \left[m_b\, x_{20} + m_b\, x_{20}\, t \left(-\frac{d}{dt}\tilde{f}_V k_{z0}, t_0\right)\right] \times \exp\left(-\frac{d}{dt}\tilde{f}_V(k_{z0}, t_0) t\right) = 0\ . \tag{A37}$$

Hence the terms $m_b x_{20}$ and $m_b x_{20} t \Theta_V$ in (A29) cancel, and (A30) becomes 1, equal to (A28). This is why the two sets of form factors $f_1(0) = A_0(0)$ and $V(0) = A_2(0)$ are equal.

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