Cluster-based Informed Agents Selection for Flocking with a Virtual Leader

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***Abstract*—Recent literature on flocking in multi-agent systems show that a minority of informed agents in a group of dynamic agents can influence a majority to follow a virtual leader. However, it is not reported how to select these informed agents from the group in order to increase the number of agents which will eventually follow the virtual leader. In this paper, we propose a cluster-based informed agents selection method to achieve this objective. The proposed method enables us to select informed agents such that they are spatially evenly distributed within the group of agents. We carried out extensive simulations to analyze performances of the proposed method against the traditional random-based informed agents selection method. Simulation results show that the proposed method can increase the number of agents which eventually follow the virtual leader for a given number of informed agents. Therefore, the proposed cluster-based informed agents selection method is useful for leading the majority of a group with less number of informed agents.**

***Index Terms*—Flocking, distributed control, informed agents, virtual leader, clusters.**

I. Introduction

Flocking is a collective behaviour that can be commonly observed when groups of animals migrating or foraging. In such groups, only few individuals have information about a food source or a migrating route [1], [2]. It has been studied that a few individuals in a fish school are adequate to control the foraging behavior of the group [3]. These individuals use different methods to exchange information within the group. Couzin et al. showed that how group objectives can be achieved when informed individuals have different preferences [4].

The consensus problem in multi-vehicle systems with a virtual leader was studied by Ren [5]. He showed the necessary and sufficient conditions to achieve the consensus with a time-varying virtual leader when only a fraction of vehicles have access to the virtual leader. Later, Cao and Ren proposed two distributed coordinated tracking algorithms for multi-agent systems [6]. Their algorithms guaranteed global exponential tracking with only a fraction of informed agents. In [7], Su et al. modified the Olfati-Saber's second flocking algorithm [8] by providing navigational feedback only to a few randomly selected agents. The uninformed agents which do not have direct access to the information of the virtual leader, can still follow the virtual leader if they are influenced by the informed agents occasionally. Simulation results given in [7] also show that the proportion of informed agents required to guide a given fraction of agents decreases as the size of the group increases. However, it is not obvious how to select a given number of informed agents such that majority of the group will follow a virtual leader.

We studied how informed agents selections can affect the number of agents that eventually follow the virtual leader. We conducted simulations for different fractions of informed agents with different initial densities of randomly distributed agents. We observed that an even selection of informed agents based on their spatial distribution can help driving majority of the agents to track a virtual leader effectively. This led us to propose a novel method for selecting informed agents in a group of dynamic agents based on their initial clusters. Simulation results show that the proposed cluster-based selection method can drive a larger fraction of agents to follow a virtual leader compared to the random-based selection method proposed in [7].

The rest of the paper is organized as follows. Section II recalls relevant background materials. The novel cluster-based informed agents selection method for flocking with a virtual leader is proposed in Section III. Results of our simulation study are presented in Section IV. Concluding remarks are given in Section V.

II. Background

We consider a system of N mobile agents operating in $\mathbb{R}^n$. The motion of each agent is described by a double integrator form

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = u_i, i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where q_i, p_i, u_i are the position, velocity, and acceleration of the agent i, respectively. The control protocol given in [7] assumes that randomly selected M_0 $(1 < M_0 \leq N)$ agents are influenced by a virtual leader with following dynamics

$$\begin{cases} \dot{q}_\gamma = p_\gamma, \\ \dot{p}_\gamma = f_\gamma(q_\gamma, p_\gamma), \end{cases} \tag{2}$$

where $q_\gamma, p_\gamma, f_\gamma \in \mathbb{R}^n$ are the position, velocity, and acceleration of the virtual leader with $(q_\gamma(0), p_\gamma(0)) = (q_d, p_d)$ and

$\dot{q}_\gamma = p_d$. The spatial neighbors of agent i at time t are defined by

$$\mathcal{N}_i(t) = \{j : \|q_i - q_j\| < r, j = 1, 2, \ldots, N, j \neq i\}, \quad (3)$$

where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$ and $r(> 0)$ is the interaction range between two agents. We assume an identical interaction range for all the agents, thus, $j \in \mathcal{N}_i(t) \Leftrightarrow i \in \mathcal{N}_j(t)$. Due to the symmetry, interactions among agents can be represented by an undirected dynamic graph $\mathcal{G}(t)$.

The distributed control scheme for multi-agent dynamic systems proposed in [7] can be expressed as

$$u_i = f_i^g + f_i^d + f_i^\gamma. \quad (4)$$

Here, f_i^g, f_i^d, f_i^γ are the gradient-based term, velocity consensus term, and navigational feedback term, respectively. The gradient-based term is used to control the position of a agent i within its neighborhood, which can be defined as

$$f_i^g = -\sum_{j \in \mathcal{N}_i(t)} \nabla_{q_i} \psi_\alpha(\|q_j - q_i\|_\sigma). \quad (5)$$

For a parameter $\epsilon > 0$, σ-norm of a vector z is given by $\|z\|_\sigma = \frac{1}{\epsilon}\left(\sqrt{1+\epsilon\|z\|^2} - 1\right)$. Note that $\|z\|_\sigma$ is differentiable everywhere whereas $\|z\|$ is not differentiable at $z = 0$. The smooth pairwise attractive/repulsive potential function $\psi_\alpha(z)$ is given by

$$\psi_\alpha(z) = \int_{\|d\|_\sigma}^{z} \phi_\alpha(s)ds, \quad (6)$$

where d is the desired distance between agents. The potential function reaches its global minimum at $z = \|d\|_\sigma (< \|r\|_\sigma)$ and global maximum at $z = 0$, and becomes constant for $\|z\|_\sigma \geq \|r\|_\sigma$. In (6), $\phi_\alpha(z)$ is given by

$$\phi_\alpha(z) = \frac{1}{2} p_h\left(\frac{z}{\|r\|_\sigma}\right)\left[\frac{(a+b)(z - \|d\|_\sigma + c)}{\sqrt{1 + (z - \|d\|_\sigma + c)^2}} + (a - b)\right], \quad (7)$$

where $0 < a \leq b$, and $c = |a - b|/\sqrt{4ab}$. In (7), $p_h(z)$ can be expressed as

$$p_h(z) = \begin{cases} 1, & \text{if } z \in [0, h) \\ \frac{1}{2}\left[1 + cos\left(\pi\frac{z-h}{1-h}\right)\right], & \text{if } z \in [h, 1] \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

where $h \in (0, 1)$ [8].

The velocity consensus term in (4) is defined as

$$f_i^d = \sum_{j \in \mathcal{N}_i(t)} a_{ij}(q)(p_j - p_i), \quad (9)$$

where $q = [q_1, q_2, \ldots, q_N]^{\mathrm{T}} \in \mathbb{R}^{nN}$. Terms of the adjacency matrix of graph $\mathcal{G}(t)$ are given by

$$a_{ij}(q) = \begin{cases} 0, & \text{if } j = i \\ p_h(\|q_j - q_i\|_\sigma/\|r\|_\sigma), & \text{otherwise.} \end{cases} \quad (10)$$

In (4), the navigational feedback to track the virtual leader is given by

$$f_i^\gamma = -h_i[c_1(q_i - q_\gamma) + c_2(p_i - p_\gamma)], \quad (11)$$

where $c_1, c_2 > 0$ are constants. In contrast to Olfati-Saber's second flocking algorithm [8], the flocking algorithm given in [7] assumes that only few agents are informed of (q_γ, p_γ). If an agent i is an informed agent, $h_i = 1$, otherwise, $h_i = 0$. Uninformed agents are divided as types I and II based on their interactions with informed agents. A type I uninformed agent has a joint path with an informed agent across a finite sequence of nonempty, contiguous, and uniformly bounded time-intervals $[t_i, t_{i+1})$ where $t_{i+1} \geq t_i$ and $i \geq 0$. In contrast, there does not exist such a joint path between a type II uninformed agent and an informed agent. In [7], Su et al. showed that the velocities of all informed agents and the type I uninformed agents are asymptotically approaching the desired velocity p_γ, even if only a small fraction of the agents are selected as informed agents. The total number of informed agents and type I uninformed agents are defined as $M (M_0 \leq M \leq N)$. In this paper, we propose an algorithm to improve M by enhancing the number of type I uninformed agents.

III. Cluster-based Selection of Informed Agents

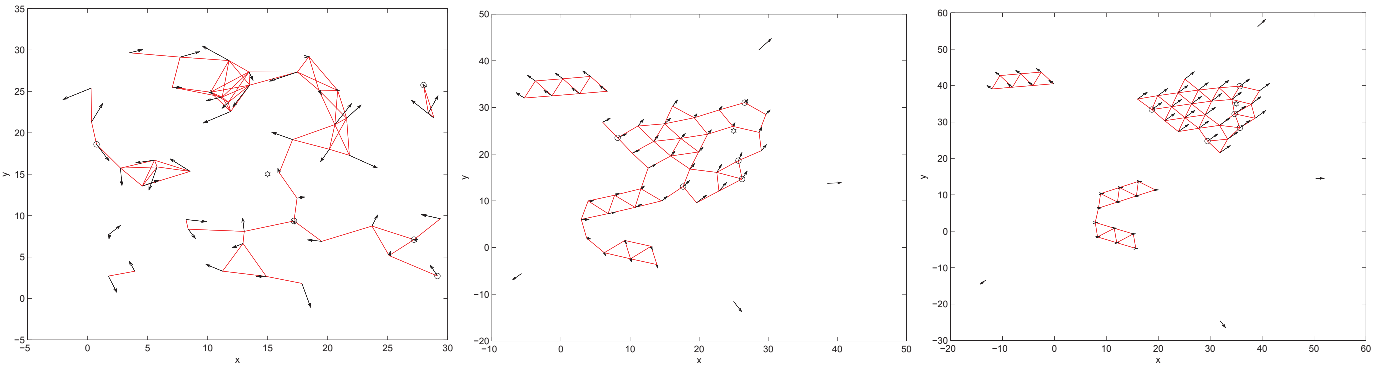
The number of type I informed agents in a group of dynamic agents clearly depends on the interactions between informed agents and the rest of the group. According to our observations, the number of such interactions varies upon the positioning of informed agents within an initial spatial distribution of the agents. If the informed agents are evenly distributed throughout the space, there is a higher chance that an uninformed agent gets inspired by an informed agents, which ultimately results in a higher number of type I uninformed agents. In order to achieve that, we propose a cluster-based informed agent selection method. Clustering groups a set of objects according to certain properties of them such that the objects in the same group show higher similarity [9]. In this work, we use clustering to partition agents based on their initial position within the group. Afterwards, we select an informed agent from each of those clusters. Here, we use a *k*-means clustering algorithm [10] once at the beginning of the simulation for clustering of the agents. Chen et al. [11] also investigated on cluster consensus of discrete-time multi-agent systems with several different subgroups.

The *k*-means algorithm can divide the agents into k partitions based on the distance from each agent to k different centroids. These centroids can be later used for selecting informed agents. In this work, we select only a single informed agent from each cluster. Hence, $k = M_0$. The *k*-means algorithm finds M_0 clusters of agents, $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{M_0}\}$, by minimizing their within-cluster sum of squares $\sum_{j=1}^{M_0} \sum_{\forall i \in \mathcal{C}_j} \|q_i - \mu_j\|^2$. Here, $\mu_j \in \mathbb{R}^n$ is the centroid of cluster $\mathcal{C}_j$.

The *k*-means algorithm starts by initializing μ_j of cluster $\mathcal{C}_j$, $\forall j (1 \leq j \leq M_0)$, such that

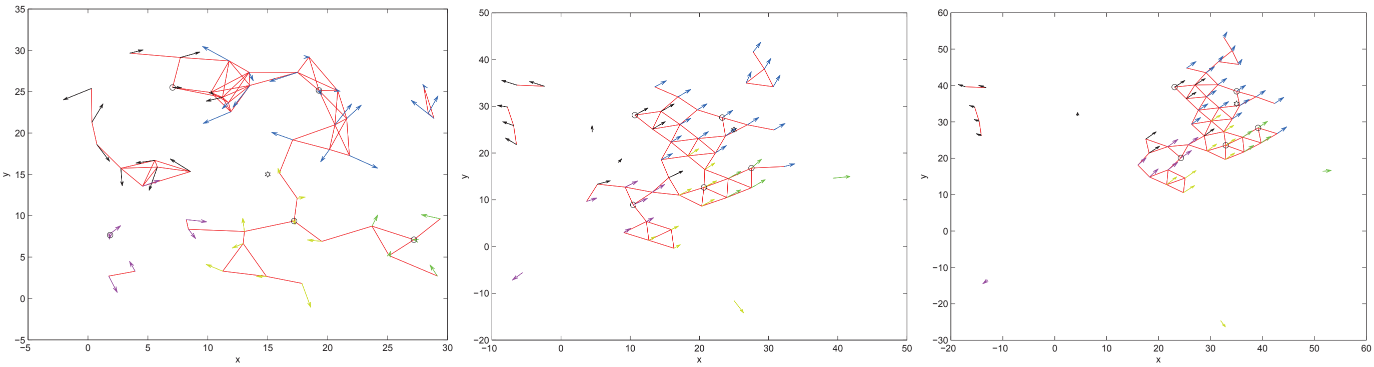
$$\mu_j^{(1)} = \{q_s : s \in \mathcal{V}, \mu_i^{(1)} \neq q_s, 1 \leq i \leq M_0, i \neq j\}. \quad (12)$$

After the initialization of centroids, the *k*-means algorithm proceeds by alternating between an *assignment step* and an *update step* as described below.



$t = 0$ $t = 10$ $t = 20$

Fig. 1. Flocking of 50 agents with 5 randomly selected informed agents. At $t = 20$, there are 26 agents following the virtual leader.



$t = 0$ $t = 10$ $t = 20$

Fig. 2. Flocking of 50 agents. All the parameters remain same as in Fig. 1, but informed agents are selected on initial clusters. Arrowheads in the same color belong to the same initial cluster. At $t = 20$, there are 42 agents following the virtual leader.

Assignment step:

Each agent s in the group is assigned to a cluster $\mathcal{C}_j$ such that

$$\mathcal{C}_j^{(l)} = \{s : \|q_s - \mu_j^{(l)}\|^2 \leq \|q_s - \mu_i^{(l)}\|^2, \forall i, 1 \leq i \leq M_0\}. \tag{13}$$

An agent can only belong to a single cluster in a particular iteration l, hence $s \in \mathcal{C}_j^{(l)} \Rightarrow s \notin \mathcal{C}_i^{(l)}$ if $i \neq j$.

Update step:

The centroids are estimated based on the positions of the agents that are assigned to each cluster in the previous step, such that

$$\mu_j^{(l+1)} = \frac{1}{|\mathcal{C}_j^{(l)}|} \sum_{\forall i \in \mathcal{C}_j^{(l)}} q_i. \tag{14}$$

For a given threshold $\xi \geq 0$, if $|\mu_j^{(l+1)} - \mu_j^{(l)}| \leq \xi$, $\forall j (1 \leq j \leq M_0)$, the algorithm terminates and the informed agents are selected as

$$h_i = \begin{cases} 1, & \text{if } i = \arg\min_{s \in \mathcal{C}_j} \|q_s - \mu_j\|, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

IV. Simulation Study

We use computer simulations to evaluate and analyze the performances of the proposed cluster-based informed agents selection method against the random selection method [7]. A set of simulations were performed on 50 agents ($N = 50$) moving in a 2-dimensional ($n = 2$) space under the influence of the control protocol (4). The number of informed agents is set to $M_0 = 5$, *i.e.* the fraction of informed agents $\delta = M_0/N = 0.1$. Initial positions and velocities of the 50 agents were randomly chosen according to uniform distribution from the boxes $[0, 30] \times [0, 30]$ and $[-2, 2] \times [-2, 2]$, respectively. The initial position of the virtual leader was set to $q_\gamma(0) = [15, 15]^{\mathrm{T}}$ and the velocity to $q_\gamma(0) = [1, 1]^{\mathrm{T}}$. The rest of the parameters are as follows: $r = 4.8$, $d = 4$, $\epsilon = 0.1$, $h = 0.7$, $a = 1$, $b = 2$, $c_1 = 0.1$, and $c_2 = 0.4$. Simulation results for random and cluster-based informed agents selection methods are shown in Figs. 1 and 2, respectively. Solid lines in the figures represent neighboring relations, arrowheads represent velocities of the agents, and hexagrams represent positions of the virtual leaders. The informed agents are marked with circles. The initial distribution of the agents (at $t = 0$) was kept the same for both simulations for a fair comparison. The number of type II uninformed agents increases in both the cases as time evolves, thus increasing the number of agents

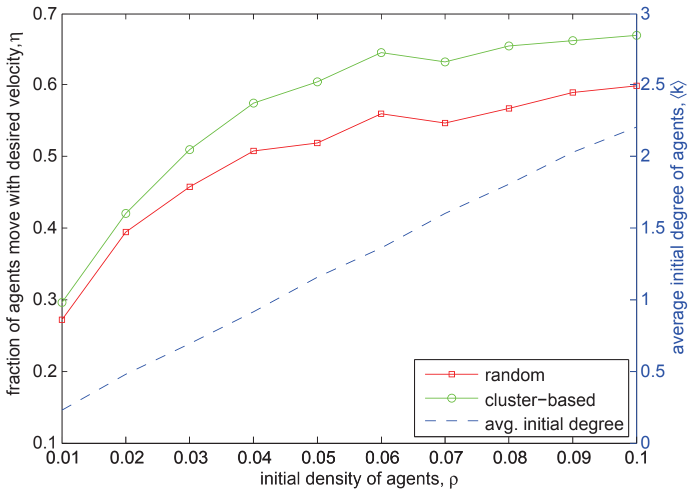


Fig. 3. Fraction of agents with desired velocity as a function of the initial density of agents. All estimates are the results of averaging over 50 realizations with $N = 100$ and $\delta = 0.1$.

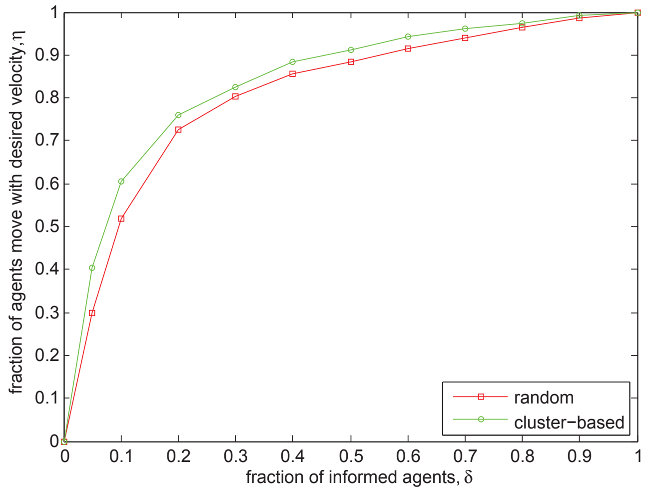


Fig. 4. Fraction of agents with desired velocity as a function of the fraction of informed agents. All estimates are the results of averaging over 50 realizations with $N = 100$ and $\rho = 0.05$.

move with the virtual leader. At $t = 20$, the fraction of agents move with the virtual leader is $\eta = 0.52$ when the informed agents are selected randomly. When the informed agents are selected using proposed cluster-based method, $\eta = 0.84$. In this particular case, the cluster-based informed agents selection method drives more number of agents to follow the virtual leader compared to the random selection method.

We carried out extensive simulations to further verify the above results by evaluating η against the initial density of the informed agents $\rho \in [0.01, 0.1]$. Following parameters remained fixed through out all simulations: $N = 100$, $r = 4$, $d = 3.3$, $\epsilon = 0.1$, $h = 0.6$, $a = 1$, $b = 2$, $c_1 = 0.1$, $c_2 = 0.4$, and $\delta = 0.1$. Initial positions and velocities of the agents were randomly selected from a $[0, L] \times [0, L]$ box ($\rho = N/L^2$) and a $[-0.5, 0.5] \times [-0.5, 0.5]$ box, respectively. The initial position and velocity of the virtual leader were set at $q_\gamma(0) = [L/2, L/2]^{\mathrm{T}}$ and $p_\gamma(0) = [2, 2]^{\mathrm{T}}$. Simulations given in Fig. 3 illustrates that the cluster-based informed agent selection method outperforms the random selection method in terms of η, for all values of ρ. As ρ decreases, networks become more sparse (e.g.: when $\rho \leq 0.2$, $\langle k \rangle \leq 0.4826$), and therefore, the uninformed agents are unlikely to get influenced by the informed agents.

More simulations were performed to evaluate performances of the proposed method by varying $\delta \in [0, 1]$. All the parameters remain similar to the simulations associated with Fig. 3, except ρ is fixed at 0.05. According to the simulation results given in Fig. 4, the cluster-based informed agent selection method is capable of driving more agents to follow the virtual leader compared to random selection of informed agents regardless of the number of informed agents.

V. Conclusion

It has been widely studied that a minority of informed agents in a group of dynamic agents can influence a majority to follow a virtual leader. Investigations were carried out to examine how a selection of informed agents can affect the proportion of agents which will eventually follow the virtual leader. Based on the observations, we propose a novel cluster-based informed agents selection method for flocking of multi-agent dynamic systems with a virtual leader. Computer simulations were performed to evaluate performances of the proposed method against an existing random-based method. Simulation results show that in several circumstances the proposed method can influence a larger proportion of agents to follow the virtual leader compared to the random selection of informed agents.

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