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Synchronization of impulsively coupled complex systems with delay

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This paper investigates the synchronization of complex systems with delay that are impulsively coupled at discrete instants only. Based on the comparison theorem of impulsive differential system, a distributed impulsive control scheme is proposed to achieve the synchronization for systems with delay. In the control strategy, the influence of all nodes to network synchronization relies on its weight. The proposed control scheme is applied to the chaotic delayed Hopfield neural networks and numerical simulations are presented to demonstrate the effectiveness of the proposed scheme. © *2011 American Institute of Physics*. [doi:10.1063/1.3633081]

Synchronization and control of complex systems are an important topic that has drawn a great deal of attentions in different forms both in nature and in man-made systems. Owe to its effectiveness, robustness and low cost, impulsive control strategy has been used to synchronize linearly or nonlinearly coupled systems, but these studies in the literature were based on continuous coupling. In this paper, we proposed a new and different coupled systems model, which are coupled only at discrete instants through impulsive connections and showed that systems with delay can reach synchronization by exchanging information at instants. Numerical simulations of the chaotic delayed Hopfield neural networks are presented to demonstrate the effectiveness of the proposed scheme.

I. INTRODUCTION

Complex systems have been shown to exist in nature[1,2] and synchronizing them is both interesting and important because there are many real-world applications.[3] Some neurons, for example, have been observed to oscillate in synchrony in mammalian brains and a similar phenomenon has also been reported to occur in other areas such as mathematics, sociology, and biology.[3–9]

One important consideration in practical networks is the existence of time delays because obstructions to the transmission of signals are inevitable in a biological neural network, in an epidemiological model, in a communications network, or in an electrical power grid. Even though it is well known about the delays at the couplings (i.e., edges),[10–14] delays at the dynamical nodes have almost seldom been explored. Delayed nodes occur frequently in nature and famous examples include the chaotic delayed Hopfield neural network and the delayed logistic differential equation.[15] Systems with delay nodes are therefore important systems that warrant further investigations.

Apart from time delays, complex systems are susceptible to sudden surges in their flows called impulses. Impulses have been observed, for example, in space programs, in control systems as well as in communications security equipment and they are usually modeled by impulsive differential equations. Interest in impulsive complex systems has grown in recent years and many effective, robust and inexpensive impulsive control strategies have been invented.[16–35] Zhou *et al.*,[16] for instance, investigated the synchronization of delayed dynamical networks with impulsive effects, and Yang and Chua[19] studied the impulsive stabilization problem for chaotic synchronization and control. Liu *et al.*[20] considered robust impulsive synchronization of uncertain dynamical network. The authors introduced impulsive control protocols for multi-agent linear continuous dynamic systems.[29,30] In the existing control schemes, some of the control strategies are based on the weighted average of the states of all nodes. This may lead to the difficulty, since it is not easy for a node to obtain information of all other nodes in a large scale network. The other control schemes are based on a special solution of an isolated node, which may be difficult to obtain in some practical applications. In order to avoid the implementation difficulty of the controller, by utilizing the information from neighboring nodes, Guan *et al.*[33] introduced the concept of control topology and investigated the problem of distributed impulsive synchronization of complex dynamical networks with system delay and multiple coupling delays. Zhang *et al.*[34] proposed an impulsive control scheme, in which each local controller shares information from partial nodes by taking the weights of great majority nodes as zero. Recently, Han *et al.*[35] proposed a model for impulsively coupled systems, i.e., systems that are coupled at discrete instants only via impulsive connections, and obtained sufficient conditions for their synchronization. However, in the existing works, in particularly, networked nonlinear dynamical systems coupled at certain discrete instants,[29,30,35] delays are not considered. Research shows that ignoring delays may lead to design flaws and incorrect analysis conclusions.

The question naturally arises as to whether it is possible to achieve synchronization for systems that have both delay and impulsive connections. The aim of this paper is to carry out an exploration of this aspect. In this paper, the synchronization of impulsively coupled systems with delay is investigated. Based on the theorem of impulsive differential equations, we designed a more feasible distributed impulsive control scheme and showed that systems with delay can reach synchronization by exchanging information at instants. Numerical simulations of the chaotic delayed Hopfield neural networks are presented to demonstrate the effectiveness of the proposed scheme.

The rest of the paper is organized as follows: in Sec. II, description of the model is given; in Sec. III, sufficient condition of global synchronization of impulsively coupled systems with delay are obtained; numerical simulations for verifying the theoretical result are presented in Sec. IV; the paper is concluded in Sec. V.

II. MODEL DESCRIPTION

Consider a complex systems consisting of N identical nodes, which is described by

$$\frac{dx_i(t)}{dt} = -Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau)) + u_i(t), \quad i = 1,2,\ldots,N \tag{1}$$

where $x_i(t) = (x_{i1}, x_{i2}, \ldots, x_{in})^T$ is the state vector of the th$i$ node, $C = diag\{c_1, c_2, \ldots, c_n\} \in R^{n\times n}$ is a diagonal matrix with positive diagonal entries $c_i > 0$, $i = 1,2,\ldots,n$, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ are weight and delayed weight matrices, respectively. $f(x_i(t)) = (f_1(x_{i1}(t)), \ldots, f_n(x_{in}(t)))^T \in R^n \in R^n$ is a continuous map satisfying

$$H_1 : |f_k(x_{im}) - f_k(x_{jm})| \le l_i |x_{im} - x_{jm}|, \quad \forall x_{im}, y_{jm} \in R.$$

$\tau \ge 0$ is the delay, $u_i(t)(i = 1,2,\ldots,N)$ is a distributed impulsive control input, which utilizes the information from neighboring nodes

$$u_i(t) = \sum_{k=1}^{\infty} B_k [x_i(t_k) - \sum_{j=1}^{N} \theta_{ij} x_j(t_k)] \delta(t - t_k), \tag{2}$$

or equivalently

$$u_i(t) = \sum_{k=1}^{\infty} B_k \sum_{j=1}^{N} \bar{\theta}_{ij} x_j(t_k) \delta(t - t_k),$$

where

$$\bar{\theta} = (\bar{\theta}_{ij})_{N\times N} = \begin{bmatrix} 1-\theta_{11} & -\theta_{12} & \cdots & -\theta_{1N} \\ -\theta_{21} & 1-\theta_{22} & \cdots & -\theta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\theta_{N1} & -\theta_{N2} & \cdots & 1-\theta_{NN} \end{bmatrix},$$

The discrete set $\{t_k\}$ satisfies

$$0 \le t_0 < t_1 < \ldots < t_k < \ldots < t_k \to \infty (t \to \infty),$$

B_k is the control gains, $\delta(\cdot)$ is the Dirac impulsive function. $\theta_{ij} \ge 0$ denotes the influence weight of the state of the th$j$ node on that of the th$i$ node, and for any $i = 1,\ldots,N$, $\sum_{j=1}^{N} \theta_{ij} = 1$. θ_{ij} is designed as follow: if the th$j$ node can obtain the information of th$i$ node, then $\theta_{ij} > 0$; otherwise, $\theta_{ij} = 0$. The weighted directed graph $\bar{\theta}$ is assumed to be irreducible, which means there is no isolated node in network, namely, every node can receives information from other nodes or sends out information to other nodes at discrete instants t_k.

Remark 1: The distributed impulsive control scheme (2) provides us a flexible method for utilizing the local information from neighboring nodes of the network. For any $\theta_{i1} = 1 (i = 1,\ldots,N)$, $\theta_{ij} = 0 (j = 2,3,\ldots,N)$, which means that all nodes receive the information of the first node, or $\theta_{ij} > 0 (j = 1,2,\ldots,N)$, which suggests that each node receives the local information from its neighboring nodes, can also be considered as a special case in this paper. It is worth noting that we can take the weights of nodes as one desires or even the weights of great majority nodes as zero and achieve network synchronization by only a few nodes. This is very significant in practical applications.

Remark 2: It is worth pointing out that in some cases when the networked systems cannot endure continuous control or it is impossible to give continuous control, the impulsive control is an effective and low-cost method.

The impulsively coupled networks with N identical nodes can then be described by

$$\begin{aligned} &\frac{dx_i(t)}{dt} = -Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau)), \\ &\qquad i = 1,2,\ldots,N, t \ne t_k, \\ &\Delta x_i = B_k \sum_{j=1}^{N} \bar{\theta}_{ij} x_j(t), \quad t = t_k, \quad k = 1,2,\ldots \\ &x_i(t) = \phi^i(t), \quad t \in [t_0 - \tau, t_0], \quad i = 1,2,\ldots,N, \end{aligned} \tag{3}$$

where $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$ and $x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$. Moreover, any solution of (3) is left continuous at each $t = t_k$, i.e., $x_i(t_k^-) = x_i(t_k)$.

Remark 3: The model is of great importance because it can be used to describe many non-continuously coupled systems such as the species-food model in the biology, the transfer and exchange of information between ants, and the integrated circuit models.

The impulsively coupled network (3) is said to be globally synchronized if for all $i,j = 1,2,\ldots,N$

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0,$$

where $\|\cdot\|$ denotes the Euclidean norm in R^n.

Let $C([-\tau, 0], R^n)$ be the Banach space of all the continuous functions that map the interval $[-\tau, 0]$ into R^n that is equipped with the norm

$$\|\phi\| = sup_{-\tau\le\theta\le 0} \|\phi(\theta)\|.$$

The initial conditions for Eq. (3) are $x_i(t) = \phi^i(t) \in C([-\tau, 0], R^n)$ and it is assumed that (3) has a unique solution with respect to these initial conditions.

Suppose $T(\epsilon)$ denotes the set of matrices such that the sum of the element in each row is equal to the real number ϵ. The set M_1 is defined as follow: if $M = (M_{ij})_{(N-1)\times N} \in M_1$, each row of M contains exactly one element 1 and one element -1, and all other elements are zeros. $j_{i1}(j_{i2})$ denotes the column indexes of the first (second) nonzero element in the th$i$ row. The set H is defined by $H = \{\{j_{11}, j_{12}\}, \{j_{21}, j_{22}\}, \ldots, \{j_{P1}, j_{P2}\}\}$. The set M_2 is defined as follow: $M_2 \subset M_1$ and if $M = (M_{ij})_{(N-1)\times N} \in M_2$, for any pair of the column indexes j_s and j_t, there exist indexes $j_1, j_2, \ldots, j_l$ with $j_1 = j_s$ and $j_t = j_l$ such that $j_m, j_{m+1} \in H$ for $m = 1, 2, \ldots, l-1$.

Lemma 1: (Ref. [33]) Let $M = M_{ij} \in M_2$ be a $(N-1)\times N$ matrix and $A \in T(\epsilon)$ be a $N \times N$ matrix, there exists a $N \times (N-1)$ matrix G_M such that $MA = \hat{A}M$, where $\hat{A} = MAG_M$, $MG_M = I_{N-1}$. Moreover, let Γ be a $n \times n$ constant matrix and $\mathbf{A}_\Gamma = A \otimes \Gamma$, then $\mathbf{MA} = \widehat{\mathbf{A}}\mathbf{M}$, where $\widehat{\mathbf{A}} = \hat{A} \otimes \Gamma$ and $\mathbf{M} = M \otimes I_n$.

Denote the function that maps A to $\hat{A}$ by S_M, i.e., the function is defined as

$$\hat{A} = S_M(A) = MAG_M.$$

Lemma 2: (Ref. [36]) Given any vectors x, y of appropriate dimensions and a positive definite matrix $P > 0$ with compatible dimensions, then the following inequality holds:

$$2x^T y \le x^T P x + y^T P^{-1} y.$$

III. MAIN RESULTS

In this section, new criteria are presented for the global synchronization of impulsively coupled systems based on the comparison theorem of impulsive differential system. Rewrite the network (3) by Kronecker product

$$\begin{aligned} \dot{x}(t) &= -\bar{C}x(t) + \bar{A}F(x(t)) + \bar{B}F(x(t-\tau)), \quad t \ne t_k, \\ \Delta x &= B_k^N \bar{\Theta} x(t_k), \end{aligned} \tag{4}$$

where $x(t) = (x_1^T, x_2^T, \ldots, x_N^T)^T$, $\bar{C} = I_N \otimes C$, $\bar{A} = I_N \otimes A$, $\bar{B} = I_N \otimes B$, $B_k^N = I_N \otimes B_k$ $\bar{\Theta} = \bar{\theta} \otimes I_n$, and $F(x(t)) = (f^T(x_1(t)), \ldots, f^T(x_N(t)))^T$.

For $M \in M_2$, let $\mathbf{M} = M \otimes I_n$, then $y(t) = \mathbf{M}x(t) = (y_1^T(t), y_2^T(t), \ldots, y_{N-1}^T(t))^T$, $y_i(t) = (y_{i1}, y_{i2}, \ldots, y_{in})^T, i = 1, \ldots, N-1$. Note that $j_{i1}(j_{i2})$ denotes the column indexes of the first (second) nonzero element in the i th row, obviously, $y_i(t) = x_{ji1}(t) - x_{ji2}(t)$. Because of the assumptions on $M \in M_2$, the crucial property of $x^T\mathbf{M}^T\mathbf{M}x$ is that $x^T\mathbf{M}^T\mathbf{M}x \to 0$ if and only if $\| x_i - x_j \| \to 0$, for all $i, j = 1, 2, \ldots, N$.

Then we establish the following theorem.

Theorem 1: Suppose H_1 holds, $\gamma = -2\lambda(C) + \lambda(AA^T) + \lambda(LL^T) + \lambda(BL^2B^T) + 1$, $\beta_k = \lambda(D^TD)$, where $D = I_{(N-1)n} + (\hat{\theta} \otimes B_k)$, $\lambda(P)$ is the largest eigenvalue of the matrix P, $L = diag\{l_1, l_2, \ldots, l_n\}$, the impulsively coupled systems with delay (3) is globally asymptotically synchronized if there exists a constant $\xi > 1$ such that

$$ln(\xi\beta_k) + \gamma\Delta_k \le 0, \quad k = 1, 2, \ldots. \tag{5}$$

Proof: Choose

$$V(t) = x^T(t)\mathbf{M}^T\mathbf{M}x(t) + \int_{t-\tau}^{t} x^T(s)\mathbf{M}^T\mathbf{M}x(s)ds$$

to be a Lyapunov function. Then, we have

$$\lambda_0 x^T(t)x(t) \le V(t) \le \lambda_0 x^T(t)x(t) + \tau\lambda_0\|x\|^2,$$

where $\|x\| = \sup_{t-\tau \le s \le t} \|x(s)\|$, λ_0 is the largest eigenvalue of $\mathbf{M}^T\mathbf{M}$. Differentiating $V(t)$ along the trajectories of (4), we have

$$\begin{aligned} \frac{dV(t)}{dt} &= 2x^T(t)\mathbf{M}^T\mathbf{M}[-\bar{C}x(t) + \bar{A}F(x(t)) + \bar{B}F(x(t-\tau))] \\ &\quad + [x^T(t)\mathbf{M}^T\mathbf{M}x(t) - x^T(t-\tau)\mathbf{M}^T\mathbf{M}x(t-\tau)], \end{aligned}$$

From H_1 and Lemma 2, one obtains

$$\begin{aligned} &2x^T(t)\mathbf{M}^T\mathbf{M}(-\bar{C})x(t) \\ &\le 2\sum_{i=1}^{N-1}(x_{j_{i1}}(t) - x_{j_{i2}}(t))^T(-C)(x_{j_{i1}}(t) - x_{j_{i2}}(t)) \\ &\le -2\lambda(C)\sum_{i=1}^{N-1}(x_{j_{i1}}(t) - x_{j_{i2}}(t))^T(x_{j_{i1}}(t) - x_{j_{i2}}(t)) \\ &\le -2\lambda(C)x^T(t)\mathbf{M}^T\mathbf{M}x(t), \end{aligned}$$

$$\begin{aligned} &2x^T(t)\mathbf{M}^T\mathbf{M}\bar{A}F(x(t)) \\ &\le 2\sum_{i=1}^{N-1}(x_{j_{i1}}(t) - x_{j_{i2}}(t))^T A[f(x_{j_{i1}}(t)) - f(x_{j_{i2}}(t))] \\ &\le \sum_{i=1}^{N-1}[(x_{j_{i1}}(t) - x_{j_{i2}}(t))^T AA^T(x_{j_{i1}}(t) - x_{j_{i2}}(t)) \\ &+ (f(x_{j_{i1}}(t)) - f(x_{j_{i2}}(t)))^T(f(x_{j_{i1}}(t)) - f(x_{j_{i2}}(t)))] \\ &\le [\lambda(AA^T) + \lambda(LL^T)]x^T(t)\mathbf{M}^T\mathbf{M}x(t), \end{aligned}$$

$$\begin{aligned} &2x^T(t)\mathbf{M}^T\mathbf{M}\bar{B}F(x(t-\tau)) \\ &\le 2\sum_{i=1}^{N-1}(x_{j_{i1}}(t) - x_{j_{i2}}(t))^T B[f(x_{j_{i1}}(t-\tau)) \\ &- f(x_{j_{i2}}(t-\tau))] \\ &\le \sum_{i=1}^{N-1}[(x_{j_{i1}}(t) - x_{j_{i2}}(t))^T BL^2B^T(x_{j_{i1}}(t) - x_{j_{i2}}(t)) \\ &+ (f(x_{j_{i1}}(t-\tau)) - f(x_{j_{i2}}(t-\tau)))^T(L^2)^{-1} \\ &\times (f(x_{j_{i1}}(t-\tau)) - f(x_{j_{i2}}(t-\tau)))] \\ &\le \lambda(BL^2B^T)x^T(t)\mathbf{M}^T\mathbf{M}x(t) + x^T(t-\tau)\mathbf{M}^T\mathbf{M}x(t-\tau), \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{dV(t)}{dt} &\le [-2\lambda(C) + \lambda(AA^T) + \lambda(LL^T) + \lambda(BL^2B^T) \\ &\quad + 1]x^T(t)\mathbf{M}^T\mathbf{M}x(t) \le \gamma[x^T(t)\mathbf{M}^T\mathbf{M}x(t) \\ &\quad + \int_{t-\tau}^{t} x^T(s)\mathbf{M}^T\mathbf{M}x(s)ds] = \gamma V(t) \end{aligned}$$

for all $t \in (t_{k-1}, t_k)$, where $L = diag\{l_1, l_2, \ldots, l_n\}$, $\gamma = -2\lambda(C) + \lambda(AA^T) + \lambda(LL^T) + \lambda(BL^2B^T) + 1$. This implies

$$V(t) \le V(t_{k-1}^{+})e^{\gamma(t-t_{k-1})}, \quad t \in (t_{k-1}, t_k] \tag{6}$$

On the other hand, when $t = t_k$, from Lemma 1, we have

$$\begin{aligned}
V(t_k^{+}) &= x^T(t_k)(I_{Nn} + B_k^N\bar{\Theta})^T\mathbf{M}^T\mathbf{M}(I_{Nn} + B_k^N\bar{\Theta})x(t_k) \\
&\quad + \int_{t_k-\tau}^{t_k} x^T(s)(I_{Nn} + B_k^N\bar{\Theta})^T\mathbf{M}^T\mathbf{M}(I_{Nn} + B_k^N\bar{\Theta})x(s)ds \\
&= x^T(t_k)\mathbf{M}^T(I_{(N-1)n} + (\hat{\theta}\otimes B_k))^T \\
&\quad \times (I_{(N-1)n} + (\hat{\theta}\otimes B_k))\mathbf{M}x(t_k) \\
&\quad + \int_{t_k-\tau}^{t_k} x^T(s)\mathbf{M}^T(I_{(N-1)n} + (\hat{\theta}\otimes B_k))^T \\
&\quad \times (I_{(N-1)n} + (\hat{\theta}\otimes B_k))\mathbf{M}x(s)ds \\
&\le \lambda(D^TD)[x^T(t_k)\mathbf{M}^T\mathbf{M}x(t_k) \\
&\quad + \int_{t_k-\tau}^{t_k} x^T(s)\mathbf{M}^T\mathbf{M}x(s)ds] \le \beta_k V(t_k),
\end{aligned} \tag{7}$$

where $D = I_{(N-1)n} + (\hat{\theta}\otimes B_k)$. When $k = 1$ in inequality (6), we have

$$V(t) \le V(t_0^{+})e^{\gamma(t-t_0)}$$

for all $t \in (t_0, t_1]$, which leads to $V(t_1) \le V(t_0^{+})e^{\gamma(t_1-t_0)}$. Similarly, from (7), we have

$$V(t_1^{+}) \le \beta_1 V(t_1) \le \beta_1 V(t_0^{+})e^{\gamma(t_1-t_0)}$$

and so we have, in general,

$$V(t) \le V(t_0^{+})\beta_1\beta_2\ldots\beta_{k-1}e^{\gamma(t-t_0)}$$

for all $t \in (t_{k-1}, t_k]$, $k = 1, 2, \ldots$. From inequality (5), we obtain that there exists a constant $\xi > 1$ such that $ln(\xi\beta_k) + \gamma\Delta_k \le 0$ for all $k = 1, 2, \ldots$ and all $t \in (t_{k-1}, t_k]$, then

$$\begin{aligned}
V(t) &\le \beta_1\beta_2\ldots\beta_{k-1}V(t_0^{+})e^{\gamma(t_k-t_0)} \\
&\le V(t_0^{+})[\beta_1 e^{\gamma(t_1-t_0)}]\ldots[\beta_{k-1}e^{\gamma(t_{k-1}-t_{k-2})}] \\
&\quad \times e^{\gamma(t_k-t_{k-1})} \\
&\le \frac{1}{\xi^{k-1}}V(e(t_0^{+}))e^{\gamma(t_k-t_{k-1})}
\end{aligned}$$

Thus, the network (4) achieves synchronization.

In practice, the gain matrices B_k and impulsive distances Δ_k are usually chosen to be constants for convenience and we have the following corollary.

*Corollary 1***:** Suppose H_1 holds, γ, β_k are defined as shown above, let the impulses be equidistant and separated by interval $\Delta_k = \Delta$, the control gains $B_k = B'$, $k = 1, 2, \ldots$, the impulsively coupled networks with delay (3) is globally asymptotically synchronized if there exists a constant $\xi > 1$ such that $ln(\xi\beta) + \gamma\Delta \le 0$.

The proof similar to that of Theorem 1, it is omitted here.

IV. NUMERICAL SIMULATIONS

In this section, the chaotic delayed Hopfield neural network is used as nodes of network (1) to show the effectiveness of above-mentioned scheme. The chaotic delayed Hopfield neural network is given by

$$\frac{dx(t)}{dt} = -Cx(t) + Af(x(t)) + Bf(x(t-\tau)), \tag{8}$$

with $x(t) = (x_1(t), x_2(t))^T \in R^2$, $f(x(t)) = (tanh(x_1(t)), tanh(x_2(t)))^T \in R^2$ and

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{bmatrix}, \quad B = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{bmatrix}.$$

The chaotic delayed Hopfield neural network (8) has a very rich complex dynamical behavior and contains, for example, a double-scroll chaotic attractor (depicted in Fig. 1) for time delay $\tau = 1$.

In this simulation, we consider 11 impulsively coupled delayed Hopfield neural networks

$$\begin{cases}
\dfrac{dx_i(t)}{dt} = -Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau)), \\
i = 1, 2, \ldots, 11, \quad t \ne t_k, \\
\Delta x_i = B_k(x_i(t_k) - \sum\limits_{j=1}^{11}\theta_{ij}x_j(t_k)), \quad k = 1, 2, \ldots \\
x_i(t) = \phi_i(t), \quad t \in [t_0 - \tau, t_0], \quad i = 1, 2, \ldots, 11,
\end{cases} \tag{9}$$

The total synchronization error is calculated by

$$Error(t) = \sum_{i=1}^{N}\sqrt{\sum_{k=1}^{2}(x_{ik}(t) - \sum_{j=1}^{N}\theta_{ij}x_{jk}(t))^2}.$$

For the chaotic delayed Hopfield neural network (9), calculations show that $|f_k(x_{im}) - f_k(x_{jm})| \le |x_{im} - x_{jm}|$, and so the Lipschitz constant is $l_i = 1, (i = 1, 2)$.

Fig. 2 shows the evolution process of the total error and the states of the network (9) with $u_i(t) = 0$. It is easy to see that the network (9) is not synchronized without impulsive control.

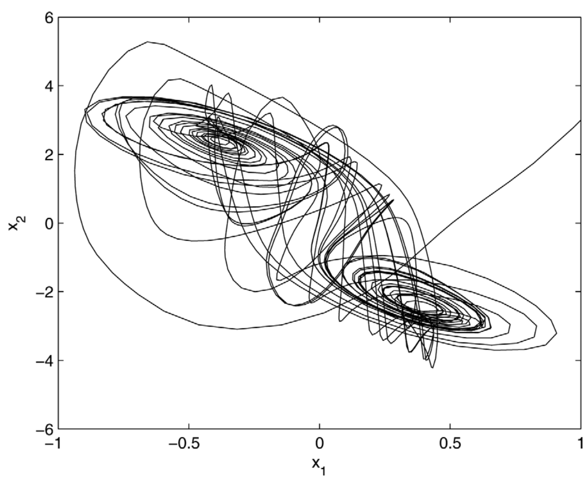

FIG. 1. A fully developed double-scroll-like chaotic attractor of delayed Hopfield neural network.

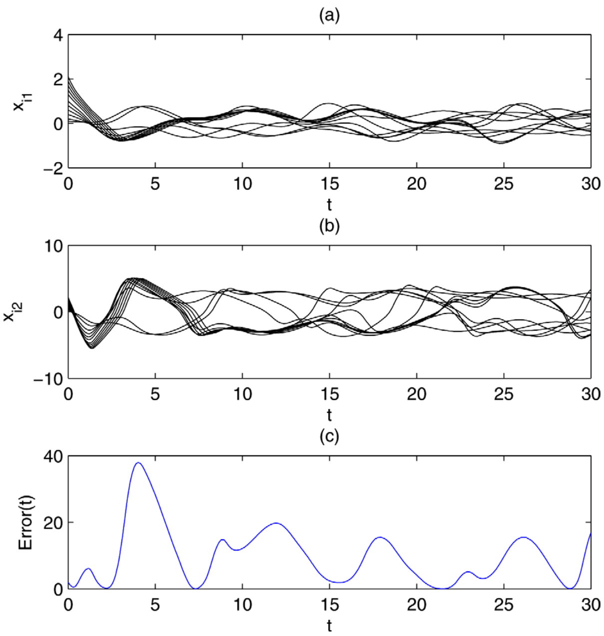

FIG. 2. (Color online) Evolution of (a) the states x_{i1}, (b) the states x_{i2}, and (c) the total Error (t) of network (9) without impulsively coupling.

Let the impulses be equidistant and separated by interval $\Delta = 0.02$ and the control gains $B' = diag\{-0.3, -0.3, -0.3\}$. The initial values of these systems are chosen randomly in the real number interval [0,3], respectively. For simplicity's sake, we firstly assume that all nodes share the same state information from common nodes, i.e., $\theta_{ij} = \theta_j$.

In Fig. 3, let $\theta_j = 1/11$, $j = 1, 2, \ldots, 11$, one can see all of the states tend to coherence asymptotically as time evolves, which implies impulsively coupled delayed Hopfield neural networks (9) achieves synchronization.

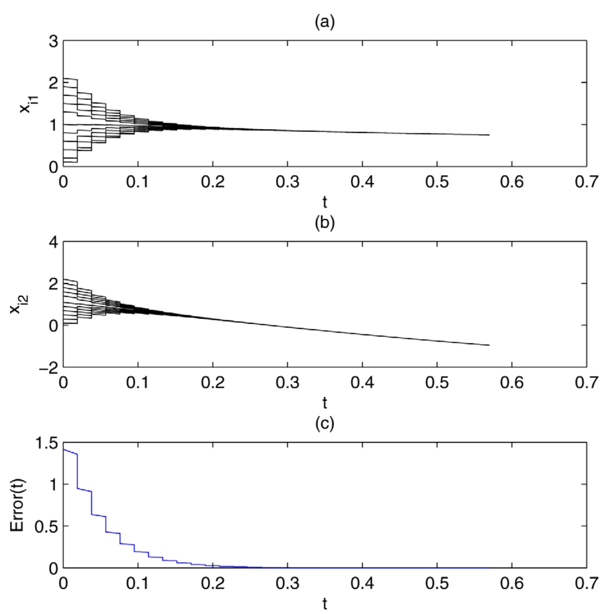

FIG. 3. (Color online) Evolution of (a) the states x_{i1}, (b) the states x_{i2} and (c) the total Error (t) of impulsively coupled systems (9) with $\Delta = 0.02$, $t_0 = 0$ and $\theta_i = 1/11$, $i = 1, 2, \ldots, 11$.

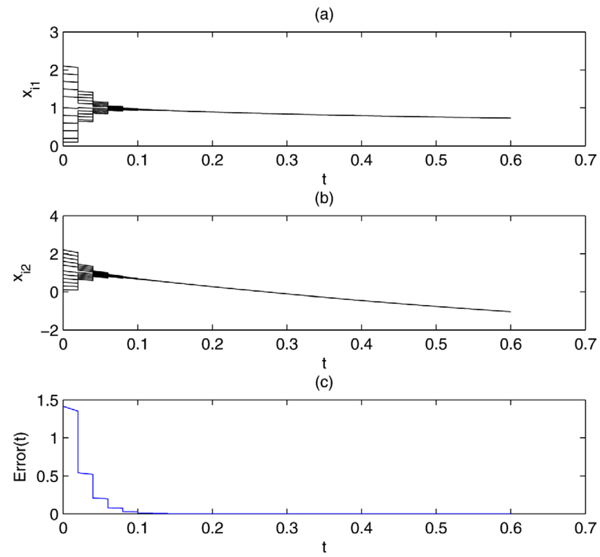

FIG. 4. (Color online) Evolution of (a) the states x_{i1}, (b) the states x_{i2} and, (c) the total Error(t) of impulsively coupled systems (9) with $\Delta = 0.02$, $t_0 = 0$ and $\theta_1 = \theta_2 = \theta_3 = 1/3$, $\theta_i = 0$, $i = 4, 5, \ldots, 11$.

When $\theta_1 = \theta_2 = \theta_3 = 1/3$, $\theta_i = 0$, $i = 4, 5, \ldots, 11$, $B' = diag\{-0.6, -0.6, -0.6\}$, network (9) can also achieve synchronization, which is shown in Fig. 4.

In Fig. 5, let $\theta_1 = 1$, $\theta_j = 0$, $j = 2, \ldots, 11$, and $B' = diag\{-0.18, -0.18, -0.18\}$, one can see all of the states tend to coherence asymptotically as time evolves, which implies impulsively coupled delayed Hopfield neural networks (9) achieves synchronization.

Let $\theta_j = 1/11$, $j = 1, 2, \ldots, 11$, and $\Delta = 2.2$, Fig. 6 shows the evolution process of the total error and the states in network (9), one can see that impulsively coupled delayed Hopfield neural networks (9) cannot achieves synchronization with large impulses interval.

Secondly, we assume that every node shares only the local information from its neighboring nodes, i.e., the weighted directed graph is chosen as

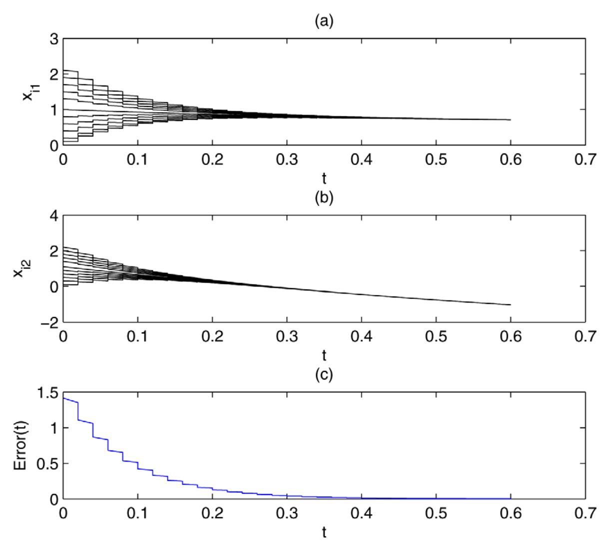

FIG. 5. (Color online) Evolution of (a) the states x_{i1}, (b) the states x_{i2}, and (c) the total Error(t) of impulsively coupled systems (9) with $\Delta = 0.02$, $t_0 = 0$ and $\theta_1 = 1, \theta_i = 0$, $i = 2, 3, \ldots, 11$.

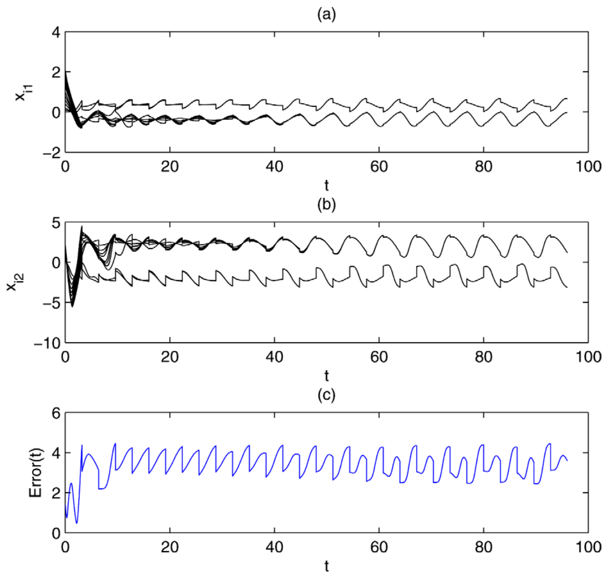

FIG. 6. (Color online) Evolution of (a) the states x_{i1}, (b) the states x_{i2} and (c) the total Error(t) of impulsively coupled systems (9) with $\Delta = 2.2$, $t_0 = 0$ and $\theta_i = 1/11$, $i = 1, 2, \ldots, 11$.

$$-\bar{\theta}_a = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

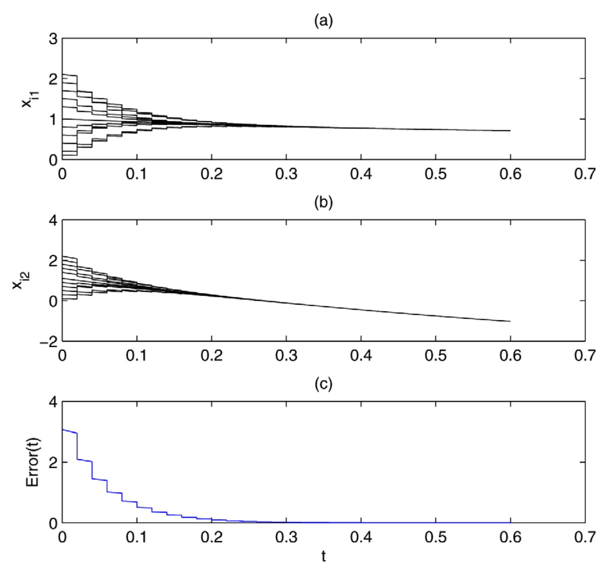

FIG. 7. (Color online) Evolution of (a) the states x_{i1}, (b) the states x_{i2}, and (c) the total Error(t) of impulsively coupled systems (9) with the weighted directed graph $\bar{\theta}_a$.

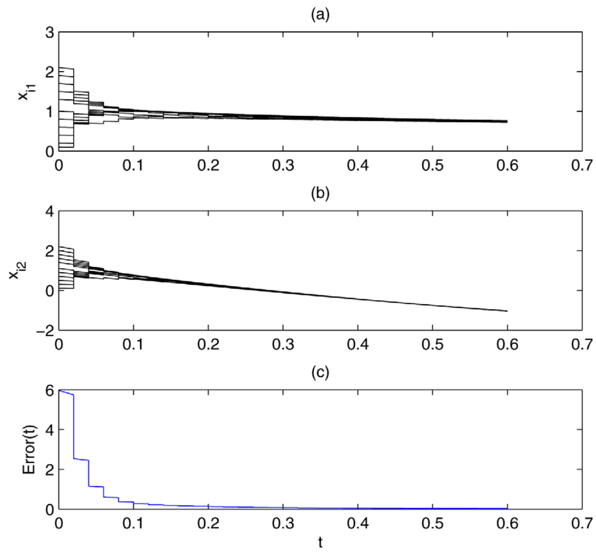

FIG. 8. (Color online) Evolution of (a) the states x_{i1}, (b) the states x_{i2} and (c) the total Error(t) of impulsively coupled systems (9) with the weighted directed graph $\bar{\theta}_b$.

or

$$\bar{\theta}_b = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1\cdots & 0 & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & \cdots & 1 \end{bmatrix}_{11\times 11}.$$

Let $B' = diag\{-0.3, -0.3, -0.3\}$, $\Delta = 0.02$, Figs. 7 and 8 show the evolution process of the states and the total error in network (9) with the weighted directed graph $\bar{\theta}_a$ and $\bar{\theta}_b$, respectively, one can see all of the states tend to coherence asymptotically as time evolves, which implies that distributed impulsively coupled delayed Hopfield neural networks (9) can achieves synchronization but at a slower speed compared with the numerical results in Fig. 3.

V. CONCLUSIONS

This paper has studied the synchronization of complex dynamical systems that are connected impulsively at only discrete instants and whose isolated node experience time delays. Based on the comparison theorem of impulsive differential systems, a new and effective distributed impulsive controller has been designed and analyzed, which ensures the dynamical networks achieve synchronization. The proposed distributed impulsive control scheme is applied to the chaotic delayed Hopfield neural network and numerical simulations are performed to test the effectiveness of the method.

Different from existing control schemes in the literature, complex dynamical systems considered in the paper are coupled only at discrete instants through impulsive connections. The designed distributed impulsive control scheme provides us a flexible and very huge freedom degree for utilizing the local information from neighboring nodes of the network according to different practical situations. The implementation difficulty of the controller is avoided. Our work is a significant extension of the current studies–continuously coupled systems to a much more general and realistic situation–non-continuously coupled systems. Therefore, this method is very meaningful in practicability.

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