Minimization of Impulse Response Using Hybrid Dynamic Vibration Absorber

Yanlung CHEUNG* Waion WONG* and Li CHENG*
*Department of Mechanical Engineering
The Hong Kong Polytechnic University, Hong Kong SAR, China
E-mail: mmwowong@polyu.edu.hk

Abstract
An active-passive dynamic vibration absorber with acceleration feedback is proposed to minimize the impulse response of a single degree-of-freedom (SDOF) system. It has been proved that the proposed hybrid vibration absorber provides better suppression of the impulse response of a single degree-of-freedom system than the traditional passive vibration absorber. The design parameters are the feedback gain, the tuning frequency ratio, the damping ratio and the mass ratio of the absorber. The effects of these parameters on the vibration absorption of the primary structure are analysed based on the analytical model. Design parameters of both passive and active elements of the HVA are optimized for minimizing the impulse response of the primary system. The proposed control system requires only one feedback signal which can be obtained directly using an accelerometer attached onto the primary mass. The control system may be implemented using an analogue circuit connecting an accelerometer, a power amplifier and an inertial actuator mounted onto the primary system. The control system of the proposed hybrid vibration absorber is much simpler than most of the control systems used on active control of vibration absorber found in the relevant literature.

Key words: Dynamic vibration absorber, Optimization

1. Introduction

Dynamic vibration absorber (DVA) is an auxiliary mass-spring-damper system which, when correctly tuned and attached to a vibrating system subject to harmonic excitation, causes to cease the steady-state motion at the point to which it is attached. The first research conduced at the beginning of the twentieth century considered an undamped DVA tuned to the frequency of the disturbing force [1]. Such an absorber is a narrow-band type as it is unable to eliminate structural vibration after a change in the disturbing frequency.

Finding the optimum parameters of a viscous friction DVA in SDOF system drew the attention of many scholars. One of the optimization methods is H_∞ optimization. The optimum design method of DVA is called "Fixed-points theory", which was well documented in the textbook by Den Hartog [2]. Another optimization method is H_2 optimization. The optimum design method of DVA is proposed by Warburton [3,4]. However, the disadvantages of DVA are that the performances of the DVA depend on the mass ratio between the absorber mass and the primary mass [5]. When the mass ratio is small, the performance of the DVA is limited.

In order to improve the performance of the DVA, some researchers rearranged the elements of the DVA such as forming a variant DVA [6-9] and some researchers applied multiple vibration absorbers and Helmholtz resonators to reduce vibration or the sound transmission in a shell [10]. The advantage of these methods is the device is still a passive device. On the other hand, some researchers [11-19] added an active force actuator to a DVA to form a hybrid vibration absorber (HVA). Various methods are proposed for the control of the active element of the HVA including neural network [11], delayed resonator [12], modal feedback control [13-17] and closed-loop poles by modal feedback [18-19]. However, all these control systems of HVA are very complicated and only their researches focus only on the best control of the active element but not the best use of the passive components including the stiffness, damping and mass of the HVA.

In this article, a HVA with an acceleration feedback is optimized for minimizing the impulse response of the primary system. The acceleration feedback is chosen because it is very popular of the engineers and the researchers using the accelerometer to measure the vibration. The feedback circuit and hence the control system can be much simpler than those found in the literature. Both active and passive elements are optimized together for the proposed HVA and its performance on suppressing the impulse response of the primary structure is compared to that of the dynamic vibration absorber optimized for the same purpose [20,21].

2. Theory

A HVA coupled with a primary system where x, M and K denote, respectively, displacement, mass, damper coefficient and stiffness of the primary system x_a, m, c and k those of the absorber is shown as figure 1. The system is mathematically described by

$$\begin{cases} M\ddot{x} = -Kx - k(x - x_a) - c(\dot{x} - \dot{x}_a) - f_a + F \\ m\ddot{x}_a = -k(x_a - x) - c(\dot{x}_a - \dot{x}) + f_a \end{cases} \tag{1}$$

where F is a disturbance and f_a an actuation force.

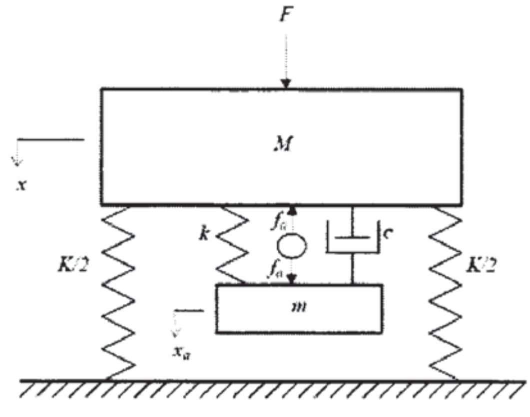

Fig. 1 Schematic diagram of the proposed hybrid vibration absorber (m-k-c-f_a system) attached to the primary (M-K) system.

Let $f_a = -a\ddot{x}$ and taking Laplace Transformation of Eq. (1), the transfer functions of the primary mass M may be written as

$$H(\lambda) = \frac{X}{F/K} = \frac{\gamma^2 - \lambda^2 + 2j\zeta\gamma\lambda}{(1-\lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2 - 2\alpha\lambda^4 + 2j\zeta\gamma\lambda(1-(1+\mu)\lambda^2)} \tag{2a}$$

$$H(\lambda) = \frac{\dot{X}}{\omega_n F/K} = \frac{-2\zeta\gamma\lambda^2 + j\lambda(\gamma^2 - \lambda^2)}{(1-\lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2 - 2\alpha\lambda^4 + 2j\zeta\gamma\lambda(1-(1+\mu)\lambda^2)} \tag{2b}$$

where $\omega_n = \sqrt{\frac{K}{M}}$, $\omega_a = \sqrt{\frac{k}{m}}$, $\lambda = \frac{\omega}{\omega_n}$, $\gamma = \frac{\omega_a}{\omega_n}$, $\zeta = \frac{c}{2\sqrt{mk}}$, $\mu = \frac{m}{M}$ and $\alpha = \frac{a\omega_n^2}{2K}$.

Crandall and Mark [23] proposed another optimization principle of the damped DVA in terms of minimizing the mean square motion of the primary structure under white noise excitation, i.e. $\min_{\gamma,\zeta_a}\left(E\left[x^2\right]\right)$ where

$$E[x^2]=\int_{-\infty}^{\infty}\left|H(\omega)\right|^2 S_{y}(\omega)\,d\omega \tag{3}$$

If the input spectrum is assumed to be ideally white, i.e. $S_{y}(\omega)=S_0$, a constant for all frequencies. Eq. (3) may be written as

$$E\left[x^2\right]=S_0\int_{-\infty}^{\infty}\left|H(\omega)\right|^2 d\omega \tag{4}$$

The non-dimensional mean square velocity of mass M may be defined as

$$E\left[x^2\right]=\frac{\omega_n S_0}{2\pi}\int_{-\infty}^{\infty}\left|H(\lambda)\right|^2 d\lambda \tag{5}$$

Substitute the equation (2a) into Eq. (5), the equation may be written as

$$E\left[x^2\right]=\frac{\omega_n S_0}{2\pi}\int_{-\infty}^{\infty}\left|\frac{\gamma^2-\lambda^2+2j\zeta\gamma\lambda}{\left(1-\lambda^2\right)\left(\gamma^2-\lambda^2\right)-\mu\gamma^2\lambda^2-2\alpha\lambda^4+2j\zeta\gamma\lambda\left(1-(1+\mu)\lambda^2\right)}\right|^2 d\lambda \tag{6}$$

A useful formula of Crandall [23] written as Eq. (7) below is used for solving Eq. (6).

$$\text{If } H(\omega)=\frac{-j\omega^3 B_3-\omega^2 B_2+j\omega B_1+B_0}{\omega^4 A_4-j\omega^3 A_3-\omega^2 A_2+j\omega A_1+A_0} \tag{7}$$

$$\text{then } \int_{-\infty}^{\infty}\left|H(\omega)\right|^2 d\omega=\pi\frac{\left[\frac{B_0^2}{A_0}(A_2A_3-A_1A_4)+A_3\left(B_1^2-2B_0B_2\right)+A_1\left(B_2^2-2B_1B_3\right)+\frac{B_3^2}{A_4}(A_1A_2-A_0A_3)\right]}{A_1(A_2A_3-A_1A_4)-A_0A_3^2} \tag{8}$$

Comparing Eqs. (2a) and (7), the equation may be written as

$$\begin{aligned}&A_0=\gamma^2,\quad A_1=2\zeta\gamma,\quad A_2=1+\gamma^2+\mu\gamma^2,\quad A_3=2\zeta\gamma(1+\mu),\quad A_4=1-2\alpha,\\&B_0=\gamma^2,\quad B_1=2\zeta\gamma,\quad B_2=1,\quad B_3=0\end{aligned} \tag{9}$$

Using Eqs. (8) and (9), Eq. (6) may be rewritten as

$$E\left[x^2\right]=\frac{\omega_n S_0}{4\zeta\gamma}\left(\frac{1+\gamma^2\left(4\zeta^2(1+\mu)-\mu-2(1-\alpha)\right)+(1+\mu)^2\gamma^4}{\mu+2\alpha}\right) \tag{10}$$

If $\frac{\partial}{\partial\gamma}E\left[x^2\right]=\frac{\partial}{\partial\zeta_a}E\left[x^2\right]=0$, the system will have a optimum tuning. The following equations may therefore been considered.

$$\frac{\partial}{\partial\zeta}E\left[x^2\right]=\frac{\omega_n S_0}{4\zeta^2\gamma}\frac{-1+\gamma^2\left(4\zeta^2(1+\mu)+\mu+2(1-\alpha)\right)-(1+\mu)^2\gamma^4}{\mu+2\alpha}=0\text{, and} \tag{11a}$$

$$\frac{\partial}{\partial\gamma}E\left[x^2\right]=\frac{\omega_n S_0}{4\zeta\gamma^2}\frac{-1+\gamma^2\left(4\zeta^2(1+\mu)-\mu-2(1-\alpha)\right)+3(1+\mu)^2\gamma^4}{\mu+2\alpha}=0. \tag{11b}$$

Using Eq. (11a), the damping ratio may be written as

$$\zeta=\sqrt{\frac{1-\gamma^2(\mu+2(1-\alpha))+(1+\mu)^2\gamma^4}{4\gamma^2(1+\mu)}}\,. \tag{12}$$

Using Eq. (11b) the damping ratio may be written as

$$\zeta = \sqrt{\frac{1+\gamma^2(\mu+2(1-\alpha))-3(1+\mu)^2\gamma^4}{4\gamma^2(1+\mu)}}. \tag{13}$$

Using Eqs. (12) and (13), the optimum tuning ratio can be solved and written as

$$\gamma_{\text{opt}} = \sqrt{\frac{\mu+2(1-\alpha)}{2(1+\mu)^2}}. \tag{14}$$

Substituting Eq. (14) into Eq. (12), the optimum damping ratio can be solved and written as

$$\zeta = \sqrt{\frac{(3\mu+2(2-\alpha))(\mu+2\alpha)}{8(1+\mu)(\mu+2(1-\alpha))}} \tag{15}$$

The performance index of the HVA is defined as

$$P.I. = E\left[x^2\right]_{\text{opt}} = \frac{1}{4\zeta_{\text{opt}}\gamma_{\text{opt}}}\left(\frac{1+\gamma_{\text{opt}}{}^2\left(4\zeta_{\text{opt}}{}^2(1+\mu)-\mu-2(1-\alpha)\right)+(1+\mu)^2\gamma_{\text{opt}}{}^4}{\mu+2\alpha}\right) \tag{16}$$

Substituting Eqs. (14) and (15) into (16), Eq. (16) may be simplified as

$$P.I. = \frac{1}{2}\sqrt{\frac{3\mu+2(2-\alpha)}{(1+\mu)(\mu+2\alpha)}} \tag{17}$$

Similarly, the optimum tuning for the HVA minimizing the mean square velocity can be obtained. The performance index of the HVA minimizing the velocity is

$$P.I. = \frac{1+2\gamma^2\left(2\zeta^2-1\right)(1-2\alpha)+\gamma^4(1-2\alpha)(1+\mu)}{4\zeta\gamma(1-2\alpha)(\mu+2\alpha)} \tag{18}$$

The optimum frequency and damping of the proposed HVA are shown in Table 1. It can be observed that the performance index of $E\left[x^2\right]_{\text{opt}}$ is decreased when the mass ratio and the feedback gain are increased. That means there is no optimum mass ratio and feedback gain for minimizing the performance index of $E\left[x^2\right]$. It can also be observed that the performance index of $E\left[\dot{x}^2\right]$ is decreased when the mass ratio is increased. However, the performance index of $E\left[\dot{x}^2\right]$ has a minimum when $\alpha = \frac{1-\mu}{4}$.

Table 1. H_2 Optimum tuning of HVA with acceleration feedback

Transfer Function	Optimum frequency	Optimum damping	Performance index
$E\left[x^2\right]$	$\sqrt{\frac{\mu+2(1-\alpha)}{2(1+\mu)^2}}$	$\sqrt{\frac{(3\mu+2(2-\alpha))(\mu+2\alpha)}{8(1+\mu)(\mu+2(1-\alpha))}}$	$\frac{1}{2}\sqrt{\frac{3\mu+2(2-\alpha)}{(1+\mu)(\mu+2\alpha)}}$
$E\left[\dot{x}^2\right]$	$\sqrt{\frac{1}{1+\mu}}$	$\sqrt{\frac{\mu+2\alpha}{4(1-2\alpha)}}$	$\frac{1}{\sqrt{(\mu+2\alpha)(1-2\alpha)(1+\mu)}}$

3. The stability of the HVA

The control system is stable if the real parts of all poles are negative. The characteristic equation of the control system may be written as

$$(1-2\alpha)p^4 + 2\zeta\gamma(1+\mu)p^3 + \left(1+\gamma^2+\mu\gamma^2\right)p^2 + 2\zeta\gamma p + \gamma^2 = 0 \tag{19}$$

where $p = \frac{s}{\omega_n}$ and $\forall \zeta, \gamma, \mu, \alpha \in R^+$

Applying the Routh's stability criterion, the array of coefficient may be written as

$$\begin{array}{c|ccc} p^4 & 1-2\alpha & 1+\gamma^2+\mu\gamma^2 & \gamma^2 \\ p^3 & 2\zeta\gamma(1+\mu) & 2\zeta\gamma & 0 \\ p^2 & \dfrac{(1+\mu)^2\gamma^2+\mu+2\alpha}{1+\mu} & \gamma^2 & \\ p & \dfrac{2\zeta\gamma(\mu+2\alpha)}{(1+\mu)^2\gamma^2+\mu+2\alpha} & & \\ 1 & \gamma^2 & & \end{array} \tag{20}$$

Using Eq. (20), the system is stable if $\alpha < \frac{1}{2}$ according to the Routh's stability criterion. The roots locus of Eq. (19) with $\mu = 0.2$ is plotted in Fig. 2 for illustration. According to Table 1, the optimum frequency and damping are real values if the feedback gain $\alpha < \frac{1}{2}$.

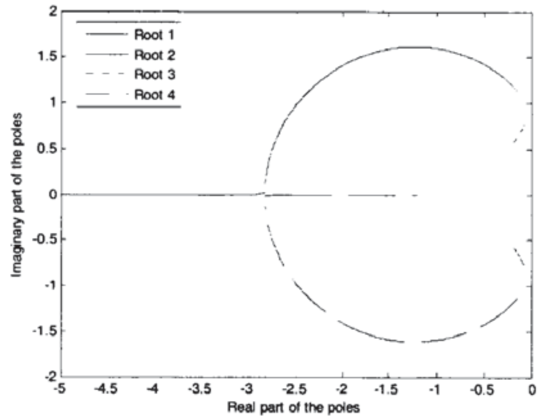

Fig. 2 Root locus of the primary structure with HVA with $\mu = 0.2$

4. Numerical Example

The mean square motion of a simply supported beam attached with the proposed HVA and excited by a uniform distributed force as shown in Fig. 3 is considered as an example. The length of the beam is $L = 1\text{m}$ and a HVA is attached at $x = x_0 = 0.5\text{m}$. The dimensions of the cross section are $0.025\text{m} \times 0.025\text{m}$. The mass ratio of the HVA is 0.05. The material of the beam is aluminium with $\rho = 2710\text{kgm}^{-3}$ and $E = 6.9\text{GPa}$.

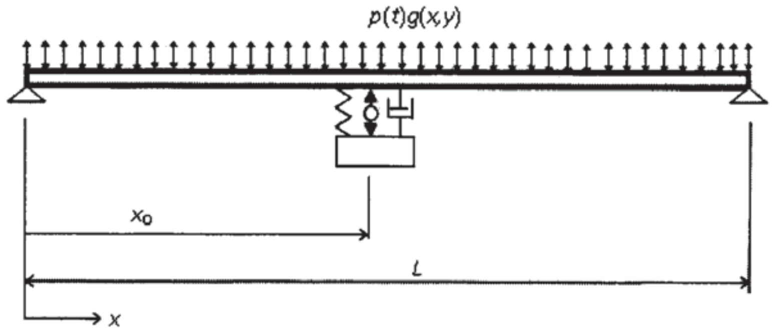

Fig. 3 Schematics of a simply supported beam with a hybrid vibration absorber excited by a uniform disturbed force.

The equations of motion of the dynamic system may be written as

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = p(t)g(x) + F_h(t)\delta(x - x_0) \tag{21a}$$

$$m\ddot{x}_a = k(w(x_0) - x_a) + c(\dot{w}(x_0) - \dot{x}_a) + f_a \tag{21b}$$

$$F_h(t) = -m\ddot{x}_a \tag{21c}$$

Here it is assumed that the externally applied forcing function is $p(t)g(x)$, where $p(t)$ is a function of time and $g(x)$ is a deterministic function of x. $F_h(t)$ is the force applied to the beam from the HVA. f_a is the active force from the HVA. The eigenfunction and the eigenvalues of the beam can be written as

$$\varphi_i(x) = \frac{2}{L}\sin(\beta_i x) \text{ where } \beta_i = \frac{i\pi}{L},\ i \in N. \tag{22}$$

The solution to this problem can be expanded in a Fourier series written as

$$w(x,t) = \sum_{p=1}^{\infty} q_p(t)\varphi_p(x) \tag{23}$$

$$\text{where } \int_0^L \varphi_i^2(x)dx = L, \text{ where } i \in N. \tag{24}$$

Similarly, the spatial part of the forcing function can be expanded as

$$g(x) = \sum_{p=1}^{\infty} a_p \varphi_p(x). \tag{25}$$

The derivative of Dirac delta functions may also be expanded as

$$\delta(x - x_0) = \sum_{p=1}^{\infty} b_p \varphi_p(x) \tag{26}$$

where the Fourier coefficients a_i and b_i are respectively

$$a_i = \frac{2}{n\pi L}, \text{ where } n \text{ is odd else } a_i = 0 \tag{27a}$$

$$b_i = \frac{2}{L}\sin\left(\frac{i\pi x_o}{L}\right). \tag{27b}$$

Here a_i depend only on the spatial distribution of the forcing function $g(x)$. If Eqs. (22), (23), (24), (25), (26) and (27) are substituted into Eq. (21a) and the Laplace transform is taken with respect to time, the result is a set of algebraic equations written as

$$\rho A s^2 Q_i(s) + EI\beta_i^4 Q_i(s) = a_i P(s) + b_i F_h(s), \text{ where } i \in N. \tag{28}$$

If this is solved for the generalized co–ordinates $Q_i(s)$ the result is

$$Q_i(s) = \frac{a_i P(s) + b_i F_h(s)}{\rho A s^2 + EI\beta_i^4}. \tag{29}$$

If $P(s)$ and $F_h(s)$ were known then the s–domain motion of the beam may be written as

$$W(x,s) = \sum_{p=1}^{\infty} \frac{a_p P(s) + b_p F_h(s)}{\rho A s^2 + EI\beta_p^4}\varphi_p(x) \tag{30}$$

where $W(x,s)$ is the Laplace transform of $w(x,t)$ with respect to time. Let the active force be $f_a = -a\ddot{w}(x_0)$. Using Eqs. (21b) and (21c), the relations between the motion of the point of attachment and the force transmitted to the beam at the point of attachment is

$$F_h(s) = -W(x_0,s)\frac{ms^2\left(k + cs - as^2\right)}{ms^2 + cs + k} \tag{31}$$

$W(x,s)$ can be obtained using Eqs. (21d), (30) and (31) and written as

$$W(x,s)=\sum_{p=1}^{\infty}\frac{a_pF(s)-b_pW(x_0,s)\dfrac{ms^2\left(k+cs-as^2\right)}{ms^2+cs+k}}{\rho As^2+EI\beta_p^4}\varphi_p(x). \tag{32}$$

Substituting $x=x_0$ into Eqs. (32), $W(x_0,s)$ can be obtained and written as

$$W(x_0,s)=\frac{\displaystyle\sum_{p=1}^{\infty}\frac{a_p\varphi_p(x_0)F(s)}{\rho As^2+EI\beta_p^4}}{1+\displaystyle\sum_{q=1}^{\infty}\frac{b_q\varphi_q(x_0)\dfrac{ms^2\left(k+cs-as^2\right)}{ms^2+cs+k}}{\rho As^2+EI\beta_q^4}} \tag{33}$$

Substituting Eq. (33) into (32), the transfer function of the beam may be written as

$$\frac{W(x,s)}{F(s)}=\sum_{p=1}^{\infty}\frac{a_p-b_p\dfrac{\displaystyle\sum_{q=1}^{\infty}\frac{a_q\varphi_q(x_0)}{\rho As^2+EI\beta_q^4}}{\dfrac{ms^2+cs+k}{ms^2\left(k+cs-as^2\right)}+\displaystyle\sum_{r=1}^{\infty}\frac{b_r\varphi_r(x_0)}{\rho As^2+EI\beta_r^4}}}{\rho As^2+EI\beta_p^4}\varphi_p(x). \tag{34}$$

Substituting $s=j\omega$ and replacing the dimensionless parameters of the Eq. (34), the frequency response function of the beam may be written as

$$\frac{W(x,\omega)}{F(s)}=\frac{1}{\rho A\omega_n^2}\sum_{p=1}^{\infty}\frac{a_p-b_p\dfrac{\mu L\displaystyle\sum_{q=1}^{\infty}\frac{a_q\varphi_q(x_0)}{\gamma_q^2-\lambda^2}}{-\dfrac{\gamma^2-\lambda^2+2j\zeta\gamma\lambda}{\lambda^2\left(\gamma^2+2j\zeta\gamma\lambda+\dfrac{2\alpha\lambda^2}{\varepsilon}\right)}+\mu L\displaystyle\sum_{r=1}^{\infty}\frac{b_r\varphi_r(x_0)}{\gamma_r^2-\lambda^2}}}{\gamma_p^2-\lambda^2}\varphi_p(x) \tag{35}$$

where $\omega_n=\sqrt{\dfrac{EI\beta_1^4}{\rho A}}$, $\omega_r=\sqrt{\dfrac{EI\beta_r^4}{\rho A}}$, $\gamma_r=\dfrac{\omega_r}{\omega_n}$, $\mu=\dfrac{m}{\rho AL}$, $\varepsilon=\mu\varphi_1^2(x_0)$

and

$$\alpha=\frac{a\omega_n^2\varphi_1^2(x_0)}{2EIL\beta_1^4}.$$

The eigenfunctions of the beam obey the orthogonality relations written as

$$\int_0^L\varphi_i(x)\varphi_j(x)dx=0,\ \text{if}\ i\neq j \tag{36a}$$

$$\int_0^L\varphi_i(x)\varphi_j(x)dx=L,\ \text{if}\ i=j \tag{36b}$$

Consider the orthogonality relations and the equation $\left|\dfrac{W(x,s)}{F(s)}\right|^2=\dfrac{W(x,s)}{F(s)}\times\overline{\left(\dfrac{W(x,s)}{F(s)}\right)}$, the mean square motion over the whole domain of the beam may be written as

$$\frac{\int_0^L \left|\frac{W(x,\lambda)}{P(\lambda)}\right|^2 dx}{L} = \left(\frac{1}{\rho A\omega_n^2}\right)^2 \sum_{p=1}^{\infty} \left| \frac{a_p - \dfrac{\mu L b_p \sum_{q=1}^{\infty} \dfrac{a_q \varphi_q(x_0)}{\gamma_q^2 - \lambda^2}}{-\dfrac{\gamma^2 - \lambda^2 + 2j\zeta\gamma\lambda}{\lambda^2\left(\gamma^2 + 2j\zeta\gamma\lambda + \dfrac{2\alpha_2\lambda^2}{\varepsilon}\right)} + \mu L \sum_{r=1}^{\infty} \dfrac{b_r \varphi_r(x_0)}{\gamma_r^2 - \lambda^2}}}{\gamma_p^2 - \lambda^2} \right|^2 \tag{37}$$

Similarly, the mean square motion of $\dot{W}(x,\lambda)$ over the whole domain of the beam may be written as

$$\frac{\int_0^L \left|\frac{\dot{W}(x,\lambda)}{\omega_n P(\lambda)}\right|^2 dx}{L} = \left(\frac{\lambda}{\rho A\omega_n^2}\right)^2 \sum_{p=1}^{\infty} \left| \frac{a_p - \dfrac{\mu L b_p \sum_{q=1}^{\infty} \dfrac{a_q \varphi_q(x_0)}{\gamma_q^2 - \lambda^2}}{-\dfrac{\gamma^2 - \lambda^2 + 2j\zeta\gamma\lambda}{\lambda^2\left(\gamma^2 + 2j\zeta\gamma\lambda + \dfrac{2\alpha_2\lambda^2}{\varepsilon}\right)} + \mu L \sum_{r=1}^{\infty} \dfrac{b_r \varphi_r(x_0)}{\gamma_r^2 - \lambda^2}}}{\gamma_p^2 - \lambda^2} \right|^2 \tag{38}$$

According to references [20] and [21], the approximated optimum frequency and damping of the HVA for a multi-degree-of-freedom primary system can be derived in a way similar to that of the SDOF primary system and they are listed in Table 2.

Table 2. Approximated H_2 Optimum tuning of HVA with acceleration feedback for a beam structure

Transfer Function	Optimum tuning	Optimum damping	Performance index
$E[w^2]$	$\sqrt{\dfrac{\varepsilon+2(1-\alpha)}{2(1+\varepsilon)^2}}$	$\sqrt{\dfrac{(3\varepsilon+2(2-\alpha))(\varepsilon+2\alpha)}{8(1+\varepsilon)(\varepsilon+2(1-\alpha))}}$	$\dfrac{1}{2}\sqrt{\dfrac{3\varepsilon+2(2-\alpha)}{(1+\varepsilon)(\varepsilon+2\alpha)}}$
$E[\dot{w}^2]$	$\sqrt{\dfrac{1}{1+\varepsilon}}$	$\sqrt{\dfrac{\varepsilon+2\alpha}{4(1-2\alpha)}}$	$\dfrac{1}{\sqrt{(\varepsilon+2\alpha)(1-2\alpha)(1+\varepsilon)}}$

Similar to the case of SDOF primary system, the active gain of the HVA for MDOF primary system can be derived and written as $\alpha = \dfrac{1-\varepsilon}{4}$. The mean square motion and velocity of the beam are calculated according to Eqs. (37) and (38) respectively and they are compared to the DVA with the corresponding optimum frequency and damping [20,21] and the results are shown in Figure 4 and Figure 5 respectively. It can be observed from Figs. 4 and 5 that the suppression using the proposed HVA is much better than the DVA. Moreover, the suppression of mean square motion and velocity of the beam using the HVA at the higher modes are also better than those using the passive DVA.

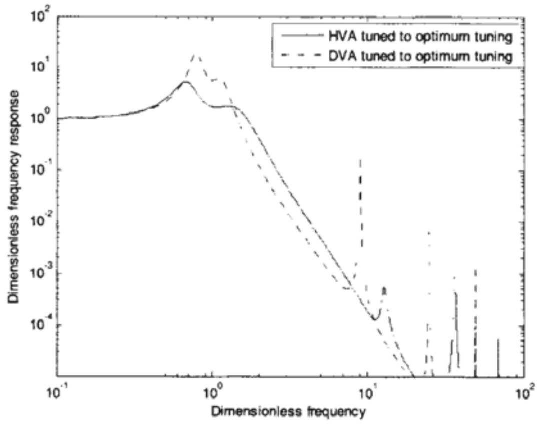

Fig. 4 Spatial average mean square motion $\int_0^L \left|\frac{W(x,\lambda)}{P(\lambda)}\right|^2 dx$ of the beam using the proposed optimal HVA compared to the optimal DVA [20,21]

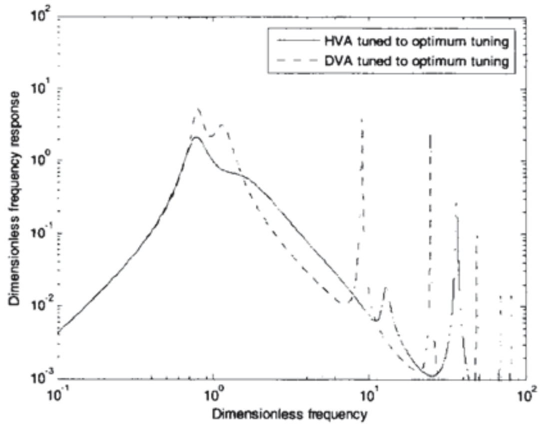

Fig. 5 Spatial average mean square velocity $\int_0^L \left|\frac{\dot{W}(x,\lambda)}{\omega_n P(\lambda)}\right|^2 dx$ of the beam using the proposed optimal HVA compared to the optimal DVA [20,21]

5. Conclusion

An active-passive dynamic vibration absorber with acceleration feedback is proposed to minimize the impulse response of a single degree-of-freedom (SDOF) system. It has been proved that the proposed hybrid vibration absorber provides better suppression of the impulse response of a single degree-of-freedom system than the traditional passive vibration absorber. The design parameters are the feedback gain, the tuning frequency ratio, the damping ratio and the mass ratio of the absorber. The effects of these parameters on the vibration absorption of the primary structure are analysed based on the analytical model. The proposed control system requires only one feedback signal which can be obtained directly using an accelerometer attached onto the primary mass. The control system may be implemented using an analogue circuit connecting an accelerometer, a power amplifier and an inertial actuator mounted onto the primary system. The proposed absorber has also been applied to a multi degree-of-freedom vibrating system and the suppression of mean square motion and velocity of the beam using the HVA at the higher modes are also better than those using the passive DVA.

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