Analysis of the Resonance Modes of PZT/Epoxy 1-3 Composite Rings

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Abstract: PZT/epoxy 1-3 composite rings with PZT volume fractions Φ ranging from 0.58 to 0.93 are fabricated. They have sufficiently small epoxy width (~77 - 81 µm) and can be treated as effective homogeneous media. The mode coupling theory and a finite element model (FEM) are applied to predict the thickness (f_T), lateral (f_L), radial (f_R) and wall-thickness (f_W) resonances of the composite rings. As the frequencies and electromechanical coupling coefficients play a significant role in the performance of composite transducer, it is important to know how these parameters change with Φ. Good agreements are found between the experimental results and the FEM simulations. It is obvious that f_R and f_W vibrations greatly deteriorate at low value of Φ. To avoid unwanted modes from coupling to f_T, the thickness and configuration of the PZT elements in the rings should be optimized.

1. INTRODUCTION

Combining a piezoelectric ceramic with a passive polymer phase can form a variety of new piezoelectric materials [1-4]. By proper selection of the ceramic, polymer, volume fractions, and PZT element aspect ratio (Width to Height, L/T), the material properties can be tailored to specific device requirements. The resonance characteristics and mode coupling of 1-3 composites have been studied [2,4], and such materials have been used effectively in hydrophone and medical ultrasonic imaging applications [8].

Piezoceramic rings are widely used in Langevin sandwich transducers for ultrasonic motor, welding tool and other high power applications. Since the dynamic characteristics of a ring are different from that of a solid disc, some models and characteristics of PZT have been studied and reported [5-7]. As the length of the Langevin transducers is comparable to the diameter of the ring, therefore, the radial and wall-thickness resonances of the ring may couple with the desired axial mode of transducer [8]. To alleviate this problem, 1-3 composite rings can be used to suppress mode coupling and maintain a pure axial mode.

In this study, PZT/epoxy 1-3 composite rings with PZT volume fractions Φ ranging from 0.58 to 0.93 are fabricated. A commercial FEM code (ANSYS version 5.6) was used to analysis the vibration characteristics of the composite rings and compare to experimental results. The effects of varying Φ on the resonant frequency F_r and effective electromechanical coupling coefficient factor k_{eff} are predicted.

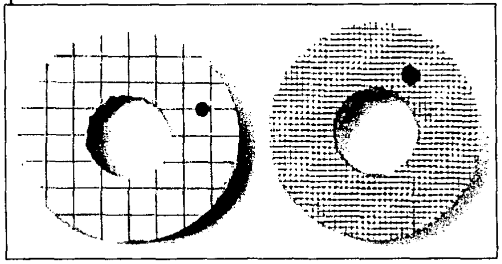

Fig. 1. Photographs showing the 1-3 composite rings with various ceramic volume fraction Φ (a) 0.91, (b) 0.58.

2. SAMPLES PREPARATION

The 1-3 composite rings were fabricated by the dice-and-fill technique [7]. Commercial PZT, APC840 (12.7 mm outer diameter, 5.05 mm inner diameter, and 2.3 mm thickness), was used as the active phase in the 1-3 structure. Araldite LY5138-2/HY5138 epoxy was used as the passive phase. Parallel grooves were cut in two perpendicular directions on the PZT rings by using a Disco DAD 321 automatic dicing saw equipped with a 70 µm thick diamond saw blade. Due to blade vibration, the resulting grooves were about 77 to 81 µm in width. By making different number of cuts per direction (cpd) from 5 to 39 cpd, composite rings with different PZT element

Table I. 1-3 piezocomposite rings with 12.7 mm outer diameter, 5.05 mm inner diameter, and ~77-81 µm epoxy width.

Type	**APC840**	**APC840/Epoxy Composite**									
Cuts per direction (cpd)	0	5	7	9	13	17	21	25	29	33	39
PZT Volume fraction Φ	1.0	0.93	0.91	0.88	0.84	0.79	0.75	0.71	0.67	0.63	0.58
PZT element aspect ratio (L/T)	---	0.99	0.75	0.57	0.47	0.38	0.32	0.28	0.25	0.18	0.15
Average thickness (mm)	2.30	2.05	2.0	2.07	1.94	1.84	1.79	1.77	1.70	1.59	1.55

widths and different ceramic volume fraction Φ from 0.93 to 0.58 were obtained. After that, epoxy was filled under vacuum, and the cured composite rings were polished. The samples of 7 and 39 cpd are shown in Fig 1. The descriptions of composite rings are tabulated in Table I. The composite rings have sufficiently small epoxy width (~78 - 81 μm) and can be treated as effective homogenous material with a set of effective material properties.

3. RESONANCE MODES IN 1-3 COMPOSITE RINGS

3.1 Resonance Characteristics of Composite Rings

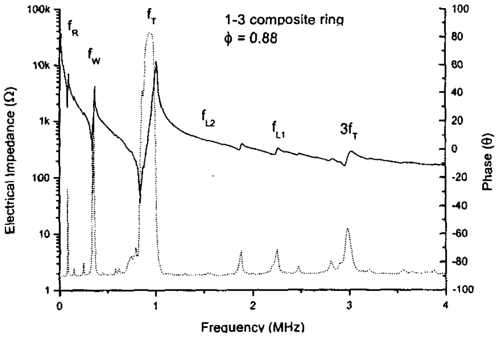

Fig. 2. Plots of electrical impedance (solid line) and phase angle (dot line) versus frequency plots for a 9 cpd composite ring. (thickness = 2.07mm)

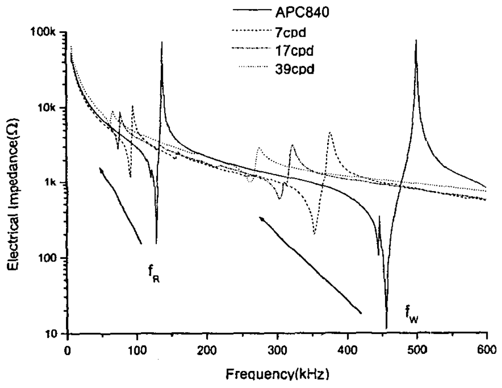

Fig. 3. The electrical impedance shows the radial (f_R) and wall-thickness (f_W) resonances of ceramic and composite rings.

The ceramic and composite rings was poled in the thickness direction and covered with silver electrode on the top and bottom surfaces. The electrical impedance spectrums of the rings were measured using a HP4194A impedance analyzer. Resonance modes of a 1-3 piezocomposite ring can be classified into two main categories (Fig. 2) as follows [4,7]:

1. Characteristics resonances of individual PZT elements inside the ring:
   (a) The thickness mode resonance f_T.
   (b) The coupled lateral mode resonances f_{L1} and f_{L2}.
2. Cooperative resonances of the whole ring:
   (a) The radial (f_R) and wall-thickness (f_W) mode resonances.
   (b) The stopband resonances f_{S1} and f_{S2}, which are resulted from the periodic structure of the composite ring.

Fig. 2 shows the resonance modes of a 1-3 composite ring with Φ = 0.88 (9cpd) and L/T = 0.57 [6]: (1) f_T(= 842.5 kHz), (2) f_R (= 85.75 kHz), (3) f_W(= 341.5 kHz), other peaks are the lateral modes f_{L1} and f_{L2}, and harmonics $3f_T$.

3.2 Radial and Wall-Thickness Resonances of Rings

The impedance vs frequency curves of the ceramic and composite rings are given in Fig. 3. The first peak corresponds to a radial motion of the ring whose walls move in the same direction. The second peak is also a radial mode but, in this case, the two walls of the ring move in opposite directions. It can be seen that for a composite ring with lower volume fraction, f_R and f_W become weaker. This implies that a polymer is effective in damping the motions in the lateral direction.

In order to understand the mode shapes, finite-element analysis have been applied to piezoelectric rings in various configurations. This technique is commonly used in analyzing vibration characteristics of piezoceramic rings, including the dependence of a variety of resonance modes on the thickness T, wall-thickness *w*, and relative displacement [5-6]. In the present works, a three-dimensional (3-D) finite-element analysis was used to study the effects of dimensions on the composite ring vibrations. Frequencies at lateral modes were calculated and compared to experimental data. The material parameters of composite rings, such as the elastic compliance ($s_{11}^E, s_{12}^E, s_{13}^E, s_{33}^E, s_{44}^E, s_{66}^E$), relative permittivity ($\varepsilon_{11}^S, \varepsilon_{33}^S$), and density (ρ), were calculated by a 1-3 composite mode [2,3], and then used as the input parameters of FEM.

The vibration motion in one direction is coupled to that in the other direction when the thickness T of the ring is comparable to wall-thickness *w*. The resonant frequency Fr and effective coupling factor k_{eff} of the ring are modeled as function of thickness T. The computed Fr and k_{eff} are shown in Fig. 4. All the resonant frequencies decrease as T increases, giving approximately a linear function. However, the wall-thickness mode of the ceramic ring has a large change with a rate ~38.0 kHz per mm, other resonaces remain constant (their rate is < 0.5 kHz per mm). Showing an opposite trend, all k_{eff} increase as T increases and, similarly, the wall-thickness mode of the ceramic ring has a larger change. As a result, it shows the wall thickness resonance of ceramic ring is dependent to T, while this dependence disappears in composite rings, reflecting the damping effect of the polymer in the lateral direction [2,4].

All radial and wall-thickness resonant frequencies (f_R and f_W) and effective coupling factors k_{eff} are determined and plotted together with the theoretical

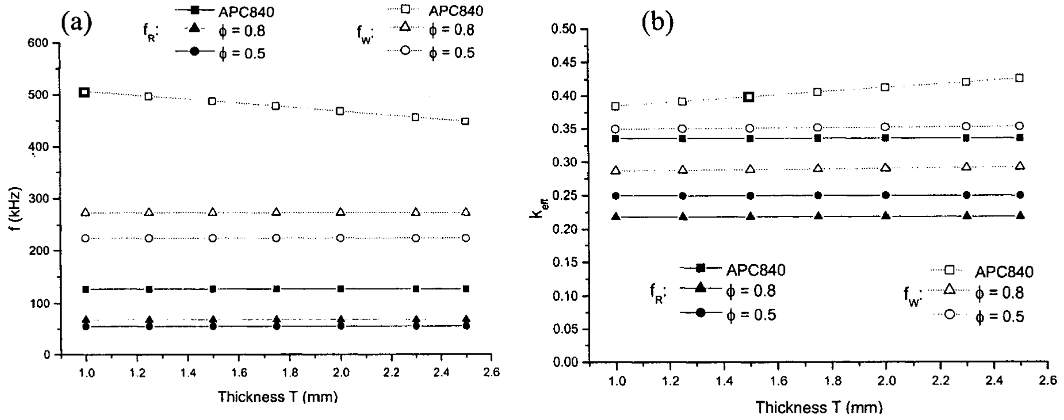

Fig. 4. The variation of (a) f_R and f_w (b) k_{eff} as a function of ring thickness T; Inner diameter is 5.1 mm and outer diameter is 12.7 mm.

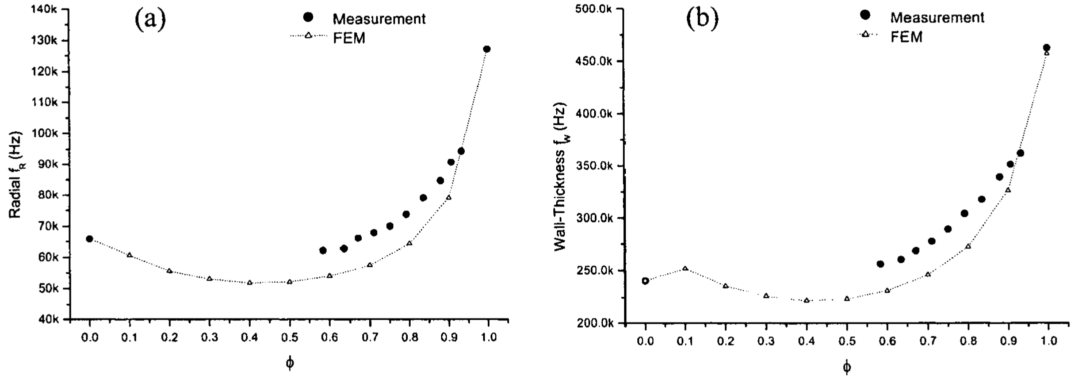

Fig. 5. The variation of (a) radial f_R and (b) wall-thickness f_w resonances as a function of Φ.

curves and FEM results in Figs. 5 and 6. One can clearly see in that these two lateral modes are suppressed significantly in the low Φ samples and trends are quite similar. The difference between FEM results and the experimental date arises because the FEM curves are calculated using parameters estimated by a simple model.

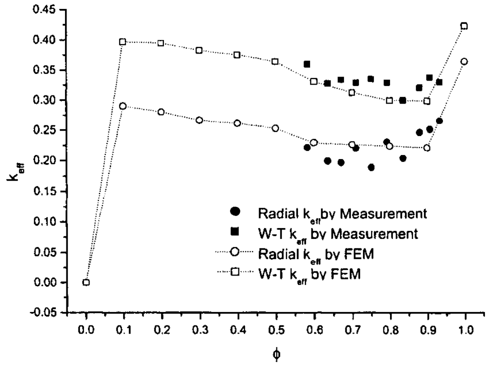

Fig. 6.. The variation of k_{eff} as a function of Φ.

3.3 Lateral Resonance of PZT elements

The mode-coupling theory can be used to explain the occurrence of lateral mode in 1-3 composite [4,7]. To consider a single element inside a 1-3 composite, it is a square pillar with width L in x and y direction (lateral direction) and with height T in the z direction (poling direction). The frequency equation of this square pillar can be expressed as [4]

$$[f^2 - f_a^2(1-\gamma)]\{f^4 - [f_a^2(1+\gamma)+f_b^2]f^2 + f_a^2 f_b^2(1+\gamma-2\alpha^2)\} = 0, \quad (1)$$

where

$$f_a = \frac{1}{2L}\sqrt{\frac{c_{11}}{\rho}} \text{ and } f_b = \frac{1}{2T}\sqrt{\frac{c_{33}}{\rho}}, \quad (2)$$

are the uncoupled lateral and longitudinal-thickness resonance frequencies, respectively; $\gamma = c_{12}/c_{11}$ and $\alpha = c_{13}/\sqrt{c_{11}c_{33}}$ are the coupling constants; c_{11}, c_{12}, c_{13}, and c_{33} are the elastic stiffness constants; and ρ is the density. Equation (2) can be factorized into a quadratic and a biquadratic equation [4], and therefore the coupled upper lateral resonance frequency f_{L1}, the coupled lower lateral resonance frequency f_{L2}, and the coupled longitudinal-thickness f_T can be determined. The biquadratic equation has two solution f_+ and f_-, which gives two frequency branches; f_{L1} and f_T are governed by these two frequency branches in terms of *G* (PZT element aspect ratio L/T). Hence, the theoretical curves for the uncoupled and coupled resonance frequencies can be derived using the material parameters of APC840.

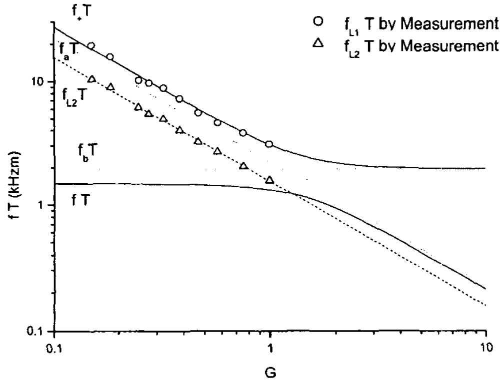

Fig. 7. Log-log plots of resonance frequency multiply by the element height $f \cdot T$ versus configuration ratio G for APC840 elements under short-circuit conditions. The line and symbols represent the theoretical and experimental results, respectively.

Fig. 2 shows the lateral modes appearing in the electrical impedance spectrum of a composite ring with Φ = 0.88 (9cpd) and L/T = 0.57. Guided by the mode-coupling theory, the two lateral modes f_{L1} and f_{L2} are weak resonances found at 1.3 MHz and 2.23 MHz. Other composite samples have similar spectra, but with different in positions of f_{L1} and f_{L2}. For some samples, they are often mixed with other high frequency resonances. The general behaviors of f_{L1} and f_{L2} explanted by mode coupling using thinning test have been reported [12]. The observed f_{L1} and f_{L2} for all the composite samples are plotted in Fig. 9 as the short-circuit cases together with the mode-coupling theory predictions, and good agreements are obtained. Although the samples have different thickness, there is no influence on fT, reflecting no dependence to *T*. At $G > 0.8$, significant mode coupling of f_T with f_{L1} and f_{L2} occurs, so the data exhibit some scattering. Oppositely, for $G < 0.8$ (low Φ), f_T exhibits very sharp and clear.

4. CONCLUSION

PZT/epoxy 1-3 composite rings with PZT volume fractions Φ ranging from 0.58 to 0.93 are fabricated, and lateral resonance characteristics have been studied experimentally with comparison to FEM simulation. They have sufficiently small epoxy width (~77 - 81 μm) and can be treated as effective homogeneous media. As the volume fraction of ceramic Φ decreases, f_R and f_W become weaker. Their behavior will change when the thickness T is comparable to wall-thickness *w*. The lateral modes f_{L1} and f_{L2} can be accurately predicted by the mode-coupling theory even though composite rings with different Φ and T were used. Overall, in designing various transducers, f_T should not be coupled to the lateral and spurious modes. The current results can be used to optimize the fabrication of composite ring.

5. ACKNOWLEDGMENTS

The authors would like to acknowledge the supports by the Center for Smart Materials of the Hong Kong Polytechnic University, the Innovation and Technology Fund (ITF UIM/29) of the HKSAR Government and ASM Assembly Automation Ltd.

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