Numerical study on the dynamics of driven disordered colloids

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(Received 18 October 2002; revised manuscript received 25 June 2003; published 17 October 2003)

Using Langevin simulations, we numerically investigate the dynamics of driven two-dimensional colloids subject to randomly distributed pointlike pinning centers. Increasing the strength of pinning centers, we find a crossover from elastic to plastic depinnings, where a substantial increase in the depinning force is observed. The influence of temperature is examined, and we find a dynamic melting transition from the moving smectic to the moving liquid at high driving forces. A peak is found in the dynamic critical driving force across the transition, accompanied by a crossing of velocity-force dependence curves.

DOI: 10.1103/PhysRevB.68.134209 PACS number(s): 82.70.Dd, 64.60.Ht

There has been considerable interest in colloidal crystals during the past two decades, since they provide ideal model systems for studies of various problems in material science,[1] physical chemistry,[2] and condensed matter physics.[3] In particular, the confined colloidal crystals have been used to investigate the general problems of ordering and dynamics in two-dimensions.[4–8] Much attention has focused on the melting of two-dimensional (2D) colloidal crystals in the absence of disorder,[4,5,7] and a reentrant behavior in the orientational order has recently been found.[7] More recently, it has been demonstrated[9] that point defects can be created in 2D colloidal crystals by manipulating individual particles with optical tweezers. Thereafter, many authors studied the colloidal crystallization and melting in 2D systems with disordered substrates.[10,11]

Although the equilibrium thermodynamics of colloidal crystals in two-dimensions has been studied extensively, the nonequilibrium dynamics of driven 2D colloids interacting with quenched disorder remains relatively unexplored. When brought into nonequilibrium, physical systems may exhibit different kinds of pattern formation,[12,13] which are much richer than the traditional phase transitions in equilibrium systems. Such exotic systems also include vortices in superconductors,[14] charge density waves,[15,16] and Wigner crystals.[17] Particularly the driven disordered vortex lattice has been studied.[18–28] It has been established that for all but very weak pinning, the vortex lattice is plastically deformed and the onset of vortex motion takes place through channels. This result goes beyond the applicability of the conventional collective pinning theory for current-voltage (*I-V*) characteristics.[29] In the vortex glass picture the transport is dominated by vortex creep, leading to exponential *I-V* characteristics.[20] Increasing temperatures transform the vortex glass phase into a vortex liquid.[22] In addition, it was recently pointed out that the critical current shows a peak across the vortex glass to vortex liquid transition and the peak is accompanied by a crossing of the *I-V* curves.[28] An actual dynamic phase transition between the plastically deformed phase and a moving crystal for the vortex lattice was first proposed by Koshelev and Vinokur,[21] whose idea is that the random pinning noise diminishes with increasing the vortex velocity, allowing a high-velocity reordering.

Most recently, Reichhardt and Olson[30] examined the depinning properties of driven colloids interacting with quenched disorder using Langevin simulations. They found a crossover from elastic to plastic depinnings with an increasing strength of the disorder, where a pronounced increase in the depinning force was observed. However, the temperature dependence of the dynamics is still open. We will address this point in the present paper.

In this paper, we numerically investigate the dynamics of driven 2D colloids subject to randomly distributed pointlike pinning centers. The colloids are simulated using Langevin dynamics, and the motion of colloids is described by

$$\frac{d\mathbf{R}_i}{dt} = -\sum_{i\neq j} \nabla_i U_{c-c}(\mathbf{R}_i - \mathbf{R}_j) - \sum_{j'} \nabla_i U_{c-p}(\mathbf{R}_i - \mathbf{r}_{j'}) + \mathbf{F}_i^L(t) + \mathbf{F}_d, \tag{1}$$

with $\mathbf{R}_{i,j}$ and $\mathbf{r}_{j'}$ the colloid and pin coordinates. $U_{c-c}(\mathbf{R})$, $U_{c-p}(\mathbf{R}-\mathbf{r})$, and $\mathbf{F}_d$ are the colloid-colloid interaction potential, the colloid-pin interaction potential, and the external driving force, respectively. The Langevin force $\mathbf{F}_i^L(t)$ describes the coupling with a heat bath, and can be expressed by the correlation function[26,27]

$$\langle F_{i\alpha}^L(t) F_{j\beta}^L(t') \rangle = 2\eta T \delta_{ij} \delta_{\alpha\beta} \delta(t-t'). \tag{2}$$

The colloids interact through a screened Coulomb potential[7,8,30]

$$U_{c-c}(\mathbf{R}_i - \mathbf{R}_j) = \frac{Q^2}{|\mathbf{R}_i - \mathbf{R}_j|} \exp(-\kappa |\mathbf{R}_i - \mathbf{R}_j|), \tag{3}$$

with Q the colloid charge and $1/\kappa$ the screening length.

Different from Ref. 30, where the quenched disorder is modeled as randomly placed parabolic traps, we choose the colloid-pin interaction as a conventional attracting Gaussian potential:[18,26,27]

$$U_{c-p}(\mathbf{R}_i - \mathbf{r}_{j'}) = -A_p \exp\left(-\frac{|\mathbf{R}_i - \mathbf{r}_{j'}|^2}{r_p^2} \right), \tag{4}$$

with A_p a constant. All the lengths will be measured with respect to the lattice constant a_0 of the ideal triangular lat-

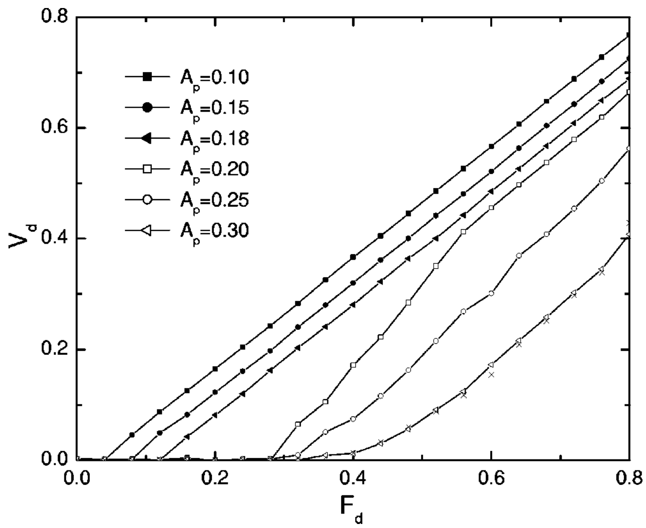

FIG. 1. Series of VFD curves for different values of A_p.

tice. We take $\kappa=2/a_0$, and the scale for temperature is chosen as the "bare" Kosterlitz-Thouless melting temperature T_{m0}.[31] The size of the pinning centers r_p is chosen to be $r_p=0.2a_0$. $N_c=400$ colloids are initially placed in a perfect triangular lattice subject to periodic triangular boundary conditions, and $N_p=1600$ pointlike pinning centers are randomly distributed. The driving force F_d is increased from zero by small increments along the horizontal symmetry axis (x axis) and the average colloid velocity $V_d=(1/N_c)\Sigma_{i=1}^{N_c}\mathbf{V}_i\cdot\hat{\mathbf{x}}$ is measured at each increment. A time integration step of $\Delta t=0.001$ is used, and averages are evaluated during 2×10^5 steps after 1×10^5 steps for equilibrium.

In Fig. 1 we present series of velocity-force dependence (VFD) curves for different values of the strength of pinning centers (through changing the dimensionless prefactor A_p) at a fixed temperature ($T/T_{m0}=0.1$). One can see that there exists a critical driving force F_c in each curve of the VFD, below which the colloids are pinned and the velocities are generated mainly by rearrangements of topological defects, producing advances of the pinned colloids so small that they can be neglected. In addition, we find a dramatic change in the gross feature of the VFD curves as A_p is varied. For weak disorder with small enough values of A_p, the VFD response is basically linear above F_c (e.g., see the curve of $A_p=0.10$). In this case, colloids depin elastically and the motion of colloids is inhomogeneous in the narrow region near F_c only. For strong disorder with large values of A_p, plastic flow takes place above F_c and the VFD curve shows a pronounced upward curvature, which is induced by defects in the flowing colloids.

We have investigated the dependence of our results upon the system size. For this we have varied the system to a larger one, i.e., one of $N_c=1600$ colloids with $N_p=6400$ randomly distributed pointlike centers (keeping $N_p/N_c=\frac{1600}{400}=\frac{6400}{1600}$ fixed). For one curve of $A_p=0.3$, results are shown in the Fig. 1 by crosses. Comparing with that of $N_c=400$ ($N_p=1600$), we find that the finite-size effects are so small that they can be neglected.

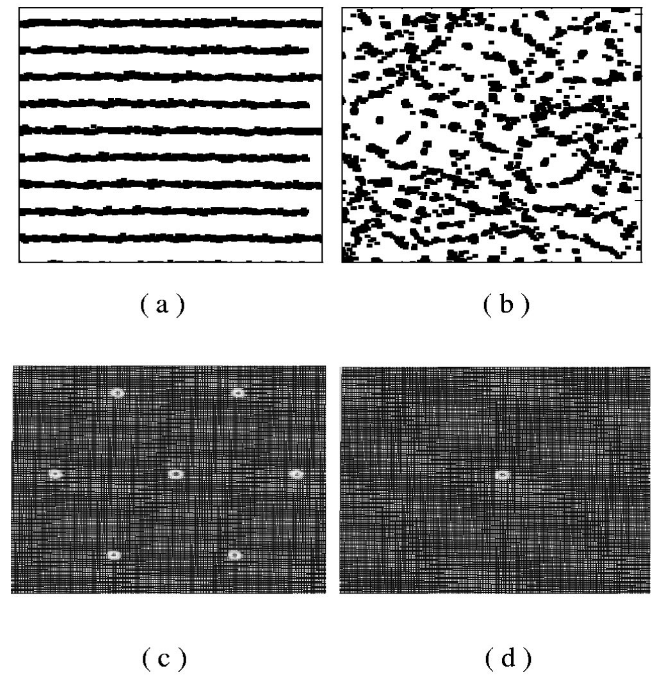

FIG. 2. Colloid trajectories [(a), (b)] and structure factors [(c), (d)] above depinning ($F_d/F_c=1.2$). (a) and (c) are for $A_p=0.15$, (b) and (d) are for $A_p=0.25$.

To identify the elastic to plastic crossover further, we have plotted the colloid trajectories near depinning. Figure 2(a) shows the elastic colloid flow for $A_p=0.15$ at a certain F_d above depinning ($F_d/F_c=1.2$). Here we find a set of periodic elastic channels, and each colloid keeps the same neighbors as it moves, consistent with recent simulations.[30] Sixfold coordinated Bragg peaks are found in the structure factor $S(\mathbf{k})=\langle|(1/N_c)\Sigma_{i=1}^{N_c}\exp[i\mathbf{k}\cdot\mathbf{R}_i(t)]|^2\rangle$ in this case, as shown in Fig. 2(c). For a comparison, we present the plastic colloid flow for $A_p=0.25$ ($F_d/F_c=1.2$) in Fig. 2(b), from which we can see that colloids move in channels between pinned regions. The channels are not static but changeable over time, so that any one colloid is only temporarily trapped in a pinning site. The correlation among colloids does not exist in this case, and the sixfold coordinated Bragg peaks in $S(\mathbf{k})$ disappear and only one peak at the center ($k_x=k_y=0$) is found, as seen in Fig. 2(d). The features of plastic flow have been observed in colloidal experiments[11] and simulations[30] as well as in vortex simulations.[25,27]

An obvious feature in the Fig. 1 is that F_c increases with A_p, and a substantial increase is found in F_c within the crossover from elastic to plastic depinnings, also shown in Fig. 3, where we present the F_c versus A_p curve. The corresponding differential curve is plotted in the inset of Fig. 3 where we find a peak. This is in good agreement with recent simulations.[30]

We have examined the temperature dependence of VFD curves. F_c is found to decrease with increasing temperatures, in agreement with simulations on vortex matter.[26,27] Furthermore, the gross feature of VFD curves in the plastic regime is strongly influenced by the temperature, as shown in Fig. 4,

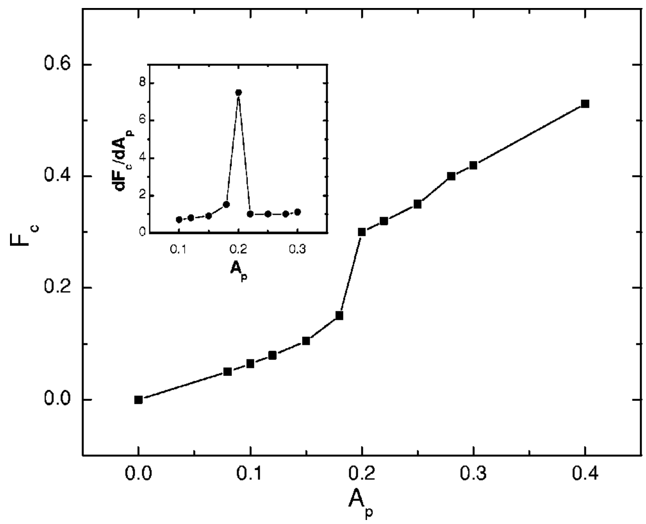

FIG. 3. Depinning force F_c vs A_p. Inset: Corresponding dF_c/dA_p.

where we give a set of VFD curves at different temperatures (the pinning strength is fixed by $A_p=0.3$). The log-log plot displays a change in the gross feature: The VFD curve is approximately linear at high temperatures (above $T^*=0.1T_{m0}$). This is the behavior of liquid. However, at low temperatures (below T^*), the VFD curve turns to a profoundly nonlinear form, consistent with the exponential glass expression.[20] Another distinguishing feature is that the VFD curves start to cross below T^*, indicating the occurrence of an order.[23,27,28] The idea that an applied driving force can cause colloids to order in the presence of pinning is a simple expression of the fact that an applied driving force tilts the disorder potential, thereby reducing the effective pinning strength. When a large enough driving force is applied, the colloids depin and flow defectively and then order, and the dynamic friction they experience decreases.[23]

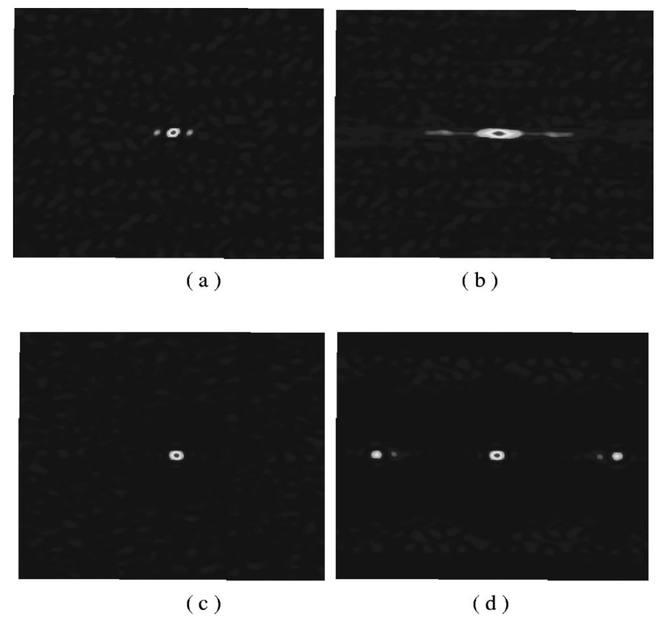

FIG. 5. Structure factors at different driving forces and different temperatures. (a) $F_d/F_c=1.2$ and $T/T_{m0}=0.5$. (b) $F_d/F_c=4.2$ and $T/T_{m0}=0.5$. (c) $F_d/F_c=1.2$ and $T/T_{m0}=0.001$. (d) $F_d/F_c=4.2$ and $T/T_{m0}=0.001$.

To characterize this ordering, we present structure factors $S(\mathbf{k})$ at different driving forces and different temperatures in Fig. 5. From Figs. 5(a) and 5(c), we find that no order occurs at low driving forces, regardless of low or high temperatures. However, at high driving forces, order takes place as the temperature is decreased below T^*, as seen in Fig. 5(d). We find in Fig. 5(d) that Bragg peaks appear only in the $k_y=0$ axis, meaning the occurrence of smectic order at low temperatures, and a positional correlation along the flow direction begins to arise. Increasing temperatures transform the

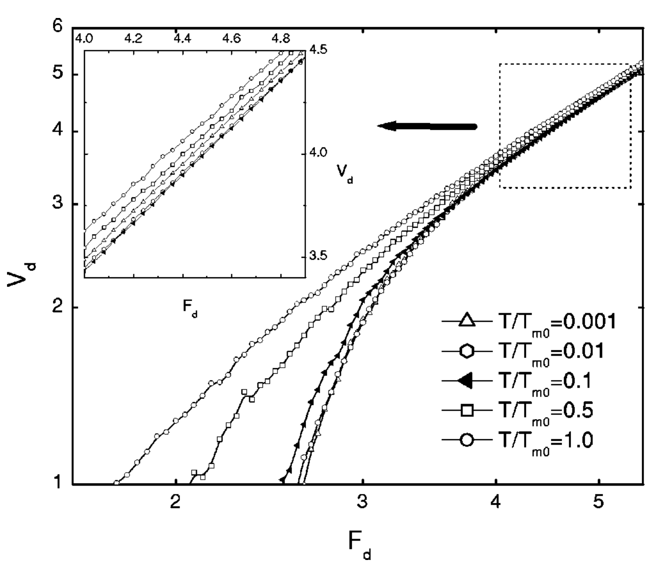

FIG. 4. Series of VFD curves at different temperatures (the pinning strength is fixed by $A_p=0.3$). Inset: Magnified VFD curves in the linear flow region.

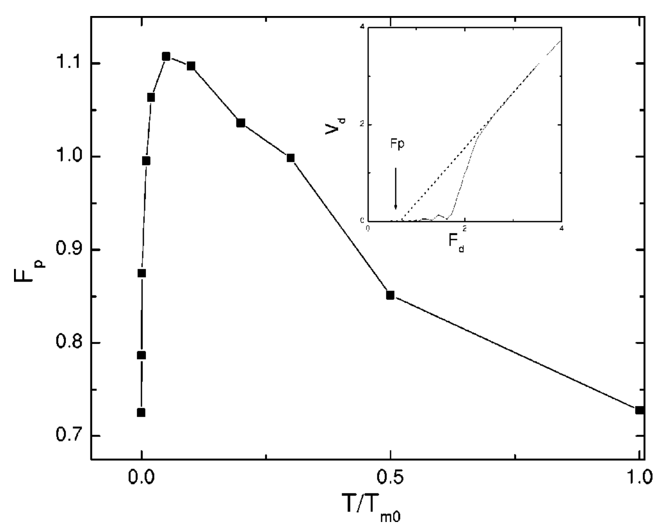

FIG. 6. Temperature dependence of the dynamic critical driving force F_p. Inset: The extrapolation from the linear flow region to obtain the dynamic critical driving force F_p.

moving smectic to the moving liquid. Comparing Figs. 5(d) and 5(b), one can see this point.

To further characterize this ordering, we reexamine the VFD curves shown in Fig. 4. At high driving forces, VFD curves all exhibit a well-defined linear flow region. If we extrapolate the linear $V_d=(F_d\text{-}F_p)R_f$ back to the x axis, as illustrated in the inset of Fig. 6, then the intercept in the x axis gives us the dynamic critical driving force F_p,[23] which is proportional to the average pinning force that colloids feel in the linear flow region. From Fig. 6 we can find that F_p exhibits a peak as the temperature is decreased across the melting temperature (near T^*). Below T^*, the collective pinning theory of Larkin and Ovchinnikov[29] can give us a qualitative idea of order in the moving configuration. In this theory, the correlation length is given by $R_c \propto \sqrt{1/F_p}$. Thus, the drop in F_p at low temperatures implies an increase in the correlation area in the linear flow region, indicating appearance of an order. From Fig. 5(d) we know the order is smectic.

In summary, we have numerically investigated the dynamics of driven disordered colloids. With an increasing strength of the pinning centers, we found a crossover from elastic to plastic depinnings, where a substantial increase in the depinning force was observed. The influence of temperature has been considered for the first time, to our knowledges and we found a dynamic melting transition from the moving smectic to the moving liquid at high driving forces. A peak was found in the dynamic critical driving force across the transition, accompanied by a crossing of the velocity-force dependence curves.

The project was partially funded by the Hong Kong Polytechnic University under account G-YW97, and partially from the ASD-Advanced Building Technology under account A-038 and the Ministry of Science and Technology of China (NKBRSF).

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