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An Improved Filter for High Order Integrated GPS/INS Based on the U-D Factorization

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ABSTRACT

In this paper, the U-D factorization-based extended Kalman filter for the 20-state integrated GPS/INS is investigated from the point of engineering realization. The Kalman filter is widely used for integration of GPS/INS, however, due to the model and numerical computation errors, the Kalman filter may diverge in engineering realization. In order to solve this problem, an extended Kalman filter based on the U-D factorization is proposed. Moreover, the high order integrated system suffers from problem of long computation time, leading to difficulties in real-time applications. An algorithmic approach is developed to improve computational speed.

A complex high dynamics aircraft trajectory is simulated to compare the improvement in the computational speed and the navigation accuracy using the conventional extended Kalman filter (EKF) and the extended Kalman filter based on the U-D factorization (UDEKF). The results indicate that the methods proposed in this paper are very effective overcoming these problems for the high dynamic integrated GPS/INS system.

1. INTRODUCTION

The Global Positioning System (GPS) and the Inertial Navigation System (INS) techniques have been widely utilized by military and civilian users. However, each system has some shortcomings when used in a stand-alone mode[2, 5, 6]. INS suffers from fast error accumulation and a relatively long initial alignment period while GPS has problem of cycle slips and low update rate. The best solution is an integration of the GPS and INS system. Integration of GPS/INS has been investigated extensively for low and moderate dynamic environments. This study is concerned on the high dynamic tactical aircraft.

The Kalman filter has been widely used in navigation systems. However, due to the model and numerical computation errors, the covariance matrix of the system may lose its properties of being positive definite and symmetric, leading to the divergence of the filter. Since the 70's, several methods have been developed to overcome this problem, such as a square-root filter and a U-D factorization filtering (Bierman,1977). In this paper, an extended Kalman filter algorithm based on the U-D factorization filtering was investigated to ensure the numerical stability and overcome the divergence of the filter. In addition, the high order integrated system suffers from problem of long computation time, leading to difficulties in real-time applications. It is therefore necessary to develop algorithmic techniques to improve the speed of the extended Kalman filter. In this paper, the structure of the system state matrices is analyzed to reduce the computational effort.

To test the developed method, a typical aircraft trajectory for a complex high dynamics environment was simulated. A comparison of the navigation accuracy, using the conventional extended Kalman filter(EKF) with that using the extended Kalman filter based on U-D factorization(UDEKF), is reported. The results indicate that the proposed algorithm in this paper is effective.

2. DESCRIPTION OF INTEGRATED GPS/INS

In the use of Kalman filter for an integration of GPS/INS, both the dynamic model and observation model are required to describe the integrated system. The dynamic model is usually described by a linear differential equation involving the system errors of the INS and GPS system. The observation model is obtained from a combination of the INS and GPS measurements.

2.1 The dynamic error model of integrated GPS/INS

In this paper, a strapdown INS is considered as a local-level, north, east, and down (NED) coordinate system. The dynamic error model of an integrated GPS/INS can be written in the following standard form,

$$\dot{X}(t) = A(t)X(t) + G(t)W(t) \tag{1}$$

where the state vector X consists of the various errors such that

$$X(t) = [\phi_n, \phi_e, \phi_d, \delta V_n, \delta V_e, \delta V_d, \delta\varphi, \delta\lambda, \delta h, \varepsilon_{cx}, \varepsilon_{cy}, \varepsilon_{cz}, \varepsilon_{rx}, \varepsilon_{ry}, \varepsilon_{rz}, \nabla_x, \nabla_y, \nabla_z, \delta_{tu}, \delta_{tru}]^T$$

$$W(t) = [w_{gx}, w_{gy}, w_{gz}, w_{mx}, w_{my}, w_{mz}, w_{ax}, w_{ay}, w_{az}, w_{tu}, w_{tru}]^T$$

Where (ϕ_n, ϕ_e, ϕ_d) are the three attitude error angles, $(\delta V_n, \delta V_e, \delta V_d)$ are the three velocity error components, $(\delta\varphi, \delta\lambda, \delta h)$ are the three position error components, $(\varepsilon_{cx}, \varepsilon_{cy}, \varepsilon_{cz})$ are the three constant gyro drift components, $(\varepsilon_{rx}, \varepsilon_{ry}, \varepsilon_{rz})$ are the three first-order Markov gyro drift components, $(\nabla_x, \nabla_y, \nabla_z)$ are the three accelerometer bias components, $(\delta_{tu}, \delta_{tru})$ are the GPS clock bias and clock drift rate vector, (w_{gx}, w_{gy}, w_{gz}) are the white noise of gyro drift, (w_{mx}, w_{my}, w_{mz}) are the first-order Markov white noise of gyro drift, (w_{ax}, w_{ay}, w_{az}) are the first-order Markov white noise of accelerometer, (w_{tu}, w_{tru}) are the first-order Markov white noise of GPS clock bias and clock drift rate, respectively.

The matrix A(t) (20 by 20) is the integrated system dynamic matrix, the matrix G(t) (20 by 11) is the coefficient matrix for the random effects. For detailed discussion in the above model see Ref[3].

2.2 the observation model of integrated GPS/INS

In this integrated system, the measurement vectors consist of a combination of the INS and GPS. From the INS position, the distance ρ_{Ij} between the aircraft and jth satellite can be calculated. Let GPS pseudorange measurement be ρ_j, then the observation equation can be expressed as

$$Z_{\rho j}(t) = \rho_{Ij} - \rho_j \quad (j = 1, 2, 3, 4) \tag{2}$$

with

$$\rho_j = [(x - x_{sj})^2 + (y - y_{sj})^2 + (z - z_{sj})^2]^{\frac{1}{2}} + \delta\rho_j \tag{3}$$

where (x, y, z) are the true position of the aircraft in the Earth Centered Earth Fixed (ECEF) coordinate system, (x_{sj}, y_{sj}, z_{sj}) are the jth satellite position in the ECEF coordinate system, $\delta\rho_j$ is the distance error, mainly caused by clock biases,

$$\delta\rho_j = \delta_{tu} + v_j \tag{4}$$

where δ_{tu} is clock bias, v_j is GPS receiver measurement noise.

Assume the INS position (x_I, y_I, z_I) in the ECEF coordinate system is given by

$$\begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \delta x_I \\ \delta y_I \\ \delta z_I \end{bmatrix} \tag{5}$$

where (x, y, z) is true aircraft position in the ECEF system, $(\delta x_I, \delta y_I, \delta z_I)$ are positioning error of the INS. ρ_{Ij} is then written as

$$\rho_{Ij} = [(x_I - x_{sj})^2 + (y_I - y_{sj})^2 + (z_I - z_{sj})^2]^{\frac{1}{2}} \tag{6}$$

Linearizing equation (6) and considering equation (5), we have

$$\rho_{Ij} \approx [(x - x_{sj})^2 + (y - y_{sj})^2 + (z - z_{sj})^2]^{\frac{1}{2}} + \begin{bmatrix} e_{j1} & e_{j2} & e_{j3} \end{bmatrix} \cdot C(t) \cdot \begin{bmatrix} \delta\varphi \\ \delta\lambda \\ \delta h \end{bmatrix} \tag{7}$$

where j = 1, 2, 3, 4

$$e_{j1} = \frac{\partial\rho_{Ij}(x,y,z)}{\partial x} = \frac{(x - x_{sj})}{[(x-x_{sj})^2 + (y-y_{sj})^2 + (z-z_{sj})^2]^{1/2}}$$

$$e_{j2} = \frac{\partial\rho_{Ij}(x,y,z)}{\partial y} = \frac{(y - y_{sj})}{[(x-x_{sj})^2 + (y-y_{sj})^2 + (z-z_{sj})^2]^{1/2}}$$

$$e_{j3} = \frac{\partial\rho_{Ij}(x,y,z)}{\partial z} = \frac{(z - z_{sj})}{[(x-x_{sj})^2 + (y-y_{sj})^2 + (z-z_{sj})^2]^{1/2}}$$

$$C(t) = \begin{bmatrix} -(u+h)\sin\varphi\cdot\cos\lambda & -(u+h)\cos\varphi\cdot\sin\lambda & \cos\varphi\cdot\cos\lambda \\ -(u+h)\sin\varphi\cdot\sin\lambda & (u+h)\cos\varphi\cdot\cos\lambda & \cos\varphi\cdot\sin\lambda \\ [u(1-e^2)+h]\cos\varphi & 0 & \sin\varphi \end{bmatrix}$$

where (φ, λ, h) are the position in the NED coordinate system, and

$$u = R_e \Big/ \sqrt{\cos^2\varphi + (1-e^2)\sin^2\varphi}$$

$$R_e = 6378137\,\text{m}, \quad e = 1/298.3$$

By substitution of Eq. (3), Eq. (4) and Eq. (7) into Eq. (2), we obtain the observation equations for the integrated GPS/INS measurements

$$Z_\rho(t) = H_\rho(t)X(t) + V_\rho(t) \tag{8}$$

where

$$z_\rho(t) = [z_{\rho 1}, z_{\rho 2}, z_{\rho 3}, z_{\rho 4}]^T$$

$$V_\rho(t) = [v_1, v_2, v_3, v_4]^T$$

$$H_\rho = [O_{4\times 6}, H_{\rho a}, O_{4\times 10}, H_{\rho b}]_{4\times 20}$$

$$H_{\rho a} = \begin{bmatrix} e_{j1} & e_{j2} & e_{j3} \end{bmatrix} \cdot C(t)$$

j = 1, 2, 3, 4

$$H_{\rho b} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$

The discrete form of Eq. (1) and Eq. (8) can be respectively expressed as,

$$X(k+1) = \Phi(k)\cdot X(k) + \Gamma(k)\cdot W(k) \tag{9}$$

$$Z(k) = H(k)\cdot X(k) + V(k) \tag{10}$$

where $\Phi(k)$ is the state transition matrix, $\Gamma(k)$ is the coefficient matrix, H(k) is the measurement matrix, w(k) and v(k) are white noise vectors with associated variance matrices Q(k) and R(k), respectively.

3. U-D FACTORIZATION-BASED EXTENDED KALMAN FILTER

3.1 The principle of sequential Kalman filtering

U-D factorization method as applied to the extended Kalman filter requires only one measurement. However, in this study, there are four observation equations as a result of GPS/INS combination. In order to apply the U-D factorization method, the sequential Kalman filtering method must be employed such that observations are added to the system one by one.

For the jth observation equation, the equations of the sequential Kalman filtering are as follows (Chui, 1991)

$$K^j = P^{j-1}(H^j)^T[H^j P^{j-1}(H^j)^T + r_j]^{-1} \tag{11}$$

$$P^j = [I - K^j H^j]P^{j-1}[I - K^j H^j]^T + K^j r_j (K^j)^T \tag{12}$$

$$\hat{X}^j = \hat{X}^{j-1} + K^j[Z^j - H^j \hat{X}^{j-1}] \tag{13}$$

where H^j is the jth row in H matrix, and

$$H = [H^1, H^2, \ldots, H^m]^T$$

Z^j is the jth row in Z matrix, and

$$Z = [Z^1, Z^2, \ldots, Z^m]^T$$

r_j is the measurement noise variance of the jth observation, and measurement noise matrix

$$R(k) = diag[r_1, r_2, \ldots, r_m]$$

for j = 1, 2, ..., m, where m is the number of observation equations, based on Eq. (11) to Eq. (13), the results of filter are as follows,

$$\hat{X}(k+1/k+1) = \hat{X}^m \tag{14}$$

$$P(k+1/k+1) = P^m \tag{15}$$

initial conditions are

$$P^0 = P(k+1/k)$$

$$\hat{X}^0 = \hat{X}(k+1/k)$$

where

$$\hat{X}(k+1/k) = \Phi(k)\hat{X}(k/k)$$

3.2 U-D factorization algorithm

In order to ensure the covariance matrix of the state is positive definite and symmetric, and to ensure that the Kalman filter is numerically stable, a U-D factorization method is used. For the U-D factorization method, the predicted covariance matrix and updated covariance matrix are decomposed, respectively, as (Bierman, 1977).

$$P(k+1/k) = U(k+1/k)\cdot D(k+1/k)\cdot U^T(k+1/k) \tag{16}$$

and

$$P(k+1/k+1) = U(k+1/k+1)\cdot D(k+1/k+1)\cdot U^T(k+1/k+1) \tag{17}$$

where U is an upper triangular matrix and D is a diagonal matrix.

First the elements of matrices *D(k+1/k+1)* and *U(k+1/k+1)* are directly obtained using $\Phi(k)$ and $\Gamma(k)$ in Eq. (9). The following matrices are then formed

$$\tilde{A} = [a_1, a_2, \ldots, a_n] = \begin{bmatrix} U^T(k+1/k+1)\cdot\Phi^T(k) \\ \Gamma^T(k) \end{bmatrix} \tag{18}$$

and

$$DW = \begin{bmatrix} D(k+1/k+1) & 0 \\ 0 & Q(k) \end{bmatrix} \tag{19}$$

where *Q(k)* is the dynamic noise matrix and is considered as a diagonal matrix. For i = n, n-1, n-2, ..., 3, 2, 1, where n is the order of the system, we have

$$s = DW \cdot a_i \quad (20)$$

and

$$D_{ii}(k+1/k) = a^T{}_i \cdot s \quad (21)$$

then the matrix s is updated as

$$s \leftarrow s / D_{ii}(k+1/k) \quad (22)$$

For j = 1, 2, 3, ..., i-2, i-1, we have

$$U_{ji}(k+1/k) = a^T{}_{jn} \cdot s \quad (23)$$

then, the matrices a_{jn} are updated as

$$a_{jn} \leftarrow a_{jn} - U_{ji}(k+1/k) \cdot a_i \quad (24)$$

Following the above process, all elements of matrices in D(k+1/k) and U(k+1/k) can be obtained with initial condition as

$$P(0/0) = U(0/0)D(0/0)U^T(0/0)$$

Because *P(0/0)* is usually diagonal, the diagonal elements of *D(0/0)* are *P(0/0)*, and *U(0/0)* is an identity matrix. The elements of matrices in *D(k+1/k+1)* and *U(k+1/k+1)* can be updated using *U(k+1/k)* and *D(k+1/k)*. For the jth row in H matrix, we have

$$F = [f_1, f_2, \ldots f_n] = U^T(k+1/k) \cdot (H^j)^T \quad (25)$$

and

$$T = [t_1, t_2, \ldots, t_n] = D(k+1/k) \cdot F \quad (26)$$

For I =1, 2, ..., n-1, n, where n is the order of system, we have

$$h_i = h_{i-1} + f_i \cdot t_i \quad (27)$$

where $h_0 = r_j$, r_j is the measurement variance of the jth observation, we then have,

$$D_{ii}(k+1/k+1) = D(k+1/k) \cdot h_{i-1} / h_i \quad (28)$$

Let

$$p = -f_i / h_{i-1} \quad (29)$$

and

$$B_i = t_i \quad (30)$$

For l=1, 2, 3, ..., i-1, we have

$$U_{lmp} = U_{li}(k+1/k) \quad (31)$$

and

$$U_{li}(k+1/k+1) = U_{li}(k+1/k) + B_l \cdot p \quad (32)$$

The matrix B_l is updated as follows,

$$B_l \leftarrow B_l + U_{lmp} \cdot t_i \quad (33)$$

The gain matrix K^j can be written as

$$K^j = B_i / h_i \quad (34)$$

Using on the basic principle of sequential Kalman filtering, for j=1, 2, 3, ..., m, following the process from the Eq. (25) to Eq. (33), when j = m, all elements of matrices in *D(k+1/k+1)* and *U(k+1/k+1)* are obtained.

U-D factorization-based Kalman filter equations are summarized in Table 1.

4. TECHNIQUES OF IMPROVING COMPUTATION SPEED

From above U-D factorization algorithms (see Table-1), it can be seen that most of the computational effort is on calculating the covariance matrix. In order to reduce the computation effort, the structure of the state transition matrix $\Phi(k)$ needs to be analyzed.

The matrix $\Phi(k)$ (20 by 20) in equation (9) is a sparse matrix. There are 69 non-zero elements and 331 zero elements. If individual elements, rather than large matrices are considered then all the non-zero elements are removed and only the multiplication with non-zero elements are performed. As a result of this the covariance prediction matrix, which requires about 16000 multiplication, now requires only 4640 multiplication. Thus, a large computational saving is obtained.

5. SIMULATION RESULTS

The simulation was carried out to test the computational effectiveness of the U-D factorization-based Kalman filter and compare the improvement in computational speed. A trajectory was designed for an actual and complex high-dynamic aircraft flight. It was assumed that an aircraft takes off, flies, and maneuvers, total time was 5300 seconds. The maneuvers included climbing, pitching, rolling, and turning.

Based on the above conditions, both Kalman filters EKF and UDEKF are simulated. The results of simulation indicate that the same navigation accuracy can be obtained using the two filters. Table 2 shows the standard deviation of navigation accuracy using the two filters. In order to display the divergence of the filter, we enlarge system dynamics noise. The covariance matrix using EKF lost the properties of positive definiteness and symmetric in 105 step. However, each time the covariance matrix using UDEKF remains positive definite and symmetric.

A comparison of the UDEKF with and without the special technique of reduction of computation is performed on a IBM PC with a clock frequency of 133 MHz. It is observed that operation of each step without special technique takes about 0.142 second, and operation of each step with special technique takes about 0.044 second.

Table 1 — U-D factorization-based Kalman filter

Prediction equation

$$\hat{X}(k+1/k) = \Phi(k)\hat{X}(k/k)$$

$$P(k+1/k) = U(k+1/k) \cdot D(k+1/k) \cdot U^T(k+1/k)$$

Gain equation

$$K^j = B_i / h_i$$

Update equation

$$\hat{X}^j = \hat{X}^{j-1} + K^j[Z^j - H^j \hat{X}^{j-1}]$$

$$\hat{X}(k+1/k+1) = \hat{X}^m$$

$$P(k+1/k+1) = U(k+1/k+1) \cdot D(k+1/k+1) \cdot U^T(k+1/k+1)$$

Initial condition

$$\hat{X}^0 = \hat{X}(k+1/k)$$

where

i= 1, 2, 3, ..., n

j=1, 2, 3, ..., m

$\hat{X}(k+1/k)$ = predicted state vector

$\Phi(k)$ = the state transition matrix

P(k+1/k) = predicted covariance matrix

K^j = the jth row in gain matrix K(k)

$\hat{X}(k+1/k+1)$ = updated state vector

P(k+1/k+1) = updated covariance matrix

Table 2 — Navigation accuracy compared using two Kalman filter (Standard Deviation)

Navigation Error	Initial State Error	Conventional Extended Kalman Filter	U-D factorization Extended Kalman Filter
Pitch angle error(arcsec)	300.0	61.80	61.80
Roll angle error(arcsec)	300.0	62.50	62.50
Heading angle error(arcsec)	600.0	90.30	90.30
Longitude error(m)	100.0	5.40	5.40
Latitude error(m)	100.0	5.40	5.40
Height error(m)	100.0	5.10	5.10
N-Velocity error(m/s)	1.0	0.03	0.03
E-Velocity error(m/s)	1.0	0.03	0.03
D-Velocity error(m/s)	1.0	0.03	0.03

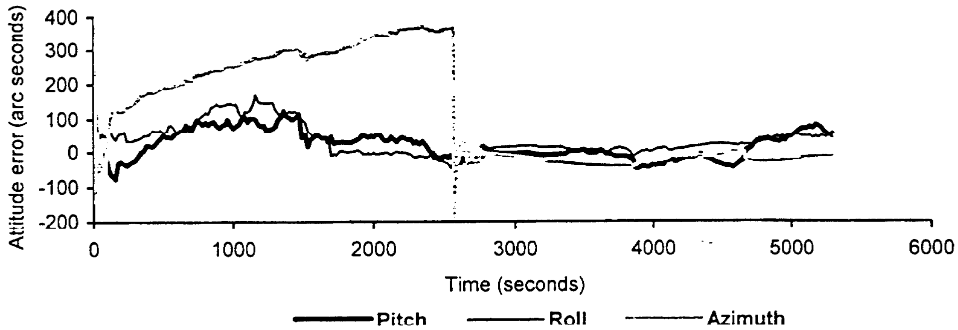

Fig. 1 Aircraft attitude error curve

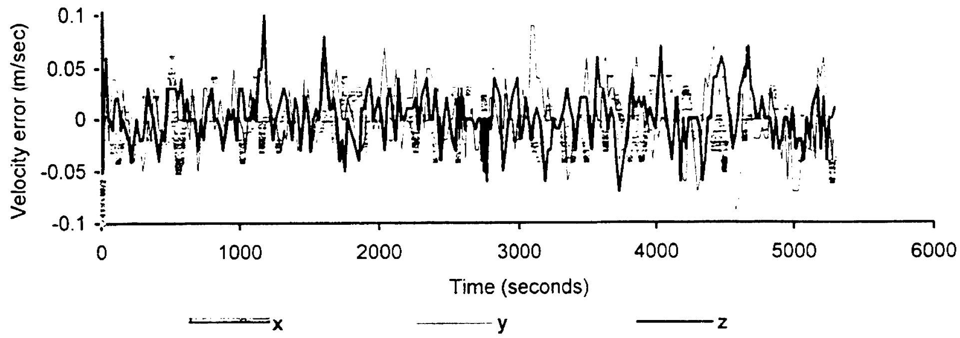

Fig. 2 Aircraft velocity error curve

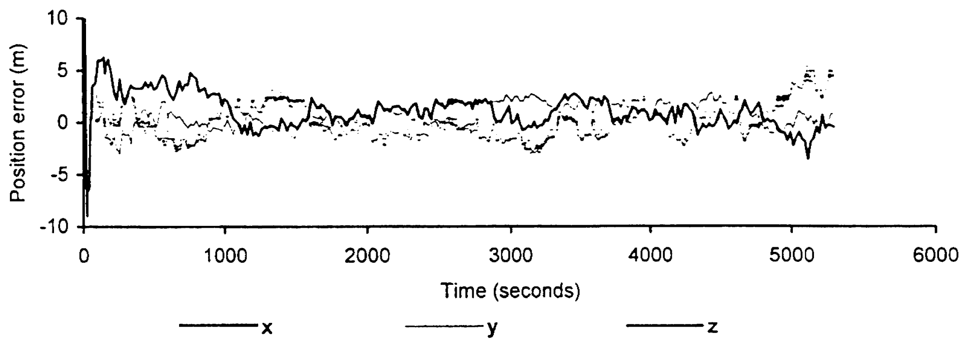

Fig. 3 Aircraft position error curve

Moreover, using UDEKF, aircraft attitude errors are displayed in Figure 1; position errors and velocity errors are in Figure 2 and Figure 3, respectively. It can be seen that aircraft navigation error, except heading angle error, can be estimated with good accuracy. Aircraft heading angle error increases in time at first, after aircraft makes more maneuvers flight, heading angle error becomes small. The integrated GPS/INS has higher observability because of more maneuvers, and the error estimation quality has been improved.

6. CONCLUSIONS

In this paper, the U-D factorization-based Kalman filter for integrated GPS/INS has been discussed in detail, and the structure of the integration navigation system matrices was analyzed to reduce the computation efforts. Both Kalman filter EKF and UDEKF are simulated. The following conclusions can be drawn.

- The algorithm presented in this paper is effective for the 20-state integration of GPS/INS. It provides sufficient accuracy for aircraft navigation.
- As is shown in Table 2, the navigation accuracy using EKF is the same as that of the UDEKF. From the point of view of control theory, the UDEKF is an optimal filter, the both filters have the same filter accuracy.
- The UDEKF guarantees the positive definiteness and the symmetry of the covariance matrices $P(k/k)$ and $P(k+1/k)$, therefore, it overcomes the problem of the divergence of filter and has excellent numerical stability.
- The UDEKF needs less computation time when use of the special technique to calculate individual element rather than to treat large matrices. Use of special technique reduced the iteration time approximately 69 percent.

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