Experimental observation of FPU recurrence in a fiber ring laser

B. Zhao and D. Y. Tang

School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore
b_zhao@pmail.ntu.edu.sg

H. Y. Tam

Department of Electrical Engineering, Hong Kong Polytechnic University, Hong Kong

Abstract: We report on the experimental observation of Fermi-Pasta-Ulam (FPU) recurrence in a passively mode-locked fiber ring laser. The spectral sidebands of a CW component generated due to the modulation instability grow up periodically as a result of the reversibility of the effect, demonstrating the existence of FPU recurrence.

OCIS codes: (190.3100) Instabilities and chaos; (140.3510) Fiber lasers

References and Links

1. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, New York, 2001), Chap. 5.
2. K. Tai, A. Hasegawa, and A. Tomita, “Observation of the modulation instability in optical fibers,” Phys. Rev. Lett. **56**, 135-138 (1986).
3. G. P. Agrawal, “Modulation instability in erbium-doped fiber amplifiers,” IEEE Photon. Tech. Lett. **4**, 562-564 (1992).
4. D. Y. Tang, W. S. Man, H. Y. Tam, and M. S. Demokan, “Modulation instability in a fiber laser induced by periodic dispersion variation,” Phys. Rev. A **61**, 023804 (2000).
5. E. Fermi, J. Pasta, and H. C. Ulam, “ Studies of nonlinear problems,” in Collected Papers of Enrico Fermi, E. Segre, ed. Vol **2**, 977-988 (1965).
6. Nail N. Akhmediev, “Déjà vu in optics,” Nature **413**, 267 (2001).
7. G. V. Simaeys, P. Emplit, and M. Haelerman, “Experimental demonstration of the Fermi-Pasta-Ulam recurrence in a modulationally unstable optical wave,” Phys. Rev. Lett. **87**, 033902 (2001).
8. S. Namiki, E. P. Ippen, H. A. Haus, and K. Tamura, “Relaxation oscillation behavior in polarization additive pulse mode-locked fiber ring lasers,” Appl. Phys. Lett. **69**, 3969-3971 (1996).
9. C. J. Chen, P. K. A. Wai and C. R. Menyuk, “Soliton fiber ring laser,” Opt. Lett. **17**, 417-419 (1992).
10. W. S. Man, H. Y. Tam, M. S. Demokan, P. K. A. Wai, and D. Y. Tang, “Mechanism of intrinsic wavelength tuning and sideband asymmetry in a passively mode-locked soliton fiber ring laser,” J. Opt. Soc. Am. B **17**, 28-33 (2000)
11. B. Zhao, D. Y. Tang, P. Shum, W. S. Man, H. Y. Tam, Y. D. Gong, and C. Lu, “Passively harmonic mode locking of twin-pulse solitons in an erbium-doped fiber ring laser,” Opt. Comm. (to be published).

Modulation instability (MI) is a universal effect existing in many nonlinear systems as a result of the interplay between the nonlinear and dispersive effects. It has been widely studied in diverse fields such as fluid dynamics, plasma physics and nonlinear optics since 1960s [1]. In the context of optical fibers, MI is referred to as a process in which due to the interplay between the self-phase modulation and the anomalous dispersion, weak perturbations grow exponentially into pulse trains from a steady CW state. Tai *et al*. reported the first experimental observation of MI in optical single-mode fibers in 1985 [2]. They found that the MI could be interpreted as a special case of the four-wave mixing whose phase-matching condition is self-generated by the nonlinear change of the refractive index and the anomalous dispersion. Agrawal has studied the MI in an erbium-doped fiber amplifier (EDFA) [3]. He found that the threshold for MI could be considerably reduced by the introduction of the internal gain, comparing with the case of the undoped fibers. Tang *et al*. have reported the

#3211 - $15.00 US
Received October 27, 2003; Revised November 20, 2003

observation of MI of the dispersive waves in fiber ring lasers [4]. They found that when the dispersive waves in the lasers are strong enough, they become unstable as a result of MI.

However, MI is a process that would not lead to a thermalization of energy. Instead, theoretical studies have shown that after a certain length of propagation, the energy that spread to different frequencies due to MI eventually returns to the initial mode, and consequently the initial waveform is reconstructed or almost reconstructed. This reversible behavior of MI is now well known as the Fermi-Pasta-Ulam (FPU) recurrence [5]. FPU recurrence has been predicted for a large number of conservative nonlinear systems, including those described by the nonlinear Schrödinger equation [6]. As also predicted by theory, MI could appear to be of the limit cycle type, simply because the recurrence time is much longer than the time scale of interest. For this reason, although a lot of theoretical study has been done on FPU recurrence, there are rare experimental observations in optics. Only until very recently, Simaeys et al experimentally demonstrated the FPU recurrence in optical fibers using a specially designed laser source [7].

In this letter, we report on an experimental observation of FPU recurrence in a passively mode-locked fiber soliton ring laser. With an appropriate laser parameter setting, CW lasing can coexist with the solitons in the laser [8]. We show that when the strength of the CW component is strong, it becomes unstable due to MI and exhibits modulation sidebands in its spectrum. Benefiting from the ring cavity configuration of our laser, we observed that the intensity of the MI sidebands changes periodically even with the fixed experimental conditions, demonstrating the FPU recurrence nature of the phenomenon. To our knowledge this is the first experimental observation of FPU recurrence in fiber lasers.

The fiber ring laser used in our experiments is schematically shown in Fig. 1. It has a ring cavity of about 5.5 meter long, which comprises of a 3.5-meter long 2000ppm erbium-doped fiber, a piece of 1 meter long single-mode dispersion-shifted fiber and another piece of 1 meter long standard single-mode fiber (SMF-28). The nonlinear polarization rotation technique is used to achieve the self-started mode locking in the laser. To this end a polarization dependent isolator together with two polarization controllers, one is made of two quarter-wave-plates and the other two quarter-wave-plates and one half-wave-plate, is used to adjust the polarization of light in the cavity. The polarization dependent isolator and the polarization controllers are mounted on a 7 cm long fiber bench to easily and accurately control the polarization of the light. The laser is pumped by a pigtailed InGaAsP semiconductor diode at the wavelength of 1480 nm. The output of the fiber laser is taken via a 10% fiber coupler and analyzed with an optical spectrum analyzer (OSA).

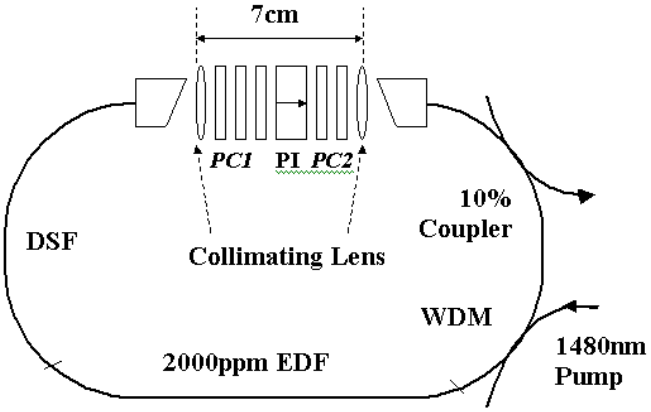

Fig. 1. Schematic of the laser setup. PI: polarization dependent isolator; PC: polarization controller; WDM: wavelength division multiplexer; EDF: erbium-doped fiber; DSF: dispersion-shifted fiber.

Received October 27, 2003; Revised November 20, 2003

Similar to those reported previously, soliton operation is an intrinsic feature of the laser, which is obtained by appropriately setting the polarization controllers and increasing the pump power beyond the threshold. The soliton of the laser has a pulse width (FWHM) about 340fs. Another characteristic of passively mode-locked fiber soliton lasers is that CW lasing can coexist with the soliton operation under a certain laser parameter setting [8]. This effect is especially pronounced in the fiber soliton lasers mode locked by using the nonlinear polarization rotation technique. Chen *et al.* have derived an analytic formula to calculate the cavity transmission of such a laser [9]. Based on their formula and also taking into account the cavity dispersion effect, it is found that the linear cavity loss of the laser is a sinusoidal function of the wavelength. In a previous paper Man et al. have demonstrated that the experimentally observed soliton sideband asymmetry is in fact caused by this property of the linear cavity loss [10]. Although ideally due to the existence of the saturable absorber effect in the laser, under an optimized soliton operation all CW components would experience negative effective gain and therefore be suppressed, in practice if the minimum linear cavity loss position is not appropriately set and/or the pump power is strong, CW lasing can still build up and coexist with the solitons. Here we present in Fig. 2 a typical optical spectrum of the laser output when the CW lasing coexists with the solitons in the cavity. The position of the CW component on the soliton spectrum is determined by the linear cavity phase bias and laser gain profile. Because of the soliton energy quantization, the strength of the CW component can be experimentally controlled by the pump power. Simply increasing the pump power, the CW strength can be significantly increased. As all linear waves are intrinsically unstable in the laser due to the modulation instability, when the strength of the CW component becomes strong, it eventually becomes unstable with symmetric spectral sidebands appearing in the spectrum.

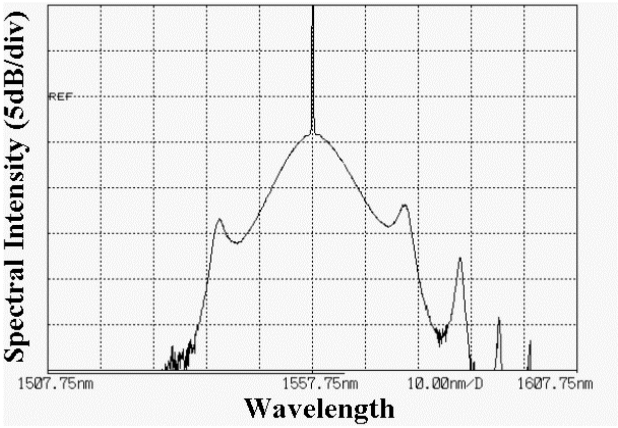

Fig. 2. A typical optical spectrum of the laser output when the CW lasing coexists with the single-pulse solitons in the cavity.

In Fig. 3 we show the enlarged views of the spectral image of the CW component under the same polarization setting of Fig. 2. In Fig. 3(a), the CW component is a very clear single mode wave with a wavelength of 1557.74nm, in which the MI sidebands are too weak to be seen. Fixing all the other parameters, when we increase the pump power from 100mW in (a) to 157mW, the MI of the CW now becomes so strong that the modulation sidebands are clearly to see in symmetric positions to the original mode as shown in Fig. 3(b). However, we noticed experimentally that the sidebands are not stable. With all the experimental conditions fixed, two different spectral images, either a symmetric spectrum with sidebands as shown in Fig. 3(b) or a clear higher single-mode spike as shown in Fig. 3(c) is observed. These two spectra appear alternately, exhibiting that the appearance of the sidebands is a periodic-like

#3211 - $15.00 US
Received October 27, 2003; Revised November 20, 2003

process. When the sidebands exist, the initial mode has obviously lower spectral strength, while without the sidebands it has higher spectral strength, showing clear energy exchange between the sidebands and the initial mode.

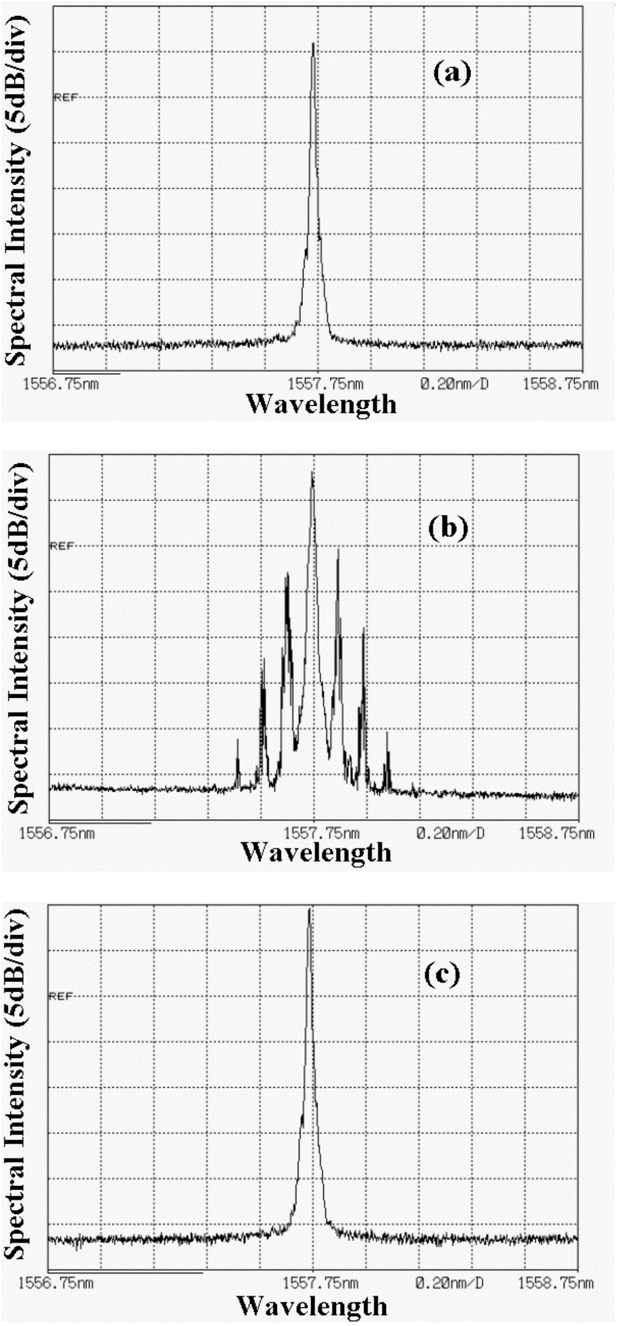

Fig. 3. Enlarged views of the CW component under different pump power. (a): with P=100mW; (b) and (c): P=157mW. All the three conditions are measured under the same polarization setting.

We have also simultaneously monitored the laser output with a 25GHz Photo-detector and a 50GHz high-speed sampling oscilloscope. We observed that whenever the spectral sidebands appear on the optical spectrum, the high-speed oscilloscope traces exhibit a noisy background [11], indicating that the CW has become unstable.

Our experimental results clearly show that the MI of the CW component of the laser does have a reversible behavior, which has been theoretically predicted in nonlinear systems as the FPU recurrence. As we described above, FPU recurrence is a general feature of MI in conservative nonlinear systems. When a CW propagates in the nonlinear media, initially all the energy is concentrated in the central mode as shown in Fig. 3(c). But since the CW is intrinsically unstable against MI, over time, it becomes modulated characterized as the appearance of the symmetrical spectral sidebands as shown in Fig. 3(b). Thus the energy flows from the central mode into the sidebands, and as a result, the energy in the central mode decreases. This process continues until a certain point is reached at which the energy starts to flow back from the sidebands to the central component. And after some time, the energy is totally back to the central mode, and the CW is reconstructed. This reciprocal energy exchange between the central mode and the sidebands is the FPU recurrence. As a result, in experiment we can observe either the CW mode as shown in Fig. 3(c) or the sidebands mode as shown in Fig. 3(b).

From Fig. 3(b), we get from the spectral pattern that the modulation frequency of the first-order sidebands is about 0.01Thz, which corresponds to the maximum-gain frequency $f_{\max}$ of the MI. Theoretically in the anomalous-dispersion regime of optical fibers the maximum gain of the MI is described by

$$\Omega_{\max} = 2\pi f_{\max} = \left(\frac{2\gamma P_0}{|\beta_2|} \right)^{1/2} ,$$

where β_2 is the GVD parameter, γ is the nonlinear coefficient, and P_0 is the peak power. In our laser, the average dispersion $\beta_2 \approx -5.15 ps^2 km^{-1}$ as estimated from the soliton sidebands, and the nonlinear coefficient $\gamma \approx 3W^{-1}km^{-1}$. Thus we get that the peak power of the CW component is about 3.5mW. We use Simaeys et al.'s analysis result ($\gamma P_0 z \approx 8$) to estimate the FPU recurrent period in our condition, and get the recurrence distance is around 780km, which corresponds to a period of around 2.6ms [7]. In experiment, the OSA we used has a minimum recovery time of about 1s between two sequential measurements, which is much bigger than the recurrent period. For this reason we cannot measure the actual recurrent period directly from the OSA, but only observe the recurrent behavior phenomenally.

By using a DC coupled photo detector and a low-speed oscilloscope, we have measured the actual recurrence period of the MI of the CW component under the same condition of Fig. 3(b) and (c) in our laser. Figure 4 shows the corresponding oscilloscope trace. In obtaining Fig. 4, we used a Fiber Bragg Grating ($\lambda_0 \approx 1557.74; \Delta\lambda \le 0.08nm; r \ge 30dB$) to suppress the center CW component after the output, so that the observed intensity change would totally due to the appearance of the spectral sidebands. A light intensity variation with a period of around 7ms can be clearly seen on the oscilloscope trace, which confirms that the intensity of the spectral sidebands varies periodically. This result is very close to the 2.6ms that we theoretically estimated above. It is to note that the high frequency components of the output such as the mode-locked pulses and the high frequency modulations of MI cannot be displayed on this oscilloscope trace. Obviously, our experimental observations consist well with the analytical estimation.

The experimental observation of the FPU recurrence benefits from two aspects of our fiber soliton laser, one is the existence of erbium-doped fiber in the cavity, and the other is the

Received October 27, 2003; Revised November 20, 2003

ring structure of the laser. The existence of the erbium-doped fiber greatly lowers the threshold for MI, which makes the MI easier to be observed. When the gain is equal to the loss in the cavity, the laser ring can be regarded as a conservation system, and then the light can travel in the cavity round after round. The long propagation path and periodical insertion of amplifiers needed to observe the FPU recurrence in transmission systems are therefore avoided in our experiment, which makes it much easier to observe the phenomenon.

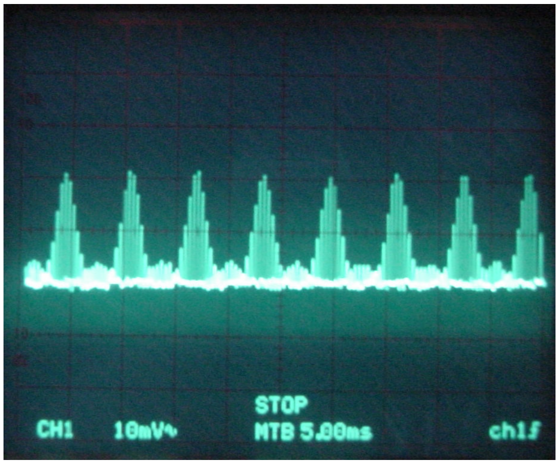

Fig. 4. The oscilloscope trace of the output under the same operation condition as Fig. 3(a) and Fig. 3(c). A recurrence period of around 7ms is clearly observed.

In conclusion, we have experimentally observed that the CW component in a fiber soliton ring laser could become unstable due to the MI, and its MI manifests the FPU recurrence. From the modulated optical spectrum, we theoretically estimated that the recurrence period is around 2.6ms. We also experimentally measured that the real recurrence period is around 7ms, which consists well with the estimation. To the best of our knowledge, this is the first experimental observation of FPU recurrence in fiber lasers.

Acknowledgments

H. Y. Tam acknowledges financial support from a university research grant from the Hong Kong Polytechnic University.