A Constraint-Aware Heuristic Path Planner for Finding Energy-Efficient Paths on Uneven Terrains

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***Abstract*—Motions of mobile robots need to be optimized to minimize their energy consumption in order to ensure long periods of continuous operations. Shortest paths do not always guarantee the minimum energy consumption of mobile robots. Also, they are not always feasible due to climbing constraints of mobile robots, especially on steep terrains. We utilize a heuristic search algorithm to find energy-optimal paths on hilly terrains using an established energy-cost model for mobile robots. The terrains are represented using grid-based elevation maps. Similar to A*-like heuristic search algorithms, the energy-cost of traversing through a given location of the map depends on a heuristic energy-cost estimation from that particular location to the goal. By using zigzag-like path patterns, the proposed heuristic function can estimate heuristic energy-costs on steep terrains that cannot be estimated using traditional methods. We proved that the proposed heuristic energy-cost function is both admissible and consistent. Therefore, the proposed path planner can always find feasible energy-optimal paths on any given terrain without node revisits, provided that such paths exist. Results of tests on real-world terrain models presented in this paper demonstrate the promising computational performance of the proposed path planner in finding energy-efficient paths.**

***Index Terms*—Mobile robot, outdoor, path planning, energy-efficient, heuristic search, uneven terrains.**

I. Introduction

In mobile robotics, motion techniques can be classified into two broad categories: motion planning algorithms and reactive navigation algorithms [1]. Motion planning algorithms use a priori information about the robot and its environment to do off-line planning. On the other hand, reactive navigation algorithms use real-time sensory information to control the motion of mobile robots according to their environment. While both are equally important in goal-driven navigation and cannot be replaced by one another, off-line mobile robot path planning algorithms fell into the former category.

Path planning can be identified as a process of finding an optimum path between two given locations on a terrain such that predefined requirements are satisfied [2]. With their theoretical foundation in network science, path planning algorithms are well adopted in robotics [3]–[9]. In previous work, the most common requirement has been minimizing the path length [10]–[14]. Shortest paths are very effective criteria for mobile robot path planning as the path length is often proportional to the traversal time, especially when the terrain is flat. Recently, mobile robots have been commonly utilized in outdoor applications [15]–[23], where terrains are mostly uneven. On such terrains, shortest paths can be physically infeasible due to motion power limitations of the mobile robots. On the other hand, traversing shortest paths can be highly energy inefficient on steep terrains. Most of the mobile robots are powered with portable energy sources, such as batteries. Therefore, the operation duration of such robots very much depends on their energy efficiency.

The focus of this paper is on finding energy-efficient paths for mobile robots which are physically feasible in a given environment. The energy consumption of the robot depends on its inclination angle as well as the traversal distance along the path. Some inclination angles are impermissible as the robot cannot generate enough power to overcome the effect of external forces on such surfaces. Therefore, these physical constraints need to be considered when finding energy minimizing paths. Geodesic shortest paths often fail to capture physical properties of the environment, such as gravity, friction, maximum driving force of the robot, and robot's stability on steep hills, which limit their applicability and usefulness in real world applications.

A. Related Work

Despite the vast range of potential application areas, such as surveillance, rescue, and mining in hostile areas, very few attentions have been devoted for energy-efficient path finding problem. One of the early attempts on mobile robots path planning on terrain maps can be found in [24]. They represented the terrain as polygonalized isolines. Minimum-time trajectories of motion on these maps were calculated using the elevation changes between adjacent isolines. Vertices of the polygons are used as nodes in the graph search. The proposed algorithm was tested using a simulated model of an unmanned robot. Several years later, Rowe and Ross introduced an energy-cost model for mobile robots navigating in uneven terrains [25]. They considered the external forces imposed on the mobile robots in their energy-cost model, and the cost of the traversal between two arbitrary points is defined as the energy requirement to overcome the effect of the friction and gravity. They also introduced anisotropism to their model by considering impermissible traversal directions due to overturn dangers and power limitations. Their energy-

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model has been adopted lately in many other works [26]–[28] as it captures most of the physical characteristics of the environment. In [25], Rowe and Ross also proposed a method to find cost-efficient paths by using A* search algorithm to pick appropriate path segments from path subspaces. They showed that there were only four ways available for traversing heading-dependent homogeneous regions optimally.

Lanthier *et al.* [27] introduced a terrain face weight idea, which apprehends the nature of the terrain, slope of each terrain face, and friction. In [27], they discretize the terrain by placing Steiner points on boundaries of the terrain faces and connect them with weighted edges. Dijkstra's algorithm [29] is then used to find a path with minimum total weight in a graph. Based on the terrain face weight concept introduced in [27], Sun and Reif [28] proposed an energy minimizing path planning method on uneven terrains for mobile robots. They also derived upper and lower bounds of the combinatorial size of energy-efficient paths on uneven terrains under certain assumptions. Their approximation algorithm outperforms the path planner introduced in [27] in terms of time complexity. Even though the terrain face concept adopted in [27], [28] can reduce the computational complexity by approximating the uneven ground level with flat surfaces, such approximations cause to degrade the accuracy of the generated paths. Therefore, it is difficult to claim the global optimality of path planners as they do not fully consider the elevation changes of terrain surfaces.

Plonski *et al.* [30] considered energy-efficient time-constrained path planning of a solar-powered robot navigating on uneven terrains. First, they obtain a solar map using Gaussian process regression. Then the energy-efficient paths are found based on this map and an empirical model of the robot. Choi *et al.* [31] proposed an energy-efficient path planning method (A*-$\mathrm{E}_{\mathrm{opt}}$) for mobile robots. They represented the terrain using simulated grid-based elevation maps. An A* heuristic search algorithm [32] was used to find the energy minimizing paths on the elevation maps. Their heuristic energy-cost function is based on the Euclidean distance between current location and the destination on the terrain. In [31], the authors showed that A* with such a heuristic function is unable to find physically feasible paths on steep terrains because heuristics can sometime be infinitely large depending on the gradient of the straight line connecting current location and goal location. Recently, we proposed a heuristic search algorithm (Basic Z*) for energy-efficient path planning on simulated grid-based elevation maps [33]. Its heuristics can overcome the impermissible traversal headings due to power limitations of a mobile robot. Results given in [33] show that the Basic Z* can find energy-efficient paths on uneven terrains where A*-$\mathrm{E}_{\mathrm{opt}}$ may fail due to infinitely large heuristic energy-cost. However, the computational efficiency of the Basic Z* algorithm may vary due to regular node revisits while searching for an optimal solution.

B. Contributions and Organization of the Paper

In this paper, we further investigate the mobile robot energy-efficient path planning problem on uneven terrains. In contrast to our previous work, we perform our tests on real world terrain models. We propose a novel energy-efficient path planning algorithm, Z*, by improving the computational efficiency of the Basic Z* algorithm. Notably, the heuristic function used in Z* algorithm generates *zigzag*-like paths to overcome the impermissible traversal headings resulted from climbing constraints of a mobile robot. We prove that this heuristic energy-cost function is *admissible* and *consistent*. Therefore, Z* can find energy-optimal paths on any terrain without node revisiting previously visited nodes. Most interestingly, the energy-efficient paths generated by the proposed approach is always physically feasible for the robots involved.

The rest of the paper is organized as follows. Section II explains how to construct a graph for finding energy-efficient paths in a given terrain. It also briefly discusses the energy-cost model used in this paper. The proposed energy-efficient path planner for outdoor mobile robots is explained in details in Section III. A physical interpretation of proposed heuristic energy-cost function is included. Results of the proposed path planner are presented and performances of the proposed path planner are analyzed in Section IV. Concluding remarks and future work are given in Section V. In addition to main contents of this paper, we provide proofs of the admissibility and consistency of the heuristic energy-cost function in APPENDIX A.

II. Preliminaries

A. Terrain Representation and Notations

With the recent advancements in geographical information systems, high resolution digital elevation models (DEMs) are available for many geographical locations of the earth [34]–[36]. They are useful in accurate representation of the terrain surface elevation. A DEM of a section of Anderson Canyon in Arizona is shown in Fig. 1. To facilitate the path planning process in this work, DEMs of terrains are transformed into weighted graphs with 8-connected neighborhoods. Each node on the graph is corresponding to a point on the terrain surface.

If n is a node in the aforementioned graph (see Fig. 1), then $(n.x, n.y, n.z)$ are the terrain surface coordinates of that node. Let n_c be the node corresponding to the current location of the robot on a given terrain, and n_n be a neighboring node that the robot will move to in next time step. The length of the projection of the straight line connecting n_c and n_n on the underlying x-y plane is defined as

$$d(n_c, n_n) = \sqrt{(n_c.x - n_n.x)^2 + (n_c.y - n_n.y)^2}. \quad (1)$$

Let the elevation difference between n_c and n_n as,

$$\Delta(n_c, n_n) = n_n.z - n_c.z. \quad (2)$$

Then the Euclidean distance s between n_c and n_n in a three-dimensional (3D) space can be defined as

$$s(n_c, n_n) = \sqrt{d(n_c, n_n)^2 + \Delta(n_c, n_n)^2} \quad (3)$$

and the angle of inclination (positive for uphilling, negative for downhilling) as

$$\phi(n_c, n_n) = \tan^{-1}\left[\frac{\Delta(n_c, n_n)}{d(n_c, n_n)}\right]. \quad (4)$$

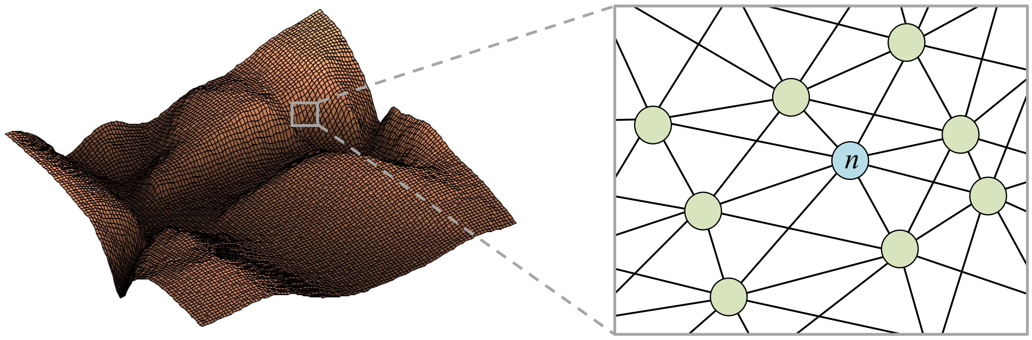

Fig. 1. A digital elevation model of a 1 km × 1 km section of Anderson Canyon in Arizona (left-hand side). Such terrain model can be converted to a graph with 8-connected neighborhoods (right-hand side) in order to facilitate the path planning process.

Edge costs of a graph depends on the optimization criteria of the paths. For a shortest path problem, they can be 3D Euclidean distances between nodes. Since our focus is on finding the energy-efficient paths, edge costs of the graph need to be defined in terms of energy-costs.

B. Energy-Cost Model

Here we adopt the energy-cost model developed by Rowe and Ross [25] to calculate the edge costs. Their model not only explains how to calculate the energy-cost, but also how to decide impermissible traversal headings. This energy-cost model assumes a constant velocity v for the entire traversal. Therefore, the two major external forces applying on the mobile robot are gravity and friction [25]. The resultant of these two forces on the mobile robot is given as $mg(\mu\ \cos\phi + \sin\phi)$. This formula has been confirmed experimentally within 1% of error for wheeled vehicles on shallow slopes in [37]. Here, m is the mass of the robot, μ is the friction coefficient, and g is the gravitational field strength. Hence, the energy-cost of $n_c n_n$ traversal is defined as $mgs(n_c, n_n)(\mu\ \cos\phi(n_c, n_n) + \sin\phi(n_c, n_n))$.

In its uphill traversal, a mobile robot may be unable to climb steep inclination due to the motion power limitations. If the maximum force available to overcome gravity and friction is $F_{\max}$, according to the physical model considered here, the maximum inclined angle that the robot can overcome is

$$\phi_f = \sin^{-1}\left(\frac{F_{\max}}{mg\sqrt{\mu^2+1}}\right) - \tan^{-1}(\mu). \tag{5}$$

Here, $F_{\max}$ can be defined as $P_{\max}/v$, where $P_{\max}$ is the maximum available motion power of the robot. This has again been experimentally confirmed within 2% of error for wheeled vehicles on shallow slopes in [37]. Furthermore, the traction is governed by the static friction coefficient μ_s between the surfaces. It can be proved that an anisotropic traction-loss phenomena will arise if the inclined angle is greater than ϕ_s [25], which is defined as

$$\phi_s = \tan^{-1}(\mu_s - \mu). \tag{6}$$

After counting on all aforesaid cases, the critical impermissible angle for the uphill traversal can be defined as

$$\phi_m = \min(\phi_f, \phi_s). \tag{7}$$

Therefore, ϕ_m is the maximum inclined angle that the robot is capable of overcoming.

For a downhill traversal, there is zero resultant external force on the mobile robot when $\phi = \phi_b$, which is defined as the critical breaking angle. It can be easily shown that

$$\phi_b = -\tan^{-1}(\mu). \tag{8}$$

A special consequence can be perceived when $\phi < \phi_b$, *i.e.* $mg(\mu\ \ \cos\phi + \sin\phi) < 0$. In such a situation, the robot normally starts to accelerate as it gains energy. However, since we assume constant velocity for the robot, it has to apply breaking force to avoid being accelerated. Generally speaking, breaking requires negligible energy. Therefore, the energy consumption of the robot in the breaking region ($\phi \leq \phi_b$) is negligible [25], [28]. Thus, the energy-cost for traversing $n_c n_n$ can be summarized as

$$k(n_c, n_n) \tag{9}$$
$$= \begin{cases} \infty, & \text{if } \phi(n_c, n_n) > \phi_m. \\ mgs(n_c, n_n)(\mu\ \cos\phi(n_c, n_n) & \\ \qquad\qquad + \sin\phi(n_c, n_n)), & \text{if } \phi_m \geq \phi(n_c, n_n) > \phi_b. \\ 0, & \text{otherwise.} \end{cases}$$

The energy-cost model given in (9) assumes that the energy-cost for making turns is negligible. The same assumption has been made in previous works on energy minimizing path planning on uneven terrains [25]–[28]. Changes in energy-consumption with different velocity profiles have been studied in [38], [39].

III. Energy-Efficient Path Planning

Assume that the starting point and goal location of the mobile robot are represented by nodes n_s and n_g, respectively. Here, the search problem is to find a minimum energy-cost path connecting n_s and n_g. An obvious solution to such a problem can be obtained using brute-force search algorithms, which consider all possible candidates by checking whether they satisfies a given set of requirements. Therefore, these algorithms guarantee to find an optimum solution for a problem if such a solution exists. Even though they are simple to implement and grantees to find an optimal solution if such exists, on the down side, brute-force algorithms are computationally expensive. Their computational complexity

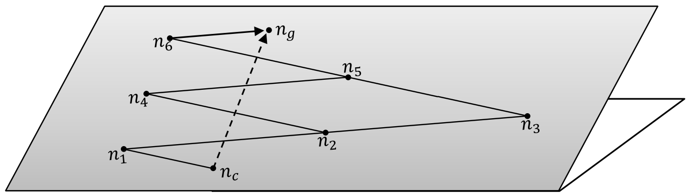

Fig. 2. The robot is unable to climb straightly from n_c to n_g on a steep surface (dashed line) since $\phi(n_c, n_g) > \phi_m$. However, it may be able to reach n_g by following a series of zigzag movements.

grows with the number of candidate solutions and therefore, they can be impractical if the size of the problem is large.

A. Heuristic Search Algorithms

Heuristic search algorithms overcome the high computational cost of brute-force algorithms by using heuristics. A* search algorithm is a typical example for this category of algorithms which achieves better computational complexity over Dijkstra's search algorithm in the shortest path finding problem. In A*-like heuristic search algorithms, the expected energy-cost of traversing to n_g through n_c is defined as

$$f(n_c) = g(n_c) + h(n_c), \tag{10}$$

where $g(n_c)$ is an energy-cost of traveling from n_s to n_c which can be calculated using (9) for each step. Here $h(n_c)$ is a heuristic estimate of the energy-cost of traveling from n_c to n_g.

1) Estimating the Heuristic Energy-Cost: If the heuristic function always returns zero, then these algorithms reduce to brute-force searches. The ideal case is the heuristic function estimating the exact cost of reaching goal from a particular node. Unfortunately, it is unrealistic to find such heuristics in most of the real world problems, but something falls between above two cases [40]. Hence, the computational cost of such algorithms depends on the quality of the heuristics.

Definition 1: The heuristic function, $h(n_c)$ is said to be admissible if it never overestimates the cost of reaching n_g from a given node [40].

A*-like heuristic search algorithms are only guaranteed to find optimal solutions if $h(n_c)$ is admissible [32]. Finding an admissible heuristic function is challenging in most of the applications, except some obvious cases such as in shortest path finding problem which can utilize the Euclidean distance to the goal from any given node as its heuristic.

Here, if the $h(n_c)$ is calculated based on the Euclidean distance by connecting n_c and n_g with a virtual straight line, *i.e.* $h(n_c) = k(n_c, n_g)$, $h(n_c)$ can sometime be infinitely large depending on the gradient of this straight line with respect to the x-y plane. Since the value of $h(n_c)$ eventually affects the value of $f(n_c)$, such situations can result in false impermissible traversal headings. An attempt made by Choi *et al.* [31] further verifies it. Even though the mobile robot is unable to traverse a straight line connecting one point to another if $\phi > \phi_m$, it may still reach the target by following a series of zigzag movements as illustrated in Fig. 2. The headings in the zigzag pattern is permissible if $\phi \leq \phi_m$. There

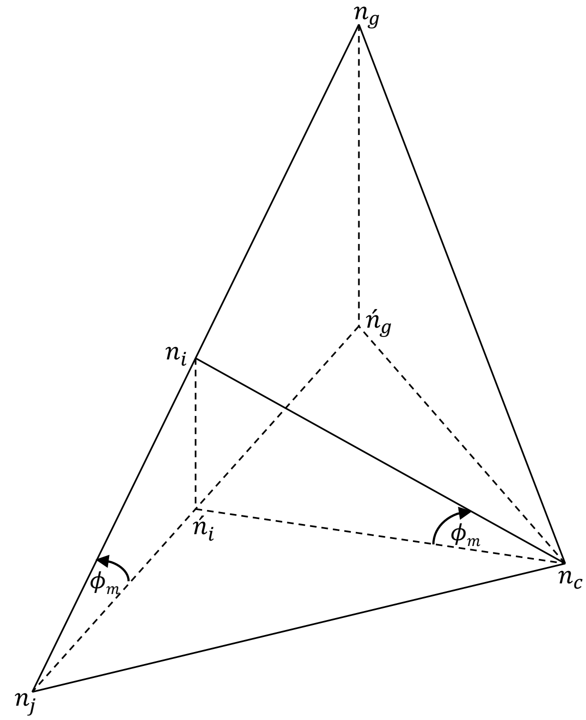

Fig. 3. It is unable to estimate the heuristic energy-cost of the $n_c n_g$ traversal since $\phi(n_c, n_g) > \phi_m$. Therefore, the proposed method selects a heading for the robot such that $\phi = \phi_m$, which results in a finite heuristic energy-cost.

are two possible permissible paths as shown in Fig. 2 to reach n_g: $n_c n_1 n_2 n_4 n_5 n_6 n_g$ and $n_c n_1 n_2 n_3 n_5 n_6 n_g$. Similarly, many other paths can be found with the same heuristic energy-cost and have $\phi \leq \phi_m$. Based on such zigzag movements, we proposed a heuristic energy-cost function [33] which can be summarized as

$$h(n_c) = \begin{cases} \frac{mg\Delta(n_c,n_g)}{\sin\phi_m}(\mu \ \cos\phi_m \\ \qquad\qquad + \sin\phi_m), & \text{if } \phi(n_c,n_g) > \phi_m. \\ mgs(n_c,n_g)(\mu \ \cos\phi(n_c,n_g) \\ \qquad\qquad + \sin\phi(n_c,n_g)), & \text{if } \phi_m \geq \phi(n_c,n_g) > \phi_b. \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Here, the last two cases are straightforward to understand. In the second case, when $\phi_m \geq \phi(n_c, n_g) > \phi_b$ is calculated by connecting n_c and n_g with a virtual straight line, *i.e.* $h(n_c) = k(n_c, n_g)$, as the robot is able to traverse the angles less than the critical impermissible angle. In the last case, the gradient of the virtual straight line that connects n_c and n_g with respect to the x-y plane, falls in the braking range. Therefore, according to the energy-cost model given in (9), the heuristic energy-cost reduces to zero. We now illustrate the physical meaning of the $h(n_c)$ when $\phi(n_c, n_g) > \phi_m$ with the help of Fig. 3. Since $\phi(n_c, n_g) > \phi_m$, it is impossible to estimate a finite value for $h(n_c)$ using $k(n_c, n_g)$. Hence, we define a new path to n_g via n_i for estimating a finite value of $h(n_c)$ which ultimately results in a zigzag-like path pattern. Let n_j be a point on the x-y plane which goes through n_c, such that $\angle n_g n_c n_j = \pi/2$ and $\angle \acute{n}_g n_j n_g = \phi_m$. Here, $\acute{n}_g$ is the projection of n_g on the same x-y plane. Therefore, $\angle \acute{n}_g n_c n_j = \pi/2$ as well. We select point n_i on the straight line connecting n_g and n_j such that $s(n_c, n_i) = s(n_j, n_i)$. In Fig. 3, $\acute{n}_i$ is the projection of n_i on the x-y plane which goes through n_c, n_j, and $\acute{n}_g$. Therefore, using similar triangles $n_j n_i \acute{n}_i$ and $n_c n_i \acute{n}_i$, we can show that $\phi(n_c, n_i) = \phi(n_j, n_i) = \phi_m$. The total heuristics energy-cost is equal to the summation of energy expenditure of traversing $n_c n_i$ and $n_i n_g$. Hence, we can define the heuristic to be the energy-cost of traversing $n_j n_i n_g$. By definition, $s(n_g, \acute{n}_g) = \Delta(n_c, n_g)$ and $s(n_j, n_g) = \Delta(n_c, n_g)/\sin\phi_m$. Therefore, $h(n_c) = mg(\mu \ \cos\phi_m + \sin\phi_m)\Delta(n_n, n_g)/\sin\phi_m$ when $\phi(n_c, n_g) > \phi_m$. Here $\phi = \phi_m$ is not the only inclination angle which generates a permissible heading for the scenario under study. It is possible to find many other paths to reach n_g with $\phi \leq \phi_m$. Nevertheless, a proof given in APPENDIX A shows that the heuristic is admissible when $\phi = \phi_m$.

2) Generating Energy-Efficient Paths: Z* uses best-first search to find the energy-efficient paths. It starts by visiting node n_s and calculating its energy-cost using (10). Obviously, $f(n_s) = h(n_s)$. Once a node is visited, it is added to an OPEN set. The OPEN set can be implemented as a sorted priority queue based on each node's $f(n_s)$ cost. In addition to the OPEN set used in Basic Z* algorithm, here Z* uses another set called CLOSED set. Similar to Basic Z* algorithm, in each iteration, a node with minimum expected energy-cost, n_c is taken out from the OPEN set and added to the CLOSED set. All the neighbors of n_c are added to OPEN set. Basic Z* omits the CLOSED set due to its uncertainty over the consistency of its heuristics.

Definition 2: A heuristic function is said to be consistent (or *monotonic*) [40] if it satisfies

$$h(n_c) \leq k(n_c, n_n) + h(n_n). \tag{12}$$

If the heuristic function is consistent, then $f(n_c)$ is monotonically nondecreasing along any path connecting n_s and n_g. Since the heuristics used in Basic Z* are not proven to be consistent, it may add certain nodes back to the OPEN set, which are taken out before. We call this as node revisiting, which is necessary to guarantee an optimal solution. However, such revisits degrade the computational efficiency of search algorithms. In APPENDIX A, we prove that the heuristic function given in (11) is consistent. With the proven consistency of the proposed heuristic function, Z* heuristic search algorithm can find a optimal solution without revisiting nodes in the CLOSED set, thus, it is computationally more efficient than Basic Z*.

The sequence of costs of the visited nodes by Z* starts with $f(n_s) = h(n_s)$, which is the minimum and are guaranteed to stay same or increase until it hits the cost of the optimal solution $f(n_g) = g(n_g)$, which is the maximum. When $n_c = n_g$, the algorithm has reached the goal and the iterative procedure will be terminated. Here, $f(n_g)$ is the cost of the energy-efficient path. The energy-efficient path will be created by traversing back from n_g to n_s using their parent connections. The resultant graph of the nodes in CLOSED set obviously gives a tree structure without any cycles as each node is only visited once.

TABLE I
PARAMETERS OF EACH TEST.

Test	Terrain	n_s (m)	n_g (m)	v (ms^{-1})	Payload (kg)
I	Anderson Canyon	(130,850)	(920,270)	0.5	0
II	Lowe Peak	(800,90)	(130,920)	1.0	25
III	Matheny Ridge	(60,250)	(960,920)	0.8	70
IV	Lowe Peak with obstacles	(800,90)	(130,920)	1.0	25

IV. RESULTS AND DISCUSSION

In this section, we evaluate Z* algorithm against Basic Z* algorithm for energy-efficient path planning on uneven terrains. We use results of the Dijkstra's algorithm with the energy-cost function defined in (9) as a reference (D-$\mathrm{E_{opt}}$), because it guarantees to provide a energy-optimal path between two given points, if it exists. We also demonstrate the difference between energy-efficient paths and shortest paths on hilly terrains by using Dijkstra's algorithm with 3D Euclidean distance (D-$\mathrm{D_{opt}}$) to find shortest paths on hilly terrains. Extensive tests were carried out to verify the completeness, optimality, and search efficiency of the Z* search algorithm.

A. Test Setup

Tests were carried out using a simulated model of a Seekur mobile robot platform [41]. Seekur is capable of operating at about a maximum motion power of $P_{\max} = 1280$ W. The mass of the Seekur is 300 kg. We used different payloads with Seekur in different tests. A complete overview of the parameter setup in four selected tests is given in Table I. Here the gravitational field strength (g) is assumed to be 9.81 ms^{-2}. In contrast to previous work on energy-efficient path planning [31], [33], which were conducted on simulated hilly landscapes, tests were carried out on several DEMs of real terrains with an area of 1 km $\times$ 1 km. All these DEMs were fed to the algorithms under test as square shaped graphs with 100 nodes on each side (*i.e.* 10,000 nodes in each graph). The friction coefficients between the terrain and the robot wheels are $\mu = 0.1$ and $\mu_s = 1.0$, respectively. All the path planning methods under test were evaluated using MATLAB on a portable computer with 2.20 GHz Intel Core i7-4702HQ CPU using a single core and 16 GB memory. Statistical results on the averaged run time of the algorithms were acquired using 20 tests.

B. Performance Analysis

The first test was conducted on a part of the Anderson Canyon in Arizona. Paths generated from the algorithms under test are shown in Fig. 4 (a). The energy-efficient paths generated using D-$\mathrm{E_{opt}}$, Basic Z*, and Z* are slightly different from each other. This is due to the fact that a mobile robot consumes no energy when $\phi(n_c, n_n) < \phi_b$. If there is more than one such n_n, the algorithms arbitrarily select one of them. It may result in different routes, but the paths are associated with the same energy-cost. This can be verified using the

TABLE II
TEST RESULTS.

Test	Algorithm	Energy-cost (kJ)	Path length (m)	Number of nodes		Run time	
				visited	revisited	average (s)	variance (s^2)
I	D-D_{opt}	447.076	1076.498	9435	0	125.796	0.752
	D-E_{opt}	425.046	1122.099	9271	0	126.284	0.756
	Basic Z*	425.046	1273.495	3320	0	71.375	0.120
	Z*	425.046	1273.495	3320	0	43.506	0.027
II	D-D_{opt}	N/A	1236.949	9782	0	131.000	0.109
	D-E_{opt}	1662.660	2058.765	7855	0	105.9117	0.383
	Basic Z*	1662.660	2059.341	6673	84	148.367	0.155
	Z*	1662.660	2059.341	6673	0	90.871	0.088
III	D-D_{opt}	N/A	1241.229	9922	0	127.653	0.274
	D-E_{opt}	1394.153	1549.596	9862	0	132.241	0.186
	Basic Z*	1394.153	1550.500	6114	3	136.762	0.193
	Z*	1394.153	1550.500	6114	0	83.814	0.100
IV	D-D_{opt}	N/A	1241.784	9697	0	132.161	0.626
	D-E_{opt}	1687619	2113.941	7905	0	107.393	0.506
	Basic Z*	1687.619	2113.969	6944	85	154.532	0.763
	Z*	1687.619	2113.969	6944	0	94.578	0.115

results summarized in Table II. The shortest path generated using D-D_{opt} is clearly distinguishable from other two paths. According to the results given in Table II, it is shorter than the other two. However, our objective is to find a minimum energy-cost path. The results clearly show that the energy-cost of the shortest path is higher than those minimum energy-cost paths. Z* finds optimal paths much faster by visiting a least number of nodes. In this test, D-E_{opt} has visited 92.71% of the map to find an energy-efficient path, whereas Basic Z* and Z* achieve that by just exploring 33.20% of the map. Basic Z* have not revisited any nodes to find the optimum solution and end up visiting same number of nodes as Z*. Since the energy-efficient path generated in this test has comparatively less turns, there is a lower possibility that a node which had been removed from the OPEN set, to become a neighbor of a node visited later down the path. In contrast, there is a higher possibility of such node revisits in the energy-efficient paths generated in other tests. This depends on the nature of the terrain as well as the starting point and the goal of a path.

The tests II and III were conducted on regions of the Lowe Peak in Utah and Matheny Ridge in Washington, respectively. Resulted paths are shown in Fig. 4 (b) and (c). In both tests, the shortest path generated by D-D_{opt} is significantly different from the other two paths. Unfortunately, such a path is practically impermissible on the given terrain due to the climbing constraints of the robot. Therefore, the energy-cost of the shortest path is incalculable. It is interesting to see that the energy minimizing algorithms utilize zigzag-like path patterns to climb the steep hills in the terrains. Such paths can be predicted by the heuristic energy-cost function used in this paper which helps to overcome the impermissible traversal headings due to the climbing constraints of the robots in uphilling. The results presented in Table II clearly illustrate the optimal nature of the paths generated by Basic Z* and Z* algorithms. Furthermore, Z* is computationally efficient as it is guaranteed to visit the least number of nodes since its heuristic is consistent. This can be easily observed when we compare the number of visited nodes by the algorithms under test in each test. Z* does not revisit nodes, which makes it more desirable than Basic Z*. Moreover, Z* is complete as it is guaranteed to find an energy-efficient path to a given goal location if that exists.

In the first three tests, it is assumed that terrains are obstacle free. However, in most of the real-world scenarios, mobile robots have to deal with dense environments and their motion need to be planned in such a way that they avoid collisions with known obstacles present in the environment. This work can be easily extended to any general real-world terrain by setting infinite energy-cost for traversing those nodes coincide with obstacles. In order to demonstrate performances of the proposed path planner in such environments, the test IV was conducted with the same test parameters used in test II, but with some artificial obstacles imposed on the previously found energy-efficient path. New paths generated by the algorithms under test are shown in Fig. 4 (d). White areas on the terrain represents obstacle areas which should be avoided by the robot. As we can observe from the given results, all the path planners under test are capable of finding collision free paths. Similar to the previous tests, Z* finds an energy-efficient path by exploring a minimum number of nodes. In this work, accuracy and optimally of all grid-based path planning methods are discussed and evaluated in given discrete domains. However, these results may differ in real-world continuous domains. Therefore, the results generated by Z* on DEMs can slightly deviate from the optimal paths on their real-world counterparts. This problem can always be alleviated by having maps with higher resolutions.

V. CONCLUSION

In this paper, we present a computationally efficient heuristic search algorithm Z* for energy-efficient path planning on hilly terrains. Traditional approaches cannot find feasible

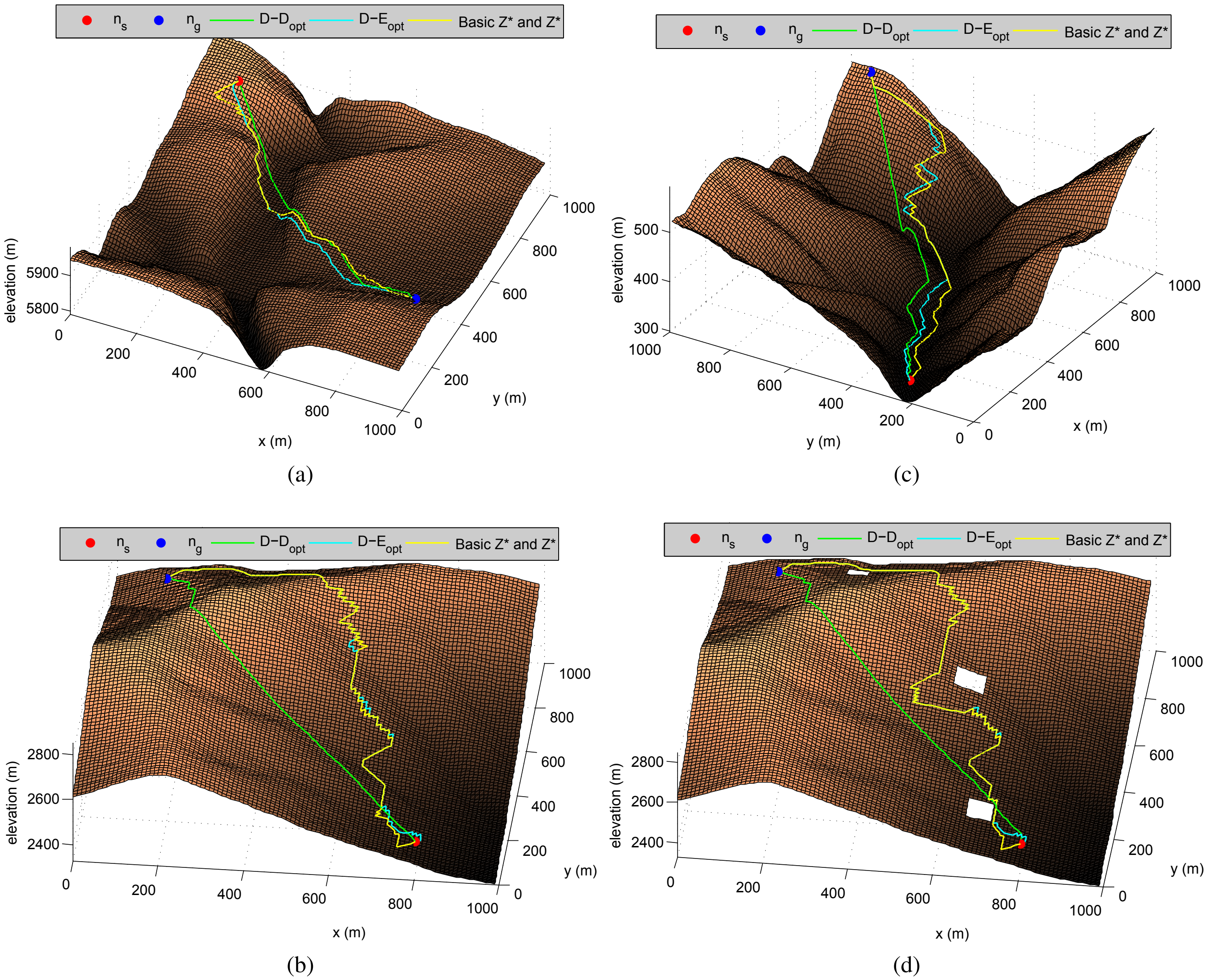

Fig. 4. Results of four selected tests showing the path generated by D-D$_{\rm opt}$, D-E$_{\rm opt}$, and Basic Z*: (a) Test I: Anderson Canyon, (b) Test II: Lowe Peak, (c) Test III: Matheny Ridge and (d) Test II: Lowe Peak with artificial obstacle regions.

energy-efficient paths on steep terrains due to motion power limitations of robots. Here, hilly terrains are represented using grid-based elevation maps. The cost of traversing from a start point to a goal point through a chain of connected nodes is calculated as the summation of the energy-cost of traveling to an intermediate node and the heuristic energy-cost of traveling from the intermediate node to the goal. The heuristic energy-cost function introduced in this paper enables Z* to find energy-efficient paths in steep terrains using zigzag-like path patterns. We prove that the proposed heuristic function is admissible and consistent. Therefore, Z* can find a physically feasible energy-efficient path on any given terrain, if it exists. Moreover, Z* is computationally efficient as it avoids any node revisits during its search. The resulted energy-efficient paths can prolong the lifetime of mobile robot systems, which are very useful in applications such as surveillance, rescue, military, mobile wireless sensor networks, and mining in hostile areas.

Appendix A
Proofs of the Admissibility and Consistency of the Proposed Heuristic Function

Proposition 1: The heuristic energy-cost function $h(n_c)$ proposed in (11) is admissible.

Proof: Suppose we need to determine a heuristic function $h(n_c)$, for calculating heuristic energy-cost for the $n_c n_g$ traversal shown in Fig. 5. Here, $h(n_c)$ is admissible if it estimates the possible minimum energy-cost for the $n_c n_g$ traversal. However, determining an admissible heuristic energy-cost function is far from obvious in the current problem, because, not all the paths connecting n_c and n_g are physically feasible for a mobile robot to achieve.

Let us consider a general path connecting n_c and n_g through $N-1$ intermediate nodes as illustrated in Fig. 5. A vector

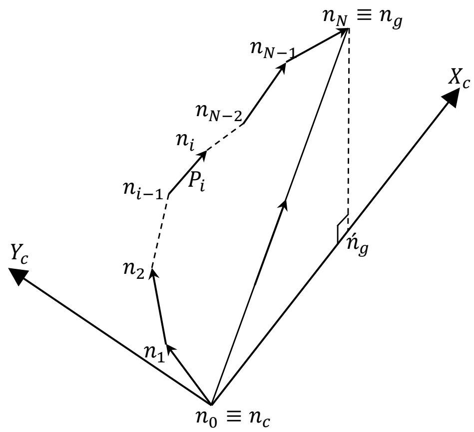

Fig. 5. For the derivation of heuristic energy-cost of $n_c n_g$ traversal, we consider a general path connecting n_c and n_g through $N-1$ intermediate nodes.

connecting nodes n_{i-1} and n_i is denoted by,

$$\mathbf{P} = s(n_{i-1}, n_i) \begin{bmatrix} \cos\phi(n_{i-1}, n_i) \cos\theta(n_{i-1}, n_i) \\ \cos\phi(n_{i-1}, n_i) \sin\theta(n_{i-1}, n_i) \\ \sin\phi(n_{i-1}, n_i) \end{bmatrix}, \qquad (13)$$

where $i = 1, 2, \ldots, N$, and $\theta(n_{i-1}, n_i)$ is the angle between X_c and the projection of $\mathbf{P}$ on X_cY_c plane. For simplicity, we denote $\phi(n_{i-1}, n_i) = \phi_i(\leq \phi_m)$, $\theta(n_{i-1}, n_i) = \theta_i$, and $s(n_{i-1}, n_i) = s_i$, $s(n_c, n_g) = s$, and $\phi(n_c, n_g) = \phi$. Note that the X_cY_c plane is defined in the horizontal plane such that $\theta(n_c, n_g) = 0$. Using the vector summation, we can write

$$s \begin{bmatrix} \cos\phi \\ 0 \\ \sin\phi \end{bmatrix} = \sum_{i=1}^{N} s_i \begin{bmatrix} \cos\phi_i \cos\theta_i \\ \cos\phi_i \sin\theta_i \\ \sin\phi_i \end{bmatrix}. \qquad (14)$$

Using (14), we define functions f_1, f_2, and f_3 as

$$f_1(s_i, \phi_i, \theta_i) = \sum_{i=1}^{N} s_i \cos\phi_i \cos\theta_i - s\cos\phi = 0 \qquad (15)$$

$$f_2(s_i, \phi_i, \theta_i) = \sum_{i=1}^{N} s_i \cos\phi_i \sin\theta_i = 0 \qquad (16)$$

$$f_3(s_i, \phi_i, \theta_i) = \sum_{i=1}^{N} s_i \sin\phi_i - s\sin\phi = 0 \qquad (17)$$

Using (9), the total heuristic energy-cost for $n_0, n_1, \ldots, n_N$ traversal can be defined as

$$H = \sum_{i=1}^{N} k(n_{i-1}, n_i). \qquad (18)$$

Our objective is to find s_i, ϕ_i, and θ_i such that H is minimized. Here, we use the method of Lagrange multipliers to solve this optimization problem and the Lagrangian is defined by,

$$\Lambda(s_i, \phi_i, \theta_i) = H - \sum_{j=1}^{3} \lambda_j f_j(s_i, \phi_i, \theta_i), \qquad (19)$$

where λ_j is the j^{th} Lagrange multiplier. When H is minimum, $\nabla\Lambda(s_i, \phi_i, \theta_i) = 0$. Therefore, using (19), we can obtain

$$\nabla_{s_i}\Lambda(s_i, \phi_i, \theta_i) = \nabla_{s_i}H - \lambda_1 \cos\phi_i \cos\theta_i - \lambda_2 \cos\phi_i \sin\theta_i - \lambda_3 \sin\phi_i = 0 \qquad (20)$$

$$\nabla_{\phi_i}\Lambda(s_i, \phi_i, \theta_i) = \nabla_{\phi_i}H + \lambda_1 s_i \sin\phi_i \cos\theta_i + \lambda_2 s_i \sin\phi_i \sin\theta_i - \lambda_3 s_i \cos\phi_i = 0 \qquad (21)$$

$$\nabla_{\theta_i}\Lambda(s_i, \phi_i, \theta_i) = \nabla_{\theta_i}H + \lambda_1 s_i \cos\phi_i \sin\theta_i - \lambda_2 s_i \cos\phi_i \cos\theta_i = 0 \qquad (22)$$

From (22), $\cos\phi_i \cos\theta_i \neq 0$ since $\nabla_{\theta_i}H = 0$. Therefore, using (22), we can obtain

$$\lambda_2 = \lambda_1 \tan\theta_i. \qquad (23)$$

Using (20), (23), and (21) we can show that

$$\nabla_{s_i}H = \lambda_1 \frac{\cos\phi_i}{\cos\theta_i} + \lambda_3 \sin\phi_i, \qquad (24)$$

$$\nabla_{\phi_i}H = -\lambda_1 s_i \frac{\sin\phi_i}{\cos\theta_i} + \lambda_3 s_i \cos\phi_i. \qquad (25)$$

Here, we need to consider three different situations to obtain the final solution.

Case 1: $\phi \leq \phi_b$

According to (9), the minimum possible energy-cost is $k(n_c, n_g) = 0$ as it is assumed the robot cannot gain energy. It is obvious, $\phi_i = \phi$ and $\theta_i = 0$. *i.e.* the minimum energy path coincide with the straight line connecting n_c and n_g. Therefore,

$$h(n_c) = 0, \text{ when } \phi \leq \phi_b \qquad (26)$$

Case 2: $\phi_m \geq \phi > \phi_b$

As the solution is not as obvious as in the previous case, we need to consider two different possibilities of ϕ_i to obtain a solution for $h(n_c)$.

Case 2.1: $\phi_i \leq \phi_b$: According to (9), $k(n_{i-1}, n_i) = 0$ when $\phi_i \leq \phi_b$. Thus $\nabla_{s_i}H = \nabla_{\phi_i}H = 0$. Using (24) and (25), we can show that

$$\frac{\lambda_1}{\lambda_3} = \frac{-\sin\phi_i \cos\theta_i}{\cos\phi_i} = \frac{\cos\phi_i \cos\theta_i}{\sin\phi_i}, \qquad (27)$$

$$\Rightarrow \tan^2\phi_i = -1. \qquad (28)$$

This is not possible for any real ϕ_i. Therefore, this case does not exist ($\phi_i \nleq \phi_b$).

Case 2.2: $\phi_b < \phi_i \leq \phi_m$: Using (9) and (18), we can obtain

$$\nabla_{s_i}H = mg(\mu\cos\phi_i + \sin\phi_i), \qquad (29)$$

$$\nabla_{\phi_i}H = mgs_i(-\mu\sin\phi_i + \cos\phi_i). \qquad (30)$$

Using (24) and (29), we have

$$mg(\mu\cos\phi_i + \sin\phi_i) = \lambda_1 \frac{\cos\phi_i}{\cos\theta_i} + \lambda_3 \sin\phi_i, \qquad (31)$$

and, with (25) and (30) we have

$$mgs_i(-\mu\sin\phi_i + \cos\phi_i) = -\lambda_1 s_i \frac{\sin\phi_i}{\cos\theta_i} + \lambda_3 s_i \cos\phi_i. \qquad (32)$$

By multiplying (31) with $\sin\phi_i$ and (32) with $\cos\phi_i$, and adding them together, we can obtain

$$\lambda_3 = mg, \tag{33}$$
$$\lambda_1 = mg\mu\cos\theta_i. \tag{34}$$

Using (23) and (34), we have

$$\lambda_2 = mg\mu\sin\theta_i. \tag{35}$$

Using (16) and (35), we can show that

$$\left(\frac{\lambda_2}{mg\mu}\right)\sum_{i=1}^{N} s_i\cos\phi_i = 0 \tag{36}$$

As $\sum_{i=1}^{N} s_i\cos\phi_i \neq 0$, $\lambda_2 = 0 \Rightarrow \theta_i = 0$, $\lambda_1 = mg\mu$, and $\lambda_3 = mg$. Therefore, (15) and (17) can be written as

$$\sum_{i=1}^{N} s_i\cos\phi_i = s\cos\phi, \tag{37}$$
$$\sum_{i=1}^{N} s_i\sin\phi_i = s\sin\phi. \tag{38}$$

Using (18), (37), and (38), we can obtain

$$h(n_c) = \sum_{i=1}^{N} mgs_i(\mu\cos\phi_i + \sin\phi_i) \tag{39}$$
$$= mgs(\mu\cos\phi + \sin\phi), \text{when } \phi_m \geq \phi > \phi_b. \tag{40}$$

This is equivalent to the energy consumption of traversing the straight line connecting n_c and n_g.

Case 3: $\phi > \phi_m$

Similar to the second case, we need to consider the two possibilities of ϕ_i to obtain a solution for $h(n_c)$.

Case 3.1: $\phi_i < \phi_b$: Similar to Case 2.1, this case does not exist.

Case 3.2: $\phi_b < \phi_i \leq \phi_m$: Unlike Case 2.2, here $\theta_i \neq 0$ as $\phi_i < \phi$. *i.e.* if $\theta_i = 0$, $h(n_c) = mgs(\mu\cos\phi + \sin\phi)$. But since $\phi > \phi_m$, it is impossible to estimate the energy-cost of traversing the straight line connecting n_c and n_g. Therefore, in order to satisfy (16), let

$$\theta_i = (-1)^i \sin^{-1}\left(\frac{\lambda_2}{mg\mu}\right). \tag{41}$$

This satisfy (16) given that N is even. Let, $\sin^{-1}\left(\frac{\lambda_2}{mg\mu}\right) = \theta_c$. Hence, (15) and (17) can be written as

$$\sum_{i=1}^{N} s_i\cos\phi_i = s\left(\frac{\cos\phi}{\cos\theta_c}\right), \tag{42}$$
$$\sum_{i=1}^{N} s_i\sin\phi_i = s\sin\phi. \tag{43}$$

Using (18),

$$H = \sum_{i=1}^{N} mgs_i(\mu\cos\phi_i + \sin\phi_i), \tag{44}$$
$$= mgs\left(\mu\frac{\cos\phi}{\cos\theta_c} + \sin\phi\right). \tag{45}$$

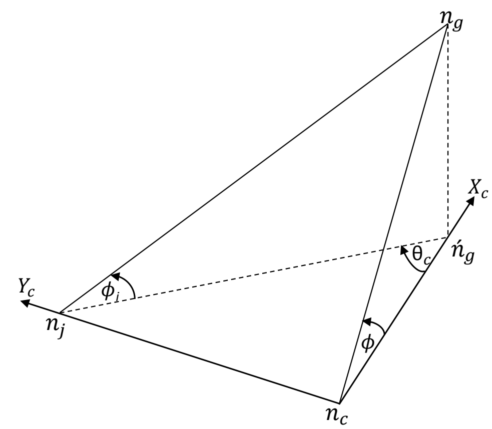

Fig. 6. A physical representation of θ_c

A physical meaning of (45) is illustrated in Fig. 6. Here, n_j is a point on Y_c axis. H is equivalent to the energy-cost of traversing $n_j n_g$. H is minimized when θ_c is maximized. However, θ_c depends on ϕ_i, which can be defined as

$$\cos\theta_c = \frac{\tan\phi_i}{\tan\phi}, \tag{46}$$

using Fig. 6. Therefore,

$$\max(\cos\theta_c) = \frac{\tan\phi_m}{\tan\phi}. \tag{47}$$

Using (45) and (47), we can define,

$$h(n_c) = \frac{mgs\sin\phi}{\sin\phi_m}(\mu\cos\phi_m + \sin\phi_m). \tag{48}$$

However, $s\sin\phi = \Delta(n_c, n_g)$. Therefore, when $\phi(n_c, n_g) > \phi_m$,

$$h(n_c) = \frac{mg\Delta(n_c, n_g)}{\sin\phi_m}(\mu\cos\phi_m + \sin\phi_m). \tag{49}$$

Therefore the heuristic energy-cost function can be summarized as in (11). ■

Proposition 2: The heuristic energy-cost function $h(n_c)$ proposed in (11) is consistent.

Proof: If the proposed $h(n_c)$ is consistent, it should satisfy (12). Suppose,

$$h(n_c) > k(n_c, n_n) + h(n_n) \tag{50}$$

Here, $k(n_c, n_n) \geq 0$. If (50) is true, the cost of traversing from n_c to n_g via n_n should be lower than $h(n_c)$. However, according to *Proposition 1*, $h(n_c)$ is the minimum possible heuristic energy-cost for $n_c n_g$ traversal. Therefore, (50) cannot stand. *i.e.* $h(n_c)$ is consistent. ■

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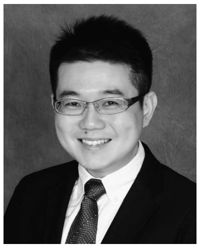

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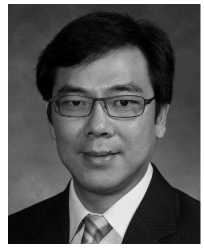

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Currently Dr. Tse serves as Editor-in-Chief for the *IEEE Circuits and Systems Magazine*, and was an Associate Editor for the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS PART I—FUNDAMENTAL THEORY AND APPLICATIONS from 1999 to 2001 and again from 2007 to 2009. He has also been an Associate Editor for the IEEE TRANSACTIONS ON POWER ELECTRONICS since 1999. He is an Editor of the *International Journal of Circuit Theory and Applications* and is on the editorial board of several other journals. Dr. Tse also received a number of paper prizes, research awards and invention awards, and was conferred honorary titles by several institutions in Australia, Canada and China.