

Tomographic Dynamic Imaging and System Identification with Partially Sampled Noisy Radio-active Signal

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ABSTRACT: *In this paper, we propose a novel approach to perform the tomographic dynamic imaging of the cross-section of humans, which is time-varying in nature. The approach requires only the partially sampled noisy radio-active signal recorded by a conventional rotating camera system, without the need of knowing any additional information or prior knowledge. Based on the reconstructed dynamic images, we use a new system identification technique to quantify the dynamic processes in humans and to estimate the physiological parameters. The reliability of the method has been tested by computer simulations and the results show that using the proposed approach, dynamic imaging and system identification by a conventional rotating camera system with partially sampled noisy radio-active signals are comparable with those using a more expensive system where fully sampled data are needed.*

1. INTRODUCTION

Advances in the computer technology and signal processing techniques have meant that tomographic imaging methods could be applied to determine in vivo body functioning. It is now possible to record the bio-distribution of radiopharmaceuticals within the body and the change of these distributions with time [1]. This information provides an important tool for diagnosis and monitoring of the effects of treatment of patients. However, such imaging methods require the use of the very expensive PET (Positron Emission Tomography) system. Hence, for reasons of availability and cost, it would be beneficial to have tomographic dynamic imaging and modeling for conventional rotating camera SPECT (Single Photon Emission Computer Tomography) system.

In fact, the use of rotating camera system for quantitative uses has been extensively studied by the Research Institute for Brain and Blood Vessels, in Akita, Japan [2]. Besides, Hansen et al [3] recently applied conventional image reconstruction methods to a rotating SPECT camera for dynamic analysis of myocardial functions. On the other hand, alternative approaches which perform system identification directly from the projection data has been proposed in [4-5].

The major problem of using the rotating camera system for tomographic dynamic imaging is that,

while the camera is rotating with the varying of the object, the directly measured projection data are inconsistent with its internal distribution. Hence, only partial information can be recorded in each rotation.

In this paper, we propose a new approach to perform tomographic dynamic imaging and system identification with rotating camera system, without the need of knowing any additional information or prior knowledge. Based on the partially sampled projection data recorded by a typical single-head rotating camera, we use a recently developed WPO method (Weighted Parabola Overlapped Integration Method) [6] to estimate the full projection data. Then, we employ a normalisation step to reduce the estimation error and the projection noise. Finally, based on the constructed dynamic images, we use a new system identification technique to quantify the dynamic process. To evaluate the performance, we use PET FDG (^{18}F)fluoro-2-deoxy-D-glucose model dynamics as an example to compare the dynamic images reconstructed and physiological parameters estimated from the partially sampled projection data recorded by the single-head rotating camera by using our new approach, with those obtained from the fully sampled projection data, such as in PET system.

2. OPERATIONS AND PROBLEMS OF SINGLE-HEAD ROTATING GAMMA CAMERA SYSTEM

The operation of a single-head rotating camera SPECT system is shown in Figure 1. The object located in the center of the figure is to be scanned. This object contains three regions in which their activity changes with time. The camera which rotates around the object can only take projection data from one angle at a time.

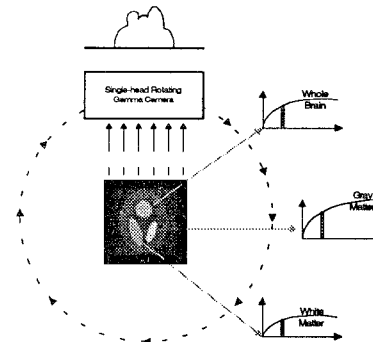


Figure 1 The Single-head Rotating Camera

In order to accurately reconstruct the image of a time-varying object from its projection data at a particular time, it is required that the projection data of this object from all angles are available at the same time. Obviously, the single head rotating camera system described above cannot fulfil this requirement.

For the traditional Flat model method, the object scanned is assumed static during the scanning period and the full projection data are obtained by first simply extending the recorded data to the whole time frame and then combining the projection data from different views together. After that, the FBP (Filtered-Backprojection Algorithm) is used to reconstruct the dynamic images.

The traditional approach has two problems. Firstly, the reconstructed dynamic images suffer from motion artifacts because the combined full projection data are inconsistent with the internal distribution of the object under scanned. Secondly, on using the conventional modeling technique for physiological parameter estimation, we need the full details of the tracer time activity curve within the object. This requirement is extremely difficult for single-head rotating camera system because of the limitation on the camera rotating speed and the trade-off between image counts and the temporal resolution. Hence, the above system, when using the conventional approach, is not suitable for physiological parameter estimation and it is mainly used for qualitative study.

3. THEORY FOR THE PROPOSED METHOD

Our proposed approach tends to solve the above two problems in three steps: 1) estimate the full projection data accurately; 2) reduce the estimation error and projection noise; 3) quantify the dynamic process using a new system modeling technique.

3.1 Estimation Of Full Projection Data

The simplest technique to estimate the full projection data is by linear interpolation. This method, which may outperform the traditional Flat model approach, is still not good enough in our situation because of the small number of samples directly measured in each angle. The estimated projection, as well as the reconstructed dynamic images, using simple linear interpolation approach is biased. To obtain a better estimation, we use a recently developed WPO method (Weighted Parabola Overlapped Integration Method) to approximate the full projection data. The WPO method can be applied to uniformly sampled data as well as non-uniformly sampled data. Its main idea is given as follows:

Assume there are n sampled data for an integrated function $y(t)$. For any four consecutive points, $t_{i-1}, t_i, t_{i+1}, t_{i+2}$, they can only be fitted with one third order polynomial. Integrating this third order function, $y_i^{(3)}$, from t_i to t_{i+1} , we obtain

$$S_i = \int_{t_i}^{t_{i+1}} y_i^{(3)}(t) dt$$

The value S_i obtained has third order algebraic accuracy. On the other hand, for the three points, t_{i-1}, t_i, t_{i+1} , they can be fitted with only one second order polynomial. Integrating this second order polynomial, $y_i^{(2)}$, from t_i to t_{i+1} , we obtain

$$S_i^b = \int_{t_i}^{t_{i+1}} y_i^{(2)}(t) dt$$

Here, the S_i^b obtained has second order algebraic accuracy. Similarly, for the points, t_i, t_{i+1}, t_{i+2} , another second order polynomial can be fitted. Integrating this polynomial, $y_{i+1}^{(2)}$, we obtain

$$S_{i+1}^f = \int_{t_i}^{t_{i+1}} y_{i+1}^{(2)}(t) dt$$

Again, the value S_{i+1}^f obtained has second order algebraic accuracy. Calculate the weighting coefficients as

$$\alpha_i = (t_{i+1} + t_{i+2} - t_i) / [2(t_{i+2} - t_{i-1})]$$

we can overlap the weighted S_i^b and S_{i+1}^f to obtain S_i as

$$S_i = (1 - \alpha_i) S_i^b + \alpha_i S_{i+1}^f$$

By using the method above, we obtain an approximation integration with third order algebraic accuracy during every interval from t_i to t_{i+1} where $0 < i < n$. For the interval t_0 to t_1 and t_{n-1} to t_n , second order algebraic accuracy can be obtained by setting $\alpha_0 = 1$ and $\alpha_{n-1} = 0$.

The first step of the proposed approach is illustrated more intuitively in Figure 2, in which the WPO method is applied to obtain an estimation of

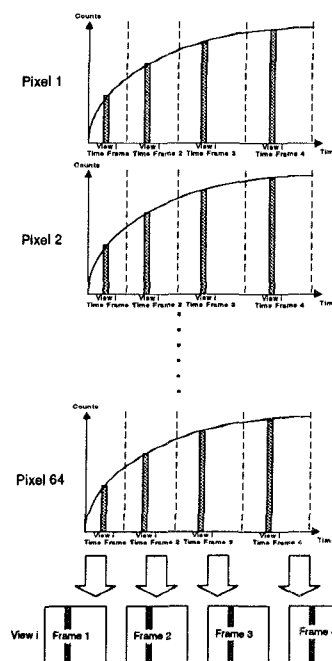


Figure 2 The first step of the proposed approach using the WPO method

the total activity (area under the curve) during each time frame for each of the pixel. The estimated total activity is then divided by the duration of the corresponding time frame to obtain an average value. These averages which represent the average projection counts are used as estimates to fill into the full projection data as shown at the bottom of the diagram, which can then be used to reconstruct the dynamic images after normalisation.

3.2 Reduction Of Projection Noise And Estimation Error

After performing the WPO, the estimated profiles are quite irregular because the WPO is carried out on pixel by pixel basis, whose the measurement is very noisy. Therefore, we employ a normalisation step to filter out such error before applying the FBP. The underlying assumption of the normalisation is that within the same time frame, the projections of the total activity within the object from different angles should be the same, i.e., the profiles at different angles should have the same total sum. This total sum is computed by averaging the total sums of the directly measured profiles at different angles for the same time frame. This total sum average can then be utilised to normalise the estimated profiles so that all profiles within the same time frame will uniformly have the same total sum. Finally, these normalised profiles at different angles within the same time frame are combined together to form the full projection data which are then used to reconstruct the dynamic images as shown at the bottom of Figure 3.

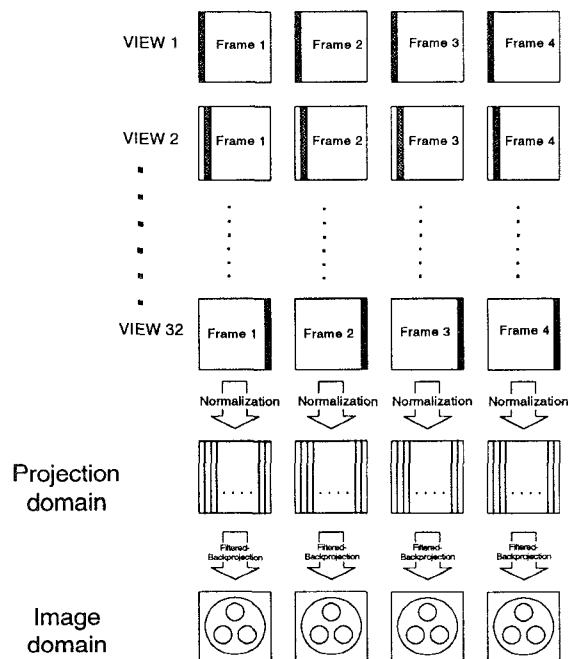


Figure 3 The second step of the proposed approach.

3.3 Quantification Of Dynamic Process

After the above two steps, we obtain a sequence of consistent dynamic images. The next step is to perform the system identification to extract the physiological parameters. In this step, in order to solve the problem of insufficient sampling as mentioned in Section 2, we make use of a new modeling approach proposed in [7]. This new modeling approach differs from the traditional modeling technique in that a new weighted least squares cost function or Rss (Residual sum of squares) is defined as follow:

$$R_{ss} = \sum_{k=1}^K w_k [\overline{y(t_k)} - \overline{z(t_k)}]^2$$

$$\text{where } \overline{y(t_k)} = \frac{1}{\Delta t_k} \int_{\Delta t_k} y(t) dt, \quad \overline{z(t_k)} = \frac{1}{\Delta t_k} \int_{\Delta t_k} z(t) dt,$$

w_k is the weighting, $y(t_k)$ is the model instantaneous predicted output values, $z(t_k)$ are the measurements obtained within certain interval Δt_k centred at t_k .

By using this newly defined Rss, the curve fitting is performed based on the average area within certain time interval instead of instantaneous value. Therefore, the number of samples needed does not depend on the changes of the object under scanned. Furthermore, Feng et al have shown in [7] that, by using the new system identification technique, the minimum number of samples needed to identify the parameters is equal to the number of model parameters if they are identifiable. As a result, we can greatly reduce the number of directly measured samples needed for performing system identification and the problem of insufficient samples can be solved.

4. Simulation Method

Due to the problem mentioned earlier, there are not many SPECT quantitative studies available. Therefore, in the simulation, we use a well-known PET FDG model [1], which is used for the analysis of glucose metabolic rates of human brain, to evaluate the performance of our proposed approach. The PET system is used for comparison because the projection data for a PET system are obtained from all angles at the same time, hence the problems for the rotating camera system to perform dynamic imaging should not exist.

The simulated brain model is shown at the center of Figure 1. It contains three ROI's (Regions of Interest). The left ellipse, right ellipse and the circle represent the brain white matter, gray matter and the average of whole brain. Their tracer activity curves are shown, respectively, at the right bottom, right middle and right top of the diagram. The outer largest ellipse is used as the background and is static. The transport rate constants, k_1 - k_4 , for the three matters are obtained from [1] and are listed below:

	k_1	k_2	k_3	k_4	$K = \frac{k_1 k_3}{(k_2 + k_3)}$
Gray Matter	0.102	0.130	0.062	0.0068	0.0329
White Matter	0.054	0.109	0.045	0.0058	0.0157
Whole Brain	0.078	0.119	0.053	0.0063	0.0241

For simplicity, K , which is equivalent or more precisely proportional to the LCMRGlc (local cerebral metabolic rates of glucose) is used as our final estimation result to compare the performance for different methods.

To obtain the projection data, the whole scanning period (120 min.) is divided into 4 time frames so as to construct four dynamic images for system identification. According to [8], the four time frames are of different length and they are, in minutes, [0,2.733], [2.733,15.683], [15.683,77.067], [77.067,120]. For each of these time frames, 32 view angles are taken respectively. Hence, a total of 128 view angles is obtained sequentially from 0° to 180° for the first time frame, then from 180° to 360° for the second time frame. And then repeat the same fashion for the third and fourth time frames. For each of the view angles, 64 raysum are projected.

The noise problem of the projection data is also studied in the simulation. Poisson noise with zero mean and variance given below are added to the data.

$$\text{noise variance} = C * \Gamma(i,j) / \Delta t(i)$$

where $0 \leq i \leq 127$, $0 \leq j \leq 63$, C is the noise constant, Γ represents the directly measured projection data and Δt means the measurement time for each angle. Six noise levels corresponding to $C=0.1, 0.2, 0.5, 1.0, 2.0$ and 4.0 are added to the directly measured projection data to simulate different noise environment.

In the simulation, we have compared 4 different methods of dynamic imaging: 1) the full projection method which is based on the full information, such as those in PET systems; 2) the Flat model method which is the traditional approach; 3) the Linear Interpolation method and 4) the normalised WPO method which are based on the partial information that can be measured by a single-head rotating camera system. After reconstructing the dynamic images, the new system identification technique is used to quantify the dynamic process.

5. Results

Figure 4 is a comparison of the directly recorded projection data and the estimated projection data, as well as the reconstructed dynamic images, at the four different time frames using the four different methods. These projection data and dynamic images are taken from the noise free case which is performed with only one trial. As shown, the projection data obtained from the normalised WPO method are very close to those obtained from the full

measurement and the reconstructed images do not suffer from motion artifacts especially for the first time frame during which the activity changes rapidly.

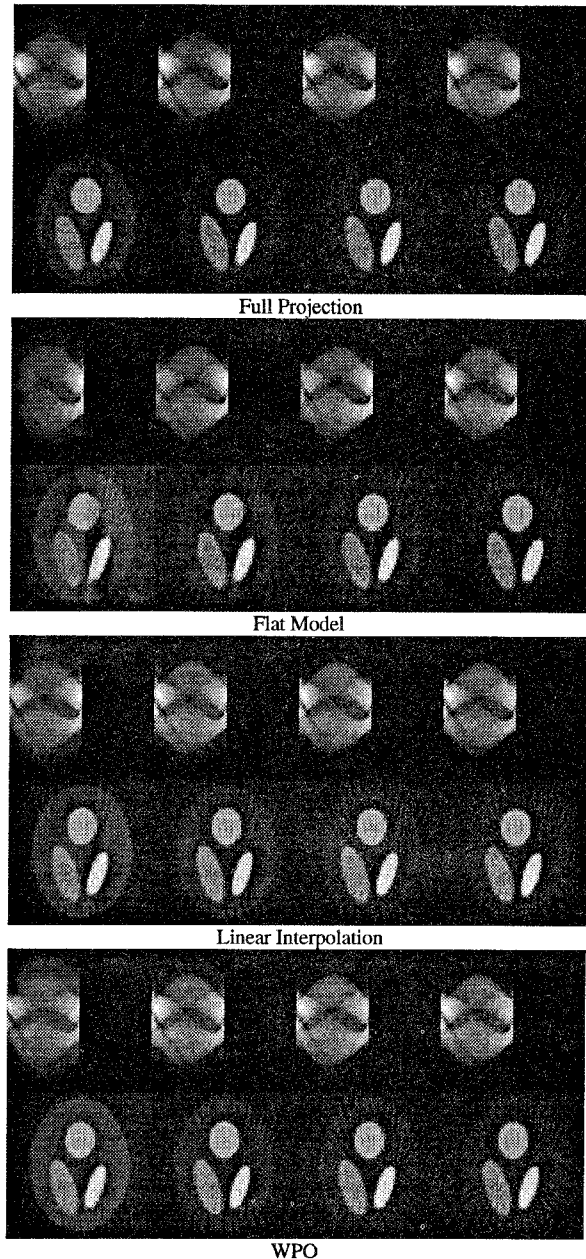


Figure 4 The projection data and the reconstructed dynamic images from four different methods.

The curves in Figure 5 are the cross sections of the images on the right, which are taken in the first time frame and are for the noise free case. For the left column of Figure 5 (i.e., (a), (c) and (e)), the higher peak of the curves represents the activity of the upper ellipse which simulates the average activity of the whole brain. The left peak represents the activity of the lower left ellipse which simulates the average activity of the brain white matter. For the right column of Figure 5 (i.e., (b), (d) and (f)), the

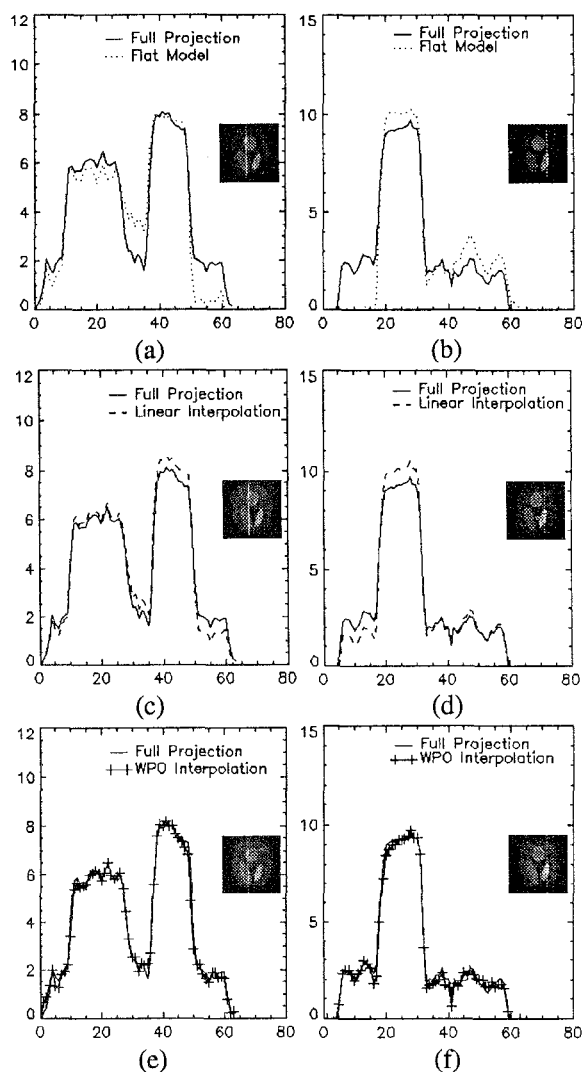


Figure 5 Comparisons of the images reconstructed from the full projection data with those from the partial projection data.

highest peak represents the activity of the lower right ellipse which simulates the average activity of the brain gray matter.

For all of the six figures, the solid line is obtained from the image reconstructed using the full projection data, which is assumed to be the most correct curve we can get as full information is recorded. The dotted lines in Figure 5 (a) and (b) are obtained from the Flat model. The dashed lines in Figure 5 (c) and (d) are obtained from the Linear interpolation and, lastly, the cross lines in Figure 5 (e) and (f) are obtained from the reconstructed image using the normalised WPO method. As can be seen, the dotted line deviates from the solid line significantly. As a result, the parameters estimated from such dynamic images are very poor. On the other hand, the cross line matches the solid line very well. It shows that by using the normalised WPO method, comparable dynamic images can be obtained and, hence, the physiological parameter estimation would be more accurate.

Based on the dynamic images reconstructed using the four different methods, the new system identification technique described earlier is used for parameter estimation. Figure 6 shows the simulation result of K 's estimated using the four different methods under different noise levels. The case with noise constant equals to 0 represents the noise free case which is performed for just one trial. For other values of noise constant, the simulations are carried out for one hundred trials to obtain average performances of each method. The left column of the figure shows the absolute percentage errors and the right column shows the standard derivations of the estimated K 's of different matters. It shows that the parameters estimated from the proposed approach using the normalised WPO method are comparable with those obtained with fully recorded projection data at all different noise levels. Besides, the overall performance of the normalised WPO method is far better than those of the Flat model and the Linear Interpolation methods.

6. Conclusions

In this paper, we have proposed a novel approach to accurately reconstruct dynamic images from the partially sampled projection data recorded by a typical rotating gamma camera system, without the need of knowing any additional information or prior knowledge. The detail theories of the proposed approach are given. The reliability of the proposed method has been tested by computer simulations at various noise levels. The results have proven that, the dynamic images reconstructed and the parameters estimated from the proposed approach using the normalised WPO interpolation method are comparable with those obtained with fully recorded projection data at all different noise levels, including the noise free condition.

In our simulation, we just use a single-head rotating camera system to illustrate the idea of the proposed approach. Indeed, the proposed approach can be easily extended to multi-head rotating camera system, which are commonly used in clinical environment. This new approach has a great potential for clinical applications by making use of the conventional rotating camera SPECT for quantitative study of physiological functions in human body with high accuracy.

7. Acknowledgment

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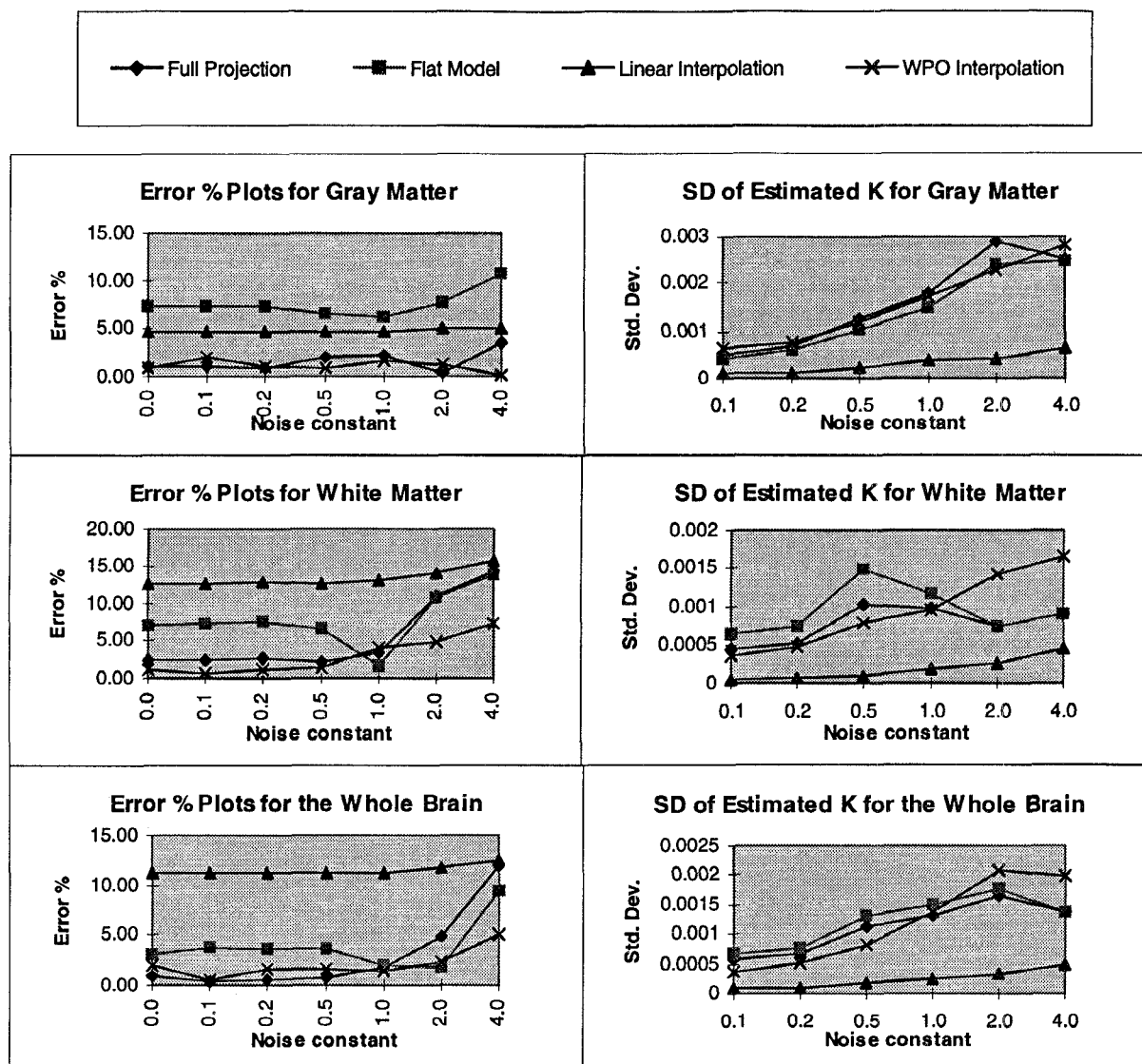


Figure 6 The percentage errors and the standard derivations of the means of estimated K 's under the various noise conditions for Gray matter, White matter and the Whole brain.

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