Computation of the AC Resistance of Multistranded Conductor Inductors with Multilayers for High Frequency Switching Converters

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***Abstract*—The design of high frequency resonant circuits is difficult because it requires the calculation of ac resistance of an inductor. The inductor suffers from skin effect and proximity effect and hence its ac resistance increases with frequency. The inductor is therefore multi stranded to minimize the ac resistance and multilayered to increase the inductance. This paper presents a method of calculating the ac resistance. The model used in the paper also takes care of the eddy current reaction field. Experimental results confirm the effectivity of the calculation method.**

***Index Terms*—Eddy current loss, high frequency inductor, multilayer, multistranded wire, switching converters.**

I. INTRODUCTION

THE MAIN purpose of high frequency operation of power electronics circuit is to reduce physical size and cost. The implication being that losses and efficiency become of paramount importance because of the need to remove heat from small surface. In the application of power electronics, the switching frequency of converters varies from tens of kHz to MHz. The prediction of eddy current loss permits the optimization of an inductor and hence the power electronic converters. The eddy current loss calculation of single strand conductors has been reported [1]. Reference [2] has analyzed the stranded conductor inductor however it has not considered the eddy current reaction field hence it is only suitable for low frequency operation. Reference [3] has presented modeling of stranded wire and how eddy current loss can be calculated from leakage fields. In the computation of the loss of multilayer inductor using multistranded conductors for high frequency operation, there is no publication.

In high frequency operation, powder iron–Ni mixed core is usually used because it offers lower core loss, however the permeability of the magnetic core is low. The inductors are usually constructed using more than one layer of turns to increase the inductance and multistranded conductor to decrease the ac resistance. The loss of an inductor will contribute 20–40% of the total loss of a power converter. The winding loss is about half of the total loss of an inductor. The computation of the inductor loss is therefore very important in order to optimize a design.

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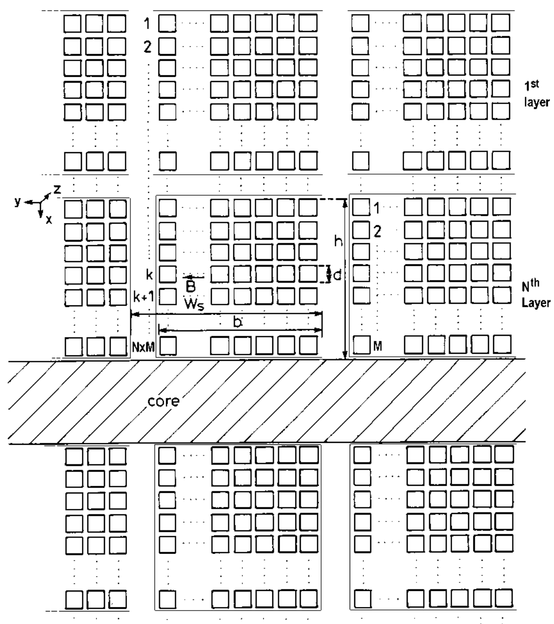

Fig. 1. Model of multistranded and multilayered inductor.

This paper is to present the calculation method of the ac resistance of an inductor. The method used here is an extension of the Dowell's theory [4] which only the winding loss of the single strand wire of transformer was calculated.

This result presented here is a generalized form for all types of inductors because the results can be reduced to single strand, single layer or any combinations of multilayer and multistranded wire construction. The method presented here is first to model the inductor so that the eddy current leakage field can be calculated. It follows that the induced voltage drops and resistive voltage drops due to the eddy current are also calculated. The calculation must also take care of the stranded conductor in each of the layer. This complicated model and calculation method is presented below. Characteristics resulted from the computation are shown. The results can be used to optimize an inductor design.

II. CALCULATION ALGORITHM

A. Model of the Inductor

Fig. 1 shows the model of the inductor. The conductor is assumed to have a rectangular section to make the calculation tractable. Each multistranded bundle of wires is assumed to be perfectly interleaved so that any given strand experiences the same electromagnetic conditions as any other strand in the same bundle. Each bundle is assumed to have M^2 strands and there are N layers.

B. Leakage Field

The current distribution in the conductors can be solved by using Faraday's law and Ampere's Law.:

$$E = \rho J \tag{1}$$

$$\nabla \times E = \frac{-dB}{dt} \tag{2}$$

J is the current density and is z-directed, and B is y-directed. For single frequency ω

$$\frac{dJ}{dx} = j\omega B\rho \tag{3}$$

where ρ is the resistivity of the conductor. Using Ampere's Law, B at the kth row can be shown to be

$$B = \frac{\mu_0 b}{W_s}\eta\left\{\overline{J}\eta h\left(\frac{k-1}{N}\right) + \int_o^x J\,dx\right\} \tag{4}$$

where μ_0 is the permeability of free space, W_s is the turn pitch, $\overline{J}$ is the average current density and η is the internal packaging factor of the stranded conductor and is defined by

$$\eta = \frac{Md}{b} \tag{5}$$

C. The Impedance

Combining (3) and (4) gives

$$\frac{dJ}{dz} = \frac{j\omega\mu_o b}{\rho W_s}\eta\left\{\overline{J}\eta h\left(\frac{k-1}{N}\right) + \int_o^x J\,dx\right\} \tag{6}$$

If J_d is the current density in a strand at $x = d$, (6) gives:

$$J_d = \overline{J}\alpha d\left\{\coth \alpha d + (k-1)\tanh\frac{\alpha d}{2}\right\} \tag{7}$$

where $\alpha = (j\omega\mu_o b\eta/\rho W_s)^{1/2}$. The resistive voltage drop per unit length at $x = d$ in the kth row is

$$V_{r_k} = \rho\overline{J}\alpha d\left\{\coth \alpha d + (k-1)\tanh\frac{\alpha d}{2}\right\} \tag{8}$$

According the Faraday's Law, the voltage induced in row k is

$$V_{ind_k} = 2\overline{J}\alpha d\rho\left(k - \frac{1}{2}\right)\tanh \alpha d \tag{9}$$

The voltage drop per unit length at $x = d$ in the kth row is the sum of the resistive voltage drop due to (7) and the induced voltage due to the flux linkage resulted from (4). Because of perfect interleaving of the strands, the resistive voltage drop across a strand is derived from the average value of the resistive voltages in a strand in each row, and the induced voltage across a strand is derived from the average of the voltages induced in a strand in each row. It follows that the total voltage per unit length at qth layer is

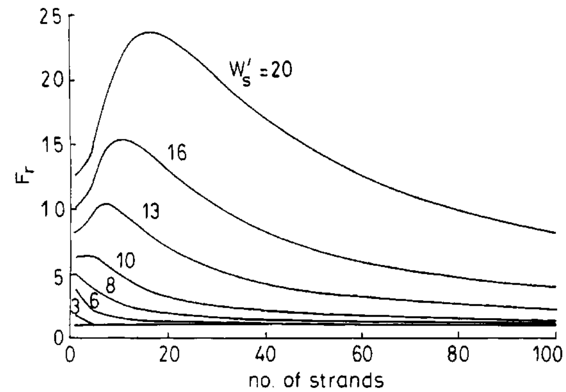

Fig. 2. Computed F_r characteristics against number of strands with one layer.

$$V_{T_q} = \frac{1}{M}\sum_{k=(q-1)M+1}^{qM}\left(\overline{J}\alpha d\rho\left\{\coth \alpha d + (k-1)\tanh\frac{\alpha d}{2}\right\}\right) + \frac{1}{M}\sum_{k=(q-1)M+1}^{qM}\sum_{k+1}^{NM} 2\overline{J}\alpha d\rho\left(k-\frac{1}{2}\right)\tanh \alpha d \tag{10}$$

Hence, the total voltage per unit length across the wire for one turn pitch is

$$V_{total} = \sum_{q=1}^{N} V_{T_q} \tag{11}$$

The effective impedance Z_n can be obtained as:

$$Z_n = \frac{V_{total}}{\overline{J}\rho N} \tag{12}$$

The ac resistance is therefore

$$R_{ac} = \Re(Z_n)R_{dc} \tag{13}$$

The ratio of the ac resistance to the dc resistance, F_r, is

$$F_r = \frac{R_{ac}}{R_{dc}} = \Re(Z_n) \tag{14}$$

III. Characteristics Results

A. Ratio of AC Resistance to DC Resistance, F_r

The ac loss can be computed by using (10)–(14). The results have been presented in normalized forms. All the dimensions can be expressed in terms of skin depth δ:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_o}} \tag{15}$$

The characteristic curve is therefore independent of dimension. Fig. 2 shows the computed results of the F_r against the number of strands. It can be seen that F_r increases as the number of strands increases especially with relatively low number of strands, typically less than ten and with a large normalized turn pitch W_s', typically more than 10 (W_s' is the normalized W_s with

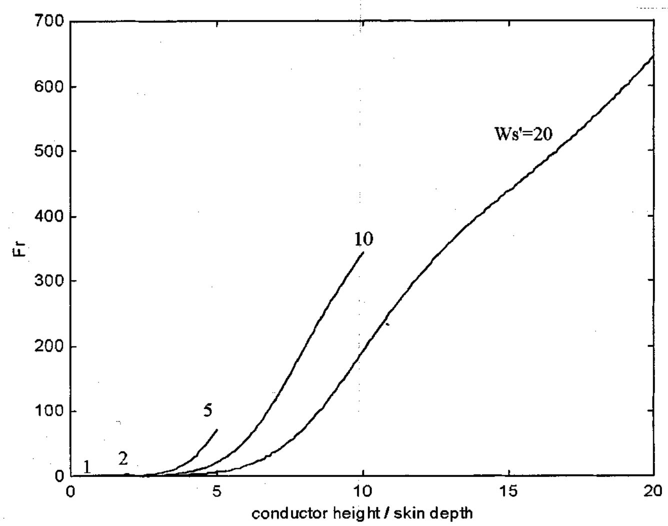

Fig. 3. F_r against bundle size with various turn pitches—3 layers and 36 strands.

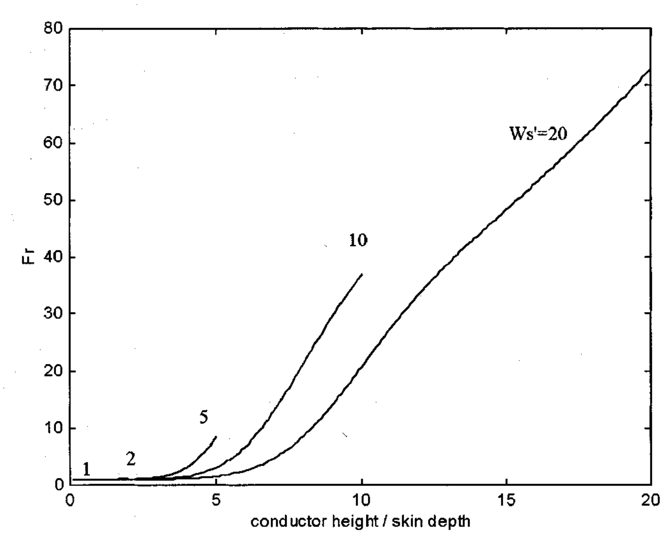

Fig. 4. F_r against bundle size with various turn pitches—1 layer and 36 strands.

base of skin depth). As the number of strands increases further, F_r decreases rapidly.

Figs. 3–5 show the characteristics of F_r against the bundle size, h, of the conductor. It can be seen that as the turn pitch W_s or the number of layers increase, F_r increases.

B. AC Resistance, R_{ac}

F_r characteristic is indicative of winding loss but does not give a clear picture of the actual loss parameter of the inductor. R_{ac} is therefore computed and is shown in Figs. 6–8. It can be seen that as the number of layers increases or the number of strands decreases, R_{ac} increases. There are minimum values of R_{ac} at certain wire size. These minima give optimum wire sizes for given turn pitches.

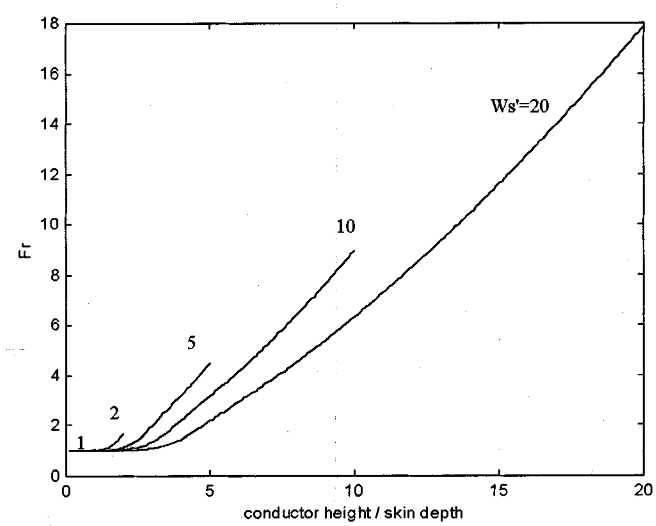

Fig. 5. F_r against bundle size with various turn pitches—1 layer and 1 strand.

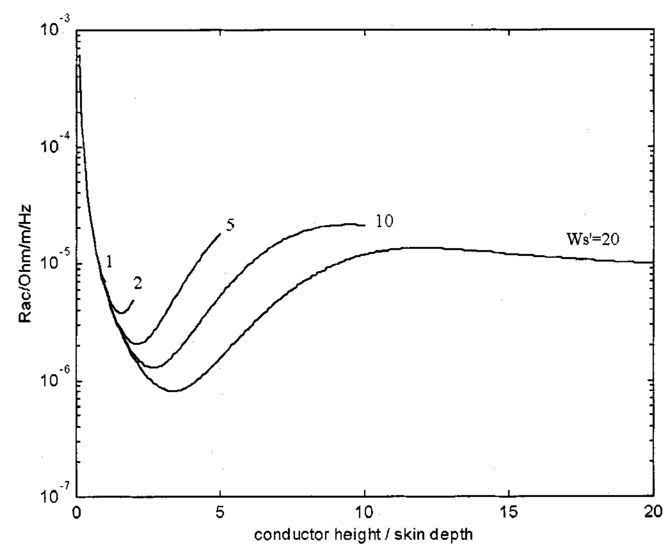

Fig. 6. R_{ac} against bundle size with various turn pitches—3 layers and 36 strands.

IV. Experimental Verification

The above results have been verified with the experimental testing. Inductors with powder iron core were constructed and tested from 10 kHz to 1 MHz. Fig. 9 presented results in the style of Fig. 2 in which F_r is plotted against the number of strands for a range of turn pitches. To obtain these experimental results, inductors of 29–190 μH, with power iron core, are constructed. The results have shown a good agreement with the computed characteristics.

Fig. 10 shows the experimental results of the R_{ac} in the style of Fig. 6. Inductors of 20–200 μH were constructed with power iron core. Again, the experimental results agreed very well with the computed R_{ac}. The minima are clearly shown in the AC resistance characteristics. This confirms the optimization of inductor design is possible. For a given turn pitch, the bundle size of the stranded wire can be chosen to obtain a minimum ac

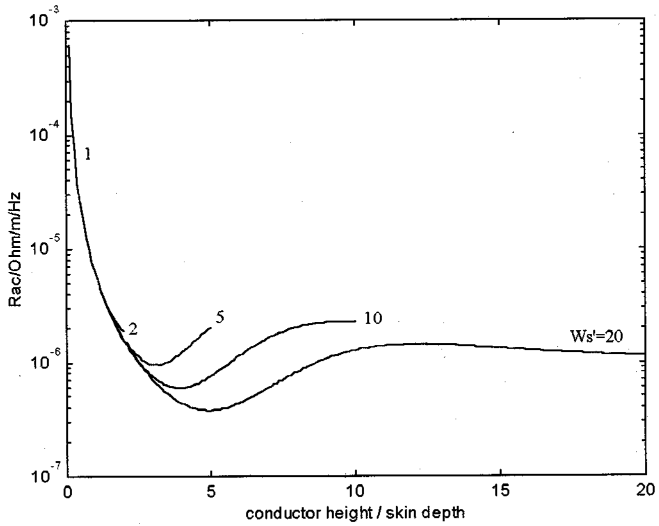

Fig. 7. R_{ac} against bundle size with various turn pitches—1 layer and 36 strands.

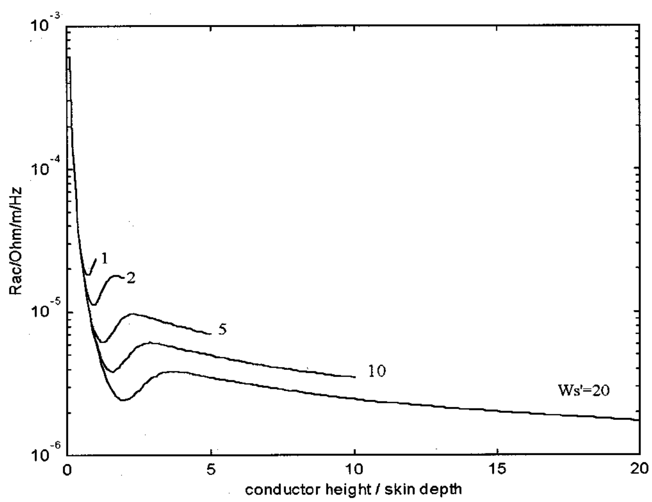

Fig. 8. R_{ac} against bundle size with various turn pitches—1 layer and 1 strand.

resistance. The optimum size of the bundle is about one-quarter of the turn pitch rather than equal to the turn pitch.

V. Conclusions

The paper has presented useful results for ac resistance of high frequency inductor. The computed results have been generalized for the multilayer and multistranded conductors. The results can also be used for simple structure such as single layer inductor $(N = 1)$, single strand conductor $(M = 1)$. The computed results show that an optimal design point could be

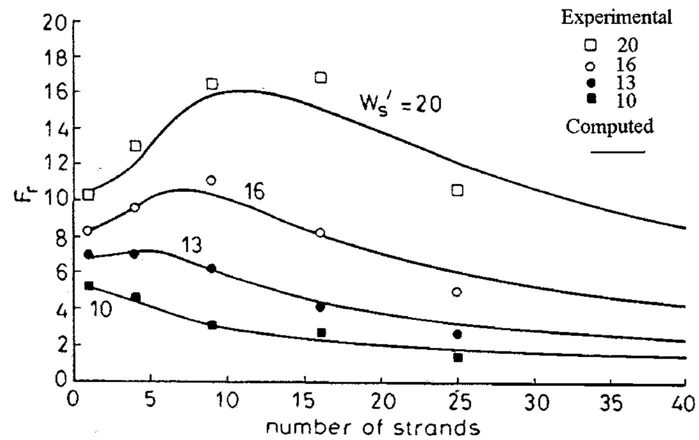

Fig. 9. Experimental and computed F_r against number of strands with various turn pitches.

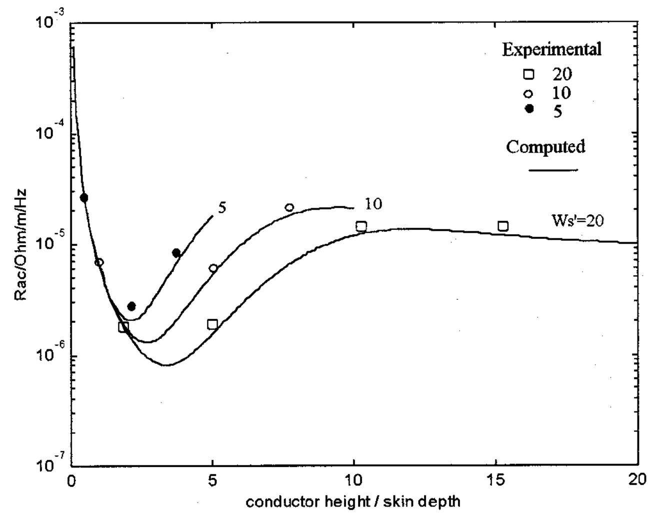

Fig. 10. Experimental and computed R_{ac} against conductor size with various turn pitches—3 layers and 36 strands.

obtained. The generalized results presented are important tools for the design of inductor for power electronics application.

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