Study of Bifurcation and Chaos in the Current-Mode Controlled Buck-Boost DC-DC Converter

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Abstract— This paper derives an iterative map for the Buck-Boost converter under current-mode control. On the basis of the model, the bifurcation phenomena under variation of a range of circuit parameters including input voltage, reference current and load resistance, have been investigated. The simulation results including the strange attractors, bifurcation diagrams and waveforms are presented.

***Index Terms*—Buck-Boost converter, chaos, bifurcation, power electronics.**

I. INTRODUCTION

In power supply design, power electronics engineers frequently observed some strange phenomena such as noise-like oscillation. However, they do not recognize what they are and what causes them and just think of them as 'noise' or 'disturbance'. Usually, what they try to do is to change some parameters in the circuit, for example the input voltage and feedback gain, so as to avoid the strange phenomena. Actually, the strange that they encounter are typical nonlinear phenomena which widely occur in nonlinear dynamic systems-bifurcation and chaos.

Profound study and analysis about the nonlinear phenomena in power electronics are profitable and even mandatory. The main reason is that studying bifurcation and chaos can help people to understand the change of behavior in power electronics when some parameters are varied. Furthermore, a complete knowledge about the domains of bifurcation and chaos in the parameter space is particular important for the power electronics engineers because they must choose the parameter values in order to obtain the desirable behavior. Moreover, the engineers will consciously avoid the bifurcation and chaos domains if they thoroughly understand when the nonlinear phenomena occur.

It is well know that the topologies of DC-DC converters are changed due to the switching operation. This results in a nonlinear time-varying system. Hence, DC-DC converters exhibit a wide range of bifurcation and chaos behavior under some conditions. The nonlinear phenomena, which appear to behavior randomly in a deterministic system even though there is no random input, are particular interesting and have interested power electronics, circuit and system, mathematics and control. Recently, the research of bifurcation and chaos in DC-DC converters has been extensively developed. In [1], detailed information about recent developments is available. Some results that have been reported are mainly about the bifurcation and chaos study in Buck converter [2]~[8], Boost converter [9]~[12] and Cuk converter [13]~[16]. However, Buck-Boost converter, one of the important converters that have wide industrial applications, has yet been reported. Our paper studies the bifurcation and chaos in the current- mode Buck-Boost DC-DC converter. It should be noted that all the published papers only presents results at 10kHz or less [11]. The paper presents the results at 20kHz which are the highest published results in this field. Parasitic components are not considered in this paper because at 20kHz operation, the authors found that the parasitic effect can be negligible. The parasitic effect does become important at more than 20kHz, but for this study, we concentrate on the theoretical study of bifurcation.

II. SYSTEM DESCRIPTION AND STATE EQUATIONS

The current-mode controlled Buck-Boost converter is shown in Fig.1. It is a second-order circuit that consists of a switch S, a diode D, a capacitor C, an inductor L, and with the load resistor R connected in parallel with the capacitor. Switch S is controlled by a feedback path comprising an RS trigger and a comparator. In the comparator, the inductor current is compared with a reference current I_{ref} which generates the on and off driving signal for switch S.

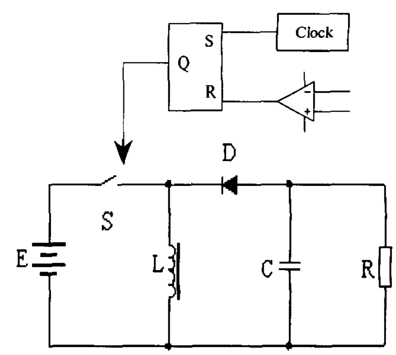

Fig.1. The circuit diagram in current-mode Buck-Boost converter with clock pulse

We assume that the converter operates in continuous current mode (CCM), where the inductance and switch period T are so chosen that the inductor current never falls to zero. Hence, there are two circuit configurations, according to whether S is closed or open. It is assumed that S is closed at the beginning of each cycle, i.e., at t = nT. The inductor current rises linearly until $i = I_{ref}$. Any clock pulse arriving during this period is ignored. When $i = I_{ref}$, switch S opens and remains open until the arrival of the next clock pulse, where it is closed again. The waveforms of inductor current and capacitor voltage are shown in Fig.2.

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With switch S closed, the diode D is blocked and the Buck-Boost converter is described by two uncoupled first-order differential equations: one for the inductor current and one for the capacitor voltage. The current linearly climbs and any clock pulses are ignored during this period. Once the inductor current i reaches the reference current I_{ref}, then the switch is opened.

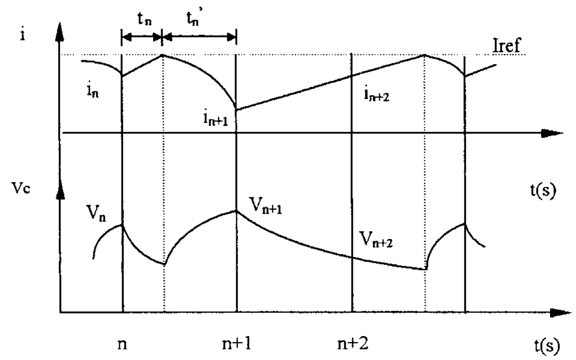

Fig.2. The waveforms of capacitor voltage and inductor current in the current-mode controlled Buck-Boost converter

With switch S open, the diode D conducts and the Buck-Boost converter is described by a pair of coupled first-order differential equations. If a clock pulse arrives, the switch S is closed again. In this paper, we use stroboscopic map, the most widely used type of discrete-time maps for modeling DC-DC converters, to obtain the Poincare section. i.e., the system states, the inductor current and capacitor voltage, are periodically sampled at time instants, n, n+1, n+2...This behavior is illustrated in Fig. 2.

When switch S is closed, the state equations are (1) and (2):

$$\begin{cases} \dfrac{di}{dt} = \dfrac{E}{L} & (1) \\ \dfrac{dv_c}{dt} = -\dfrac{1}{RC} v_c & (2) \end{cases}$$

When switch S is opened, the state equations are (3) and (4):

$$\begin{cases} \dfrac{di}{dt} = -\dfrac{1}{L} v_c & (3) \\ \dfrac{dv_c}{dt} = \dfrac{1}{C} i - \dfrac{1}{RC} v_c & (4) \end{cases}$$

where we omit the parasitic elements of inductor and capacitor so as to simplify the derivation of discrete model.

III. THE DERIVATION OF DISCRETE MODEL FOR THE BUCK-BOOST CONVERTER

The conventional state-space averaging method cannot be used in this analysis because it is only for low frequency operation rather than the operation near the switching frequency. In this paper, we use stroboscopic map, the most widely used type of discrete-time maps for modeling DC-DC converters, to obtain the Poincare section. i.e., the system states, the inductor current and capacitor voltage, are periodically sampled at time instants, t = nT.

Let (i_n, v_n) be the inductor current and capacitor voltage at a clock pulse at which the switch is closed.

From Fig.2, the converter is operated on the condition that the switch S is opened when the inductor current i reaches reference current I_{ref}. The on-state time t_n can be obtained from (1) by integration

$$\int_{i_n}^{I_{ref}} di = \frac{E}{L} \int_0^{t_n} dt$$

So the on-state time t_n is calculated by (5):

$$t_n = \frac{L}{E}(I_{ref} - i_n) \tag{5}$$

The capacitor voltage corresponding to instant t_n is calculated by (6):

$$v_c(t_n) = v_n e^{-\frac{t_n}{RC}} \tag{6}$$

The iterative model for the Buck-Boost converter can be derived as follows according to two situations, i.e., $t_n \geq T$ and $t_n < T$.

Case 1: $t_n \geq T$. S in the converter is in on-state. The values of i_n and v_n at next clock instant, i_{n+1} and v_{n+1}, can be expressed in terms of i_n and v_n .

$$\begin{cases} i_{n+1} = i_n + \dfrac{E}{L} T \\ v_{n+1} = v_n \cdot e^{-\frac{T}{RC}} \end{cases} \tag{7}$$

Case 2: $t_n < T$. S is switched from on-state to off-state during a switching period T. The values of i_n and v_n at next clock instant, i_{n+1} and $v_{,n+1}$ are calculated from (3) and (4) with I_{ref} and $v_n e^{-\frac{t_n}{RC}}$ as initial values.

According to the relationship of R, L and C, the iterative model for the Buck-Boost converter is divided into three forms:

① $1 - \dfrac{4R^2C}{L} > 0$, *i.e.* $R < \dfrac{1}{2}\sqrt{\dfrac{L}{C}}$. In this case, the roots of the characteristic equation corresponding to the original differential equations (3) and (4) are real and distinct. It describes a monotonous rising process without oscillatory motion. Hence, the iterative map for the Buck-Boost converter is given by (8) and (9).

$$i_{n+1} = c_1 e^{r_1(T-t_n)} + c_2 e^{r_2(T-t_n)} \tag{8}$$

$$v_{n+1} = -L[c_1 r_1 e^{r_1(T-t_n)} + c_2 r_2 e^{r_2(T-t_n)}] \tag{9}$$

where $r_1 = -\dfrac{1}{2RC} + \dfrac{1}{2RC}\sqrt{1 - \dfrac{4R^2C}{L}}$

$$r_2 = -\frac{1}{2RC} - \frac{1}{2RC}\sqrt{1 - \frac{4R^2C}{L}}$$

$$c_1 = I_{ref} - \frac{v_n e^{-\frac{t_n}{RC}} + L r_1 I_{ref}}{L(r_1 - r_2)}, \quad c_2 = \frac{v_n e^{-\frac{t_n}{RC}} + L r_1 I_{ref}}{L(r_1 - r_2)}$$

② $1-\frac{4R^2C}{L}=0$, *i.e.* $R=\frac{1}{2}\sqrt{\frac{L}{C}}$. In this case, the roots of the characteristic equation corresponding to the original differential equations (3) and (4) are real and equal. It also describes a monotonous rising process without oscillatory motion. Hence, the iterative map for the Buck-Boost converter is given by (10) and (11).

$$i_{n+1}=[c_1+c_2(T-t_n)]\,e^{r_1(T-t_n)} \tag{10}$$

$$v_{n+1}=-Le^{r_1(T-t_n)}[c_2+r_1c_1+r_1c_2(T-t_n)] \tag{11}$$

where $\quad r_1=r_2=-\frac{1}{2RC}$,

$$c_1=I_{ref},\quad c_2=-\frac{v_n\cdot e^{-\frac{t_n}{RC}}}{L}-r_1I_{ref}$$

③ $1-\frac{4R^2C}{L}<0$, *i.e.* $R>\frac{1}{2}\sqrt{\frac{L}{C}}$. In this case, the solutions of the characteristic equation corresponding to the original differential equations (3) and (4) are a pair of complex conjugate roots. It leads to a damped oscillatory process. Hence, the iterative map for the Buck-Boost converter is given by (12) and (13).

$$i_{n+1}=e^{\alpha(T-t_n)}[c_1\cos\beta(T-t_n)+c_2\sin\beta(T-t_n)] \tag{12}$$

$$\begin{aligned}v_{n+1}=&-Le^{\alpha(T-t_n)}[(c_1\alpha+c_2\beta)\cos\beta(T-t_n)+\\&(c_2\alpha-c_1\beta)\sin\beta(T-t_n)]\end{aligned} \tag{13}$$

where $\quad \alpha=-\frac{1}{2RC},\quad \beta=\frac{1}{2RC}\sqrt{\frac{4R^2C}{L}-1}$

$$c_1=I_{ref},\quad c_2=-\frac{1}{\beta}\left(\frac{v_n\cdot e^{-\frac{t_n}{RC}}}{L}+I_{ref}\alpha\right)$$

IV. BIFURCATION AND CHAOS STUDY IN THE BUCK-BOOST CONVERTER

Based on the iterative map in Section III, we can now study the bifurcation and chaos in the Buck-Boost converter with numerical method. Several visual aids that we can utilize to visualize chaos and bifurcation phenomena are time-domain waveform of state variables, phase portrait, Poincare or first-return maps and bifurcation diagrams. Among them, bifurcation diagram is the most powerful tool to investigate the nonlinear phenomena. In a bifurcation diagram, a periodic steady state of the system is represented as a signal point or several points equal to the periodicity of the system for a fixed parameter. For chaos, numerous points are plotted in the diagram because chaos means period infinity and the points never fall at the same position. Therefore, in such a bifurcation diagram, the change of behavior of a system is clearly shown as a parameter is varied.

From theoretical point of view, any circuit parameter can work as a bifurcation parameter. In this section, we will investigate the change of behavior in the Buck-Boost converter when a parameter, such as input voltage E, inductance L or capacitance C, is varied.

In our numerical experiments, the values of the components are chosen to ensure that the converter operates theoretically in the continuous conduction mode.

A. *E as the Bifurcation Parameter*

In the Buck-boost converter with input voltage E as bifurcation parameter, input voltage E is varied from 45V to 7V with a step of 0.1V, while other circuit parameters are fixed at the following values. i.e., I_{ref} = 4A, R=20Ω, L=0.5mH, C=4μF, T=50μs (f=20kHz). The bifurcation diagram of the converter is shown in Fig.3.

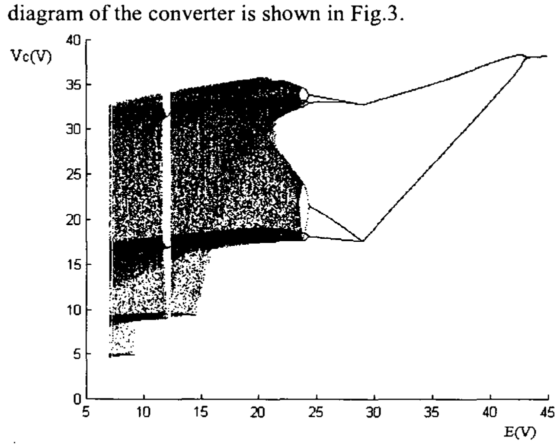

Fig.3. The bifurcation in the Buck-Boost converter with input voltage as parameter

As shown in Fig.3, the Buck-Boost converter goes through period-1, period-2, period-4 and period-8, as decribed below and eventually exhibits chaos as the input voltage E is varied from 45V to 7V. The stable period-1 is observed while the input voltage E is varied between 45V and 43.3V. The first bifurcation occurs at E=43.3V and the converter enters a stable period-2 region. As the input voltage is continuously decreased to 29.0V, the converter bifurcates to period-4. Further, peroid-4 bifurcates to period-8 at 24.4V and so on. Hence, the converter goes to chaos via period-doubling route.

In Fig.3, it can be interestingly observed that a small periodic window, which also exhibits period-doubling cascade, is embedded in the chaos region. In the periodic window, the converter experiences period-3 to period-6 and so on as the input voltage E is changed from 12.4V to 11.8V.

At E=20V, the waveforms and phase portrait of the converter are shown in Fig.4. It can be observed that the waveforms appear to behavior randomly and there is a strange attractor in the phase portrait. That a strange attractor occurs means the converter is working in the chaotic state.

Moreover, if the input voltage is equal to 50V, 35V, and 25V, which correspond to period-1, period-2, and period-4 respectively, the waveforms and phase portraits of the converter are shown in Fig.5, Fig.6, and Fig.7 respectively.

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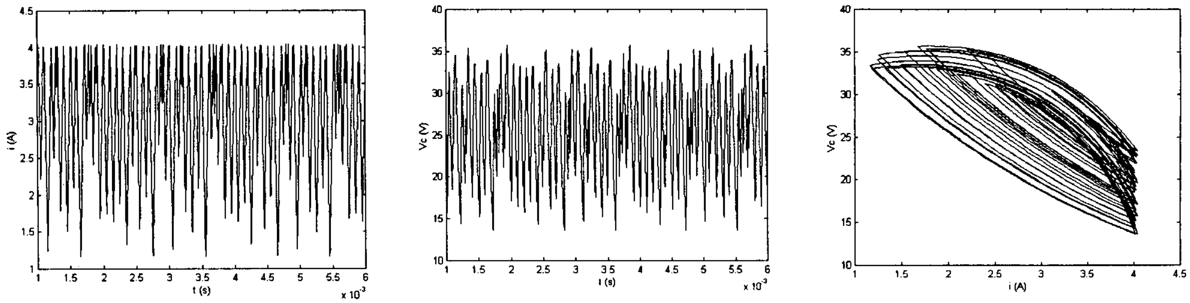
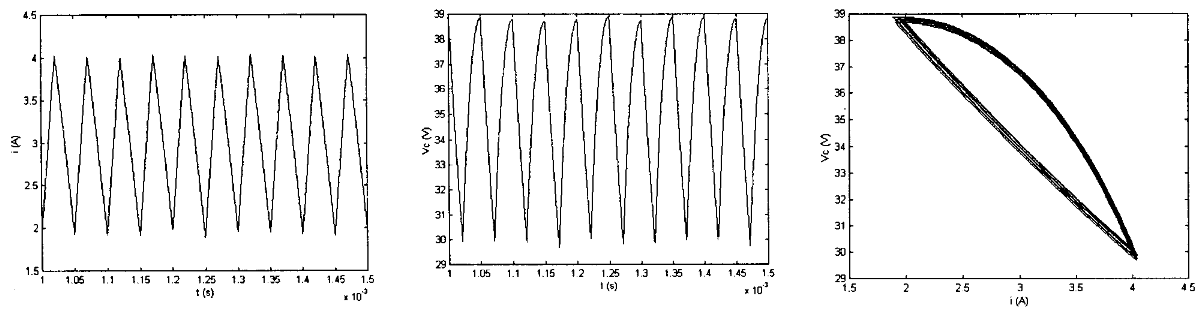


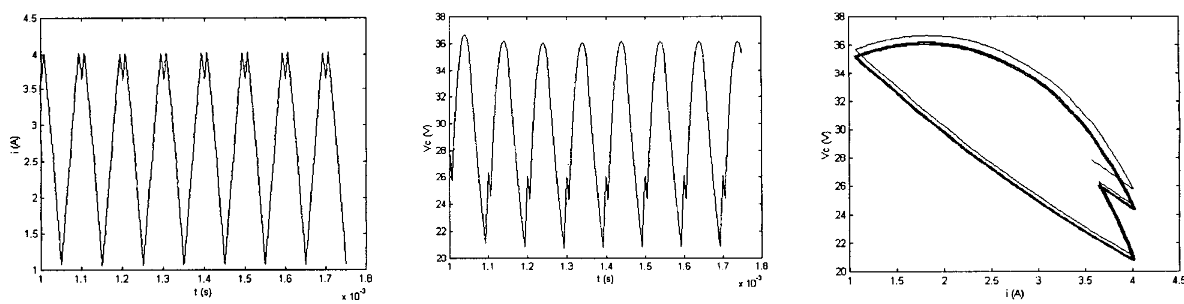
(a) (b) (c)

Fig.4. The waveforms and phase portrait in the Buck-Boost converter with E=20V



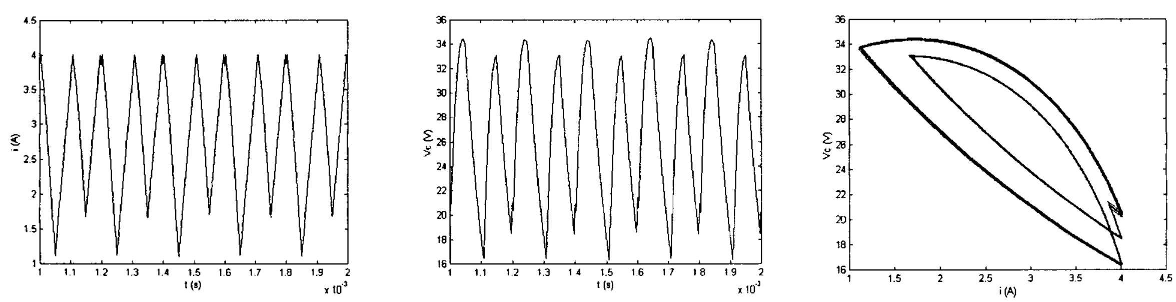
(a) (b) (c)

Fig.5. The waveforms and phase portrait in the Buck-Boost converter with E=50V



(a) (b) (c)

Fig.6. The waveforms and phase portrait in the Buck-Boost converter with E=35V



(a) (b) (c)

Fig.7. The waveforms and phase portrait in the Buck-Boost converter with E=25V

B. I_{ref} as the Bifurcation Parameter

In the Buck-boost converter with reference current I_{ref} as bifurcation parameter, the current I_{ref} is varied from 0.8A to 4.5A with a step of 0.01A, while other circuit parameters are fixed at the following values. i.e., E=12V, L=0.5mH, R=20Ω, C=4μF, T=50μs (f=20kHz). The bifurcation diagram of the converter is shown in Fig.8.

As shown in Fig.8, the Buck-Boost converter goes through period-1, period-2, period-4, period-8, and eventually exhibits chaos as reference current I_{ref} is varied from 0.8A to 4.5A. The first bifurcation occurs at I_{ref} =1.11A and the converter enters a stable period-2 region. As the input voltage is continuously increased to 1.65A, the converter bifurcates to period-4. Further, peroid-4 bifurcates to period-8 at 1.96A and so on. Hence, the converter goes to chaos via period-doubling route.

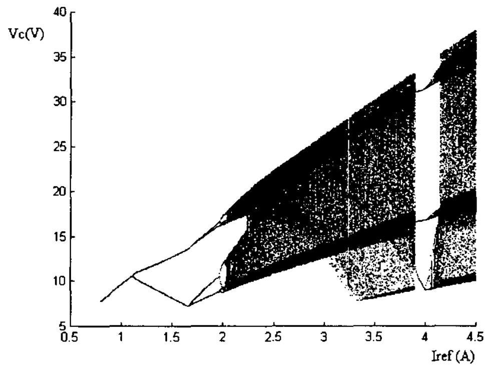


Fig.8. The bifurcation in the Buck-Boost converter with reference current as parameter

In Fig.8, it can be interestingly observed that a small periodic window, which also exhibits period-doubling cascade, is embedded in the chaos region. In the periodic window, the converter experiences period-3, period-6 and so on when reference current I_{ref} is changed from 3.8A to 4.1A. At I_{ref} =4.5A, the waveforms and phase portrait of the converter are shown in Fig.9(a)(b)(c). In Fig.9, it can be observed that the waveforms appear to behave randomly and there is a strange attractor in the phase portrait. The occurrence of the strange attractor means the converter is working in the chaotic state.

C. *R as the Bifurcation Parameter*

In the Buck-boost converter with load resistor R as bifurcation parameter, the resistance is varied from 1Ω to 25Ω with a step of 0.05Ω, while other circuit parameters are fixed at the following values. i.e., E=12V, I_{ref}= 4A, R=16Ω, L=0.5mH, T=50μs (f=20kHz). The bifurcation diagram of the converter is shown in Fig.10.

As shown in Fig.10, the Buck-Boost converter goes through period-1, period-2, period-4, period-8, and eventually exhibits chaos as resistor R is varied from 1Ω to 25Ω. The stable period-1 is observed while the resistor R is varied from 1Ω to 2.95Ω. The first bifurcation occurs at R=3.00Ω and the converter enters a stable period-2 region. As the resistor R is continuously increased to 4.80Ω, the converter bifurcates to period-4. Further, peroid-4 bifurcates to period-8 at 6.20Ω and so on. Hence, the converter goes to chaos via period-doubling route. In Fig.10, it can be interestingly observed that several small periodic windows, which also exhibit period-doubling cascade, are embedded in the chaos region. In the biggest periodic window, the converter experiences period-3, period-6 and so on when the resistance is changed from 18.9Ω to 21.2Ω.

At R=15Ω, the waveforms and phase portrait of the converter are shown in Fig.11(a)(b)(c). In Fig.11, it can be observed that the waveforms appear to behavior randomly and there is a strange attractor in the phase portrait. That a strange attractor occurs means the converter is working in the chaotic state.

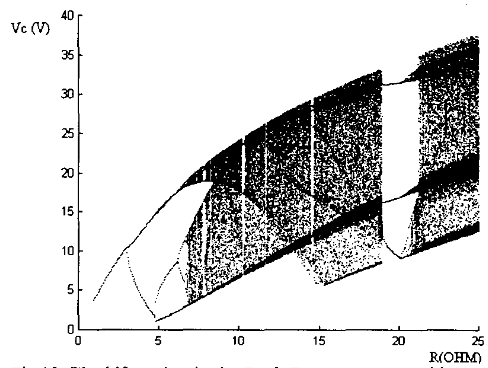


Fig.10. The bifurcation in the Buck-Boost converter with load resistance as parameter

V. CONCLUSION

It is well know that the topologies of DC-DC converters are changed due to the switching operation. This results in a nonlinear time-varying system. Hence, DC-DC converters exhibit a wide range of bifurcation and chaos behavior under some conditions. In this paper, we study the bifurcation and chaos phenomena in the current-mode Buck-Boost converter.
This paper derives an iterative map for the Buck-Boost converter under current-mode control. On the basis of the model, the bifurcation phenomena under variation of a range of circuit parameters including input voltage, reference current and resistor have been investigated.
The simulation results state that the Buck-Boost converter exhibits a wide range of nonlinear behaviors. As the bifurcation parameter is varied, the system goes to chaos via period-doubling route. It is specially pointed out that sometimes period-adding occurs in the period-doubling route. It can also be seen that the solutions of the system equations appear to behavior randomly in a deterministic system even though there is no random input. It has also found that bifurcation or chaos occurs when normal circuit parameters are used. No direction co-relation between the circuit parameters and the bifurcation parameters has been observed. Therefore analysis using Jacobian Matrix [20] is needed to predict Bifurcation points. This will be reported in the further publication.

The rich nonlinear phenomena in the Buck-Boost converter present that simple equations result in complex behaviors and different systems have similar behaviors.

The research about the domains of bifurcation and chaos in the parameter space is particular important because the power electronics engineers must choose the parameter values in order to obtain the desirable behavior. Moreover, the engineers will consciously avoid the bifurcation and chaos domains if they thoroughly understand when the nonlinear phenomena occur.

The contribution of this paper is summarised as folows :

- The iterative map of the Buck-Boost converter is developed that can be used to generate and study the bifurcation and chaos situation.
- Simulation results at 20kHz is presented so that the behaviour of the converter can be understood.

Although experimental results are not presented in this paper, in fact the above results have been confirmed by experimentment which will be presented in future publication.

ACKNOWLEDGEMENT

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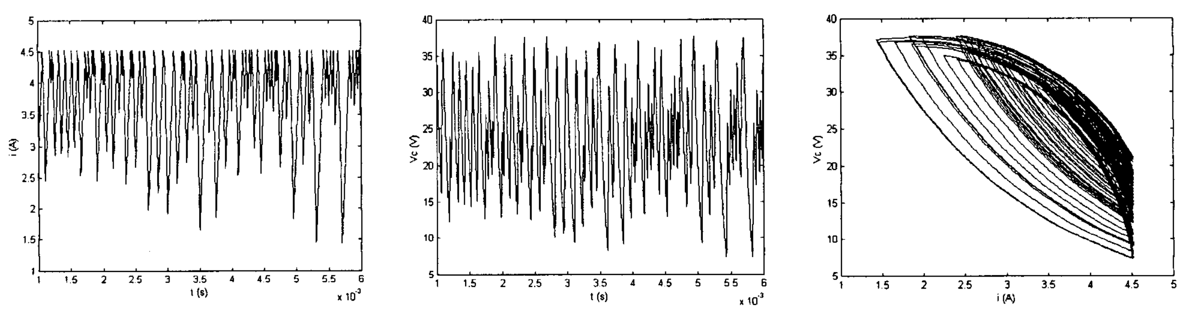
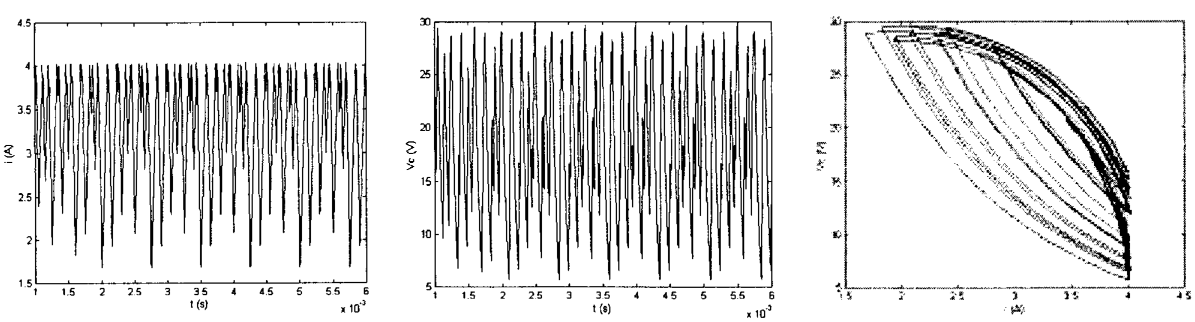


(a) (b) (c)

Fig.9. The waveforms and phase portrait in the Buck-Boost converter with I_{ref}=4.5A



(a) (b) (c)

Fig.11. The waveforms and phase portrait in the Buck-Boost converter with R = 15Ω